

Question 01

Find the sequence if $a_n - a_{n-1} = n+1$ and $a_4 = 14$. (LHR-2015)

$$a_n - a_{n-1} = n+1$$
$$a_n = a_{n-1} + n + 1$$

putting $n = 1,2,3,4,\ldots$

$$a_1 = a_0 + 1 + 1 \Rightarrow 2$$
$$a_2 = a_1 + 2 + 1 \Rightarrow 2 + 3 = 5$$
$$a_3 = a_2 + 3 + 1 \Rightarrow 5 + 4 = 9$$
$$a_4 = a_3 + 4 + 1 \Rightarrow 9 + 5 = 14$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

Hence Required Sequence is

$$2, 5, 9, 14, \ldots$$

Question 02

Find the first four terms of the sequence

$$a_n = (-1)^n n^2$$ (LHR-2021+2022)

$$a_n = (-1)^n n^2$$

putting $n = 1,2,3,4$

$$a_1 = (-1)^1 (1)^2 \Rightarrow (-1)(1) = -1$$
$$a_2 = (-1)^2 (2)^2 \Rightarrow (1)(4) = 4$$
$$a_3 = (-1)^3 (3)^2 \Rightarrow (-1)(9) = -9$$
$$a_4 = (-1)^4 (4)^2 \Rightarrow (1)(16) = 16$$

Question 03

Find the first four terms of the sequence

$$a_n - a_{n-1} = n+2, \; a_1 = 2$$ (LHR-2015+2019)

$a_n - a_{n-1} = n+2$ and $a_1 = 2$

$$a_n = a_{n-1} + n + 2$$

putting $n = 2,3,4$

$$a_2 = a_1 + 2 + 2 \Rightarrow 2 + 4 = 6$$

$a_3 = a_2 + 3 + 2 \Rightarrow 6 + 5 = 11$

$a_4 = a_3 + 4 + 2 \Rightarrow 11 + 6 = 17$

Question 04

Find the indicated terms of the following sequence

2, 6, 11, 17, ..., a_7 (LHR-2014+16+18)

$a_1 = 2$

$a_2 = 2 + 4 \Rightarrow 6$

$a_3 = 6 + 5 \Rightarrow 11$

$a_4 = 11 + 6 \Rightarrow 17$

$a_5 = 17 + 7 \Rightarrow 24$

$a_6 = 24 + 8 \Rightarrow 32$

$a_7 = 32 + 9 \Rightarrow 41$

Question 05

Find the next two terms of the following sequence

−1, 2, 12, 40, ... (LHR-2017)

$a_1 = -1 \times 2^0 \Rightarrow -1$

$a_2 = 1 \times 2^1 \Rightarrow 2$

$a_3 = 3 \times 2^2 \Rightarrow 12$

$a_4 = 5 \times 2^3 \Rightarrow 40$

$a_5 = 7 \times 2^4 \Rightarrow 112$

$a_6 = 9 \times 2^5 \Rightarrow 288$

The next two terms of the sequence are 112 & 288.

Question 06

Find the number of terms in A.P. if; $a_1 = 3, d = 7$ and $a_n = 59$. (LHR-2016)

$a_1 = 3, d = 7, a_n = 59$ and $n = ?$

By using the formula

$a_n = a_1 + (n-1)d$

$59 = 3 + (n-1)7$

$59 = 3 + 7n - 7$

$59 = 7n - 4$

$59 + 4 = 7n$

$63 = 7n$

$n = 9$ Ans

Question 07

If $a_{n-2} = 3n-11$, find n^{th} term of the sequence. (LHR-2013+2018)

$$a_{n-2} = 3n-11$$

Replace n by $n+2$

$$a_{n+2-2} = 3(n+2)-11$$
$$a_n = 3n+6-11$$
$$\boxed{a_n = 3n-5} \text{ Ans}$$

Question 08

If $a_{n-3} = 2n-5$, find n^{th} term of the sequence. (LHR-2014+16+21)

$$a_{n-3} = 2n-5$$

Replace n by $n+3$

$$a_{n+3-3} = 2(n+3)-5$$
$$a_n = 2n+6-5$$
$$\boxed{a_n = 2n+1} \text{ Ans}$$

Question 09

Which term of the A.P. 5, 2, -1, ... is -85? (LHR-2013+17+18+19)

$a_1 = 5$, $d = -3$, $a_n = -85$ and $n = ?$

By using formula

$$a_n = a_1 + (n-1)d$$
$$-85 = 5 + (n-1)-3$$
$$-85 = 5 - 3n + 3$$
$$-85 = 8 - 3n$$
$$-85 - 8 = -3n$$
$$-93 = -3n$$
$$n = \frac{-93}{-3}$$
$$\boxed{n = 31} \text{ Ans}$$

Question 10

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Which term of the A.P. -2, 4, 10, ... is 148? (LHR-2012+15)

$a_1 = -2$, $d = 6$, $a_n = 148$ and $n = ?$

By using formula

$$a_n = a_1 + (n-1)d$$

$$148 = -2 + (n-1)6$$

$$148 = -2 + 6n - 6$$

$$148 = 6n - 8$$

$$148 + 8 = 6n$$

$$156 = 6n$$

$$n = \frac{\overset{26}{\cancel{156}}}{\cancel{6}}$$

$$\boxed{n = 26} \text{ Ans}$$

Question 11

How many terms are there in A.P. in which $a_1 = 11$, $d = 3$, $a_n = 68$? (LHR-2022)

$a_1 = 11$, $d = 3$, $a_n = 68$ and $n = ?$

By using formula

$$a_n = a_1 + (n-1)d$$

$$68 = 11 + (n-1)3$$

$$68 = 11 + 3n - 3$$

$$68 = 3n + 8$$

$$68 - 8 = 3n$$

$$60 = 3n$$

$$n = 60/3$$

$$\boxed{n = 20} \text{ Ans}$$

Question 12

If the n^{th} term of the A.p is 3n-1. find A.p. (LHR-2017)

Given n^{th} term is

$$a_n = 3n-1$$

putting $n = 1,2,3,4,\ldots$

$$a_1 = 3(1)-1 \Rightarrow 2$$
$$a_2 = 3(2)-1 \Rightarrow 5$$
$$a_3 = 3(3)-1 \Rightarrow 7$$
$$a_4 = 3(4)-1 \Rightarrow 11$$
$$\vdots \quad \vdots \quad \vdots$$

So required A.p is

$$2, 5, 7, 11, \ldots$$

Question 13

Find the n^{th} term of the sequence, $\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$ (LHR-2012)

$$\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots$$

Here

$$4, 7, 10, \ldots$$

$$a_1 = 4,\ d = 3,\ n = n,\ a_n = ?$$

By using formula

$$a_n = a_1 + (n-1)d$$
$$a_n = 4 + (n-1)3$$
$$a_n = 4 + 3n - 3$$
$$a_n = 3n + 1$$

So the n^{th} term of the sequence

$$a_n = \left(\frac{3n+1}{3}\right)^2$$

Question 14

If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.p., show that $b = \frac{2ac}{a+c}$ (LHR-2015+2019)

Given

$\frac{1}{a}, \frac{1}{b}$ & $\frac{1}{c}$ are in A.p.

If $\frac{1}{a}, \frac{1}{b}$ & $\frac{1}{c}$ are in A.p, their exist common difference.

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\frac{1+1}{b} = \frac{a+c}{ca}$$

$$\frac{2}{b} = \frac{a+c}{ca}$$

$$b(a+c) = 2ac$$

$$\boxed{b = \frac{2ac}{a+c}}$$ Hence proved.

Question 15

Find A.M between $3\sqrt{5}$ & $5\sqrt{5}$. (LHR-2014)

$3\sqrt{5}$ and $5\sqrt{5}$

By using formula

$$A.M = \frac{a+b}{2}$$

$$= \frac{3\sqrt{5} + 5\sqrt{5}}{2}$$

$$= \frac{\cancel{8}^{4}\sqrt{5}}{\cancel{2}}$$

$$= 4\sqrt{5}$$

Question 16

Find A.M between $1-x+x^2$ and $1+x+x^2$. (LHR-2016)

$1-x+x^2$ and $1+x+x^2$

By using formula

$$A.M = \frac{a+b}{2}$$

$$= \frac{1-x+x^2+1+x+x^2}{2}$$

$$= \frac{2+2x^2}{2}$$

$$= \frac{\cancel{2}(1+x^2)}{\cancel{2}}$$

$$= 1+x^2$$

Question 17

Find three A.Ms between $\sqrt{2}$ & $3\sqrt{2}$. (LHR-2013)

Let A_1, A_2 and A_3 be the three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

$\sqrt{2}, A_1, A_2, A_3, 3\sqrt{2}$

$\boxed{a_1 = \sqrt{2}}$

$$a_5 = 3\sqrt{2}$$

$$a_1 + 4d = 3\sqrt{2}$$

$$\sqrt{2} + 4d = 3\sqrt{2}$$

$$4d = 3\sqrt{2} - \sqrt{2}$$

$$4d = 2\sqrt{2}$$

$$d = \frac{\cancel{2}\sqrt{2}}{\cancel{4}\,2}$$

$$d = \frac{\sqrt{2}}{2}$$

$$d = \frac{\cancel{\sqrt{2}}}{\sqrt{2}.\cancel{\sqrt{2}}}$$

$$\boxed{d = \frac{1}{\sqrt{2}}}$$

Now

$$A_1 = a_1 + 1d = \sqrt{2} + \frac{1}{\sqrt{2}}$$

$$= \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$A_2 = a_1 + 2d = \sqrt{2} + \frac{2}{\sqrt{2}}$$

$$= \frac{2+2}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

$$= \frac{2(\sqrt{2}.\cancel{\sqrt{2}})}{\cancel{\sqrt{2}}} = 2\sqrt{2}$$

$$A_3 = a_1 + 3d = \sqrt{2} + \frac{3}{\sqrt{2}}$$

$$= \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

Question 18

If 5, 8 are two A.Ms between a and b. find a and b. (LHR-2011+2018+2019)

$a, 5, 8, b$

$d = 5 - a$
$3 = 5 - a$
$a = 5 - 3$
$\boxed{a = 2}$

$d = 8 - 5$
$d = 3$

$d = b - 8$
$3 = b - 8$
$b = 3 + 8$
$\boxed{b = 11}$

Question 19

Sum the series. $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \ldots$ to n terms. (LHR-2018)

$$\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \ldots \text{ to n terms.}$$

$$a_1 = \frac{1}{1+\sqrt{x}}$$

$$a_1 = \frac{1}{1+\sqrt{x}} \times \frac{1-\sqrt{x}}{1-\sqrt{x}}$$

$$a_1 = \frac{1-\sqrt{x}}{1-x}$$

$$d = \frac{1}{1-x} - \frac{1}{1+\sqrt{x}}$$

$$d = \frac{1}{1-x} - \frac{1-\sqrt{x}}{1-x} \qquad \because \frac{1}{1+\sqrt{x}} = \frac{1-\sqrt{x}}{1-x}$$

$$d = \frac{\cancel{1} - \cancel{1} + \sqrt{x}}{(1-x)}$$

$$d = \frac{\sqrt{x}}{1-x}$$

we know that

$$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$$

$$S_n = \frac{n}{2}\left\{2\left(\frac{1-\sqrt{x}}{1-x}\right) + (n-1)\left(\frac{\sqrt{x}}{1-x}\right)\right\}$$

$$S_n = \frac{n}{2}\left\{\frac{2-2\sqrt{x}}{1-x} + \frac{n\sqrt{x}-\sqrt{x}}{1-x}\right\}$$

$$S_n = \frac{n}{2}\left(\frac{2-2\sqrt{x}+n\sqrt{x}-\sqrt{x}}{1-x}\right)$$

$$S_n = \frac{n}{2}\left(\frac{2+n\sqrt{x}-3\sqrt{x}}{1-x}\right)$$

$$\boxed{S_n = \frac{n}{2}\left\{\frac{2+\sqrt{x}(n-3)}{1-x}\right\}}$$ **Answer**

Question 20

Find the 5^{th} term of the G.P., 3,6,12,... (LHR-2014)

$a_1 = 3$, $r = 2$, $n = 5$, $a_5 = ?$

We know that

$a_n = a_1 r^{n-1}$

$a_5 = (3)(2)^4$

$\boxed{a_5 = 48}$

Question 21

Find the 5^{th} term of the sequence, $1+i, 2, \frac{4}{1+i}, \ldots$ (LHR-2011)

$a_1 = 1+i$, $r = \frac{2}{1+i}$ $n = 5$, $a_5 = ?$

$r = \frac{2}{1+i} \times \frac{1-i}{1-i} = \frac{\cancel{2}(1-i)}{\cancel{2}}$

$r = 1-i$

We know that

$a_n = a_1 r^{n-1}$

$a_5 = (1+i)(1-i)^4$

$a_5 = (1+i)[(1-i)^2]^2$

$a_5 = (1+i)(1+i^2-2i)^2$

$a_5 = (1+i)(-2i)^2$

$a_5 = (1+i)\,4(i^2)$

$a_5 = -4(1+i)$

Question 22

Find the 11^{th} term of the sequence $1+i, 2, 2(1-i), \ldots$ (LHR-2022)

$a_1 = 1+i$ $r = \frac{2}{1+i}$ $n = 11$, $a_{11} = ?$

$r = \frac{2}{1+i} \times \frac{1-i}{1-i} = \frac{\cancel{2}(1-i)}{\cancel{2}}$

$r = 1-i$

We know that

$a_n = a_1 r^{n-1}$

$a_{11} = (1+i)(1-i)^{10}$

$a_{11} = (1+i)[(1-i)^2]^5$

$a_{11} = (1+i)(1+i^2-2i)^5$

$a_{11} = (1+i)(-2i)^5$

$$a_{11} = (1+i) - 32(i^2)^2 i$$
$$a_{11} = -32(1+i)i$$
$$a_{11} = -32i - 32i^2$$
$$a_{11} = -32i + 32$$
$$a_{11} = 32(1-i)$$

Question 23

If a, b, c, d are in G.P, Prove that a−b, b−c, c−d in G.p.

(LHR-2012)

Given a, b, c, d are in G.p so their exist common ratio.

$$r = \frac{b}{a}, \quad r = \frac{c}{b}, \quad r = \frac{d}{c}$$

$$\boxed{b = ra}, \quad c = br, \quad cr = d$$

$$\boxed{c = ar^2}, \quad \boxed{d = ar^3}$$

Now

a−b, b−c, c−d will be in G.p if

$$\frac{b-c}{a-b} = \frac{c-d}{b-c}$$

$$\frac{ar - ar^2}{a - ar} = \frac{ar^2 - ar^3}{ar - ar^2}$$

$$\frac{\cancel{a}r\cancel{(1-r)}}{\cancel{a}\cancel{(1-r)}} = \frac{ar^2\cancel{(1-r)}}{ar\cancel{(1-r)}}$$

$$r = r \quad \text{(common ratio exists)}$$

So a−b, b−c, c−d will be in G.p.

Question 24

Show that the reciprocals of the terms of the geometric Sequence $a_1, a_1r^2, a_1r^4, \dots$ form another geometric sequence

(LHR-2017)

Given geometric sequence is

$$a_1, a_1r^2, a_1r^4, \dots$$

Its reciprocal is $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, \dots$

Now $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, \dots$ will be in G.P if

$$\frac{\frac{1}{a_1r^2}}{\frac{1}{a_1}} = \frac{\frac{1}{a_1r^4}}{\frac{1}{a_1r^2}}$$

$$\frac{1}{\cancel{a_1}r^2} \times \frac{\cancel{a_1}}{1} = \frac{1}{\cancel{a_1}r^{\cancel{4}}} \times \frac{\cancel{a_1}\cancel{r^2}}{1}$$

$$\frac{1}{r^2} = \frac{1}{r^2} \quad \text{(common ratio exists)}$$

So $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, \ldots$ will be in G.P.

Question 25

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P. show that the common ratio is $\pm\sqrt{\frac{a}{c}}$. (LHR-2015+16+17+19)

As $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in G.P. So their exists common ratio.

$$r = \frac{\frac{1}{b}}{\frac{1}{a}} = \frac{\frac{1}{c}}{\frac{1}{b}}$$

$$r = \frac{1}{b} \times \frac{a}{1} = \frac{1}{c} \times \frac{b}{1}$$

$$r = \frac{a}{b} = \frac{b}{c}$$

$$r = \frac{a}{b}, \quad r = \frac{b}{c}$$

Now

$$r \times r = \left(\frac{a}{\cancel{b}}\right)\left(\frac{\cancel{b}}{c}\right)$$

$$r^2 = \frac{a}{c}$$

$$r = \pm\sqrt{\frac{a}{c}}$$

Question 26

Define Geometric Mean. (LHR-2012+2019)

Geometric Mean

A Number G is said to be the geometric mean between the two numbers a, b if a, G, b are in G.P.

$G = \pm\sqrt{ab}$

Question 27

Find G.M between -2i and 8i. (LHR-2015)

we know that

$$G.M = \pm\sqrt{ab}$$
$$= \pm\sqrt{(-2i)(8i)}$$
$$= \pm\sqrt{-16i^2}$$
$$= \pm\sqrt{16}$$
$$= \pm 4$$

Question 28

Insert two G.Ms between 1 and 8. (LHR-2016+2018)

Let G_1 and G_2 the two G.Ms between 1 and 8.

$1, G_1, G_2, 8$

$\boxed{a_1 = 1}$

$$a_4 = 8$$
$$a_1 r^3 = 8$$
$$1(r^3) = 8$$
$$\boxed{r = 2}$$

Now

$$G_1 = a_1 r = (1)(2) = 2$$
$$G_2 = a_1 r^2 = (1)(2)^2 = 4$$

Question 29

Insert two G.Ms between 2 and 16. (LHR-2011+19+21)

Let G_1 and G_2 be the two G.Ms between 2 and 16.

$2, G_1, G_2, 16$

$\boxed{a_1 = 2}$

$$a_4 = 16$$
$$a_1 r^3 = 16$$
$$(2) r^3 = 16$$
$$r^3 = 8$$
$$\boxed{r = 2}$$

Now

$$G_1 = a_1 r = (2)(2) = 4$$
$$G_2 = a_1 r^2 = (2)(2)^4 = 8$$

Question 30

Find the sum of series. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ (LHR-2018+2021)

$a_1=\frac{1}{2}$, $r=\frac{1}{\cancel{4}_2}\times\frac{\cancel{2}}{1} \Rightarrow r=\frac{1}{2}$

We know that

$$S_\infty=\frac{a_1}{1-r}$$

$$S_\infty=\frac{1/2}{1-\frac{1}{2}}$$

$$S_\infty=\frac{1/\cancel{2}}{\frac{2-1}{\cancel{2}}}$$

$$S_\infty=1$$

Question 31

Find the sum of series. $\frac{9}{4}+\frac{3}{2}+1+\cdots$ (LHR-2015)

$a_1=\frac{9}{4}$ $r=\frac{3}{2}\times\frac{4}{9}$

$r=\frac{2}{3}$

We know that

$$S_\infty=\frac{a_1}{1-r}$$

$$S_\infty=\frac{9/4}{1-\frac{2}{3}}$$

$$S_\infty=\frac{9/4}{\frac{3-2}{3}}$$

$$S_\infty=\frac{9}{4}\times\frac{3}{1}$$

$$S_\infty=27/4$$

Question 32

Find the sum of series. $2+1+0.5+\cdots$ (LHR-2019)

$\boxed{a_1=2}$ $\boxed{r=\frac{1}{2}}$

we know that

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{2}{1-\frac{1}{2}}$$

$$S_\infty = \frac{2}{\frac{2-1}{2}}$$

$$S_\infty = 4$$

Question 33

Find vulgar fractions of recurring decimals: $0.\dot{7}$ (LHR-2017)

$$= 0.\dot{7}$$

$$= 0.777$$

$$= 0.7 + 0.07 + 0.007 + \cdots$$

$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \cdots$$

$$= \frac{7}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \cdots\right)$$

Here $a_1 = 1$ and $r = 1/10$

$$= \frac{7}{10}\left(\frac{1}{1-\frac{1}{10}}\right)$$

$$= \frac{7}{10}\left(\frac{1}{\frac{9}{10}}\right)$$

$$= \frac{7}{\cancel{10}}\left(\frac{\cancel{10}}{9}\right)$$

$$= 7/9$$

Question 34

If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \cdots$ if $0 < x < 2$. Prove that $x = \frac{2y}{1+y}$ (LHR-13)

Given infinite geometric series

$$y = \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots$$

we know that $S_\infty = \frac{a_1}{1-r}$

Here $a_1 = \frac{x}{2}$, $r = \frac{x^2}{4_2} \times \frac{\cancel{2}}{\cancel{x}}$

$$r = \frac{x}{2}$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{x/2}{1-x/2}$$

$$S_\infty = \frac{x/\cancel{2}}{\frac{2-x}{\cancel{2}}}$$

$$S_\infty = \frac{x}{2-x}$$

$$y = \frac{x}{2-x}$$

$$2y - xy = x$$

$$2y = x + xy$$

$$2y = x(1+y)$$

$$\boxed{x = \frac{2y}{1+y}}$$ Hence proved.

Question 35

If $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \ldots$ Prove that $x = 2\frac{(y-1)}{y}$. (LHR-2011)

Given infinite geometric series is

$$y = 1 + \frac{x}{2} + \frac{x^2}{4} + \ldots$$

we know that $S_\infty = \frac{a_1}{1-r}$

Here $a_1 = 1$, $r = \frac{x}{2}$

Now

$$S_\infty = \frac{a_1}{1-r}$$

$$y = \frac{1}{1-\frac{x}{2}}$$

$$y = \frac{1}{\frac{2-x}{2}}$$

$$y = \frac{2}{2-x}$$

$$2y - xy = 2$$

$$2y - 2 = xy$$

$$2(y-1) = xy$$

$$\boxed{x = \frac{2(y-1)}{y}}$$ Hence Proved

Question 36

Define Harmonic Progression? (LHR-2016)

Harmonic Progression

A Sequence $\{a_n\}$ is called harmonic progression if the reciprocals of its terms are in arithmetic Progression.

Question 37

Find the 9th term of the harmonic sequence. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ (LHR-2014)

Given $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ are in H.P.

Their reciprocals $3, 5, 7, \ldots$ are in A.P

$a_1 = 3$, $d = 2$ $n = 9$, $a_9 = ?$

we know that

$a_n = a_1 + (n-1)d$

$a_9 = (3) + (8)(2)$

$a_9 = 3 + 16$

$a_9 = 19$

So the 9th term of H.P is $\frac{1}{19}$.

Question 38

Find the 12th term of the harmonic sequence, $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \ldots$ (LHR-16+18)

Given $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \ldots$ are in H.P.

Their reciprocals $3, \frac{9}{2}, 6, \ldots$ are in A.P.

$a_1 = 3$, $d = \frac{9}{2} - 3$, $n = 12$, $a_{12} = ?$

$d = \frac{9-6}{2}$

$d = 3/2$

we know that

$a_n = a_1 + (n-1)d$

$a_{12} = 3 + (11)\left(\frac{3}{2}\right) \Rightarrow a_{12} = \frac{6+33}{2}$

$a_{12} = 39/2$

So the 12th term of H.P is $2/39$.

Question 39

Find the H.M between $-\frac{2}{5}$ and $-\frac{8}{5}$. (LHR-2012)

Here $a = -\frac{2}{5}$ and $b = -\frac{8}{5}$

we know that

$$H.M = \frac{2ab}{a+b}$$

$$H.M = \frac{2(-2/5)(-8/5)}{-\frac{2}{5}-\frac{8}{5}}$$

$$H.M = \frac{32/25}{\frac{-2-8}{5}}$$

$$H.M = \frac{32/25}{-10/5}$$

$$H.M = \frac{\cancel{32}^{16}}{\cancel{25}_{5}} \times -\frac{\cancel{5}}{\cancel{10}_{5}}$$

$$H.M = -\frac{16}{25}$$

Question 40

The first term of an H.P is $-\frac{1}{3}$ and the fifth term is $\frac{1}{5}$. Find its 9^{th} term. (LHR-2013 +2017)

Given $a_1 = -\frac{1}{3}$ and $a_5 = \frac{1}{5}$ are in H.P.

Their reciprocals $a_1 = -3$ and $a_5 = 5$ are in A.P.

$\boxed{a_1 = -3}$, $a_5 = 5$, $n = 9$, $a_9 = ?$

$$a_1 + 4d = 5$$

$$-3 + 4d = 5$$

$$4d = 8$$

$$\boxed{d = 2}$$

we know that

$$a_n = a_1 + (n-1)d$$

$$a_9 = -3 + 8 \times 2$$

$$a_9 = +13$$

So the 9^{th} term of H.P is $1/13$.

Question 41

If 5 is the harmonic mean between 2 and b. Find b. (LHR-2012+17+18+22)

Given $a=2$, $H.M=5$, $b=?$

we know that

$$H.M=\frac{2ab}{a+b}$$

$$5=\frac{2(2)b}{2+b}$$

$$10+5b=4b$$

$$5b-4b=-10$$

$$\boxed{b=-10}$$

Question 42

If the numbers $\frac{1}{k}$, $\frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in H.P. find k. (LHR-2019)

Given $\frac{1}{k}$, $\frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in H.P

Their reciprocals $k, 2k+1, 4k-1$ are in A.P.

So their exist common difference.

$$2k+1-k=4k-1-2k-1$$

$$k+1=2k-2$$

$$k-2k=-2-1$$

$$-k=-3$$

$$\boxed{k=3}$$

Question 43

Find A, G, H and show that $G^2=A.H.$ if $a=2, b=-6$ (LHR-2014)

Given $a=-2$ and $b=-6$

$$A=A.M=\frac{a+b}{2}=\frac{-2-6}{2}=-4$$

$$G=G.M=\pm\sqrt{ab}=\pm\sqrt{(-2)(-6)}=\pm\sqrt{12}$$

$$H=H.M=\frac{2ab}{a+b}=\frac{2(-2)(-6)}{-2-6}=\frac{24^{3}}{-8}=-3$$

we have to prove that $G^2 = AH$

$G^2 = (\pm\sqrt{12})^2 = 12$ —— (i)

$AH = (-4)(-3) = 12$ —— (ii)

From (i) and (ii)

$G^2 = AH$

Question 44

Find A, G, H and verify that A<G<H, if a=-2, b=-8. (LHR-2013)

Given $a = -2$ and $b = -8$

$A = A.M = \frac{a+b}{2} = \frac{-2-8}{2} = -5$

$G = G.M = -\sqrt{ab} = -\sqrt{(-2)(-8)} = -\sqrt{16} = -4$

$H = H.M = \frac{2ab}{a+b} = \frac{2(-2)(-8)}{-2-8} = \frac{\cancel{32}^{16}}{-\cancel{10}_{5}} = \frac{-16}{5} = -3.2$

Hence

$A < G < H$

Question 45

Find three A.Ms between 3 and 11. (LHR-2019)

Let A_1, A_2, A_3 be the three A.Ms between 3 and 11.

$3, A_1, A_2, A_3, 11$

$a_1 = 3$, $a_5 = 11$,

$a_1 + 4d = 11$

$3 + 4d = 11$

$4d = 8$

$d = 2$

Now

$A_1 = a_1 + d = 3 + 2 = 5$

$A_2 = a_1 + 2d = 3 + 4 = 7$

$A_3 = a_1 + 3d = 3 + 6 = 9$

Question 46

Define Geometric Progression? (LHR-2011)

Geometric Progression

A sequence $\{a_n\}$ is called geometric progression if $\frac{a_n}{a_{n-1}}$ is the same non-zero number for all $n \in N$ and $n > 1$.

Long Questions

Question 01

Find the 18th term of the A.P. if $a_6 = 19$ & $a_9 = 31$. LHR-201

$a_6 = 19$, $a_9 = 31$, $a_{18} = ?$

$a_1 + 5d = 19$ —(i) , $a_1 + 8d = 31$ —(ii)

Substracting (i) from (ii)

$$\cancel{a_1} + 8d = 31$$
$$-\cancel{a_1} \pm 5d = 19$$
$$3d = 12$$
$$\boxed{d = 4}$$

putting $d = 4$ in (i)

$$a_1 + 5(4) = 19$$
$$a_1 = 19 - 20$$
$$\boxed{a_1 = -1}$$

we know that

$$a_n = a_1 + (n-1)d$$
$$a_{18} = -1 + (18-1)4$$
$$a_{18} = -1 + 17(4)$$
$$a_{18} = -1 + 68$$
$$\boxed{a_{18} = 67} \text{ Ans}$$

Question 02

Find 4 A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$. LHR-2021

Let A_1, A_2, A_3 and A_4 be the four A.Ms between $\sqrt{2}$ and $12/\sqrt{2}$.

$\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$

$$\boxed{a_1 = \sqrt{2}}$$

$$a_6 = \frac{12}{\sqrt{2}}$$
$$a_1 + 5d = \frac{12}{\sqrt{2}}$$
$$\sqrt{2} + 5d = \frac{12}{\sqrt{2}}$$
$$5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$
$$5d = \frac{12-2}{\sqrt{2}}$$
$$d = \frac{\cancel{10}^2}{\sqrt{2} \times \cancel{5}}$$

$$d = \frac{2}{\sqrt{2}}$$
$$d = \frac{\sqrt{2}\cdot\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$
$$\boxed{d = \sqrt{2}}$$

Now

$A_1 = a_1 + 1d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$A_2 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

$A_3 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$A_4 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$

Question 03

Find n so that $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ may be the A.M between a & b. LHR-2012

Given

$$A.M_s = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$\frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$(a+b)(a^{n-1}+b^{n-1}) = 2(a^n+b^n)$$

$$a\cdot a^{n-1}+b\cdot a^{n-1}+a\cdot b^{n-1}+b\cdot b^{n-1} = 2a^n+2b^n$$

$$a^n+b\cdot a^{n-1}+a\cdot b^{n-1}+b^n = 2a^n+2b^n$$

$$b\cdot a^{n-1}+a\cdot b^{n-1} = 2a^n-a^n+2b^n-b^n$$

$$b\cdot a^{n-1}+a\cdot b^{n-1} = a^n+b^n$$

$$b\cdot a^{n-1}+a\cdot b^{n-1} = a\cdot a^{n-1}+b\cdot b^{n-1}$$

$$b\cdot a^{n-1}-a\cdot a^{n-1} = b\cdot b^{n-1}-a\cdot b^{n-1}$$

$$\cancel{(b-a)}\,a^{n-1} = \cancel{(b-a)}\,b^{n-1}$$

$$\frac{a^{n-1}}{b^{n-1}} = 1$$

$$\left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^{0}$$

$$n-1 = 0$$

$\boxed{n=1}$ Answer

Question 04

If $S_n = n(2n-1)$, then find the series. (LHR-2021)

Given $\quad S_n = n(2n-1)$

$$S_1 = 1(2-1) = 1$$

$$S_2 = 2(4-1) = 6$$

S_1 is the sum upto 1^{st} term.

$\boxed{a_1 = 1}$ —— (i)

S_2 is the sum upto 2^{nd} term.

$\boxed{a_1 + a_2 = 6}$ —— (ii)

put $a_1 = 1$

$1 + a_2 = 6$

$a_2 = 5$

Hence

$d = 5 - 1$

$\boxed{d = 4}$

Hence required series is

$= a_1 + (a_1 + d) + (a_1 + 2d) + \cdots$

$= 1 + (1 + 4) + (1 + 2(4)) + \cdots$

$= 1 + 5 + 9 + \cdots$

Question 05

The ratio of the sums of n terms of two series in A.P is 3n+2:n+1. Find the ratio of their 8th terms. (LHR-2014)

Let S_n and S_n' be the two required series.

1st Series

First term $= a_1$

common difference $= d$

$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$

2nd series

First term $= a_1'$

common difference $= d'$

$S_n' = \frac{n}{2}\{2a_1' + (n-1)d'\}$

According to the given condition

$S_n : S_n' = 3n+2 : n+1$

$$\frac{\cancel{\frac{n}{2}}\{2a_1 + (n-1)d\}}{\cancel{\frac{n}{2}}\{2a_1' + (n-1)d'\}} = \frac{3n+2}{n+1}$$

$$\frac{a_1 + \left(\frac{n-1}{2}\right)d}{a_1' + \left(\frac{n-1}{2}\right)d'} = \frac{3n+2}{n+1}$$

put $\frac{n-1}{2} = 7$

$\boxed{n = 15}$

$$\frac{a_1 + 7d}{a_1 + 7d'} = \frac{3n+2}{n+1}$$

$a_8 : a_8' = 3n+2 : n+1$

put $n = 15$

$\boxed{a_8 : a_8' = 47 : 16}$ **Answer**

Question 06

If S_2, S_3, S_5 are the sums of $2n, 3n, 5n$ terms of an A.p, show that $S_5 = 5(S_3 - S_2)$. (LHR-2011)

We know that

$$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$$

S_2 = Sum of $2n$ terms

$$S_2 = \frac{2n}{2}\{2a_1 + (2n-1)d\}$$

S_3 = Sum of $3n$ terms

$$S_3 = \frac{3n}{2}\{2a_1 + (3n-1)d\}$$

S_5 = Sum of $5n$ terms

$$S_5 = \frac{5n}{2}\{2a_1 + (5n-1)d\}$$

Given

$$S_5 = 5(S_3 - S_2)$$

$$R.H.S = 5(S_3 - S_2)$$

$$= 5\left[\frac{3n}{2}\{2a_1 + (3n-1)d\} - \frac{2n}{2}\{2a_1 + (2n-1)d\}\right]$$

$$= 5 \cdot \frac{n}{2}(6a_1 + 9nd - 3d - 4a_1 - 4nd + 2d)$$

$$= \frac{5n}{2}(2a_1 + 5nd - d)$$

$$= \frac{5n}{2}\{2a_1 + (5n-1)d\}$$

$$= S_5 = R.H.S$$

Hence proved.

Question 07

The sum of 9 terms of an A.p is 171 and its 8 term is 31. Find the series. (LHR-2016)

Given $a_8 = 31$ and $S_9 = 171$

$$S_9 = 171$$

$$\frac{9}{2}\{2a_1 + 8d\} = 171$$

$$\frac{9}{\cancel{2}} \cdot \cancel{2}(a_1 + 4d) = 171$$

$$9a_1 + 36d = 171 \quad \text{— (i)}$$

$$a_8 = 31$$

$$a_1 + 7d = 31 \quad \text{— (ii)}$$

Multiply by 9

$$9a_1 + 63d = 279 \quad \text{— (iii)}$$

Substracting (i) by (iii)

$$\cancel{9a_1} + 63d = 279$$
$$-\cancel{9a_1} + 36d = 171$$
$$27d = 108$$

$$\boxed{d = 4}$$

put $d = 4$ in (ii)

$$a_1 + 7(4) = 31$$

$$a_1 = 31 - 28$$

$$\boxed{a_1 = 3}$$

So required series is

$= a_1 + (a_1+d) + (a_1+2d) + \cdots$

$= 3 + (3+4) + (3+2(4)) + \cdots$

$= 3 + 7 + 11 + \cdots$

Question 08

The sum of three numbers in an A.P is 24 and their Product is 440. Find the numbers. (LHR-2015+2022)

Let a_1-d, a_1, a_1+d be the three numbers in A.P.

According to the conditions

$a_1 - \cancel{d} + a_1 + a_1 + \cancel{d} = 24$

$3a_1 = 24$

$\boxed{a_1 = 8}$

$(a_1-d)(a_1)(a_1+d) = 440$

$a_1 - d(a_1^2 + a_1 d) = 440$

$a_1^3 + \cancel{a_1^2 d} - \cancel{a_1^2 d} - a_1 d^2 = 440$

$(8)^3 - 8d^2 = 440$

$-8d^2 = 440 - 512$

$-8d^2 = -72$

$d^2 = 9$

$\boxed{d = \pm 3}$

when $a_1 = 8$ and $d = 3$ then numbers are

$a_1 - d = 8 - 3 = 5$

$a_1 = 8$

$a_1 + d = 8 + 3 = 11$

when $a_1 = 8$ and $d = -3$ then numbers are

$a_1 - d = 8 + 3 = 11$

$a_1 = 8$

$a_1 + d = 8 - 3 = 5$

So the numbers are 5, 8, 11 or 11, 8, 5.

Question 09

Find four numbers in A.P whose sum is 32 and the sum of whose square is 276 (LHR-2018)

Let $a_1-3d, a_1-d, a_1+d, a_1+3d$ be the four numbers in A.P.

According to the conditions

$a_1 - \cancel{3d} + a_1 + \cancel{d} + a_1 - \cancel{d} + a_1 + \cancel{3d} = 32$

$4a_1 = 32$

$\boxed{a_1 = 8}$

$(a_1-3d)^2 + (a_1-d)^2 + (a_1+d)^2 + (a_1+3d)^2 = 276$

$a_1^2 + 9d^2 - \cancel{6a_1 d} + a_1 + d^2 - \cancel{2a_1 d} + a_1^2 + d^2 + \cancel{2a_1 d} + a_1^2 + 9d^2 + \cancel{6a_1 d} = 276$

$4a_1^2 + 20d^2 = 276$

Dividing both sides by 4

$a_1^2 + 5d^2 = 69$

$(8)^2 + 5d^2 = 69$

$$5d^2 = 69 - 64$$
$$5d^2 = 5$$
$$\boxed{d = \pm 1}$$

when $a_1 = 8$ and $d = 1$ then numbers are

$a_1 - 3d = 8 - 3 = 5$
$a_1 - d = 8 - 1 = 7$
$a_1 + d = 8 + 1 = 9$
$a_1 + 3d = 8 + 3 = 11$

when $a_1 = 8$ and $d = -1$ then numbers are

$a_1 - 3d = 8 + 3 = 11$
$a_1 - d = 8 + 1 = 9$
$a_1 + d = 8 - 1 = 7$
$a_1 + 3d = 8 - 3 = 5$

So the numbers are $5, 7, 9, 11$ or $11, 9, 7, 5$.

Question 10

If a, b, c, d are in G.P, Prove that $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P. (LHR-2017)

If a, b, c, d are in G.P so their exists common ratio.

$$r = \frac{b}{a}, \quad r = \frac{c}{b}, \quad r = \frac{d}{c}$$

$$\boxed{b = ar}, \quad c = br, \quad d = cr$$

$$\boxed{c = ar^2}, \quad \boxed{d = ar^3}$$

Now $a^2 - b^2, b^2 - c^2$ and $c^2 - d^2$ will be in G.P if

$$\frac{b^2 - c^2}{a^2 - b^2} = \frac{c^2 - d^2}{b^2 - c^2}$$

$$\frac{a^2r^2 - a^2r^4}{a^2 - a^2r^2} = \frac{a^2r^4 - a^2r^6}{a^2r^2 - a^2r^4}$$

$$\frac{\cancel{a^2}r^2\cancel{(1 - r^2)}}{\cancel{a^2}\cancel{(1 - r^2)}} = \frac{\cancel{a^2}r^4\cancel{(1 - r^2)}}{\cancel{a^2}r^2\cancel{(1 - r^2)}}$$

$$r^2 = r^2 \quad \because \text{Common ratio exists}$$

So $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

Question 11

Find three consecutive numbers in G.P whose sum is 26 and their Product is 216. (LHR-2016+2019)

Let the three consecutive numbers in G.P are $\frac{a_1}{r}, a_1, a_1 r$.

According to the given conditions

$$\left(\frac{a_1}{r}\right)(a_1)(a_1 r) = 216$$
$$a_1^3 = 216$$
$$\boxed{a_1 = 6}$$

$$\frac{a_1}{r} + a_1 + a_1 r = 26$$

put $a_1 = 6$

$$\frac{6}{r} + 6 + 6r = 26$$

$$\frac{6+6r+6r^2}{r} = 26$$

$$6r^2+6r+6-26r=0$$

$$6r^2-20r+6=0$$

$$2(3r^2-10r+3)=0$$

$$3r^2-10r+3=0$$

$$3r^2-9r-r+3=0$$

$$3r(r-3)-1(r-3)=0$$

$$(r-3)(3r-1)=0$$

$$\boxed{r=3} \qquad \boxed{r=1/3}$$

When $a_1=6$ and $r=3$ then numbers are

$$\frac{a_1}{r} = \frac{6}{3} = 2$$

$$a_1 = 6$$

$$a_1 r = (6)(3) = 18$$

When $a_1=6$ and $r=1/3$ then numbers are

$$\frac{a_1}{r} = \frac{6}{\frac{1}{3}} = 18$$

$$a_1 = 6$$

$$a_1 r = 6\left(\frac{1}{3}\right) = 2$$

So the numbers are $2, 6, 18$ or $18, 6, 2$.

Question 12

For what value of n, $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive G.M between a and b? (LHR-2013+15+18)

Given

$$G.M = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$\sqrt{ab} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$a^{1/2}b^{1/2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$a^n+b^n = (a^{n-1}+b^{n-1})(a^{1/2}b^{1/2})$$

$$a^n+b^n = a^{n-1}a^{1/2}b^{1/2} + b^{n-1}a^{1/2}b^{1/2}$$

$$a^{1/2}\cdot a^{n-\frac{1}{2}} + b^{1/2}\cdot b^{n-\frac{1}{2}} = a^{n-\frac{1}{2}}b^{1/2} + b^{n-\frac{1}{2}}a^{1/2}$$

$$a^{1/2}a^{n-1/2} - a^{n-1/2}b^{1/2} = b^{n-1/2}a^{1/2} - b^{n-1/2}b^{1/2}$$

$$a^{n-1/2}\cancel{(a^{1/2}-b^{1/2})} = b^{n-1/2}\cancel{(a^{1/2}-b^{1/2})}$$

$$\left(\frac{a}{b}\right)^{n-\frac{1}{2}} = \left(\frac{a}{b}\right)^{0}$$

$$n-\frac{1}{2} = 0$$

$$\boxed{n = 1/2}$$

Question 13

Find the sum to infinity of the series. $r+(1+k)r^2+(1+k+k^2)r^3+\cdots$ (LHR-2011)

Given infinite geometric series is

$$S_\infty = r+(1+k)r^2+(1+k+k^2)r^3+\cdots$$

Multiplying and dividing by $(1-k)$

$$S_\infty = \frac{1}{1-k}\left[r(1-k)+r^2(1-k^2)+r^3(1-k^3)+\cdots\right]$$

$$S_\infty = \frac{1}{1-k}\left(r-kr+r^2-k^2r^2+r^3-k^3r^3+\cdots\right)$$

$$S_\infty = \frac{1}{1-k}\left[(r+r^2+r^3+\cdots)-(kr+k^2r^2+k^3r^3+\cdots)\right]$$

$$S_\infty = \frac{1}{1-k}\left[\left(\frac{r}{1-r}\right)-\left(\frac{kr}{1-kr}\right)\right] \qquad \because S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{1}{1-k}\left(\frac{r(1-kr)-kr(1-r)}{(1-r)(1-kr)}\right)$$

$$S_\infty = \frac{1}{1-k}\left(\frac{r-\cancel{kr^2}-kr+\cancel{kr^2}}{(1-r)(1-kr)}\right)$$

$$S_\infty = \frac{1}{\cancel{1-k}}\left(\frac{r\cancel{(1-k)}}{(1-r)(1-kr)}\right)$$

$$S_\infty = \frac{r}{(1-r)(1-kr)}$$

Question 14

The sum of an infinite geometric series is 9 and the sum of the squares of its terms is $\frac{81}{5}$. Find the series. (LHR-2022)

Let the infinite geometric series is

$$a_1+a_1r+a_1r^2+a_1r^3+\cdots$$

According to the given conditions

$$a_1+a_1r+a_1r^2+\cdots = 9$$

we know that

$$S_\infty = \frac{a_1}{1-r}$$

$$9 = \frac{a_1}{1-r}$$

$$9-9r = a_1$$

$$a_1^2+a_1^2r^2+a_1^2r^4+\cdots = \frac{81}{5}$$

we know that

$$S_\infty = \frac{a_1}{1-r}$$

$$\frac{81}{5} = \frac{a_1^2}{1-r^2}$$

$a(1-r) = a_1$

put $r = 2/3$

$9\left(1-\frac{2}{3}\right) = a_1$

$a_1 = 9\left(\frac{3-2}{3}\right)$

$\boxed{a_1 = 3}$

$5a_1^2 = 81(1-r^2)$

$5(81(1-r)(1-r)) = 81(1-r)(1+r)$

$5(1-r) = (1+r)$

$5-5r = 1+r$

$5-1 = r+5r$

$6r = 4$

$\boxed{r = 2/3}$

So the infinite geometric series is

$= 3 + 3\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + \cdots$

$= 3 + 2 + \frac{4}{3} + \cdots$

Question 15

If the 7^{th} and 10^{th} terms of an H.P are $\frac{1}{3}$ and $\frac{5}{21}$. Find the 14^{th} term. (LHR-2019)

Given $a_7 = \frac{1}{3}$ and $a_{10} = \frac{5}{21}$ are in H.P.

Their reciprocals $a_7 = 3$ and $a_{10} = \frac{21}{5}$ are in A.P.

$a_7 = 3$

$a_1 + 6d = 3$ —(i)

Multiply (i) by 5.

$5a_1 + 30d = 15$ —(iii)

$a_{10} = \frac{21}{5}$

$a_1 + 9d = \frac{21}{5}$

$5a_1 + 45d = 21$ —(ii)

Substracting (iii) by (ii)

$$\begin{array}{r} 5a_1 + 45d = 21 \\ -5a_1 \pm 30d = -15 \\ \hline 15d = 6 \end{array}$$

$\boxed{d = \frac{2}{5}}$

put $d = 2/5$ in (i)

$a_1 + 6\left(\frac{2}{5}\right) = 3$

$a_1 + \frac{12}{5} = 3$

$a_1 = 3 - \frac{12}{5}$

$a_1 = \frac{15-12}{5}$

$\boxed{a_1 = 3/5}$

we know that

$a_n = a_1 + (n-1)d$

$a_{14} = \frac{3}{5} + 13\left(\frac{2}{5}\right)$

$a_{14} = \frac{3}{5} + \frac{26}{5}$

$a_{14} = \frac{29}{5}$

So the 14^{th} term of H.P is $\frac{5}{29}$.

Question 16

Find n, so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be H.M between a & b. (LHR-2013+2014)

Given

$$H.M = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$$\frac{2ab}{a+b} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$$2ab(a^n+b^n) = (a^{n+1}+b^{n+1})(a+b)$$

$$2a^{n+1}b+2ab^{n+1} = a^{n+2}+b\cdot a^{n+1}+b^{n+1}\cdot a+b^{n+2}$$

$$2a^{n+1}b-b\cdot a^{n+1}+2ab^{n+1}-b^{n+1}a = a^{n+2}+b^{n+2}$$

$$a^{n+1}b+ab^{n+1} = a\cdot a^{n+1}+b\cdot b^{n+1}$$

$$a^{n+1}b-a\cdot a^{n+1} = b\cdot b^{n+1}-ab^{n+1}$$

$$a^{n+1}\cancel{(b-a)} = b^{n+1}\cancel{(b-a)}$$

$$\left(\frac{a}{b}\right)^{n+1} = \left(\frac{a}{b}\right)^0$$

$$n+1=0$$

$$\boxed{n=-1} \text{ Answer}$$