1st Year Physics

Chapter # 06

Fluid Dynamics

Written by:

Mr. Shahroze Saleem

M. Phil (Physics), B. Ed.

Lecturer, Chenab College Jhang.

Ch # 06

Fluid Dynamics

The branch of physics in which we study about the motion, effects and application of fluid's motion is called fluid dynamics.

Viscosity:

The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that donot flow easily, such as thick tar and honey etc, have large coefficients of viscosity, usually denoted by greek letter 'η'. Substances which flow easily, like water, have small coefficient of viscosities. It SI unit is Nsm^{-2}.

Drag force / Stoke's law:

An object moving through a fluid experiences a retarding force called a drag force. The drag force increases as the speed of the object increases.
The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is given as stoke's law as under:

$$F = 6\pi\eta r v$$

At high speeds the force is no longer simply proportional to speed.

Terminal velocity

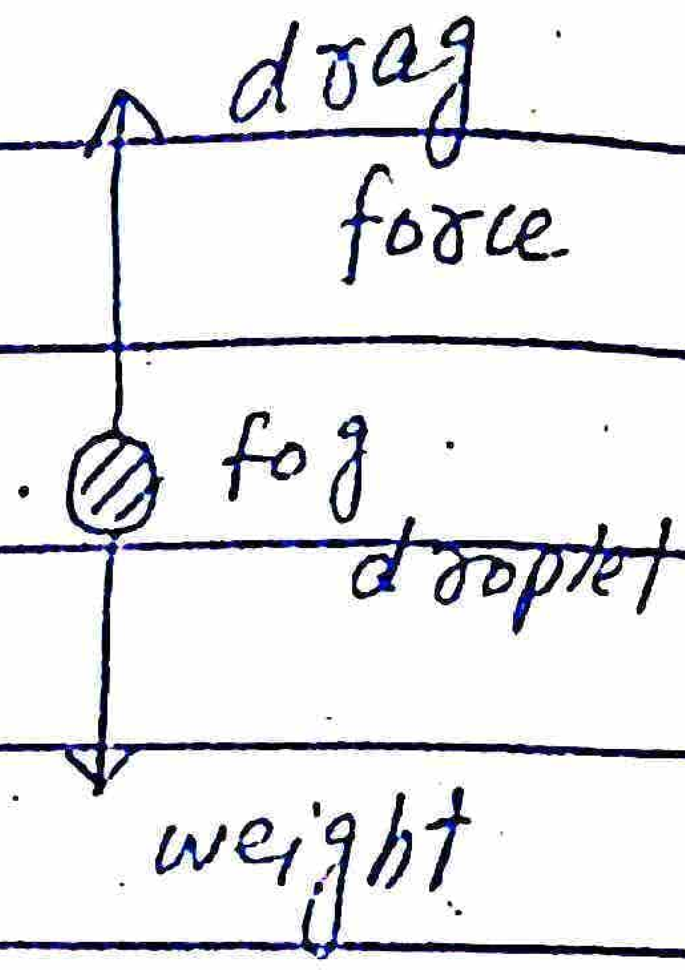
Defination:
When an object falls through a fluid and its weight becomes equal to the drag

force. Then the small constant velocity of the object is called terminal velocity.

mathematically,

$$V_t = \frac{2gr^2\rho}{9\eta}$$

Explanation:



Consider a fog droplet of mass 'm' falling through air. Its weight is acting downward while drag force due to air is acting upward. The net force is given by

net force = weight − drag force

For the terminal velocity, the weight and drag force will become equal and net force will be zero. So,

weight = drag force

$$mg = 6\pi\eta r V_t$$

$$\frac{mg}{6\pi\eta r} = V_t$$

$$\boxed{V_t = \frac{mg}{6\pi\eta r}} \longrightarrow (1$$

we know that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$\rho V = m$$

$$m = \rho V$$

For spherical droplet:

$$V = \frac{4}{3}\pi r^3$$

so,

$$m = \frac{4}{3}\pi r^3 \rho$$

Put this expression in eq. (1

$$V_t = \frac{\frac{4}{3}\cancel{\pi} r^{\cancel{3}\,2} \rho g}{6\cancel{\pi}\eta \cancel{r}}$$

$$= \frac{\cancel{4}^{\,2} r^2 \rho g}{3(\cancel{6}_3)\eta}$$

$$\boxed{V_t = \frac{2gr^2\rho}{9\eta}}$$

$$V_t \propto r^2$$

The above is the equation for terminal velocity. It shows that terminal velocity is directly proportional to the square radius of droplet.

Streamline Flow:

The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

Stream line: In stream line particle of the fluid moves along a smooth path called streamline.

Steady flow condition: The different streamlines can not cross each other. This condition is called steady flow condition. This condition is called steady flow condition. The direction of the streamline is the same as the direction of the velocity of the fluid at that point.

Turbulent Flow: The irregular or unsteady flow of the fluid is called turbulent flow.

Conditions of Ideal Fluid:

1: The fluid is non-viscous i.e. there is no internal frictional force between adjacent layers of fluid.

2: The fluid is incompressible, i.e. the density is constant.

3: The fluid motion is steady.

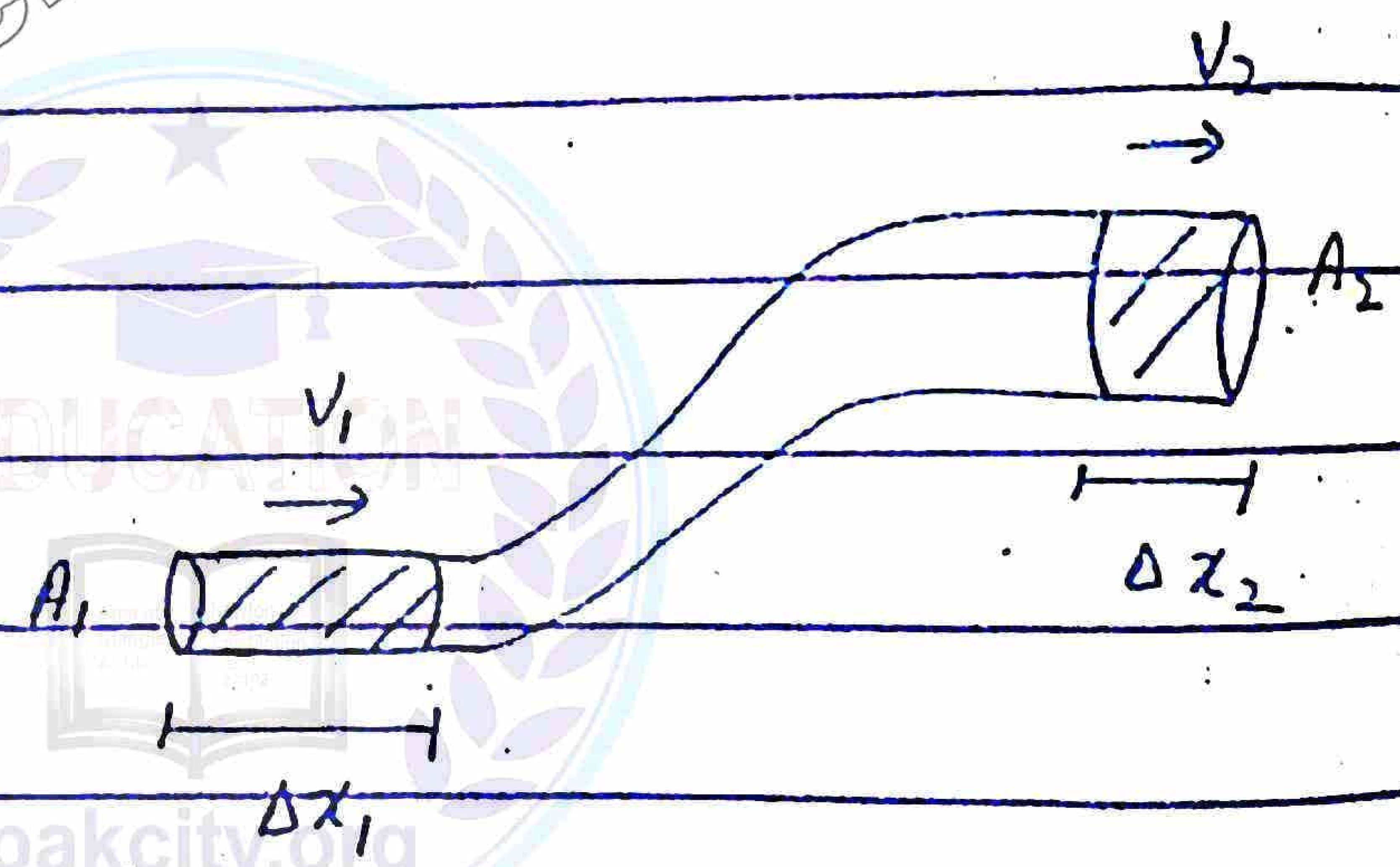
Equation Of Continuity:

Statement:

The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate.

Explanation:

Consider a non-uniform pipe through which a



non-viscous and incompressible fluid is moving as shown in the figure. At the lower end, cross sectional area is A_1, speed is v_1 and distance covered in time Δt is Δx_1. Similarly, at upper end. cross sectional area is A_2. Speed is v_2 and distance covered in time Δt is Δx_2.

we know that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$m = V\rho$$

For lower end.

$$\Delta m_1 = V_1 \rho_1$$

$$\Delta m_1 = A_1 \Delta x_1 \rho_1$$

As

$$S = vt$$

so,

$$\Delta x_1 = v_1 \Delta t$$

so, mass becomes

$$\Delta m_1 = A_1 v_1 \Delta t \rho_1$$

Similarly. for upper end

$$\Delta m_2 = A_2 v_2 \Delta t \rho_2$$

As the mass of fluid will remain same for lower and upper end, so

$$\Delta m_1 = \Delta m_2$$

$$A_1 v_1 \cancel{\Delta t} \rho_1 = A_2 v_2 \cancel{\Delta t} \rho_2$$

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

As the fluid is incompressible. So, its density will remain same.

$$\rho_1 = \rho_2 = \rho$$

so,

$$A_1 v_1 \cancel{\rho} = A_2 v_2 \cancel{\rho}$$

$$A_1 v_1 = A_2 v_2$$

This is the mathematical expression for equation of continuity. Also,

$$Av = \text{constant} = \frac{\text{Volume}}{\text{time}}$$

$$Av = \text{flow rate}$$

The unit of flow rate is $m^3 s^{-1}$.

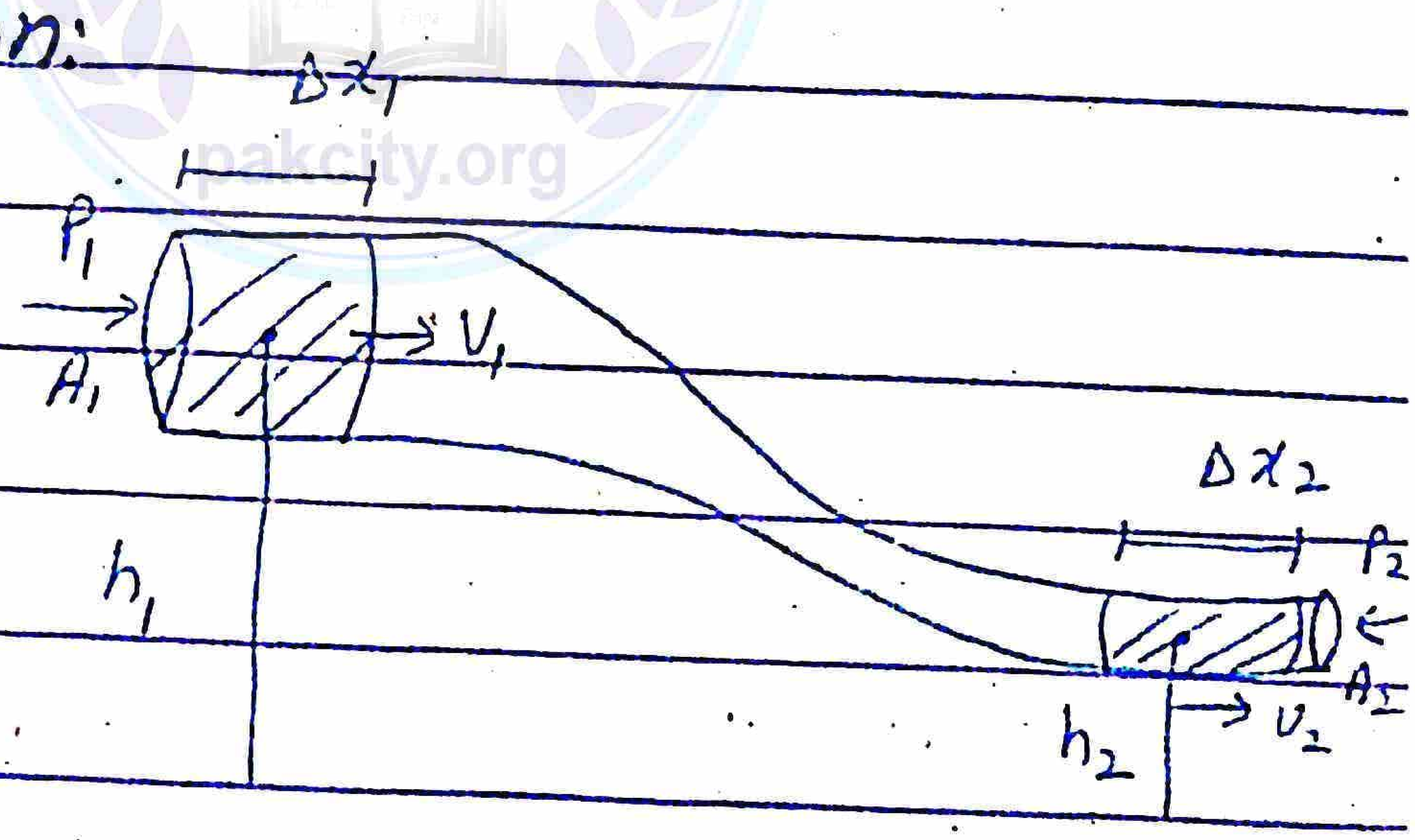
Bernoulli's Equation

Statement:

Bernoulli's equation is a fundamental equation in fluid dynamics that relates pressure to speed and height. It states that sum of pressure, K.E per unit volume and P.E per unit volume for a fluid moving through pipe remains constant. mathematically,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Explanation:



Consider a non uniform pipe through which a non viscous and incompressible fluid is moving as shown in figure. At upper end, the cross sectional area is A_1, speed is v_1, atmospheric pressure is P_1, height is h_1 and the distance covered in time t is Δx_1. while at lower end. the cross sectional area is A_2, speed is V_2, atmospheric pressure is P_2, height is h_2 and the distance covered in time t is Δx_2. The work done by the fluid particles to move fluid is given by:

$$W_1 = F_1 \Delta x_1$$

As

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$P_1 = \frac{F_1}{A_1}$$

$$P_1 A_1 = F_1$$

So,

$$W_1 = P_1 A_1 \Delta x_1$$

As $S = vt$

$$\Delta x_1 = v_1 t$$

so,

$$W_1 = P_1 A_1 v_1 t$$

Similarly, the work done at the lower end is:

$$W_2 = -P_2 A_2 v_2 t$$

Negative sign shows that work is done against the pressure.

Net work done $= W = W_1 + W_2$

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t$$

According to equation of continuity:

$$A_1 v_1 = A_2 v_2 = \frac{V}{t}$$

$$A_1 v_1 t = A_2 v_2 t = V$$

so,

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V$$

We know that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho}$$

So,

$$W = (P_1 - P_2)\frac{m}{\rho} \rightarrow (1$$

This amount of work done for fluid will be changed into K.E and P.E.

change in K.E $= \Delta K.E = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

change in P.E $= \Delta P.E = mgh_2 - mgh_1$

According to law of conservation of energy,

Work done $= \Delta K.E + \Delta P.E$

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2)\frac{\cancel{m}}{\rho} = \cancel{m}\left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

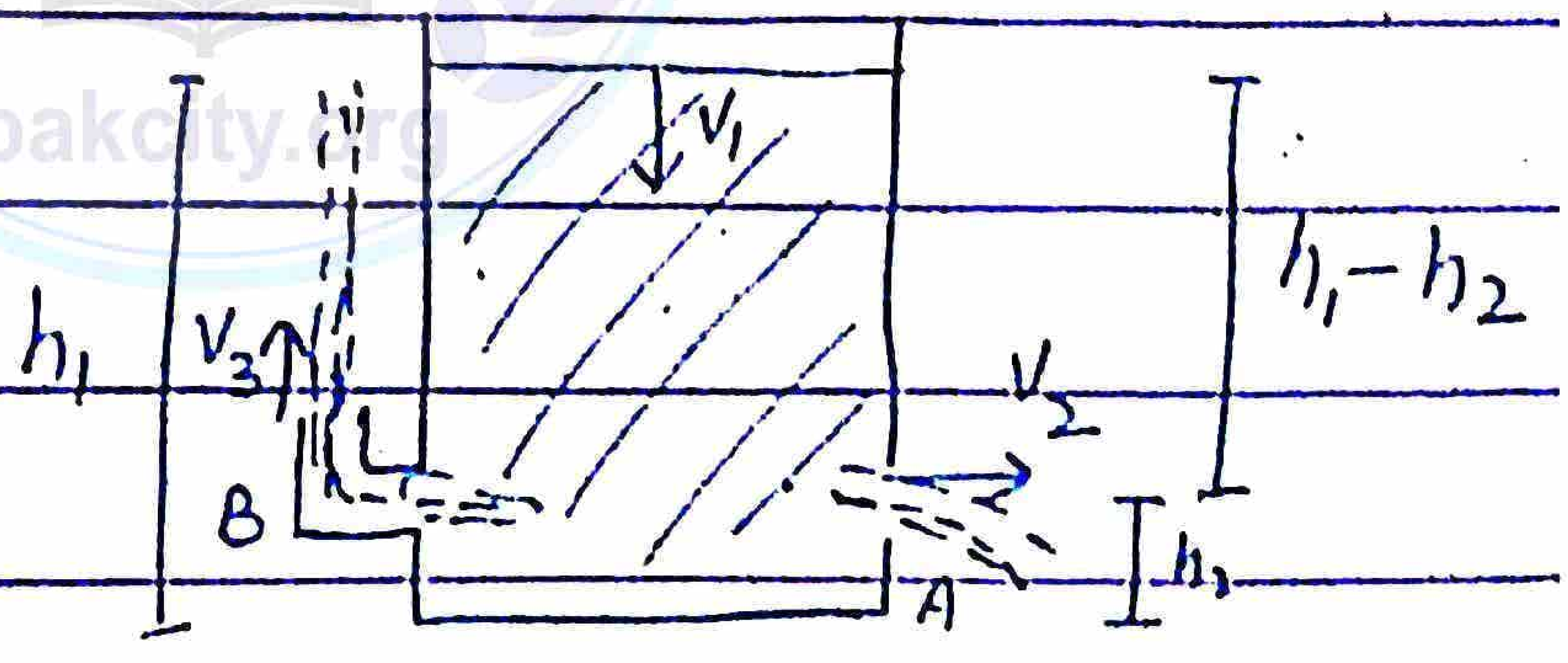
Torricelli's Theorem

Statement:

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Explanation:

Consider a tank of fluid having two orifices A and B



near the bottom of tank. The fluid moving through orifice A has speed v_2, through orifice B has speed v_3 while the surface is moving down with speed V_1. According to Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

The atmospheric pressure for positions 1 and 2 will be same.

$$P_1 = P_2 = P$$

so,

$$\cancel{P} + \frac{1}{2}\rho V_1^2 + \rho g h_1 = \cancel{P} + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\cancel{\rho}\left(\frac{1}{2}V_1^2 + g h_1\right) = \cancel{\rho}\left(\frac{1}{2}V_2^2 + g h_2\right)$$

$$\frac{1}{2}V_1^2 + g h_1 = \frac{1}{2}V_2^2 + g h_2$$

The speed of efflux through orifice A is much greater than speed at surface

$$V_2 >> V_1$$

Now we neglect V_1. i.e $V_1 = 0$

so,

$$g h_1 = \frac{1}{2}V_2^2 + g h_2$$

$$gh_1 - gh_2 = \frac{1}{2} v_2^2$$

$$2g(h_1 - h_2) = v_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

For the orifice B facing upward, the height of the fluid will be same as the height of fluid in the tank. Because the K.E gained through the fluid will be equal to the P.E of fluid due to height.

Relation Between Speed And Pressure Of The Fluid:

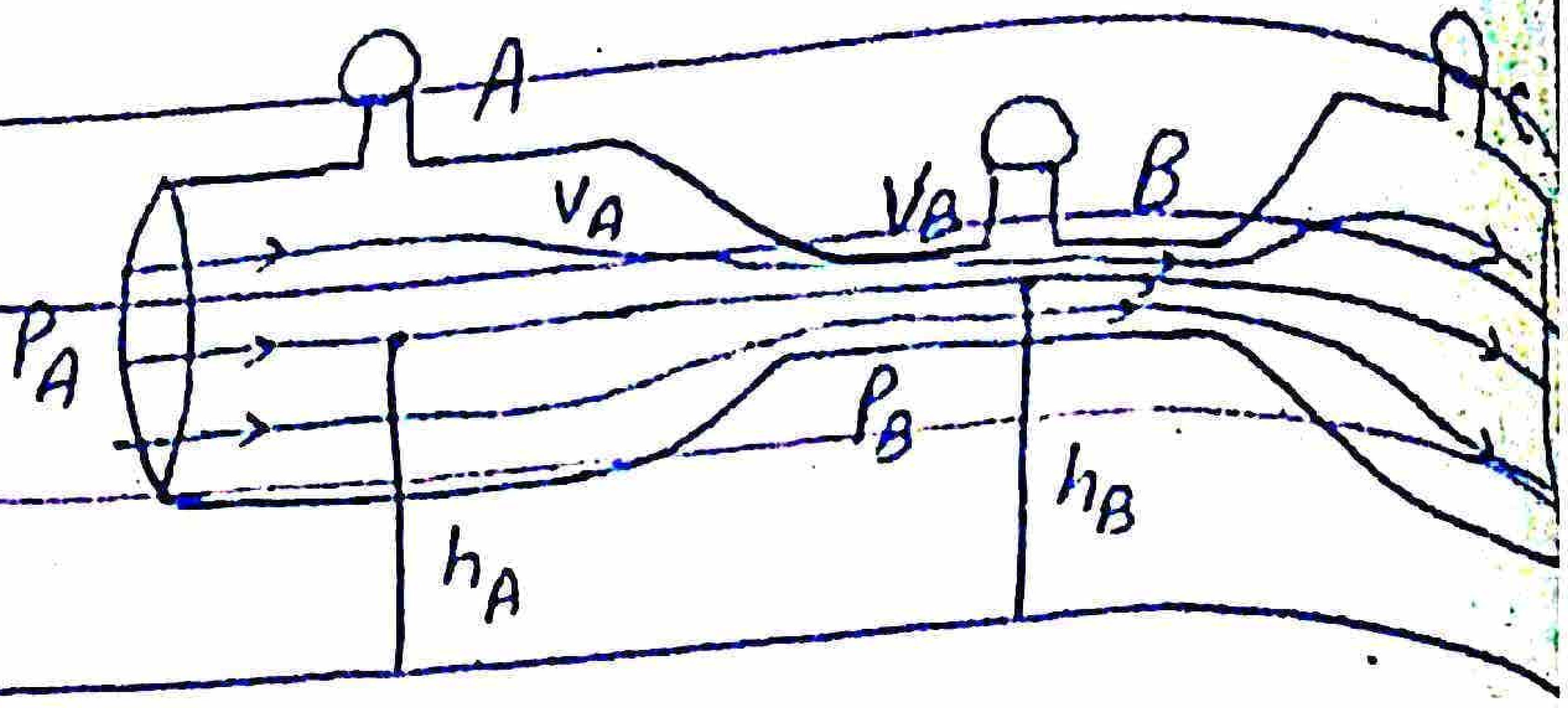
Relation:

where the speed is high the pressure will be low.

Explanation:

Consider a non-uniform

pipe having three parts of dumbel shape



through which a non viscous and incompressible fluid is moving. The speed of fluid through portion A. is V_A, pressure is P_A and height is h_A. while for portion B. speed is V_B, pressure is P_B and height is h_B. According to Bernoulli's equation.

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

As the heights for both the portions is same:

$$h_A = h_B = h$$

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h = P_B + \frac{1}{2}\rho V_B^2 + \rho g h$$

$$P_A - P_B = \frac{1}{2}\rho V_B^2 - \frac{1}{2}\rho V_A^2$$

According to equation of continuity, the speed of fluid will be greater where the cross sectional area is small. consider

$V_A = 0.2\ ms^{-1}$, $V_B = 2\ ms^{-1}$

density of water $= \rho = 1000\ Kgm^{-3}$

so,

$$P_A - P_B = \frac{1}{2}(1000)(2)^2 - \frac{1}{2}(1000)(0.2)^2$$

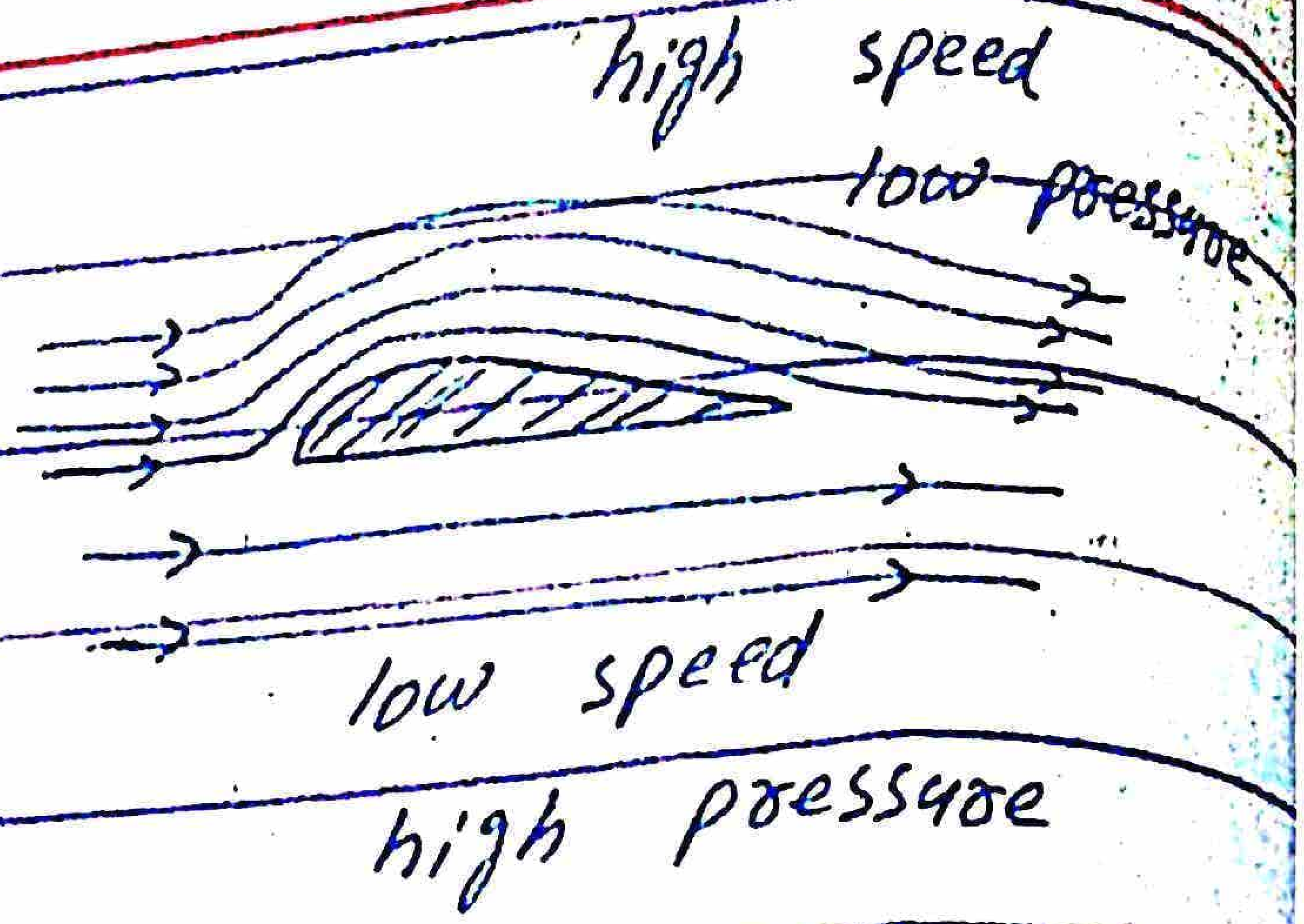
$$= 2000 - 20$$

$$P_A - P_B = 1980\ Nm^{-2}$$

So, the pressure at portion A is greater. But here speed is smaller.

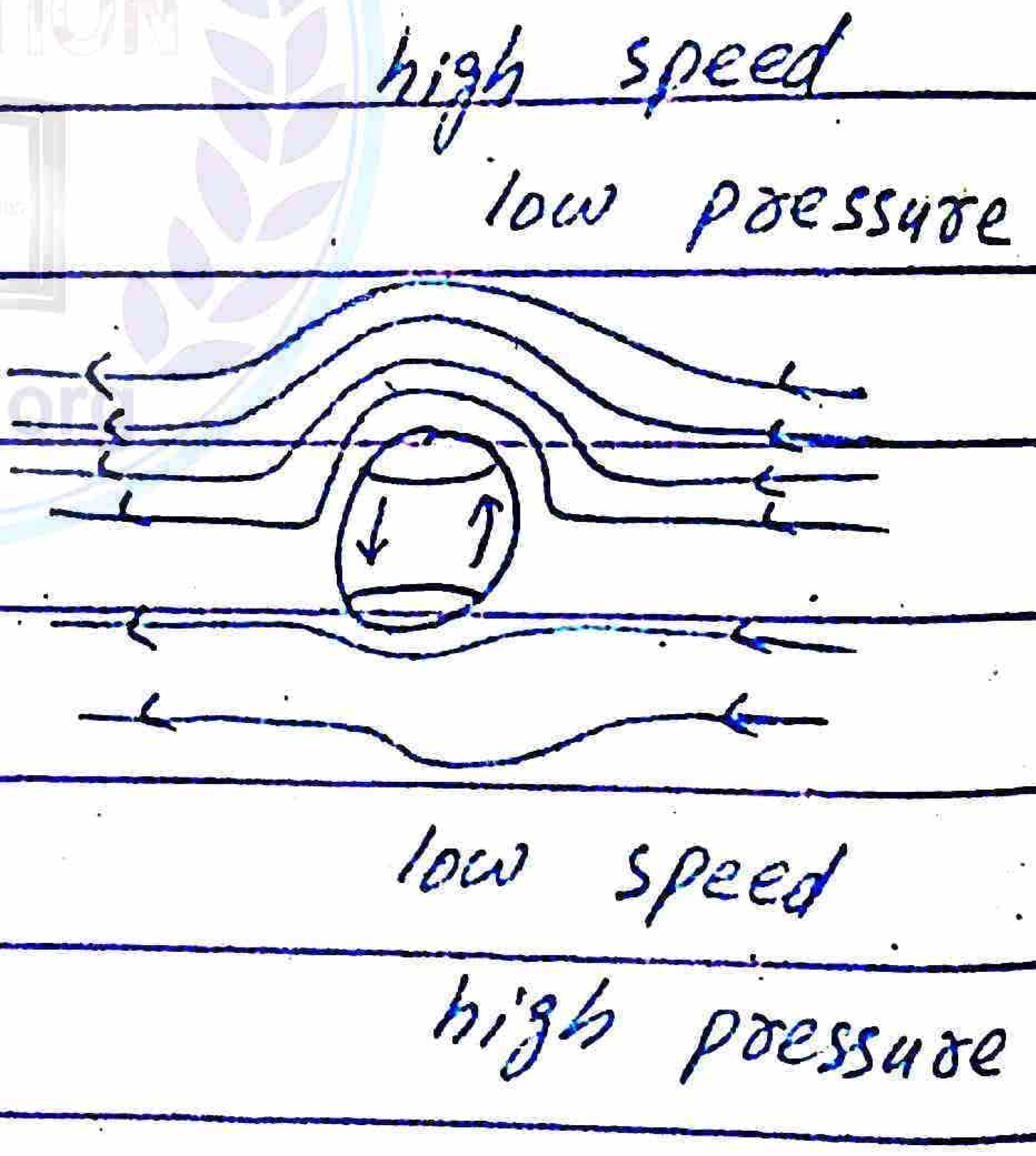
Lift of aeroplane:

The wings of aeroplane are designed such that they creater high speed and low pressure at the upper side. while there is low speed and high atmospheric pressure at the lower portion of wing. Therefore, this high pressure push the aeroplane upward.



Swing of ball:

When the ball is hit by bat or racket, it will create high speed and low pressure there. while the high pressure present at the opposite face will produce swing in the



ball. This extra curvature produced in the ball deceives the opponent player.

Venturi Relation

Defination:

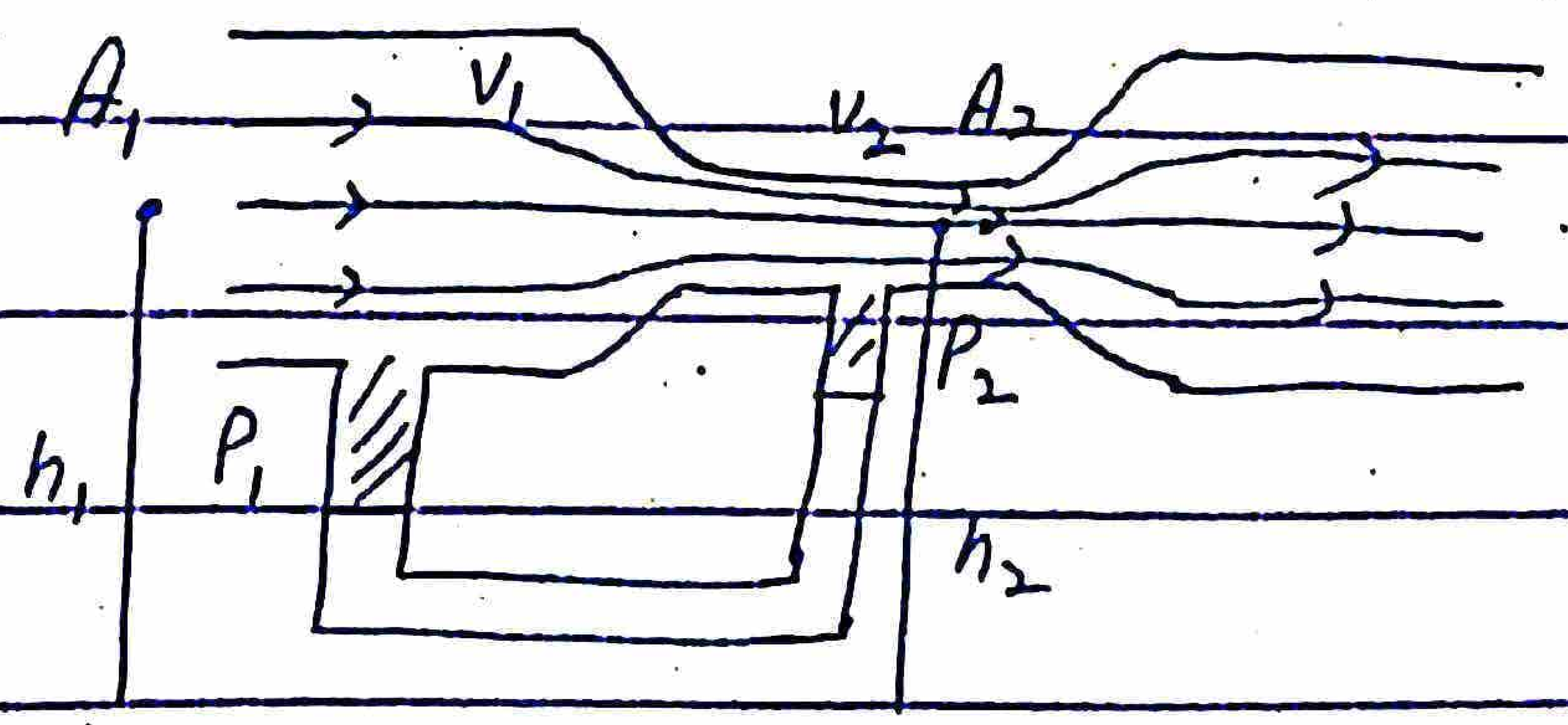
The device used to measure the speed of liquid flow is called venturi meter, the relation upon which venturi meter works is called venturi relation.

mathematically,

$$P_1 - P_2 = \frac{1}{2} \rho V_2^2$$

Explanation:

Consider a non uniform pipe of dum bel shape



through which a non viscous

an incompressible fluid is moving as shown in the figure. For position 1, cross sectional area is A_1, speed is V_1, height is h_1 and pressure is P_1. While for the middle portion, cross sectional area is A_2, speed is V_2, height is h_2 and pressure is P_2.

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

As the heights for both the portions is same.

$$h_1 = h_2 = h$$

so,

$$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g h} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g h}$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2$$

According to equation of continuity, the speed will be

low where the cross sectional area is greater. $v_1 < v_2$. So we neglect v_1. i.e. $v_1 = 0$
now

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2$$

This equation is known as venturi relation.

Blood flow:

Density of blood:

Blood is an incompressible fluid having a density nearly equal to that of water.

Viscosity of blood: A high concentration (~50%) of red blood cells increases its viscosity from three to five times that of water.

Why is pressure of blood greater than atmospheric pressure?

Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances the volume of blood is sufficient to keep the vessels inflated at all times, even in the relaxed state between heart beats. This means there is tension in the walls of the blood vessels and consequently the pressure of blood inside is greater than the external atmospheric pressure.

Normal blood pressure:

The pressure varies from a high (systolic pressure) of 120 torr ($1\ torr = 133.3\ Nm^{-2}$) to a low diastolic pressure of about 75-80 torr between beats in normal, healthy person. The number tend to increase with age, corresponding to decrease in the flexilibity of the vessel walls.

Torr:

The unit torr or mm of Hg is opted instead of SI unit of pressure because of its extensive use in medical equipments.

$$1 \text{ torr} = 133.3 Nm^{-2}$$

Sphygmomanometer:

An instruement called sphygmomanometer measures blood pressure dynamically.

Measurement of systolic pressure:

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture produces a high flow speed. As a result the flow is initially turbulent.

Measurement of diastolic pressure:

As the pressure drops the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow switches from turbulent to laminar, and gurgle in the stethoscope disappears. This is the signal to record diastolic pressure.

Short Questions

6.1: The fractional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures how much force is required to slide one layer of liquid over another. Substances that donot flow easily such as honey and thick tar have a great co-efficient of viscosity usually denoted by greek letter "η" substances that flow easily like wate have a small co-efficient of viscosity. Its SI unit is Nms^{-2}.

6.2: An object moving through a fluid experiences a drag force called retarding force. The drag force increases as the speed of object increases. The drag force F on the sphere of radius r moving slowly with speed v through a fluid of viscosity 'η' is given stokes law as under.

$$F = 6\pi\eta r v$$

At high speeds the force is no longer propotional to speed.

6.3 The mass of fog droplet is very small. when it falls down, its weight becomes equal to drag force and it falls with a very small constant terminal velocity. so, it appears to be suspended in air.

6.4:

Laminar flow	Turbulent flow
The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path as followed by the particles which passed that points earlier.	The irregular or unsteady flow of fluid is called turbulent flow.

6.5: Bernoulli's equation is a fundamental equation in fluid dynamics that relates pressure to speed and height. It states that sum of pressure, K.E per unit volume, P.E per unit volume for a fluid moving through pipe remains constant.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Applications:

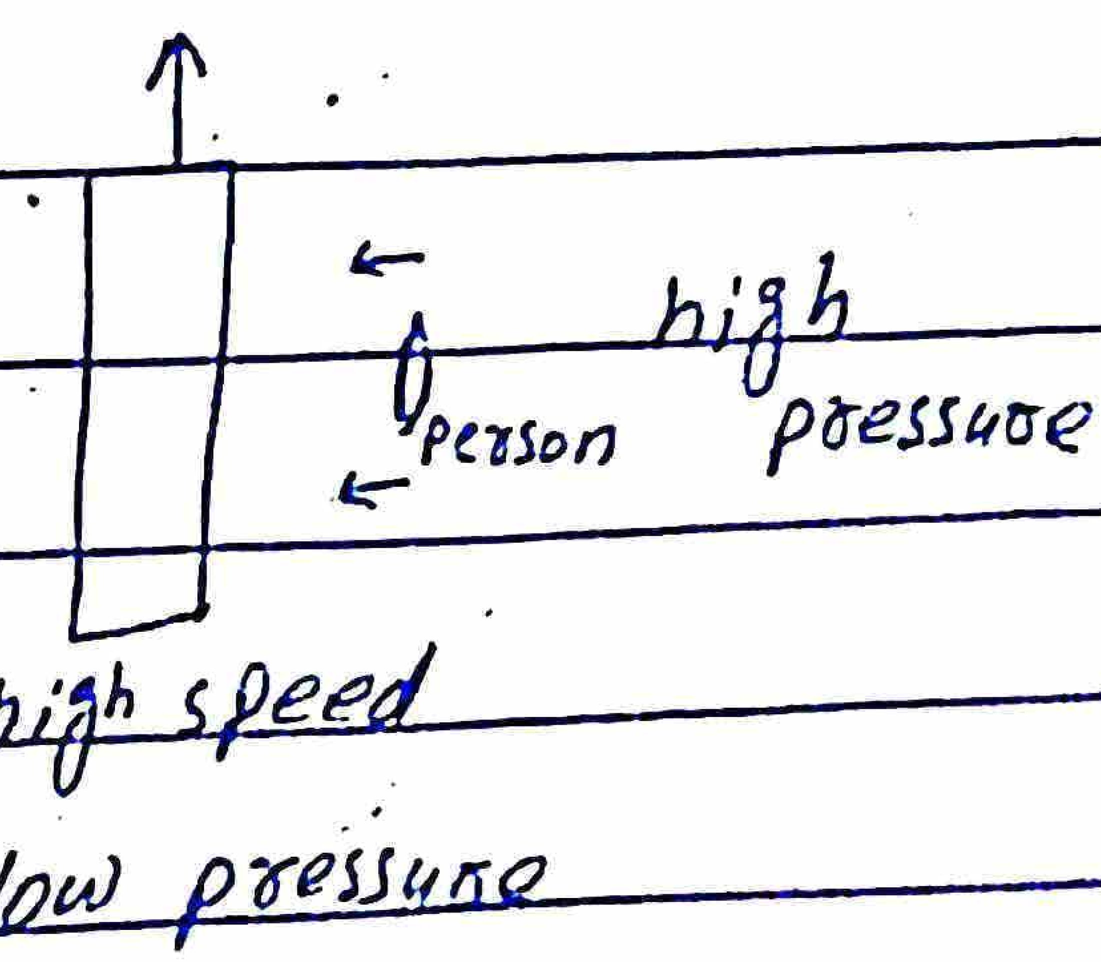
1: Lift of aeroplane

2: Swing of ball

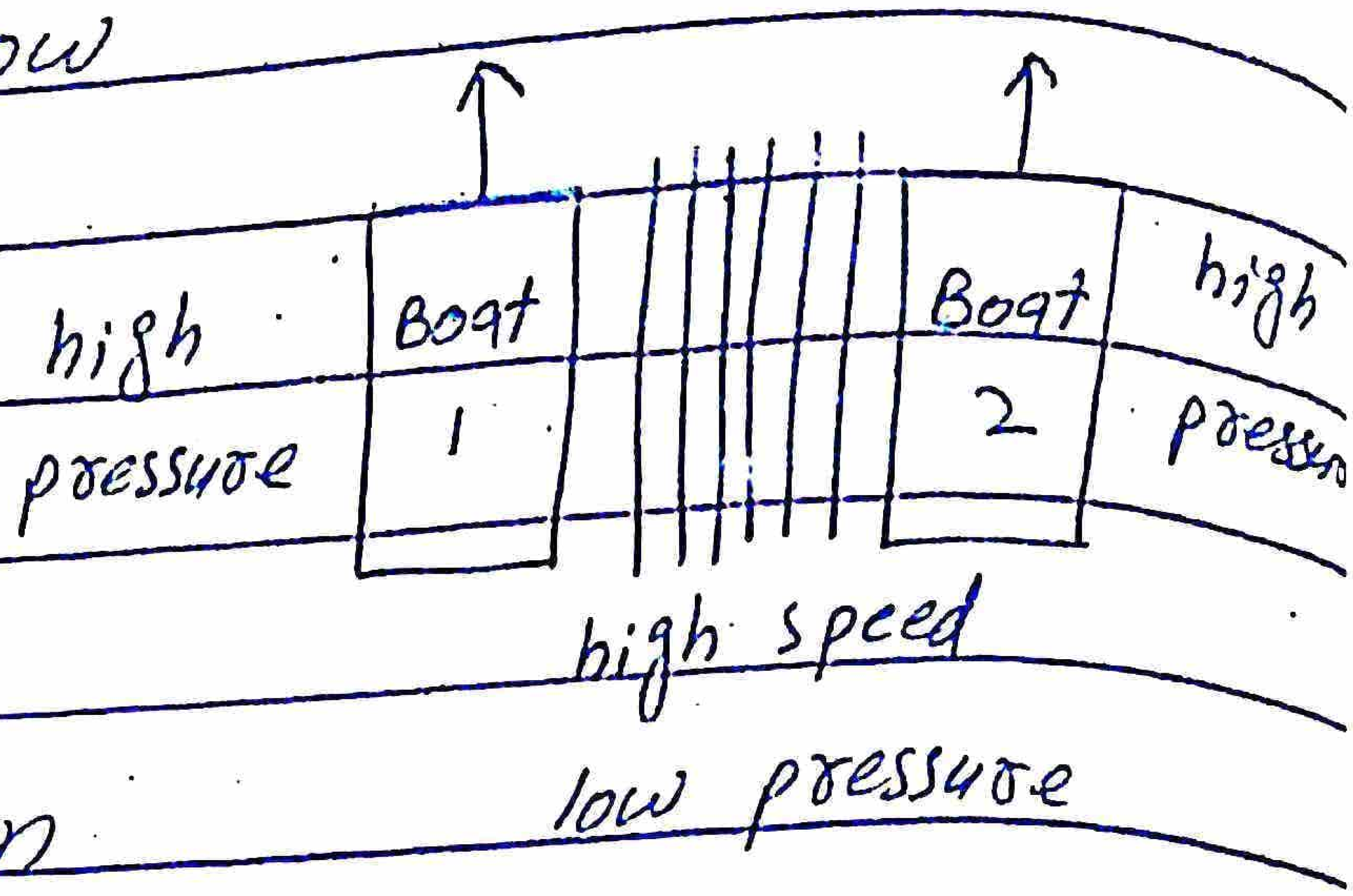
3: Venturi relation.

6.6 Yes, there is a danger that the person will fall towards the fast moving train.

Because the fast moving train will make low pressure while there is high pressure at the back of person that may push him towards the train.

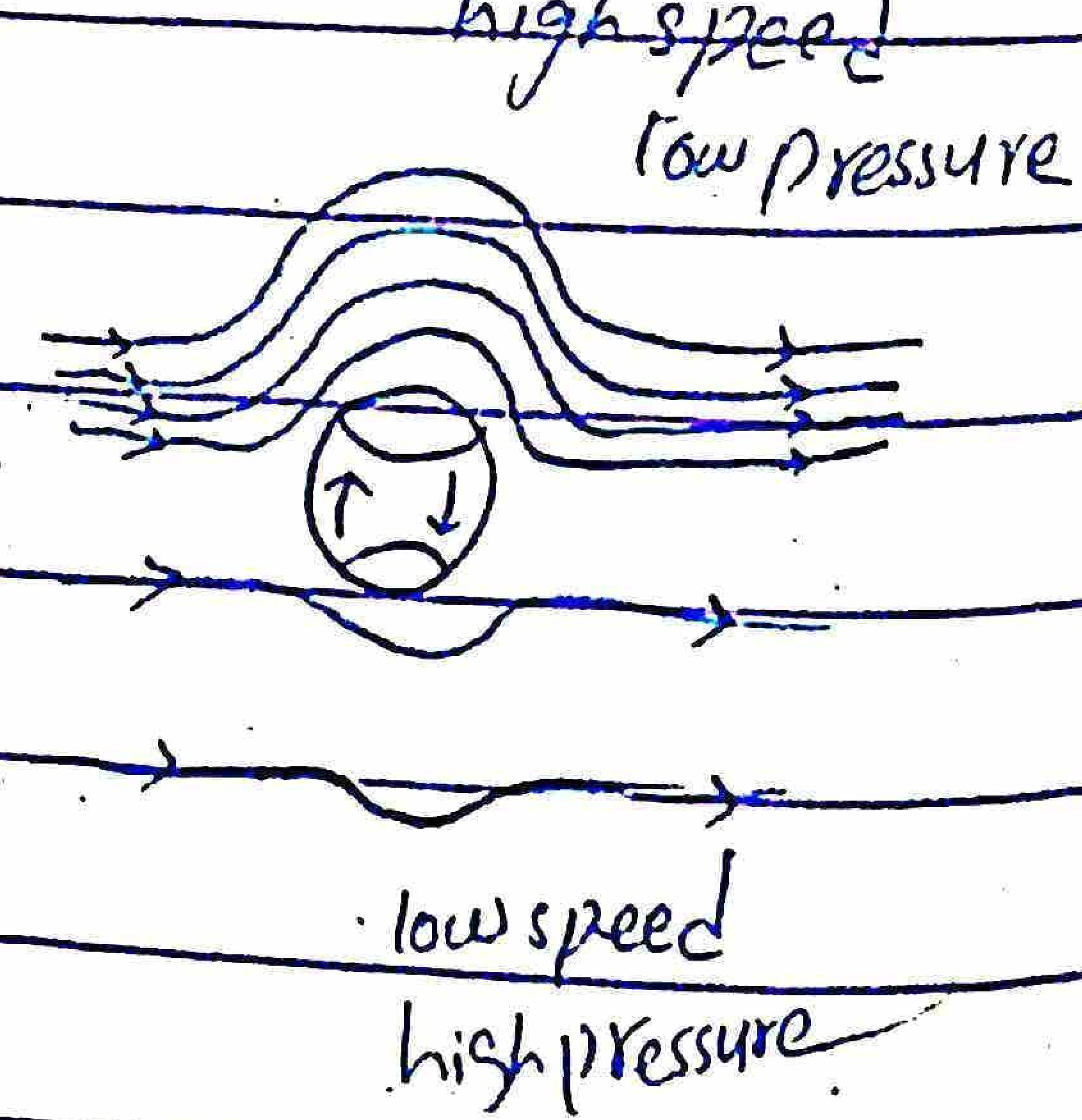


6.8: Two row boats moving parallel in the same direction are pulled towards each other due to difference of pressure created by water flow.



The speed of water will be high between the boats creating low pressure. while there is high pressure at the sides of the boats that may pull boats towards each other.

6.9: When the ball is hit by bat or rocket, it will create high speed and low pressure. While



the high pressure is present at the opposite face will produce swing in the ball. This extra curvature produced in the ball decieves in the opponent player.

6.10: The carburetor of a car engine uses a venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

6.11: Lying horizontally will be the smallest value of blood pressure in the body. Because the level of heart will be same as the level of the body.

6.12: In an orbiting space station, the blood pressure in major arteries in the leg would be same as the blood pressure in major arteries in the neck. Because in orbiting space station, there will be no gravity.

Numerical Problems

6.1 Data

density $= \rho = 1246\ kgm^{-3}$

$\eta = 8 \times 10^{-4}\ kgm^{-1}s^{-1}$

terminal speed $= v_t = 3\ cmh^{-1}$

$$v_t = \frac{3\ cm}{h} = \frac{3\ m}{(100)(3600)\ s}$$

$$v_t = 8.33 \times 10^{-6}\ ms^{-1}$$

radius $= r = ?$, $g = 9.8\ ms^{-2}$

Solution

$$v_t = \frac{2gr^2\rho}{9\eta}$$

$$9\eta v_t = 2 g r^2 \rho$$

$$\frac{9\eta v_t}{2 g \rho} = r^2$$

$$r^2 = \frac{9(8\times10^{-4})(8.33\times10^{-6})}{2(9.8)(1246)}$$

$$r^2 = 2.46\times10^{-12}$$

$$\boxed{r = 1.6\times10^{-6}\ m}$$

6.2 Data

diameter $= d_1 = 1\,cm = 10^{-2}\,m$

speed $= v_1 = 1\,ms^{-1}$

second diameter $= d_2 = ?$

second speed $= v_2 = 21\,ms^{-1}$

Solution According to equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_2}{2}\right)^2 v_2$$

$$\frac{d_1^2 v_1}{\cancel{4}} = \frac{d_2^2 v_2}{\cancel{4}}$$

$$d_1^2 v_1 = d_2^2 v_2$$

$$\frac{d_1^2 v_1}{v_2} = d_2^2$$

$$d_2^2 = \frac{(10^{-2})^2 (1)}{21}$$

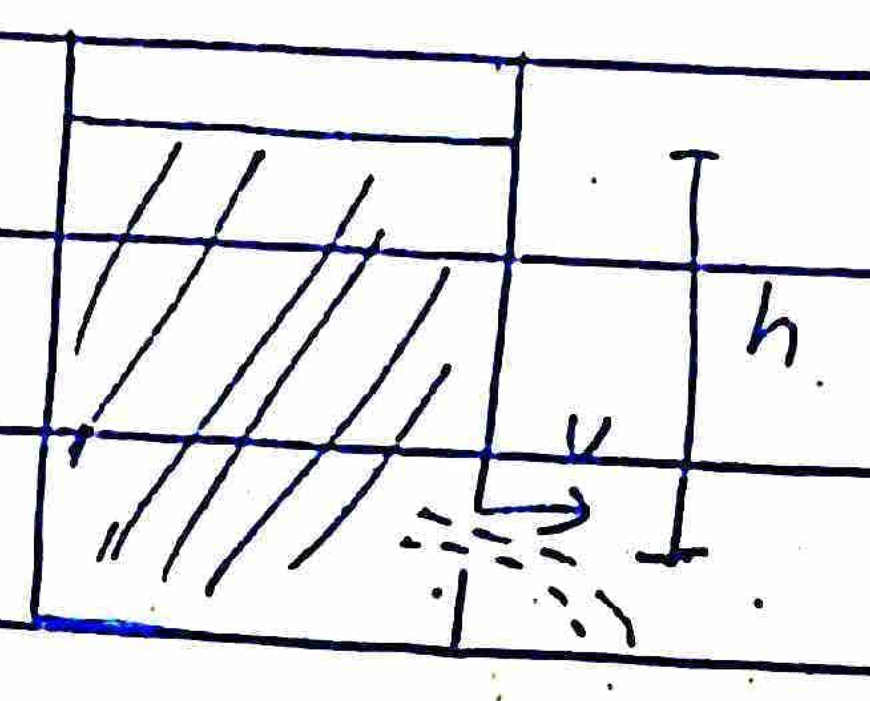
$$d_2^2 = 4.76 \times 10^{-6}$$

$$d_2 = 2 \times 10^{-3}\ m$$

$$= 2 \times 10^{-1} \times 10^{-2}\ m$$

$$\boxed{d_2 = 0.2\ cm}$$

6.3 Data



height $= h = 15\ m$

speed of water

$v = ?$, $g = 9.8\ ms^{-2}$

area $= A = 0.06\ cm^2$

$$A = \frac{0.06}{(100)(100)}\ m^2 = 6 \times 10^{-6}\ m^2$$

volume $= V = ?$, time $= t = 1\ s$

Solution

$$v = \sqrt{2gh}$$

$$= \sqrt{2(9.8)(15)}$$

$$\boxed{v = 17\ ms^{-1}}$$

$$Av = \frac{V}{t}$$

$$(6\times10^{-6})(17) = \frac{V}{1}$$

$$V = 1.02\times10^{-4}\ m^3$$

$$V = 1.02\times10^{-4}\times100\times100\times100\ cm^3$$

$$\boxed{V = 102\ cm^3}$$

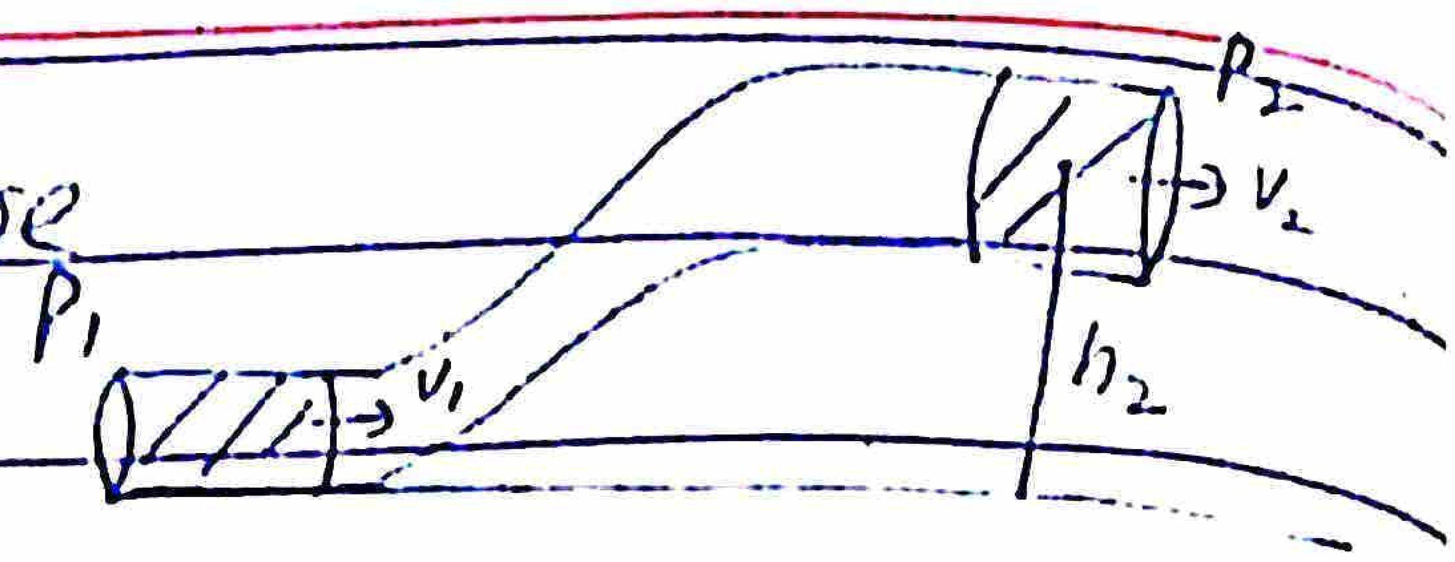
6.4 Data

speed $= v_1 = 3\ ms^{-1}$. $h_1 = 0$

height $= h_2 = 3\ m$, $g = 9.8\ ms^{-2}$

speed at other end $= v_2 = 4\ ms^{-1}$

lower point pressure



$P_1 = 80\ kPa$

$= 80000\ Pa$

upper point pressure $= P_2 = ?$

density of water $= \rho = 1000\ kg\,m^{-3}$

Solution

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$80000 + \frac{1}{2}(1000)(3)^2 + 0 = P_2 + \frac{1}{2}(1000)(4)^2 + (1000)(9.8)(3)$$

$$80000 + 4500 = P_2 + 8000 + 29400$$

$$84500 = P_2 + 37400$$

$$84500 - 37400 = P_2$$

$$P_2 = 47100\ Pa$$

$$P_2 = 47.100 \times 10^3\ Pa$$

$$\boxed{P_2 = 47\ kPa}$$

6.5 Data

speed at top $= v_1 = 450\ ms^{-1}$

speed below wing $= v_2 = 410\ ms^{-1}$

pressure difference $= P_2 - P_1 = ?$

density of air $= \rho = 1.29\ kgm^{-3}$

Solution

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

here

$$h_1 = h_2 = h$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g h} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h}$$

$$\frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = P_2 - P_1$$

$$\frac{1}{2}\rho(v_1^2 - v_2^2) = P_2 - P_1$$

$$P_2 - P_1 = \frac{1}{2}(1.29)(450^2 - 410^2)$$

$$P_2 - P_1 = 22188\ Pa$$

$$= 22.188 \times 10^3\ Pa$$

$$\boxed{P_2 - P_1 = 22\ KPa}$$

6.7 Data

radius of heating duct $= r = ?$

speed of air $= v = 3\ ms^{-1}$

volume $= V = 300\ m^3$

time $= t = 15\ min = (15)(60)\ s$

$t = 900\ s$

Solution

$$Av = \frac{V}{t}$$

$$\pi r^2 v = \frac{V}{t}$$

$$r^2 = \frac{V}{\pi v t}$$

$$r^2 = \frac{300}{\pi(3)(900)}$$

$$r^2 = 0.035$$

$$r = 0.19\ m$$

$$r = (0.19)(100)\ cm$$

$$\boxed{r = 19\ cm}$$

6.9 Data

gauge pressure $= P_1 - P_2 = ?$

height $= h_2 - h_1 = 15\ m$

density of water $= \rho = 1000\ kg\,m^{-3}$

$g = 9.8\ m s^{-2}$

Solution

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

here

$$v_1 = v_2 = v$$

$$P_1 + \cancel{\frac{1}{2}\rho v^2} + \rho g h_1 = P_2 + \cancel{\frac{1}{2}\rho v^2} + \rho g h_2$$

$$P_1 - P_2 = \rho g h_2 - \rho g h_1$$

$$= \rho g (h_2 - h_1)$$

$$= (1000)(9.8)(15)$$

$$P_1 - P_2 = 147000\ Pa$$

$$\boxed{P_1 - P_2 = 1.47 \times 10^5\ Pa}$$