Conic	Condition of tangency
Circle $x^2 + y^2 = r^2$	$c^2 = r^2(1 + m^2)$
Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c^2 = a^2m^2 + b^2$
Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c^2 = a^2m^2 - b^2$
Parabola $y^2 = 4ax$	$c = \frac{a}{m}$

Equation of tangent at point $(\mathbf{x_1}, \mathbf{y_1})$ of conic

The term to replace	Replacement
x^2	xx_1
y^2	yy_1
x	$\frac{x + x_1}{2}$
Y	$\frac{y + y_1}{2}$

Note: this method is valid only when the point lie on the conic.

EXERCISE 9.1

Q.1 Draw the following parabolas:

(i) $y^2 = 10x$ (ii) $x^2 = -12y$ (iii) $y^2 - x - 2y - 1 = 0$

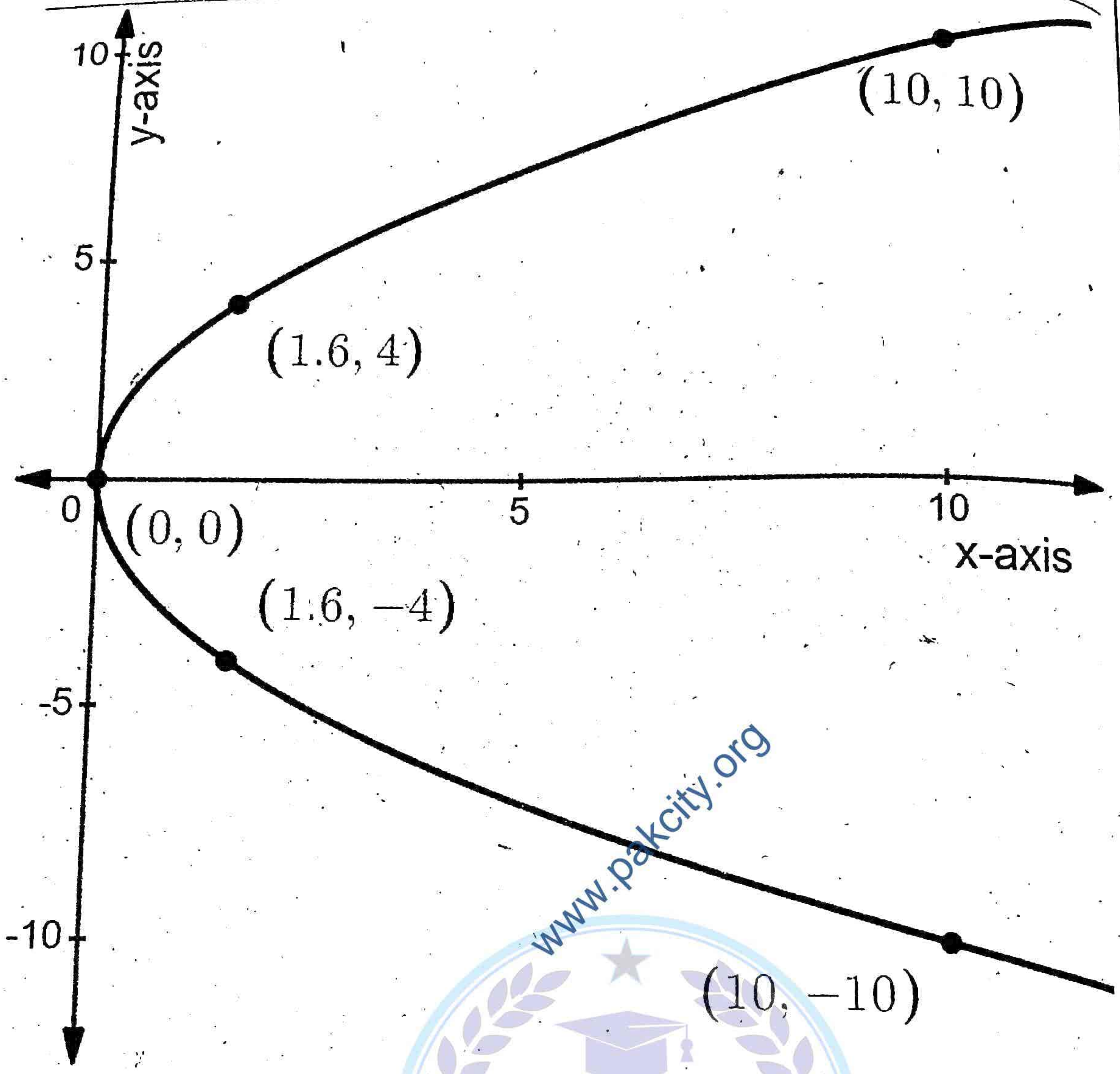
(iv) $x^2 - 6x - 2y + 5 = 0$

Solution:

(i) $y^2 = 10x$

$$x = \frac{y^2}{10}$$

y	x	(x, y)
0	0	(0,0)
± 4	1.6	$(1.6, \pm 4)$
± 10	10	$(10, \pm 10)$



(ii) $x^2 = -12y$

$y = -\frac{x^2}{12}$

x	y	(x, y)
0	0	(0,0)
±12	-12	$(\pm12, 12)$
±10	10	$(10, \pm10)$

-20

(iii) y^2

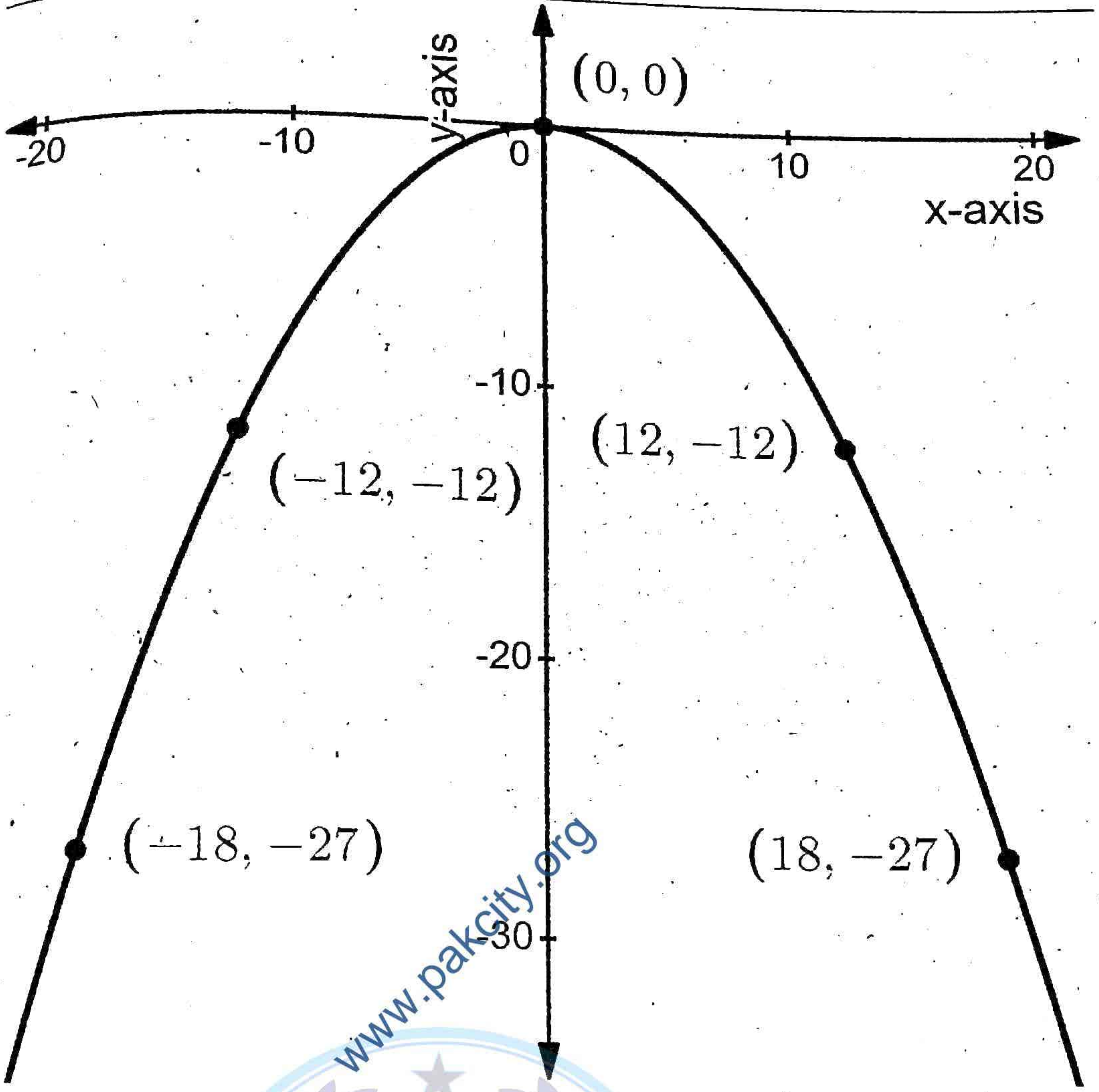
y^2 –

y^2 –

$(y$ –

$x = ($

x
−1
2
−2
2
−1



(iii) $y^2 - x - 2y - 1 = 0$

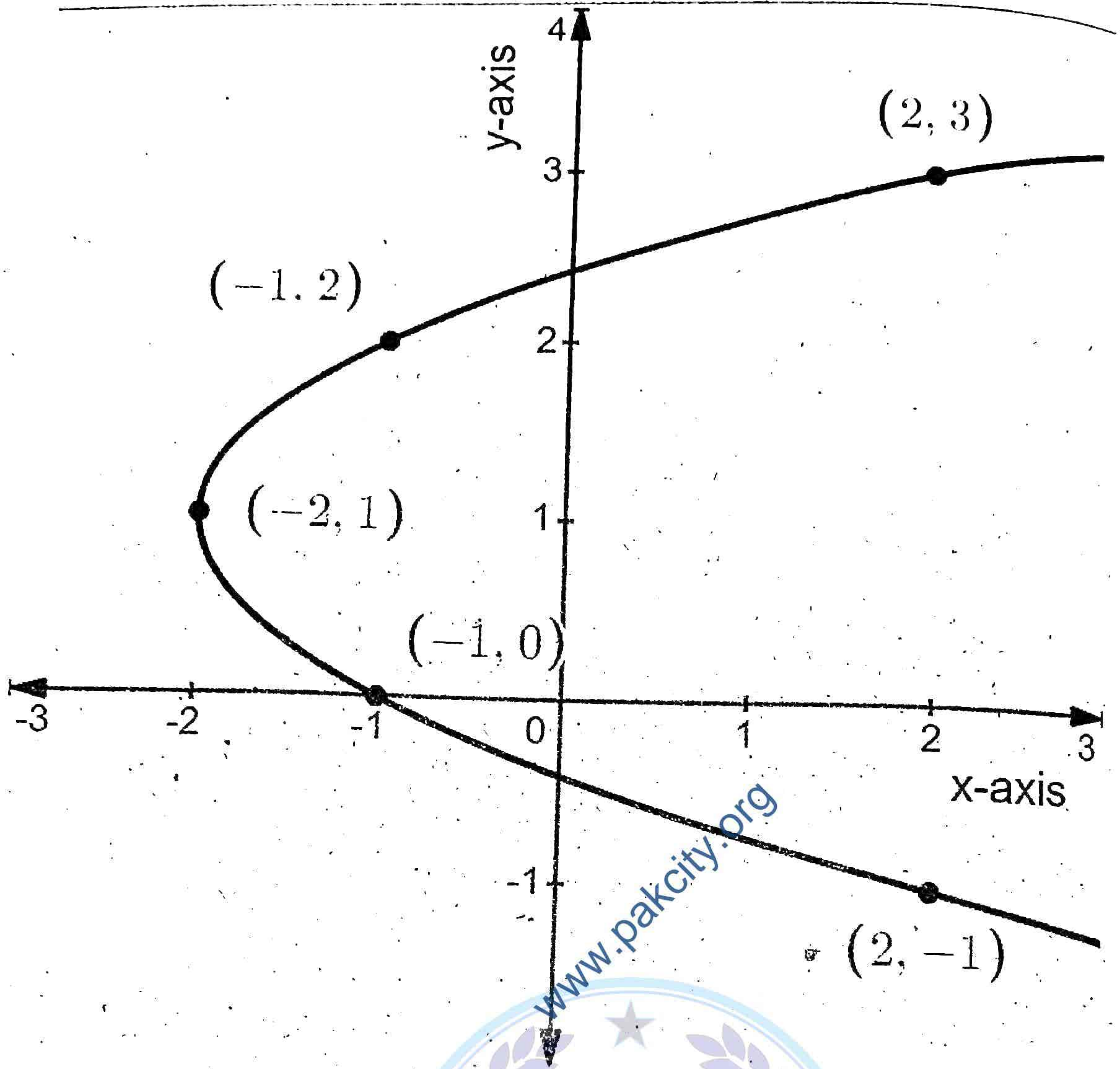
$y^2 - 2y = x + 1$

$y^2 - 2y + 1 - 1 - 1 = x$

$(y - 1)^2 - 2 = x$

$x = (y - 1)^2 - 2$

x	y	(x, y)
-1	0	$(-1,0)$
2	-1	$(2,-1)$
-2	1	$(-2,1)$
2	3	$(2,3)$
-1	2	$(-1,2)$



(iv) $x^2 - 6x - 2y + 5 = 0$

$(x^2 - 6x + 9) - 9 + 5 = 2y$

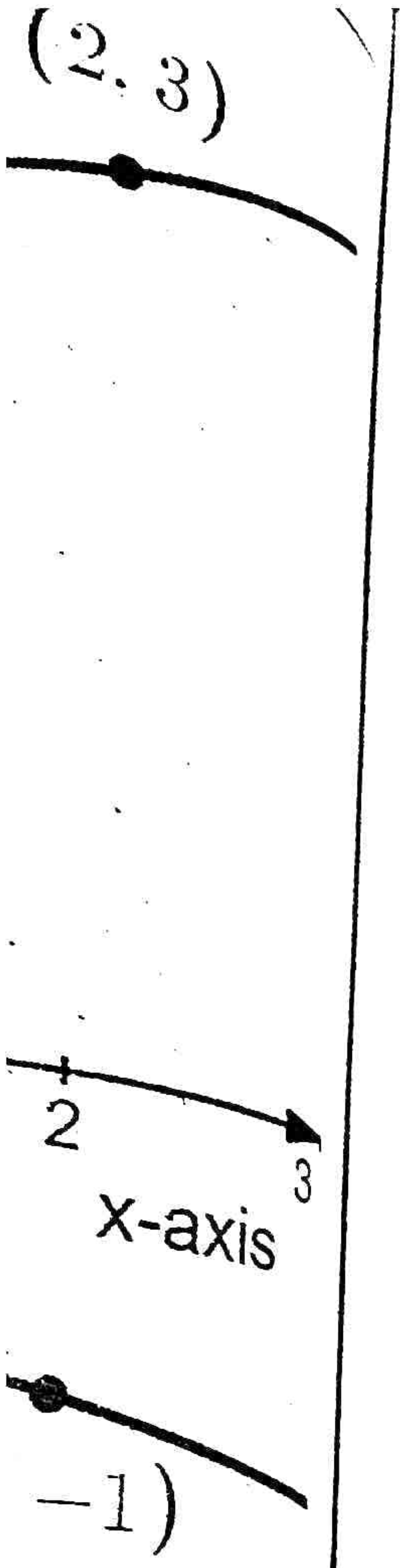
$(x - 3)^2 - 4 = 2y$

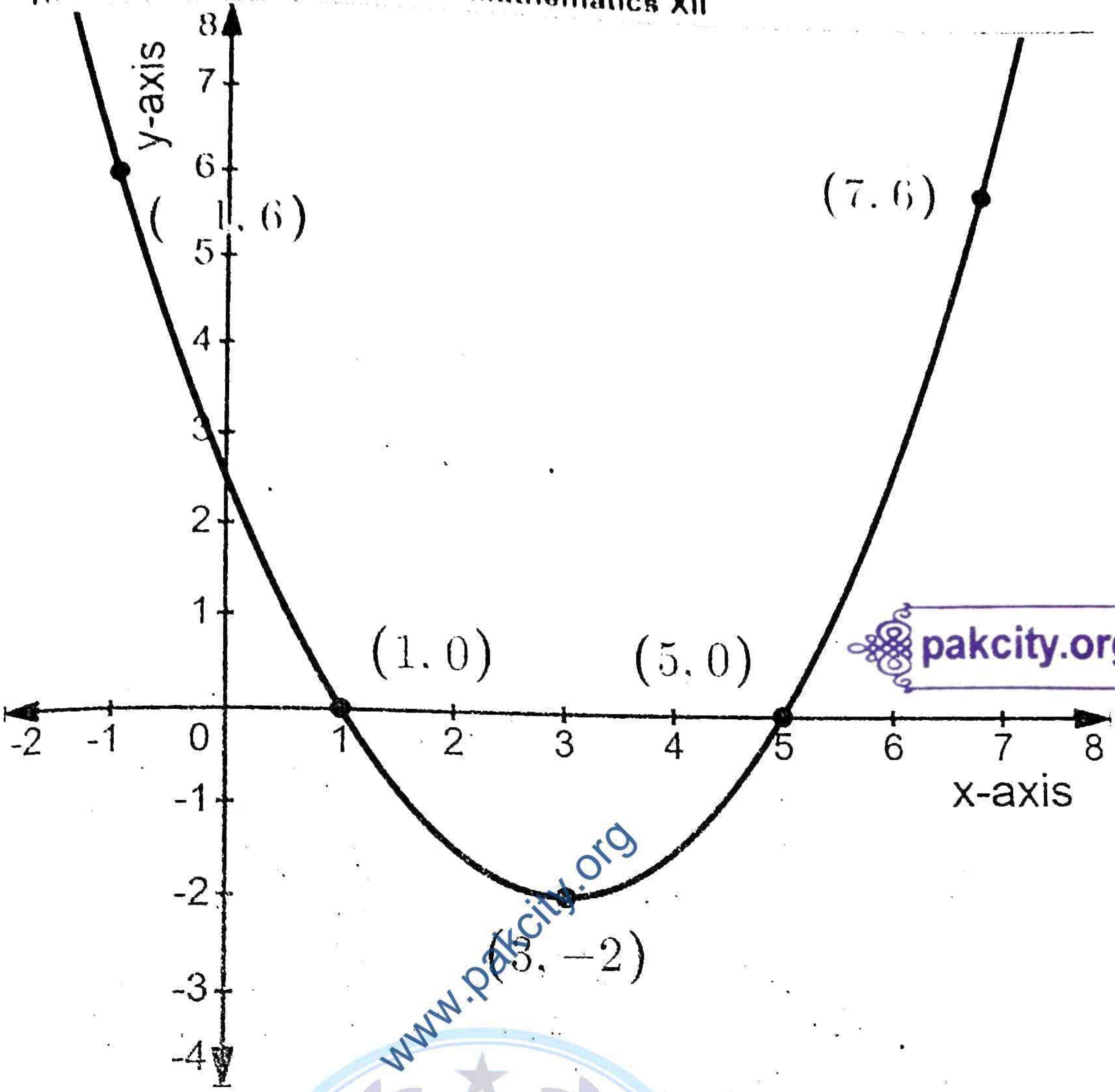
$y = \frac{1}{2}\{(x - 3)^2 - 4\}$

x	y	(x, y)
1	0	(1,0)
3	−2	(3,−2)
5	0	(5,0)
7	6	(7,6)
−1	6	(−1,6)

Q.2 Dete
following
(i) $y^2 =$
(iv) $(x +$
(vi) $y^2 -$

Solution
(i) $y^2 =$
Compari
Vertex =
$4a = -8$
Length o
Principle
Focus





Q.2 Determine vertex, focus, latus rectum and equation of directrix of the following. Also find the equation of the axis of symmetry.

(i) $y^2 = -8x$ (ii) $x^2 = -16y$ (iii) $(y+3)^2 = 12(x-2)$

(iv) $(x+5)^2 = 8(y-3)$ (v) $x^2 + 4x - y + 5 = 0$

(vi) $y^2 - 6y + 8x - 23 = 0$

Solution:

(i) $y^2 = -8x$

Comparing with $y^2 = 4ax$

Vertex = $(0,0)$

$4a = -8 \Rightarrow a = -2$

Length of latus rectum = $|4a| = |-8| = 8$

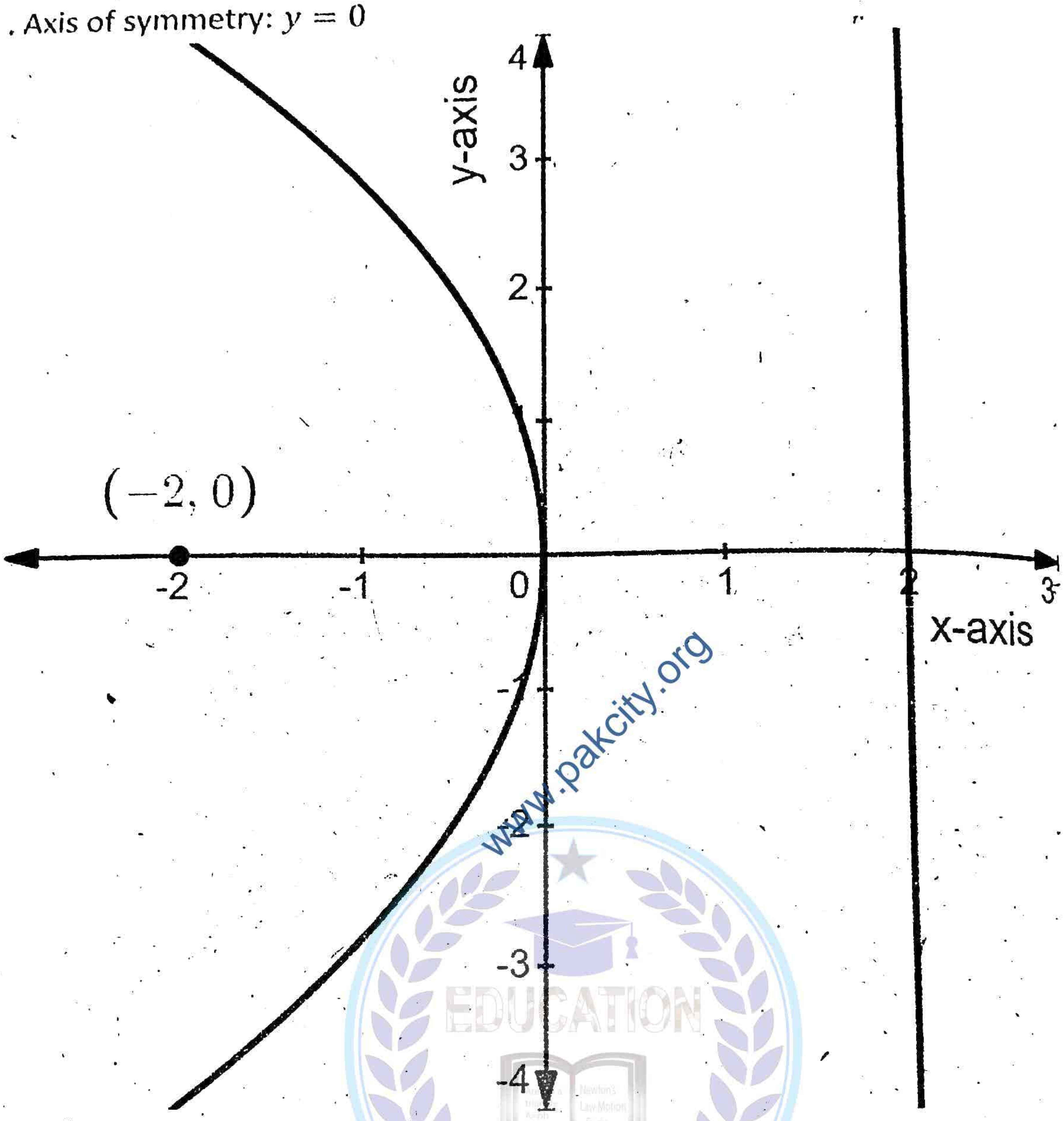
Principle axis is along x-axis

Focus $(a, 0) = (-2,0)$

Equation of directrix: $x = -a$

$x = 2$

Axis of symmetry: $y = 0$



(ii) $x^2 = -16y$

Comparing with $x^2 = 4ay$

$4a = -16 \Rightarrow a = -4$

Length of latus rectum $= |4a| = |-16| = 16$

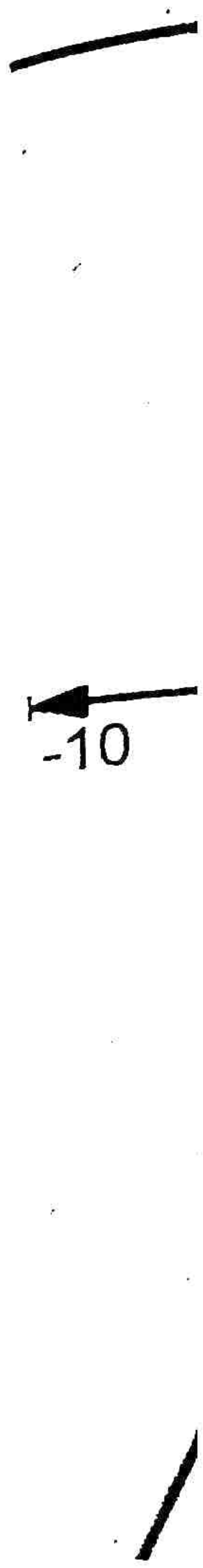
Principle axis is along y-axis

Focus $(0, a) = (0, -4)$

Equation of directrix: $y = -a$

$y = 4$

Axis of symmetry: $x = 0$



(iii) $(y$

Comp

Verte

$4a =$

Lengt

Princi

Focus

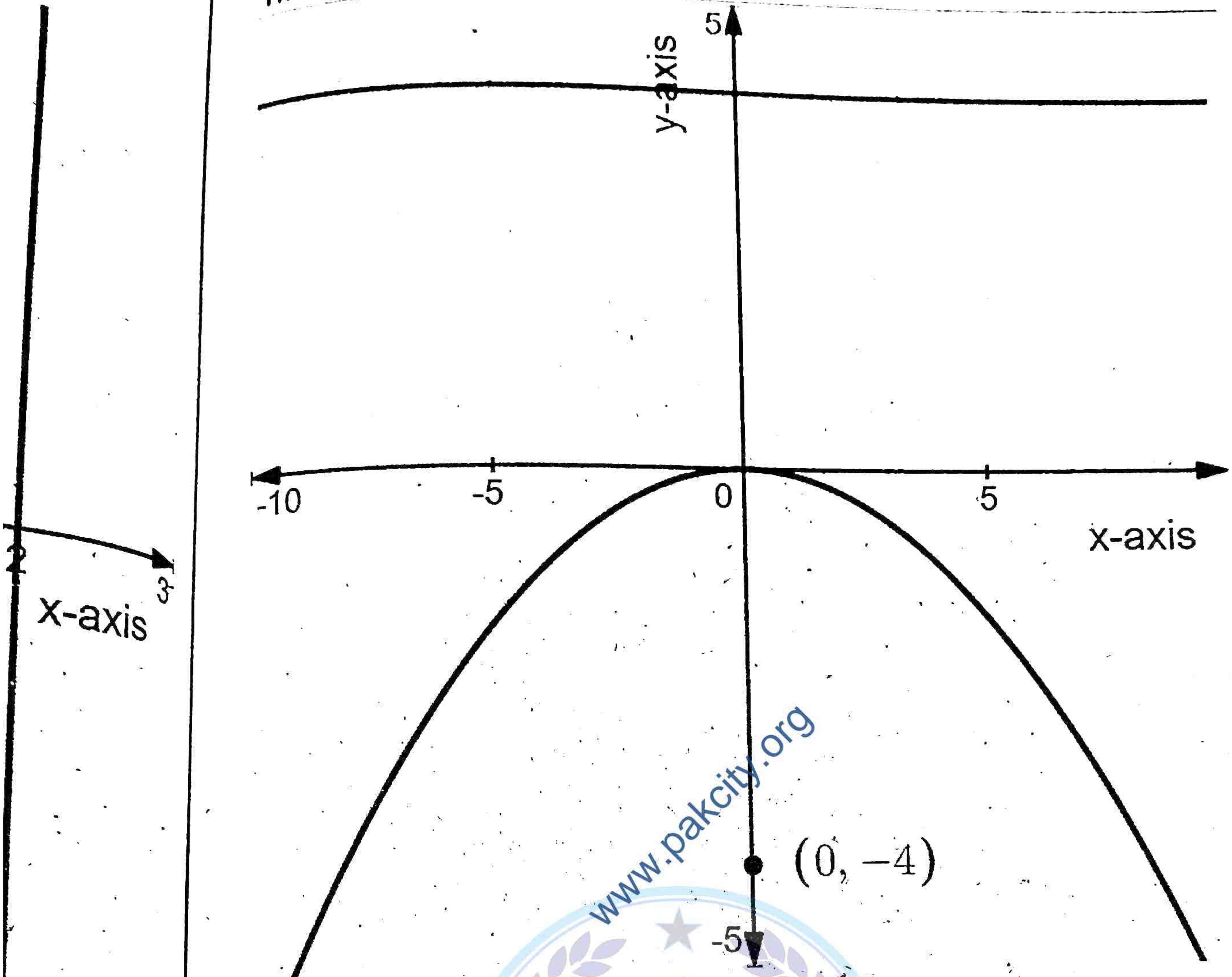
Equat

$x = 2$

$x =$

$y =$

Axis o



(iii) $(y+3)^2 = 12(x-2)$

Comparing with $(y-k)^2 = 4a(x-h)$

Vertex $(h,k) = (2,-3)$

$4a = 12 \Rightarrow a = 3$

Length of latus rectum $= |4a| = 12$

Principle axis is parallel x-axis

Focus $(a+h,k) = (3+2,-3) = (5,-3)$

Equation of directrix: $x = h - a$

$x = 2 - 3$

$x = -1$

$y = -6$

Axis of symmetry: $y = -3$

(iv) $(x+5)^2 = 8(y-3)$

Comparing with $(x-h)^2 = 4a(y-k)$

Vertex $(h,k) = (-5,3)$

$4a = 8 \Rightarrow a = 2$

Length of latus rectum $= |4a| = 8$

Principle axis is parallel y-axis

Focus $(h, k+a) = (-5, 3+2) = (-5,5)$

Equation of directrix: $y = k - a$

$y = 3 - 2$

$y = 1$

Axis of symmetry: $x = -5$

(v) $x^2 + 4x - y + 5 = 0$

$x^2 + 4x = y - 5$

$x^2 + 2(x)(2) + (2)^2 = y - 5 + 4$

$(x+2)^2 = y - 1$

$\{x - (-2)\}^2 = 1(y-1)$

Comparing with $(x-h)^2 = 4a(y-k)$

Vertex $(h,k) = (-2,1)$

$4a = 1 \Rightarrow a = \frac{1}{4}$

Length of latus rectum $= |4a| = 1$

Principle axis is parallel y-axis

Focus $(h, k+a) = \left(-2, 1+\frac{1}{4}\right) = \left(-2, \frac{5}{4}\right)$

Equation of directrix: $y = k - a$

$y = 1 - \frac{1}{4}$

$y = \frac{3}{4}$

Axis of symmetry: $x = -2$

(vi) $y^2 - 6y + 8x - 23 = 0$

$y^2 - 6y = -8x + 23$

$(y)^2 - 2(y)(3) + (3)^2 = -8x + 23 + 9$

$(y-3)^2 = -8x + 32$

$(y-3)^2 = -8(x-4)$

Comparing wit

Vertex (h,k) =

$4a = -8 \Rightarrow a$

Length of latu

Principle axis i

Focus $(a + h,$

Equation of di

$x = 4 - (-2)$

$x = 6$

Q.3 Find the e

$3x - 5 = 0.$

Solution:

Focus = $F(1,$

Equation of d

Let $P(x,y)$ b

$d = \sqrt{(x_2 -}$

$d = \left|\frac{ax_1 + b}{\sqrt{a^2}}\right.$

By definition

$\overline{PF}$ = Distanc

$\sqrt{(x-1)^2 +}$

$\sqrt{(x^2 - 2x +}$

Squaring bot

$x^2 + y^2 - 2$

$9x^2 + 9y^2 -$

$9y^2 + 12x +$

Q.4 Find the

$x + y - 1 =$

Solution:

Focus = $F(3,$

Comparing with $(y-k)^2 = 4a(x-h)$

Vertex $(h,k) = (4,3)$

$4a = -8 \Rightarrow a = -2$

Length of latus rectum $= |4a| = |-8| = 8$

Principle axis is parallel x-axis

Focus $(a+h,k) = (-2+4,3) = (2,3)$

Equation of directrix: $x = h - a$

$x = 4 - (-2)$

$x = 6$

Q.3 Find the equation of parabola whose focus is $\mathrm{F}(1,-2)$ and directrix is $3x - 5 = 0$.

Solution:

Focus $= \mathrm{F}(1,-2)$

Equation of directrix: $l: 3x - 5 = 0$

Let $P(x,y)$ be the moving point on parabola

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$$

By definition of parabola

$\overline{\mathrm{PF}}$ = Distance from moving point to directrix

$$\sqrt{(x-1)^2 + (y+2)^2} = \left|\frac{3x-5}{\sqrt{3^2+0^2}}\right|$$

$$\sqrt{(x^2-2x+1) + (y^2+4y+4)} = \left|\frac{3x-5}{3}\right|$$

Squaring both sides

$$x^2 + y^2 - 2x + 4y + 5 = \frac{9x^2 - 30x + 25}{9}$$

$$9x^2 + 9y^2 - 18x + 36y + 45 = 9x^2 - 30x + 25$$

$$9y^2 + 12x + 36y + 20 = 0$$

Q.4 Find the equation of parabola whose focus is $\mathrm{F}(3,4)$ and directrix is $x + y - 1 = 0$.

Solution:

Focus $= \mathrm{F}(3,4)$

Equation of directrix: $l: x + y - 1 = 0$

Let $P(x, y)$ be the moving point on parabola

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

By definition of parabola

$\overline{\text{PF}}$ = Distance from moving point to directrix

$$\sqrt{(x-3)^2 + (y-4)^2} = \left|\frac{x + y - 1}{\sqrt{1^2 + 1^2}}\right|$$

Squaring both sides

$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = \frac{(x + y - 1)^2}{2}$$

$$2(x^2 + y^2 - 6x - 8y + 25) = x^2 + y^2 + 1 + 2xy - 2x - 2y$$

$$x^2 + y^2 - 10x - 14y - 2xy + 49 = 0$$

Q.5 Find the equation of parabolas whose focus and vertex are as under:

(i) Vertex $(0,0)$; focus $(5,0)$ (ii) Vertex $(0,0)$; focus $(0,-2)$

(iii) Vertex $(1,-3)$; focus $(1,2)$ (iv) Vertex $(2,4)$; focus $(3,4)$

Solution:

(i) Vertex $(0,0)$; focus $(5,0)$

Focus $(a, 0) = (5,0)$

$a = 5$

Principle axis is along x-axis

$y^2 = 4ax$

$y^2 = 4(5)x$

$y^2 = 20x$

(ii) Vertex $(0,0)$; focus $(0,-2)$

Focus $(0, a) = (0,-2)$

$a = -2$

Principle axis is along x-axis

$x^2 = 4ay$

$x^2 = 4(-2)y$

$x^2 = -8y$

(iii) Vertex $(1,-3)$; focus $(1,2)$

Vertex $(h, k) = (1,-3)$

Principle axis is along y-axis as x-ordinate is same in vertex and focus

---

Focus $(h, k +$

$k + a = 2$

$-3 + a = 2$

$a = 5$

$(x - h)^2 = 4a$

$(x - 1)^2 = 4($

$x^2 - 2x + 1 =$

$x^2 - 2x - 20$

(iv) Vertex $(2,$

Vertex $(h, k) =$

Principle axis i

Focus $(a + h,$

$a + h = 3$

$a + 2 = 3$

$a = 1$

$(y - k)^2 = 4a$

$(y - 4)^2 = 4($

$y^2 - 8y + 16$

$y^2 - 8y - 4x$

$y^2 - 8y - 4x$

Q.6 Find the e

(i) focus $(3,0)$

(ii) focus $(0,4)$

(iii) focus $(-4,$

Solution:

(i) focus $(3,0)$

Focus = $F(3,0)$

Equation of dir

Let $P(x, y)$ be t

$d = \sqrt{(x_2 - x_1}$

$d = \left|\frac{ax_1 + by_1}{\sqrt{a^2 +}}\right.$

By definition of

Focus $(h, k + a) = (1,2)$

$k + a = 2$

$-3 + a = 2$

$a = 5$

$(x - h)^2 = 4a(y - k)$

$(x - 1)^2 = 4(5)(y + 3)$

$x^2 - 2x + 1 = 20y + 60$

$x^2 - 2x - 20y - 59 = 0$

(iv) Vertex $(2,4)$; focus $(3,4)$

Vertex $(h, k) = (2,4)$

Principle axis is along x-axis as y-ordinate is same in vertex and focus

Focus $(a + h, k) = (3,4)$

$a + h = 3$

$a + 2 = 3$

$a = 1$

$(y - k)^2 = 4a(x - h)$

$(y - 4)^2 = 4(1)(x - 3)$

$y^2 - 8y + 16 = 4x - 12$

$y^2 - 8y - 4x + 16 + 12 = 0$

$y^2 - 8y - 4x + 28 = 0$

Q.6 Find the equation of parabola focus and directrix are given:

(i) focus $(3,0)$ and directrix $x - 5 = 0$

(ii) focus $(0,4)$ and directrix $y + 6 = 0$

(iii) focus $(-4,3)$ and directrix $y = 6$

Solution:

(i) focus $(3,0)$ and directrix $x - 5 = 0$

Focus $= F(3,0)$

Equation of directrix: $l: x - 5 = 0$

Let $P(x, y)$ be the moving point on parabola

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

By definition of parabola

$\overline{PF}$ = Distance from moving point to directrix

$$\sqrt{(x-3)^2+(y-0)^2} = \left|\frac{x-5}{1}\right|$$

$$\sqrt{(x^2-6x+9)+y^2} = |x-5|$$

Squaring both sides

$$x^2-6x+9+y^2 = (x-5)^2$$

$$x^2-6x+9+y^2 = x^2-10x+25$$

$$y^2+10x-6x+9-25 = 0$$

$$y^2+4x-16 = 0$$

(ii) focus $(0,4)$ and directrix $y+6=0$

Focus $= F(0,4)$

Equation of directrix: $l: y+6=0$

Let $P(x,y)$ be the moving point on parabola

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

By definition of parabola

$\overline{PF}$ = Distance from moving point to directrix

$$\sqrt{(x-0)^2+(y-4)^2} = \left|\frac{y+6}{\sqrt{0^2+1^2}}\right|$$

$$\sqrt{x^2+(y^2-8y+16)} = \left|\frac{y+6}{1}\right|$$

Squaring both sides

$$x^2+y^2-8y+16 = (y+6)^2$$

$$x^2+y^2-8y+16 = y^2+12y+36$$

$$x^2-8y-12y-36+16 = 0$$

$$x^2-20y-20 = 0$$

(iii) focus $(-4,3)$ and directrix $y=6$

Focus $= F(-4,3)$

Equation of directrix: $l: y-6=0$

Let $P(x,y)$ be the moving point on parabola

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

By definitior

$\overline{PF}$ = Distan

$\sqrt{(x+4)^2 +}$

$\sqrt{(x^2+8x}$

Squaring bo

x^2+y^2+

x^2+y^2+

x^2+8x-

x^2+8x+

OR

$y=6$

$k-a=6$

Focus $(h,k$

$h=-4$

$k+a=3$

By (1) + (

$2k=9$

$k=\frac{9}{2}$

$(2) \Rightarrow \frac{9}{2}+$

$a=3-\frac{9}{2}$

$a=-\frac{3}{2}$

Principle ax

$(x-h)^2=$

$(x-(-4)$

$(x+4)^2=$

x^2+8x+

x^2+8x+

By definition of parabola

$\overline{PF}$ = Distance from moving point to directrix

$$\sqrt{(x+4)^2+(y-3)^2}=\left|\frac{y-6}{\sqrt{1^2+0^2}}\right|$$

$$\sqrt{(x^2+8x+16)+(y^2-6y+9)}=\left|\frac{y-6}{1}\right|$$

Squaring both sides

$$x^2+y^2+8x-6y+25=(y-6)^2$$

$$x^2+y^2+8x-6y+25=y^2-12y+36$$

$$x^2+8x-6y+12y+25-36=0$$

$$x^2+8x+6y-11=0$$

OR

$y=6$

$k-a=6 \rightarrow (1)$

Focus $(h,k+a)=(-4,3)$

$h=-4$

$k+a=3 \rightarrow (2)$

By $(1)+(2)$

$2k=9$

$$k=\frac{9}{2}$$

$$(2) \Rightarrow \frac{9}{2}+a=3$$

$$a=3-\frac{9}{2}$$

$$a=-\frac{3}{2}$$

Principle axis is parallel y-axis

$$(x-h)^2=4a(y-k)$$

$$(x-(-4))^2=4\left(-\frac{3}{2}\right)\left(y-\frac{9}{2}\right)$$

$$(x+4)^2=-6\left(\frac{2y-9}{2}\right)$$

$$x^2+8x+16=-3(2y-9)$$

$$x^2+8x+16=-6y+27$$

$x^2 + 8x + 6y + 16 - 27 = 0$

$x^2 + 8x + 6y - 11 = 0$

Q.7 Find the equation of parabola whose vertex and directrix are as under:

(i) vertex $(0,0)$ and directrix $x = -6$

(ii) vertex $(0,0)$ and directrix $y = 5$

(iii) vertex $(3,4)$ and directrix $x = 5$

Solution:

(i) vertex $(0,0)$ and directrix $x = -6$

$-a = -6$

$a = 6$

Principle axis is along x-axis

$y^2 = 4ax$

$y^2 = 4(6)x$

$y^2 = 24x$

(ii) vertex $(0,0)$ and directrix $y = 5$

$-a = 5$

$a = -5$

Principle axis is along y-axis

$x^2 = 4ay$

$x^2 = 4(-5)y$

$x^2 = -20y$

(iii) vertex $(3,4)$ and directrix $x = 5$

vertex $(h, k) = (3,4)$

$h = 3, k = 4$

$x = 5$

$h - a = 5$

$h - 5 = a$

$a = 3 - 5$

$a = -2$

Principle axis is parallel x-axis

$(y - k)^2 = 4a(x - h)$

$(y - 4)^2 = 4(-2)(x - 3)$

$y^2 - 8$

$y^2 - 8$

$y^2 - 8$

$y^2 - 8$

Q.8 Fi

(i) vert

(ii) ver

Soluti

(i) ver

$y^2 =$

$x^2 =$

Point

$(1) \Rightarrow$

$4a =$

$(1) \Rightarrow$

$3y^2 =$

$(2) \Rightarrow$

$4a =$

$(2) \Rightarrow$

$4x^2 =$

(ii) ve

Verte

$(y -$

$(x -$

Point

$(1) \Rightarrow$

$36 =$

$\boxed{4a =}$

$(1) \Rightarrow$

$y^2 - 8y + 16 = -8(x - 3)$
$y^2 - 8y + 16 = -8x + 24$
$y^2 - 8y + 8x + 16 - 24 = 0$
$y^2 - 8y + 8x - 8 = 0$

Q.8 Find the equation of parabola whose vertex and point are given:
(i) vertex (0,0); point (3,4)
(ii) vertex (5,0); point (4,6)

Solution:

(i) vertex (0,0); point (3,4)
$y^2 = 4ax \rightarrow (1)$
$x^2 = 4ay \rightarrow (2)$
Point = (3,4)
$(1) \Rightarrow (4)^2 = 4a(3)$
$4a = \frac{16}{3}$
$(1) \Rightarrow y^2 = \frac{16}{3}x$
$3y^2 = 16x$

$(2) \Rightarrow (3)^2 = 4a(4)$
$4a = \frac{9}{4}$
$(2) \Rightarrow x^2 = \frac{9}{4}y$
$4x^2 = 4y$

(ii) vertex (5,0); point (4,6)
Vertex $(h,k) = (5,0)$
$(y-k)^2 = 4a(x-h) \rightarrow (1)$
$(x-h)^2 = 4a(y-k) \rightarrow (2)$

Point = (4,6), $(h,k) = (5,0)$
$(1) \Rightarrow (6-0)^2 = 4a(4-5)$
$36 = 4a(-1)$
$\boxed{4a = -36}$
$(1) \Rightarrow (y-0)^2 = -36(x-5)$

$y^2 = -36(x-5)$

Point $= (4,6),\ (h,k) = (5,0)$

$(2) \Rightarrow (4-5)^2 = 4a(5-0)$

$(-1)^2 = 4a(5)$

$\boxed{4a = \frac{1}{5}}$

$(2) \Rightarrow (x-5)^2 = \frac{1}{5}(y-0)$

$(x-5)^2 = \frac{1}{5}y$

Q.9 Find the standard equation of parabola whose latus rectum and vertex are the diameter and centre of the circle respectively $x^2 + y^2 - 4x - 8y - 5 = 0$.

Solution:

$x^2 + y^2 - 4x - 8y - 5 = 0$

$x^2 + y^2 + 2(-2)x + 2(-4)y - 5 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = -2, f = -4, c = -5$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-2)^2 + (-4)^2 - (-5)}$

$r = \sqrt{25} = 5$

Length of latus rectum = radius of circle

$4a = 5 \Rightarrow a = \frac{5}{4}$

Centre $(-g, -f) = (2,4)$

Focus of parabola = Centre of circle

$(a+h, k) = (2,4)$

$a + h = 2$

$h = 2 - \frac{5}{4} = \frac{3}{4}$

$(y-k)^2 = 4a(x-h)$

$(y-4)^2 = 4\left(\frac{5}{4}\right)\left(x - \frac{3}{4}\right)$

$(y-4)^2 = 5\left(x - \frac{3}{4}\right)$

And

$(h, a+k) = (2$

$a + k = 4$

$a = 4 - \frac{5}{4}$

$a = \frac{11}{4}$

$(x-h)^2 = 4c$

$(x-2)^2 = 4$

$(x-2)^2 = 1$

Q.10 Find the
diameter is th
the focus.

Solution:

$x^2 = 12y$

$x^2 = 4ay$

$4a = 12$

$a = 3$

Length of lat

Principle axi

Focus $(0, a)$

Centre of ci

$(a, b) = (0,$

Diameter of

Diameter of

Radius = Di

Radius $= \frac{12}{2}$

$(x-0)^2 +$

$x^2 + (y^2 -$

$x^2 + y^2 -$

$x^2 + y^2 -$

And

$(h, a+k) = (2,4)$

$a + k = 4$

$a = 4 - \frac{5}{4}$

$a = \frac{11}{4}$

$(x-h)^2 = 4a(y-k)$

$(x-2)^2 = 4\left(\frac{11}{4}\right)(y-4)$

$(x-2)^2 = 11(y-4)$

Q.10 Find the equation of circle and its circle is at the focus, whose diameter is the latus rectum of the parabola $x^2 = 12y$ and its centre is at the focus.

Solution:

$x^2 = 12y$

$x^2 = 4ay$

$4a = 12$

$a = 3$

Length of latus rectum $= |4a| = 12$

Principle axis is along y-axis

Focus $(0, a) = (0,3)$

Centre of circle = Focus of parabola

$(a, b) = (0,3)$

Diameter of circle = Length of latus rectum

Diameter of circle = 12

Radius $= \frac{\text{Diameter}}{2}$

Radius $= \frac{12}{2} = 6$

$(x-0)^2 + (y-3)^2 = 6^2$

$x^2 + (y^2 - 6y + 9) = 36$

$x^2 + y^2 - 6y + 9 - 36 = 0$

$x^2 + y^2 - 6y - 27 = 0$

Q.11 For what point of the parabola $y^2 = 10x$, the abscissa is equal to three times its ordinate.

Solution:

x = Abscissa

y = Ordinate

$x = 3y \rightarrow (1)$

$$y^2 = 10x$$
$$y^2 = 10(3y)$$
$$y^2 = 30y$$
$$y^2 - 30y = 0$$
$$y(y - 30) = 0$$
$$y = 0,30$$

$(1) \Rightarrow x = 3(0)$

$\boxed{x = 0, y = 0}$

$(1) \Rightarrow x = 3(30)$

$\boxed{x = 90, y = 30}$

Points are (0,0) and (90,30)

EXERCISE 9.2

The line $y = mx + c$ is tangent to

	$y^2 = 4ax$	$x^2 = 4ay$
Condition	$c = \frac{a}{m}$	$c = -am^2$
Common Point	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$(2am, am^2)$
Equation of tangent	$y = mx + \frac{a}{m}$	$y = mx - am^2$
Equation of tangent at (x_1, y_1)	$yy_1 = 2a(x + x_1)$	
Equation of normal at (x_1, y_1)	$y(x - x_1)$ $+ 2a(y - y_1) = 0$	

pa
tangent.

Solution:

$y = mx +$

$x^2 = 4ay$
$x^2 = 4a($
$x^2 = 4am$
$x^2 - 4am$
$B^2 - 4AC$
$(-4am)^2$
$16a^2m^2$
$16a^2m^2$
$16a^2m^2$
$\div$ by $16a$
$am^2 = -$
$c = -am$

$x = \frac{-b}{}$

$x = \frac{-(-}{}$

$x = \frac{4am}{2}$

$\boxed{x = 2a}$

$(1) \Rightarrow y$
$y = 2am$
$\boxed{y = am}$

$y = mx$
$(1) \Rightarrow y$

Q.2 Find lines an

Q.1 Find the condition when the line $y = mx + c$ is tangent to the parabola $x^2 = 4ay$. Also find point of contact and the equation of tangent.

Solution:

$y = mx + c \rightarrow (1)$

$$x^2 = 4ay$$
$$x^2 = 4a(mx + c)$$
$$x^2 = 4amx + 4ac$$
$$x^2 - 4amx - 4ac = 0$$
$$B^2 - 4AC = 0$$
$$(-4am)^2 - 4(1)(-4ac) = 0$$
$$16a^2m^2 + 16ac = 0$$
$$16a^2m^2 = -16ac$$
$$16a^2m^2 = -16ac$$

÷ by $16a$

$$am^2 = -c$$
$$c = -am^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-4am) \pm \sqrt{0}}{2(1)}$$
$$x = \frac{4am}{2}$$
$$\boxed{x = 2am}$$

$$(1) \Rightarrow y = m(2am) - am^2 \quad [\because c = -am^2]$$
$$y = 2am^2 - am^2$$
$$\boxed{y = am^2}$$

$$y = mx + c$$
$$(1) \Rightarrow y = mx - am^2$$

Q.2 Find condition of tangency and the point of tangency for the following lines and parabolas. Also find equation of tangent in each case:

(i) $2x + y = c; y^2 = 10x$
(ii) $3x + 4y = p; x^2 = 12y$
(iii) $y = cx; y^2 = 8(x - 1)$

Solution:

(i) $2x + y = c; y^2 = 10x$

$y^2 = 10x$

Comparing with $y^2 = 4ax$

$4a = 10 \Rightarrow a = \frac{5}{2}$

$2x + y = c$

$y = -2x + c \rightarrow (1)$

Comparing with $y = mx + c$

$m = -2, c = c$

Condition:

$c = \frac{a}{m}$

$c = \frac{\frac{5}{2}}{-2} = -\frac{5}{4}$

Common Point:

$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{\frac{5}{2}}{(-2)^2}, \frac{2\left(\frac{5}{2}\right)}{-2}\right)$

$= \left(\frac{1}{4}\left(\frac{5}{2}\right), -\frac{5}{2}\right)$

$= \left(\frac{5}{8}, -\frac{5}{2}\right)$

Equation of tangent:

$(1) \Rightarrow y = -2x + c$

$y = -2x - \frac{5}{4}$

(ii) $3x + 4y = p; x^2 = 12y$

$x^2 = 12y$

Comparing with $x^2 = 4ay$

$4a = 12$

$3x + 4y$

$4y = -3.$

$y = -\frac{3}{4}$

Comparir

$m = -\frac{3}{4}$

Conditio

$c = -an$

$\frac{p}{4} = -3($

$\frac{p}{4} = -3($

$p = -\frac{2}{4}$

Commor

$(2am, a$

$= \left(-\frac{9}{2},\right.$

$= \left(-\frac{9}{2},\right.$

Equation

$(1) \Rightarrow y$

$y = -\frac{3}{4}$

$y = -\frac{3}{4}$

(iii) $y = c$

$y = cx -$

$y^2 = 8(x$

$4a = 12 \Rightarrow a = 3$

$3x + 4y = p$

$4y = -3x + p$

$y = -\frac{3}{4}x + \frac{p}{4} \rightarrow (1)$

Comparing with $y = mx + c$

$m = -\frac{3}{4}, c = \frac{p}{4}$

Condition:

$c = -am^2$

$\frac{p}{4} = -3\left(-\frac{3}{4}\right)^2$

$\frac{p}{4} = -3\left(\frac{9}{16}\right)$

$p = -\frac{27}{4}$

Common Point:

$(2am, am^2) = \left(2(3)\left(-\frac{3}{4}\right), 3\left(-\frac{3}{4}\right)^2\right)$

$= \left(-\frac{9}{2}, 3\left(\frac{9}{16}\right)\right)$

$= \left(-\frac{9}{2}, \frac{27}{16}\right)$

Equation of tangent:

$(1) \Rightarrow y = -\frac{3}{4}x + \frac{p}{4}$

$y = -\frac{3}{4}x - \left(\frac{1}{4}\right)\frac{27}{4}$

$y = -\frac{3}{4}x - \frac{27}{16}$

(iii) $y = cx; y^2 = 8(x - 1)$

$y = cx \rightarrow (1)$

$y^2 = 8(x - 1)$

$(cx)^2 = 8x - 8$

$c^2x^2 - 8x + 8 = 0 \rightarrow (2)$

$B^2 - 4AC = 0$

$(-8)^2 - 4(c^2)(8) = 0$

$64 = 32c^2$

$c^2 = 2$

$c = \pm\sqrt{2}$

$(2) \Rightarrow 2x^2 - 8x + 8 = 0$

$x^2 - 4x + 4 = 0$

$(x)^2 - 4(x)(2) + (2)^2 = 0$

$(x-2)^2 = 0$

$x - 2 = 0$

$x = 2$

Common Point:

$(2, \pm 2\sqrt{2})$

Equation of tangent:

$(1) \Rightarrow y = \pm\sqrt{2}x$

Q.3 Find the equation of tangent and normal to the following parabolas at the given points:

(i) $y^2 = 8x; (2,4)$

(ii) $x^2 = 4y; (-6,9)$

(iii) $(y-1)^2 = 9(x-2); (3,4)$

Solution:

(i) $y^2 = 8x; (2,4)$

$y^2 = 8x$

Differentiate w.r.t "x"

$\frac{d}{dx}(y^2) = \frac{d}{dx}(8x)$

$2yy' = 8$

$y' = \frac{4}{y}$

m at $(2,4) = \frac{4}{4}$

$(x_1, y_1) = (2,4)$

Equation of tan

$y - y_1 = m(x$

$y - 4 = 1(x -$

$0 = x - y - 2$

$x - y + 2 = 0$

Equation of no

$y - y_1 = -\frac{1}{m}$

$y - 4 = -1(x$

$y - 4 = -x +$

$x + y - 4 - 2$

$x + y - 6 = 0$

(ii) $x^2 = 4y$; (

$x^2 = 4y \rightarrow (1$

Differentiate

$\frac{d}{dx}(x^2) = \frac{d}{dx}$

$2x = 4y'$

$y' = \frac{x}{2}$

m at $(-6,9) =$

$(x_1, y_1) = (-$

Equation of ta

$y - y_1 = m($

$y - 9 = -3($

$y - 9 = -3x$

$3x + y - 9 +$

$3x + y + 9 =$

Equation of n

m at $(2,4) = \frac{4}{4} = 1$

$(x_1, y_1) = (2,4)$

Equation of tangent:

$y - y_1 = m(x - x_1)$

$y - 4 = 1(x - 2)$

$0 = x - y - 2 + 4$

$x - y + 2 = 0$

Equation of normal:

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 4 = -1(x - 2)$

$y - 4 = -x + 2$

$x + y - 4 - 2 = 0$

$x + y - 6 = 0$

(ii) $x^2 = 4y$; $(-6,9)$

$x^2 = 4y \rightarrow (1)$

Differentiate w.r.t "x"

$\frac{d}{dx}(x^2) = \frac{d}{dx}(4y)$

$2x = 4y'$

$y' = \frac{x}{2}$

m at $(-6,9) = \frac{-6}{2} = -3$

$(x_1, y_1) = (-6,9)$

Equation of tangent:

$y - y_1 = m(x - x_1)$

$y - 9 = -3(x + 6)$

$y - 9 = -3x - 18$

$3x + y - 9 + 18 = 0$

$3x + y + 9 = 0$

Equation of normal:

ing parabolas at

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 9 = \frac{1}{3}(x + 6)$$

$$3y - 27 = x + 6$$

$$0 = x - 3y + 27 + 6$$

$$x - 3y + 33 = 0$$

(iii) $(y-1)^2 = 9(x-2)$; $(3,4)$

$(y-1)^2 = 9(x-2) \rightarrow (1)$

Differentiate w.r.t "x"

$$\frac{d}{dx}(y-1)^2 = \frac{d}{dx}\{9(x-2)\}$$

$$2(y-1)y' = 9(1-0)$$

$$y' = \frac{9}{2(y-1)}$$

$$m \text{ at } (3,4) = \frac{9}{2(4-1)} = \frac{3}{2}$$

$(x_1, y_1) = (3,4)$

<u>Equation of tangent:</u>

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{3}{2}(x - 3)$$

$$2y - 8 = 3x - 9$$

$$0 = 3x - 2y - 9 + 8$$

$$3x - 2y - 1 = 0$$

<u>Equation of normal:</u>

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 3)$$

$$3y - 12 = -2(x - 3)$$

$$3y - 12 = -2x + 6$$

$$2x + 3y - 12 - 6 = 0$$

$$2x + 3y - 18 = 0$$

Q.4 Fin

$x^2 = 4$

Solutio

$x^2 = 4$

Differe

$\frac{d}{dx}(x^2$

$2x =$

$y' =$

m at F

Equat

$y - y$

$y - y$

Equat

$y - y$

$y - y$

Q.5 A

deep

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paral

Solut

Let

Diam

Dept

The v

Q.4 Find the equation of tangent and normal at $P(x_1, y_1)$ to the parabola $x^2 = 4ay$.

Solution:

$x^2 = 4ay$

Differentiate w.r.t "x"

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(4ay)$$

$$2x = 4ay'$$

$$y' = \frac{x}{2a}$$

$$m \text{ at } P(x_1, y_1) = \frac{x_1}{2a}$$

Equation of tangent:

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{x_1}{2a}(x - x_1)$$

Equation of normal:

$$y - y_1 = -\frac{2a}{x_1}(x - x_1)$$

$$y - y_1 = \frac{2a}{x_1}(x_1 -$$

Q.5 A light house uses a parabolic reflector that is 1m in diameter. How deep should the reflector be if light source is placed halfway between the vertex and the plane of rim to produce parallel beam of light to the axis of parabola.

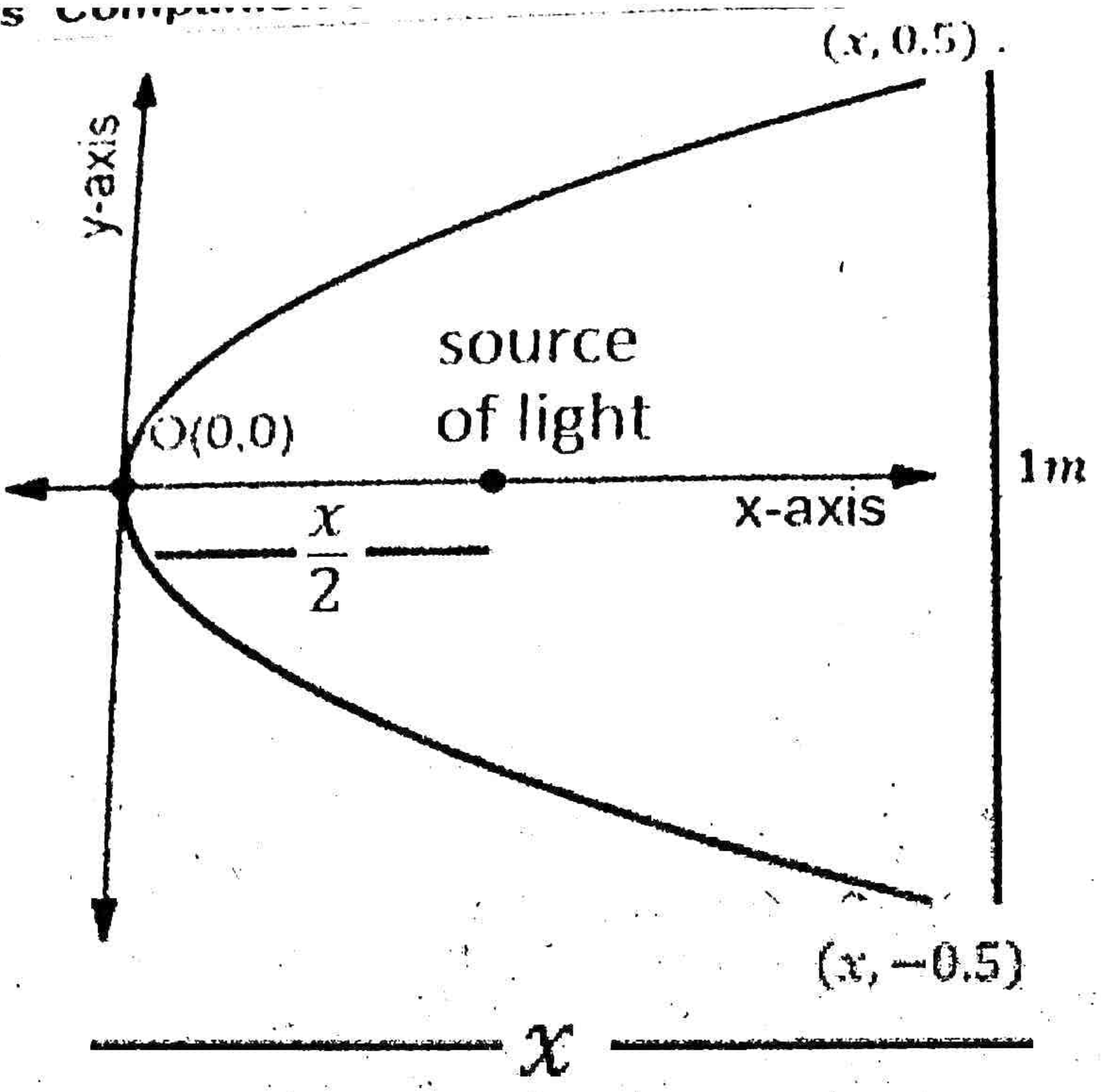
Solution:

Let

Diameter of reflector $= \text{d} = 1m$

Depth of parabolic reflector $= x = ?$

The vertex is at origin



$$y^2 = 4ax$$

$$\left(\frac{1}{2}\right)^2 = 4\left(\frac{x}{2}\right)(x)$$

$$\frac{1}{4} = 2x^2$$

$$x^2 = \frac{1}{4(2)}$$

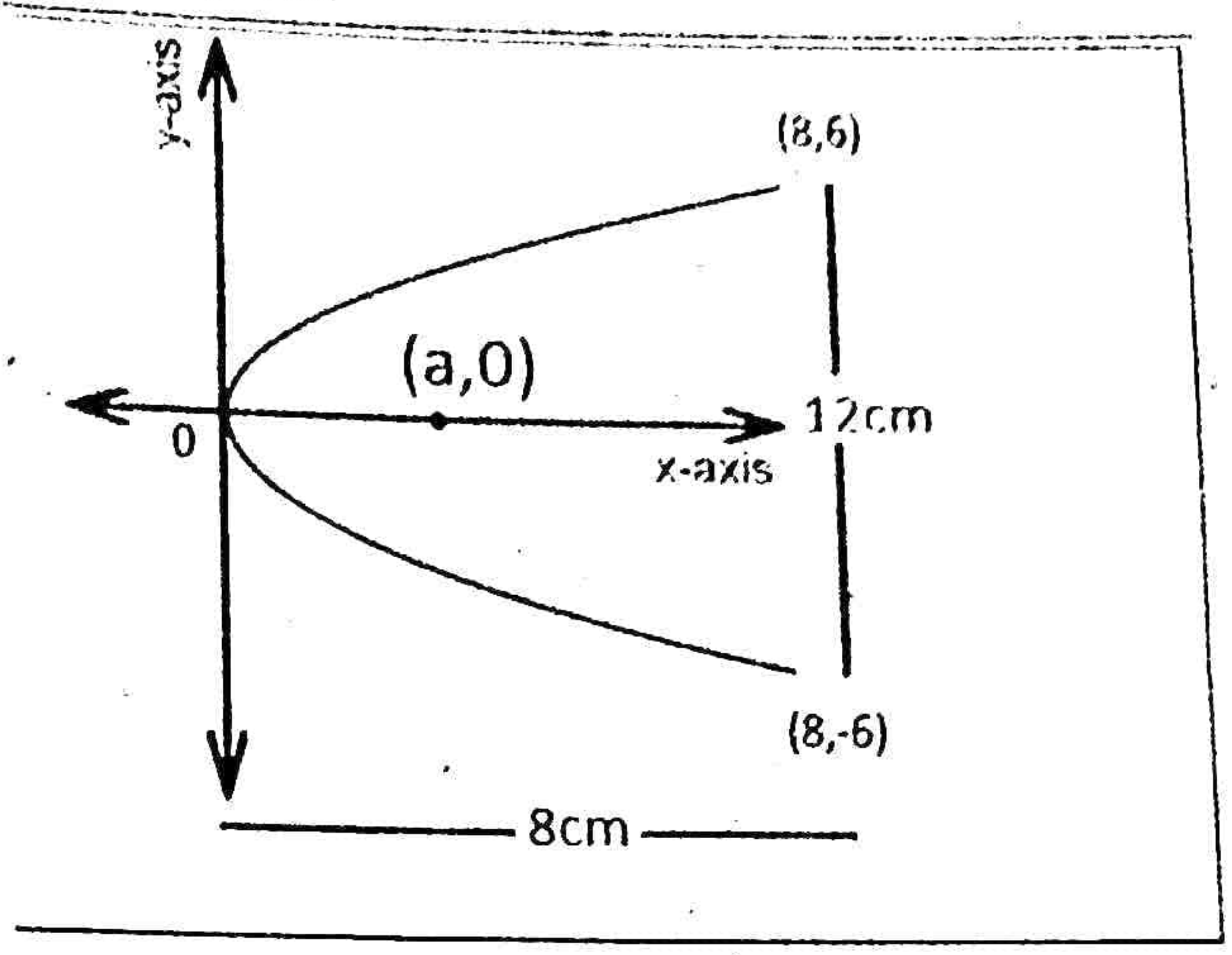
$$x = \frac{1}{\sqrt{4(2)}}$$

$$x = \frac{1}{2\sqrt{2}}$$

Depth of reflector $= \frac{1}{2\sqrt{2}}$

Q.6 There is a parabolic reflector of 12 cm in diameter that is used in a vehicle where should the light source be placed to produce parallel beam of light whereas the reflector is 8 cm deep.

Solution:



Let

Diameter of reflector = 12cm

Position of light source = $a = ?$

Vertex of parabola is at origin

$y^2 = 4ax \rightarrow (1)$

The point (8,6) is on (1)

$(1) \Rightarrow 6^2 = 4a(8)$

$4a = \frac{9}{2}$

$a = \frac{9}{2(4)}$

$\boxed{a = \frac{9}{8}}$

Q.7 The main cable of suspension bridge is suspended in the shape of parabola between the two towers are 100m apart and 30m high from the roadway. If the cable is at the height of 5 m from the roadway at the centre of the bridge, then find the equation of parabola and the distance of 10 m high suspended cable from the centre of the bridge.

Solution:

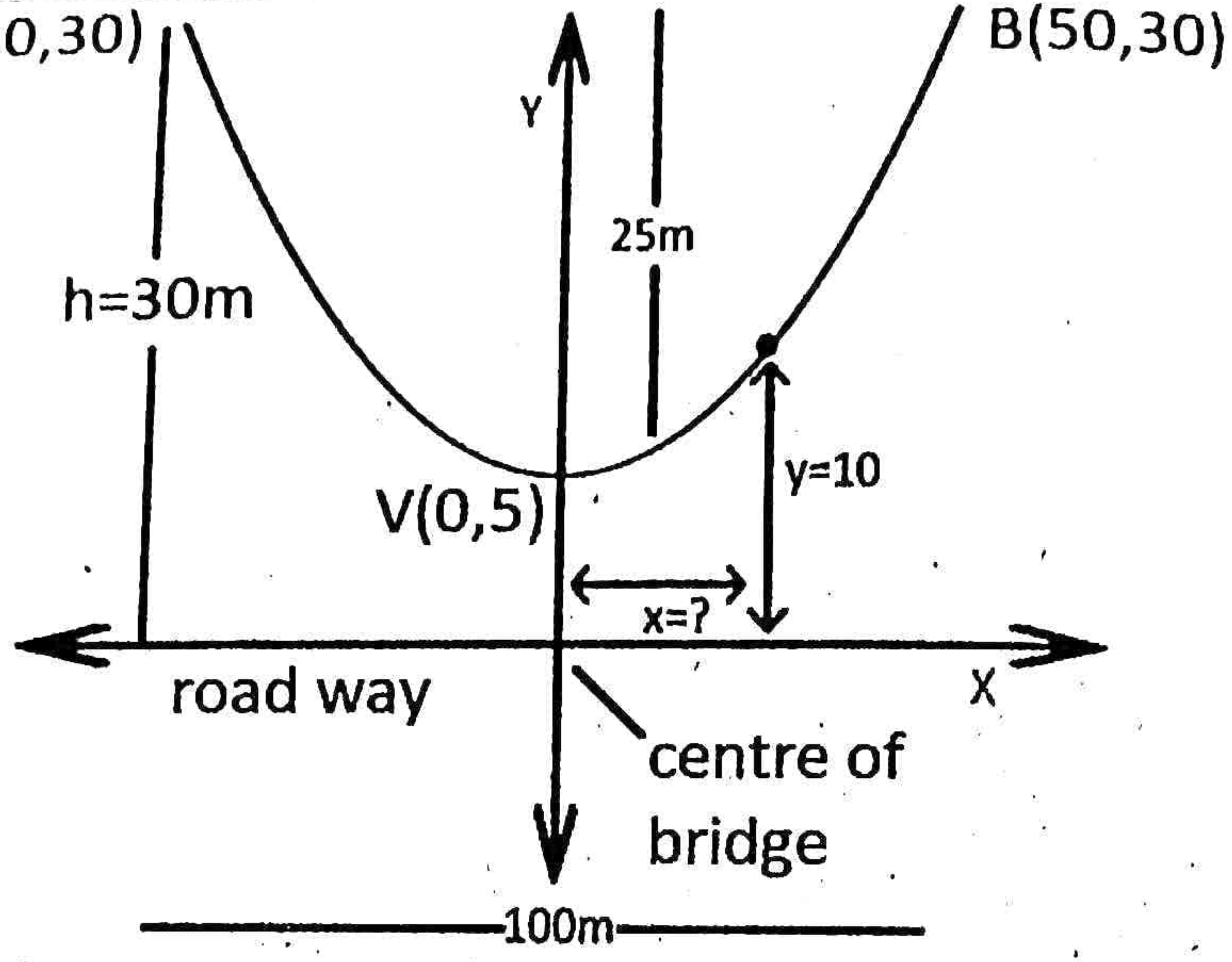
(a)

Let

Distance between two towers A and B is $100m$

Height of tower $= h = 30m$

Distance of 10m high suspended cable from centre of bridge $= x = ?$



Vertex$(h,k) = (0,5)$

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-5)$

$x^2 = 4a(y-5) \to (1)$

$\because$ (50,30) is on (1)

$(1) \Rightarrow 50^2 = 4a(30-5)$

$2500 = 4a(25)$

$4a = \dfrac{2500}{25} \Rightarrow 4a = 100$

$(1) \Rightarrow x^2 = 100(y-5) \to (2)$

(b)

$y = 10, x = ?$

$(2) \Rightarrow x^2 = 100(10-5)$

$x^2 = 100(5)$

$x = \sqrt{100(5)}$

$x = 10\sqrt{5}m$

Q.8 The main cable of a suspension bridge is in the shape of parabola. The towers are 600 feet apart and 60 feet high from the roadway. If the cable touches at the roadway at the midway between the towers. What is height of the suspended cable 150 feet from the centre of the bridge.

Solution:

The given condi

Let

Height of bridge

Distance betwee

Height of suspen

Vertex is at origi

$x^2 = 4ay \to (1)$

B(300,60) is on

$(1) \Rightarrow 300^2 = 4$

$4a = \dfrac{300^2}{60} \Rightarrow 4a$

$(1) \Rightarrow x^2 = 150$

$x = 150, y = ?$

$(1) \Rightarrow (150)^2 =$

$\dfrac{(150)^2}{1500} = y$

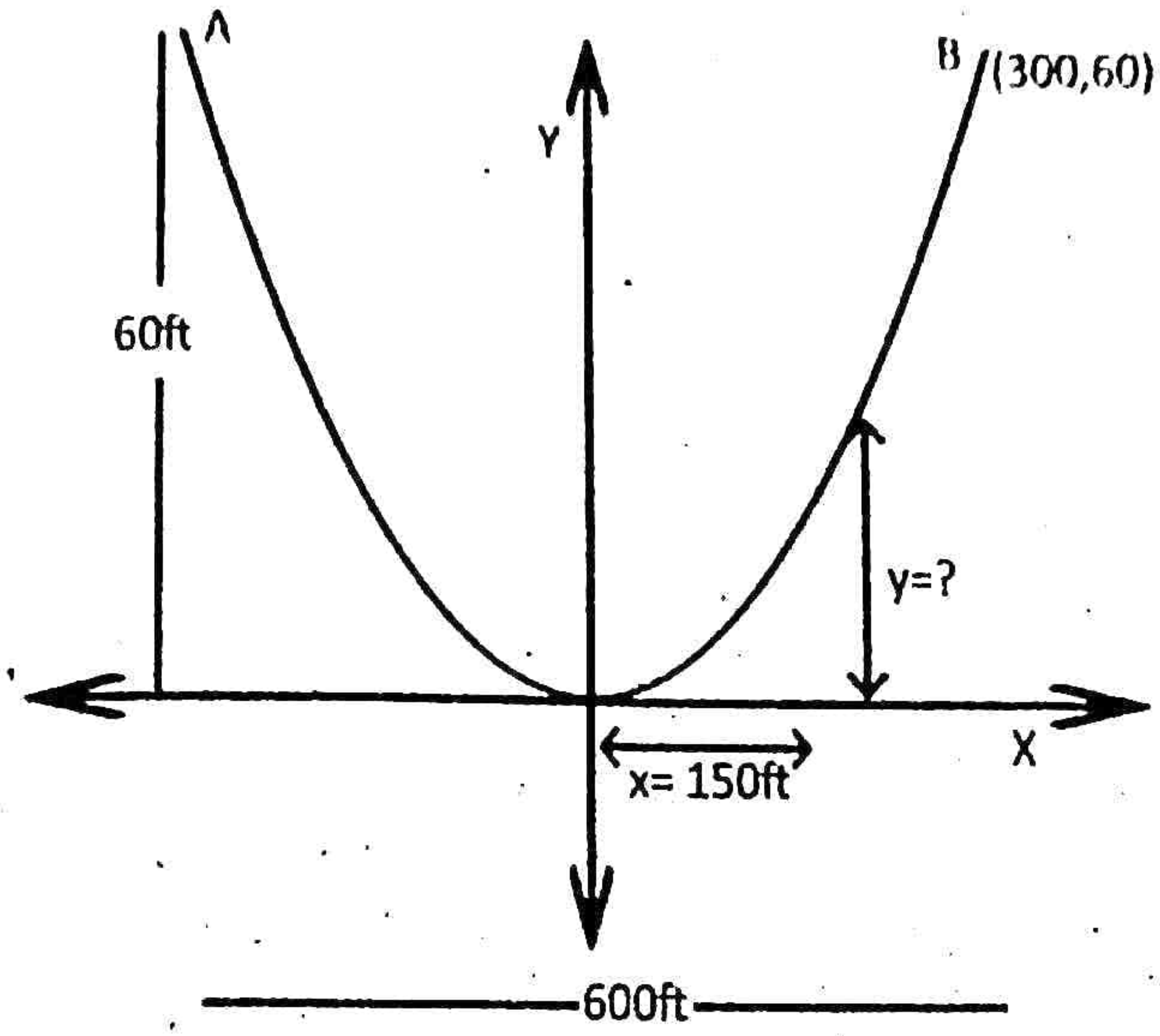
$y = 15m$

Q.1 Find the semi-
and equation of di
graphs.

(i) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

ape of parabola. Th
adway. If the cabl
owers. What is
of the bridge.

The given condition is shown



Let

Height of bridge $= 60$ft

Distance between bridge $= 600$ft

Height of suspension cable at 150ft fron cebtre of bridge $= x = ?$

Vertex is at origin

$x^2 = 4ay \rightarrow (1)$

B(300,60) is on (1)

$(1) \Rightarrow 300^2 = 4a(60)$

$$4a = \frac{300^2}{60} \Rightarrow 4a = 1500$$

$(1) \Rightarrow x^2 = 1500y$

$x = 150, y = ?$

$(1) \Rightarrow (150)^2 = 1500y$

$$\frac{(150)^2}{1500} = y$$

$y = 15m$

EXERCISE 9.3

Q.1 Find the semi-axes, eccentricity, foci, vertices, covertices, latus rectum and equation of directrices of the following ellipses. Also draw their graphs.

(i) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ (ii) $\frac{x^2}{16} + \frac{y^2}{10} = 1$

(iii) $\frac{(x-3)^2}{25}+\frac{(y+4)^2}{16}=1$ (iv) $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{16}=1$

(v) $9x^2+25y^2=225$ (vi) $4x^2-16x+25y^2+200y-306=0$

Solution:

(i) $\frac{x^2}{9}+\frac{y^2}{25}=1$

Comparing with

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

$a^2=25\Rightarrow a=5$

$b^2=9\Rightarrow b=3$

Centre $=(0,0)$

$c^2=a^2-b^2$

$c=\sqrt{25-9}=4$

$e=\frac{c}{a}=\frac{4}{5}$

Major axis is along y-axis

Vertices $(0,\pm a)=(0,\pm 5)$

Covertices $(\pm b,0)=(\pm 3,0)$

Foci $(0,\pm c)=(0,\pm 4)$

Equation of directrices:

$$y=\pm\frac{a}{e}$$

$$y=\pm\frac{5}{\frac{4}{5}}$$

$$y=\pm\frac{25}{4}$$

(ii) $\frac{x^2}{16}$ +

Compa

$\frac{x^2}{a^2}+\frac{y}{b}$

$a^2=1$

$b^2=1$

Centre

$c^2=a$

$c=\sqrt{1}$

$e=\frac{c}{a}$

Major a

Vertice

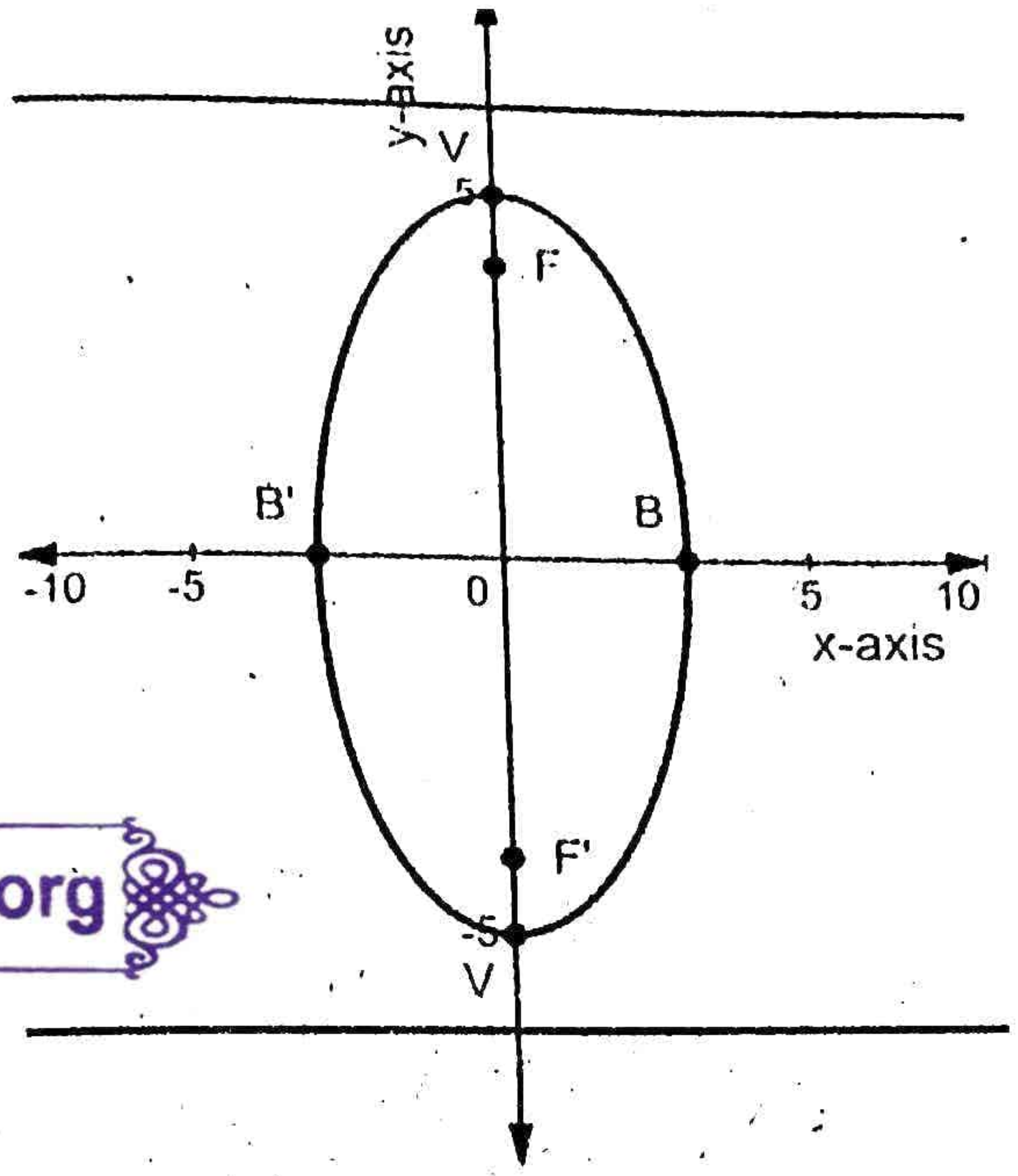
Coverti

Foci (0,

Equatio

$x=\pm$

$- 306 = 0$



(ii) $\frac{x^2}{16}+\frac{y^2}{10}=1$

Comparing with

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$a^2=16 \Rightarrow a=4$

$b^2=10 \Rightarrow b=\sqrt{10}$

Centre $=(0,0)$

$c^2=a^2-b^2$

$c=\sqrt{16-10}=\sqrt{6}$

$e=\frac{c}{a}=\frac{\sqrt{6}}{4}$

Major axis is along x-axis

Vertices $(\pm a,0)=(\pm 4,0)$

Covertices $(0,\pm b)=\left(0,\pm\sqrt{10}\right)$

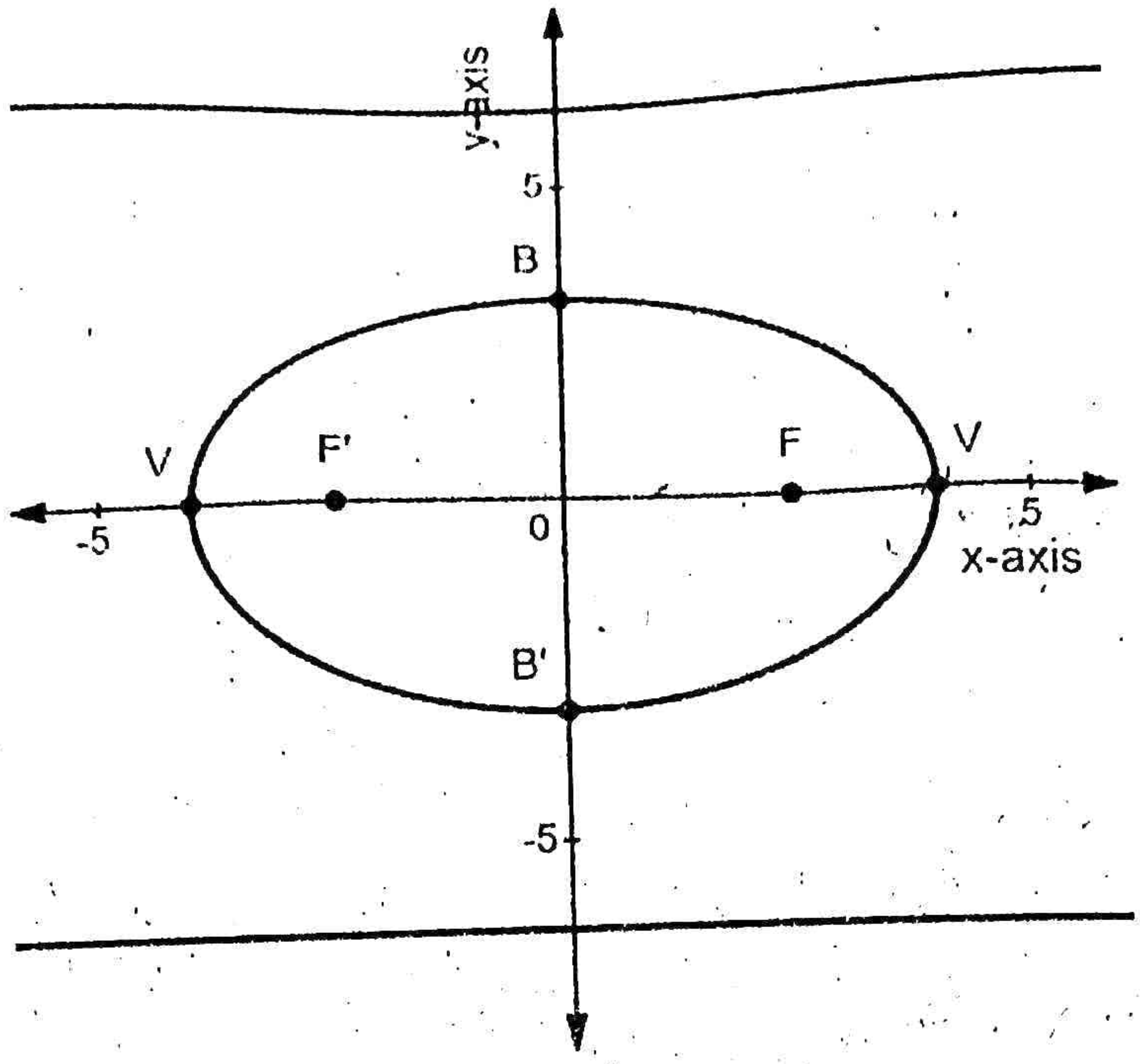
Foci $(0,\pm c)=\left(\pm\sqrt{6},0\right)$

Equation of directrices:

$x=\pm\frac{a}{e}$

$x=\pm\frac{4}{\frac{\sqrt{6}}{4}}$

$x = \pm \frac{16}{\sqrt{6}}$



(iii) $\frac{(x-3)^2}{25} + \frac{(y+4)^2}{16} = 1$

$$\frac{(x-3)^2}{25} + \frac{\{y-(-4)\}^2}{16} = 1$$

Comparing with

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 16 \Rightarrow b = 4$

Centre $(h, k) = (3, -4)$

$c^2 = a^2 - b^2$

$c = \sqrt{25 - 16} = 3$

$$e = \frac{c}{a} = \frac{3}{5}$$

Major axis is along x-axis

Vertices $(h \pm a, k) = (3 \pm 3, -4)$

$= (6, -4)$ and $(0, -4)$

Covertices $(h, k \pm b) = (3, -4 \pm 4)$

$= (3, -8)$ and $(3, 0)$

Foc

$= (6, -4$

Equation

$x = h \pm$

$x = 3 \pm$

$x = 3 \pm$

$x = \frac{34}{3}$ a

(iv) $\frac{(x+1)^2}{9} +$

$\frac{\{x-(-1)\}}{9}$

Comparing

$\frac{(x-h)^2}{b^2} +$

$a^2 = 16 \Rightarrow$

$b^2 = 9 \Rightarrow b$

Centre $(h, k$

$c^2 = a^2 - b$

Foci $(h \pm c, k) = (3 \pm 3, -4)$

$= (6, -4)$ and $(0, -4)$

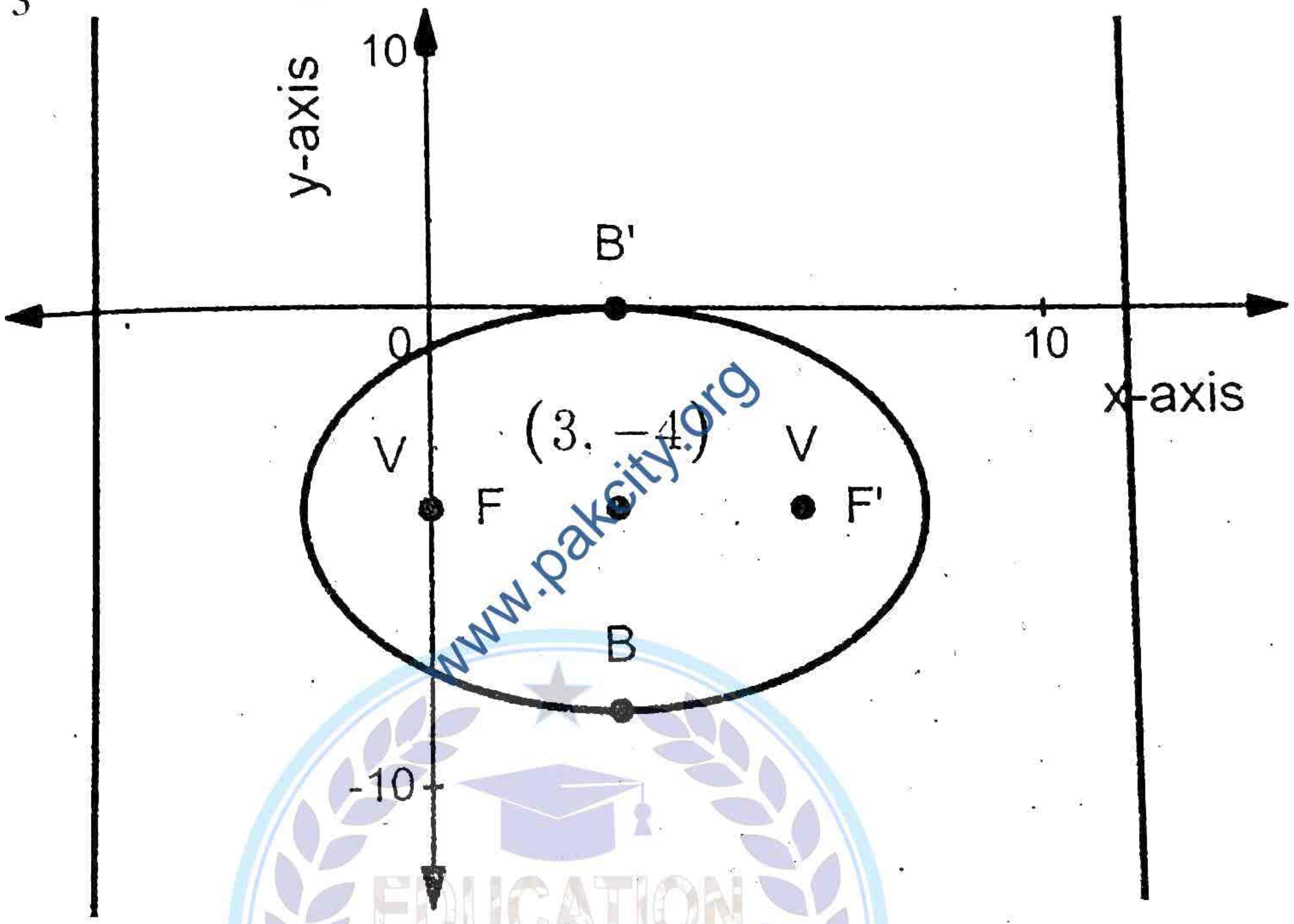
Equation of directrices:

$$x = h \pm \frac{a}{c}$$

$$x = 3 \pm \frac{5}{\frac{3}{5}}$$

$$x = 3 \pm \frac{25}{3}$$

$$x = \frac{34}{3} \text{ and } x = -\frac{16}{3}$$



(iv) $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{16} = 1$

$$\frac{\{x-(-1)\}^2}{9} + \frac{(y-2)^2}{16} = 1$$

Comparing with

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$a^2 = 16 \Rightarrow a = 4$

$b^2 = 9 \Rightarrow b = 3$

Centre $(h, k) = (-1, 2)$

$c^2 = a^2 - b^2$

$c = \sqrt{16-9} = \sqrt{7}$

$$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$$

Major axis is along y-axis

Vertices $(h, k \pm a) = (-1, 2 \pm 4)$

$= (-1, 6)$ and $(-1, -2)$

Covertices $(h \pm b, k) = (-1 \pm 3, 2)$

$= (-4, 2)$ and $(2, 2)$

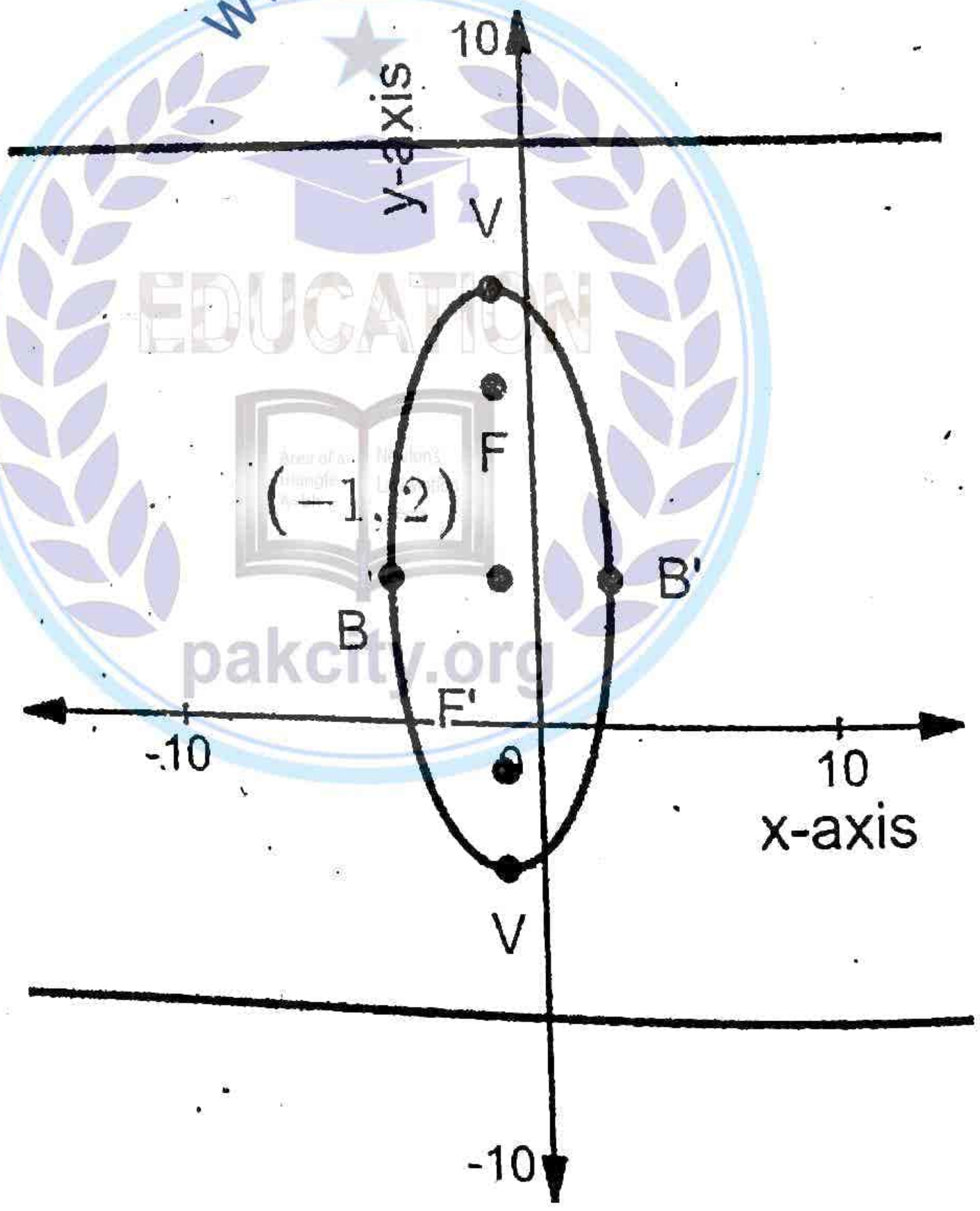
Foci $(h, k \pm c) = (-1, 2 \pm \sqrt{7})$

Equation of directrices:

$$y = k \pm \frac{a}{e}$$

$$y = 2 \pm \frac{4}{\frac{\sqrt{7}}{4}}$$

$$y = 2 \pm \frac{16}{\sqrt{7}}$$



(v) $9x^2 + 25y^2 = 225$

÷ by 225

comparing with

$\frac{y^2}{b^2} = 1$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 9 \Rightarrow b = 3$

Centre $= (0,0)$

$c^2 = a^2 - b^2$

$c = \sqrt{25-9} = 4$

$e = \frac{c}{a} = \frac{4}{5}$

Major axis is along

Vertices $(\pm 5, 0) =$

Covertices $(0, \pm b)$

Foci $(0, \pm c) = (\pm$

Equation of direct

$x = \pm \frac{a}{e}$

$x = \pm \frac{5}{\frac{4}{5}}$

$x = \pm \frac{25}{4}$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Comparing with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$a^2=25 \Rightarrow a=5$

$b^2=9 \Rightarrow b=3$

Centre $=(0,0)$

$c^2=a^2-b^2$

$c=\sqrt{25-9}=4$

$$e=\frac{c}{a}=\frac{4}{5}$$

Major axis is along x-axis

Vertices $(\pm5,0)=(\pm5,0)$

Covertices $(0,\pm b)=(0,\pm3)$

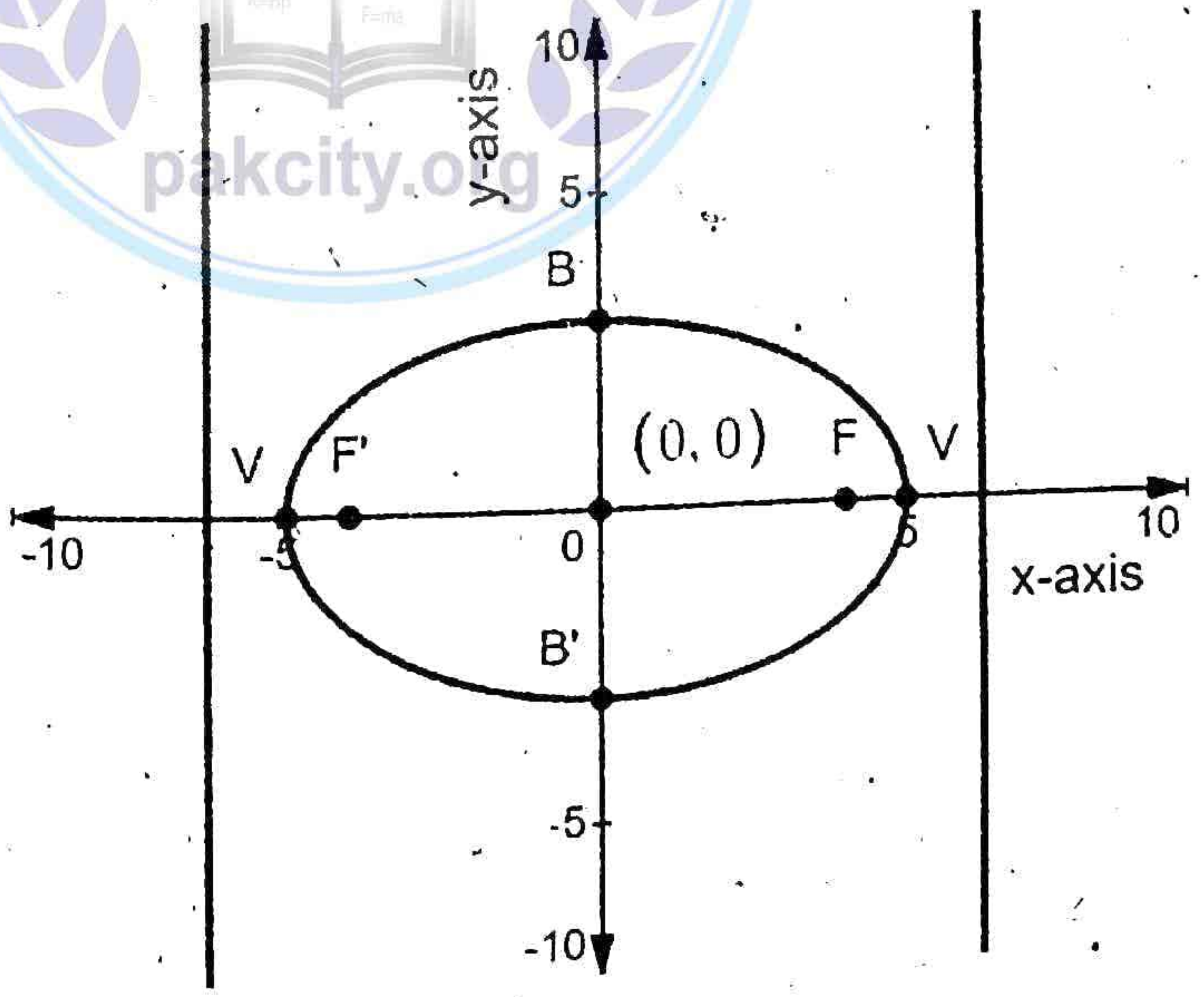
Foci $(0,\pm c)=(\pm4,0)$

Equation of directrices:

$$x=\pm\frac{a}{e}$$

$$x=\pm\frac{5}{\frac{4}{5}}$$

$$x=\pm\frac{25}{4}$$



(vi) $4x^2-16x+25y^2+200y+316=0$

$4\{x^2-4x\}+25\{y^2+8y\}=316$

$4\{x^2-4x+4\}+25\{y^2+8y+16\}=-316+16+400$

$4\{(x)^2-2(x)(2)+(2)^2\}+25\{(y)^2+2(y)(4)+(4)^2\}=100$

$4(x-2)^2+25(y+4)^2=100$

÷ by 100

$$\frac{(x-2)^2}{25}+\frac{(y+4)^2}{4}=1$$

$$\frac{(x-2)^2}{25}+\frac{\{y-(-4)\}^2}{4}=1$$

Comparing with

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$a^2=25\Rightarrow a=5$

$b^2=4\Rightarrow b=2$

Centre $(h,k)=(2,-4)$

$c^2=a^2-b^2$

$c=\sqrt{25-4}=\sqrt{21}$

$$e=\frac{c}{a}=\frac{\sqrt{21}}{5}$$

Major axis is along x-axis

Vertices $(h\pm a,k)=(2\pm 5,-4)$

$=(7,-4)$ and $(-3,-4)$

Covertices $(h,k\pm b)=(2,-4\pm 2)$

$=(2,-6)$ and $(2,-2)$

Foci $(h\pm c,k)=\left(2\pm\sqrt{21},-4\right)$

Equation of directrices:

$$x=h\pm\frac{a}{e}$$

$$x=2\pm\frac{5}{\frac{\sqrt{21}}{5}}$$

$$x=2\pm\frac{25}{\sqrt{21}}$$

Q.2 Find

and thei

(i) Major

y-axis

(ii) Ellips

(iii) foci a

(iv) foci a

(v) foci at

(vi) Mino

units long

(vii) cover

(viii) direc

Solution:

(i) Major a

y-axis

∴ Major ax

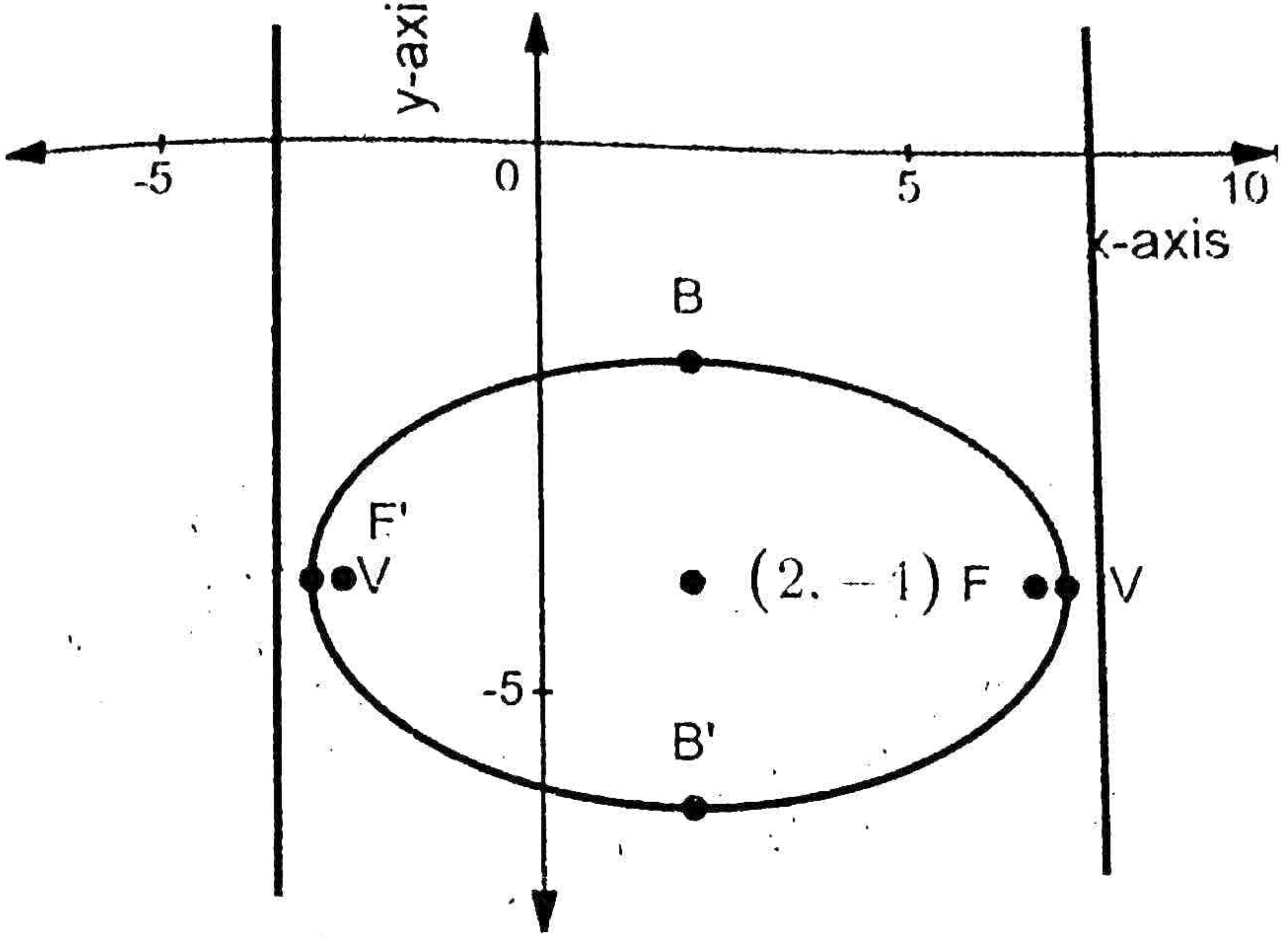
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=$

Length of r

$a=6$

400

$^2\} = 100$



Q.2 Find the equations of the following ellipse whose centres are at origin and their axes are along coordinate axes. Also verify the given conditions.

(i) Major and minor axes are 12 and 8 respectively with minor axis is along y-axis

(ii) Ellipse passes through $\left(1, \sqrt{\frac{3}{2}}\right)$ and $\left(\frac{2}{\sqrt{3}}, 1\right)$ with major axis along y-axis

(iii) foci at $(\pm 3,0)$ and vertices at $(\pm 5,0)$

(iv) foci at $(0, \pm 4)$ and latus rectum $\frac{18}{5}$

(v) foci at $(\pm 5,0)$ and minor axis is 12 units long and along y-axis

(vi) Minor axis along x-axis with length is 8 units and latus rectum are 6 units long and along y-axis

(vii) covertices are $(0, \pm 3)$ and distance between foci $= 10$ units

(viii) directrix $y = \frac{25}{3}$ and latus rectum $\frac{32}{5}$

Solution:

(i) Major and minor axes are 12 and 8 respectively with minor axis is along y-axis

$\therefore$ Major axis is along x-axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow (1)$$

Length of major axis $= 2a = 12$

$a = 6$

$a^2 = 36$

Length of minor axis $= 2b = 8$

$b^2 = 16$

$(1) \Rightarrow \frac{x^2}{36} + \frac{y^2}{16} = 1$

(ii) Ellipse passes through $\left(1, \sqrt{\frac{3}{2}}\right)$ and $\left(\frac{2}{\sqrt{3}}, 1\right)$ with major axis along y-axis

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \rightarrow (1)$

$\left(1, \sqrt{\frac{3}{2}}\right)$ and $\left(\frac{2}{\sqrt{3}}, 1\right)$

$\left(1, \sqrt{\frac{3}{2}}\right)$ is on (1)

$$(1) \Rightarrow \frac{(1)^2}{b^2} + \frac{\left(\sqrt{\frac{3}{2}}\right)^2}{a^2} = 1$$

$\frac{1}{b^2} + \frac{3}{2a^2} = 1 \rightarrow (2)$

$\left(\frac{2}{\sqrt{3}}, 1\right)$ is on (1)

$$(1) \Rightarrow \frac{\left(\frac{2}{\sqrt{3}}\right)^2}{b^2} + \frac{(1)^2}{a^2} = 1$$

$\frac{4}{3b^2} + \frac{1}{a^2} = 1 \rightarrow (3)$

Multiply by $\frac{3}{2}$ and subtract from (2)

$$\frac{2}{b^2} + \frac{3}{2a^2} = \frac{3}{2}$$

$$\underline{\pm\frac{1}{b^2} \pm \frac{3}{2a^2} = \pm 1}$$

$$\frac{2}{b^2} - \frac{1}{b^2} = \frac{1}{2}$$

$$\frac{1}{b^2} = \frac{1}{2}$$

$b^2 = 2$

$(3) \Rightarrow$

$\frac{2}{3} + \frac{1}{a^2}$

$\frac{1}{a^2} = 1$

$\frac{1}{a^2} = \frac{1}{3}$

$a^2 = 3$

$(1) \Rightarrow$

(iii) foc

Vertice

$a = 5$

Foci (

$c = 3$

$c^2 = a$

$3^2 =$

$b^2 =$

$b^2 =$

Major

$\frac{x^2}{a^2} +$

$\frac{x^2}{25} +$

(iv) fo

Foci (

$ae =$

Length

$\frac{2b^2}{a} =$

$\frac{b^2}{a} =$

$(3) \Rightarrow \frac{4}{3(2)} + \frac{1}{a^2} = 1$

$\frac{2}{3} + \frac{1}{a^2} = 1$

along y-axis

$\frac{1}{a^2} = 1 - \frac{2}{3}$

$\frac{1}{a^2} = \frac{1}{3}$

$a^2 = 3$

$(1) \Rightarrow \frac{x^2}{2} + \frac{y^2}{3} = 1$

(iii) foci at $(\pm 3,0)$ and vertices at $(\pm 5,0)$

Vertices $(\pm a, 0) = (\pm 5,0)$

$a = 5$

Foci $(\pm c, 0) = (\pm 3,0)$

$c = 3$

$c^2 = a^2 - b^2$

$3^2 = 5^2 - b^2$

$b^2 = 25 - 9$

$b^2 = 16$

Major axis is along x-axis

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

(iv) foci at $(0, \pm 4)$ and latus rectum $\frac{18}{5}$

Foci $(0, \pm c) = (0, \pm ae) = (0, \pm 4)$

$ae = 4 \rightarrow (1)$

Length of latus rectum $= \frac{18}{5}$

$\frac{2b^2}{a} = \frac{18}{5}$

$\frac{b^2}{a} = \frac{9}{5}$

$$b^2 = \frac{9a}{5} \rightarrow (2)$$

$$\because b^2 = a^2(1 - e^2)$$
$$\frac{9a}{5} = a^2(1 - e^2)$$
$$\frac{9}{5} = a(1 - e^2)$$
$$\frac{9}{5} = \left(\frac{4}{e}\right)(1 - e^2)$$
$$9e = 20(1 - e^2)$$
$$9e = 20 - 20e^2$$
$$20e^2 + 9e - 20 = 0$$
$$20e^2 - 16e + 25e - 20 = 0$$
$$4e(5e - 4) + 5(5e - 4) = 0$$
$$(5e - 4)(4e + 5) = 0$$
$$5e - 4 = 0 \Rightarrow e = \frac{4}{5}$$
$$4e + 5 = 0 \Rightarrow e = -\frac{5}{4} \quad \text{[not acceptable]}$$

$$(1) \Rightarrow a = \frac{4}{e}$$
$$a = \frac{4}{\frac{4}{5}} \Rightarrow a = 5$$

$$(2) \Rightarrow b^2 = \frac{9(5)}{5} = 9$$

Major axis is along y-axis
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$
$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

(v) foci at $(\pm 5,0)$ and minor axis is 12 units long and along y-axis

Foci $(\pm c, 0) = (\pm ae, 0) = (\pm 5,0)$

$$ae = 5 \rightarrow (1)$$

$b =$

$\because b^2$
$b^2 =$
$6^2 =$
$a^2 =$
$a^2 =$

Minc
$\frac{x^2}{a^2} +$
$\frac{x^2}{61} +$

(vi) M
units
Leng
$b =$
$b^2 =$

Leng
$\frac{4^2}{a} =$
$\frac{16}{3} =$

Majo
$\frac{x^2}{a^2} +$
$\frac{x^2}{\frac{256}{9}}$
$\frac{9x^2}{256} +$

(vii) c
Cove

Length of minor axis $= 2b = 12$

$b = 6$

$\because b^2 = a^2(1 - e^2)$

$b^2 = a^2 - a^2e^2$

$6^2 = a^2 - 5^2$

$a^2 = 36 + 25$

$a^2 = 61$

Minor axis is along x-axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{61} + \frac{y^2}{36} = 1$$

(vi) Minor axis along x-axis with length is 8 units and latus rectum are 6 units long and along y-axis

Length of minor axis $= 2b = 8$

$b = 4$

$b^2 = 16$

Length of latus rectum $= \frac{2b^2}{a} = 6$

$$\frac{4^2}{a} = 3$$

$$\frac{16}{3} = a \Rightarrow a^2 = \frac{256}{9}$$

Major axis is along x-axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{\frac{256}{9}} + \frac{y^2}{16} = 1$$

$$\frac{9x^2}{256} + \frac{y^2}{16} = 1$$

(vii) covertices are $(0, \pm 3)$ and distance between foci $=$ 10 units

Covertices $(0, \pm b) = (0, \pm 3)$

axis

$b = 3$

Distance between foci $= 2c = 10$

$c = 5$

$\because c^2 = a^2 - b^2$

$5^2 = a^2 - 3^2$

$a^2 = 25 + 9$

$a^2 = 34$

Major axis is along x-axis

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{34} + \frac{y^2}{9} = 1$

(viii) directrix $y = \frac{25}{3}$ and latus rectum $\frac{32}{5}$

Equation of directrix

$\frac{a}{e} = \frac{25}{3}$

$a = \frac{25e}{3} \rightarrow (1)$

Length of latus rectum $= \frac{32}{5}$

$\frac{b^2}{a} = \frac{16}{5}$

$b^2 = \frac{16}{5}a \rightarrow (2)$

$\because b^2 = a^2(1 - e^2)$

$\frac{16}{5}a = a^2(1 - e^2)$

$\frac{16}{5} = a(1 - e^2)$

$\frac{16}{5} = \left(\frac{25e}{3}\right)(1 - e^2) \quad \left[\because a = \frac{25e}{3}\right]$

$48 = 125e(1 - e^2)$

$48 = 125e - 125e^3$

$125e^3 - 125e + 48 = 0$

$125e$

$25e^2(5e -$

$(5e - 3)(2$

$5e - 3 \Rightarrow$

$25e^2 + 15$

$e = \frac{-15 \pm}{}$

$e = \frac{-3 \pm}{10}$

$(1) \Rightarrow a =$

$(2) \Rightarrow b^2 =$

Major axis is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{16} + \frac{y^2}{25} =$

Q.3 Find the

Solution:

$5x^2 + 7y^2$

$\div$ by 11

$\frac{5x^2}{11} + \frac{7y^2}{11}$

$\frac{x^2}{\frac{11}{5}} + \frac{y^2}{\frac{11}{7}} =$

Comparing

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = \frac{11}{5}$

The

$$125e^3 - 75e^2 + 75e^2 - 45e - 80e + 48 = 0$$

$$25e^2(5e - 3) + 15e(5e - 3) - 16(5e - 3) = 0$$

$$(5e - 3)(25e^2 + 15e - 16) = 0$$

$$5e - 3 \Rightarrow e = \frac{3}{5}$$

$$25e^2 + 15e - 16 = 0$$

$$e = \frac{-15 \pm \sqrt{15^2 - 4(25)(-16)}}{2(25)}$$

$$e = \frac{-3 \pm \sqrt{73}}{10}$$

$$(1) \Rightarrow a = \frac{25}{3}\left(\frac{3}{5}\right) = 5$$

$$(2) \Rightarrow b^2 = \frac{16}{5}(5)b^2 = 16$$

Major axis is along y-axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Q.3 Find the equation of auxiliary circle to $5x^2 + 7y^2 = 11$.

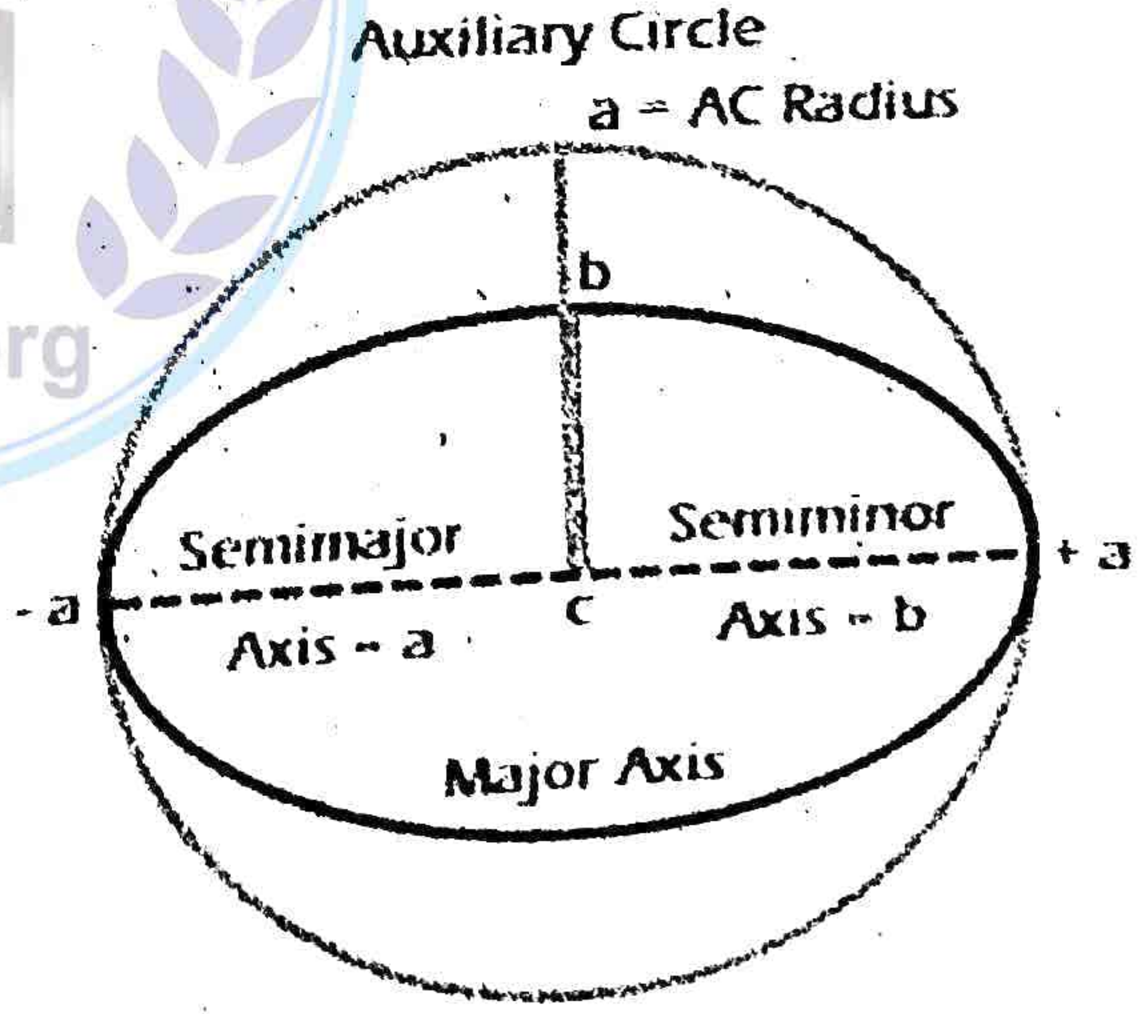
Solution:

$$5x^2 + 7y^2 = 11$$

÷ by 11

$$\frac{5x^2}{11} + \frac{7y^2}{11} = 1$$

$$\frac{x^2}{\frac{11}{5}} + \frac{y^2}{\frac{11}{7}} = 1$$



Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = \frac{11}{5}$$

$b^2 = \frac{11}{7}$

Auxiliary circle is $x^2 + y^2 = a^2$

$x^2 + y^2 = \frac{11}{5}$

Q.4 Is the point $(4,5)$ inside on or outside the ellipse $2x^2 + 3y^2 = 6$.

Solution:

$2x^2 + 3y^2 = 6$

$2x^2 + 3y^2 - 6 = 0 \rightarrow (1)$

Point $= (4,5)$

$(1) \Rightarrow \text{L.H.S} = 2(4)^2 + 3(5)^2 - 6$

$\text{L.H.S} = 32 + 75 - 6$

$\text{L.H.S} = 101 > 0$

Hence $(4,5)$ lies outside the ellipse

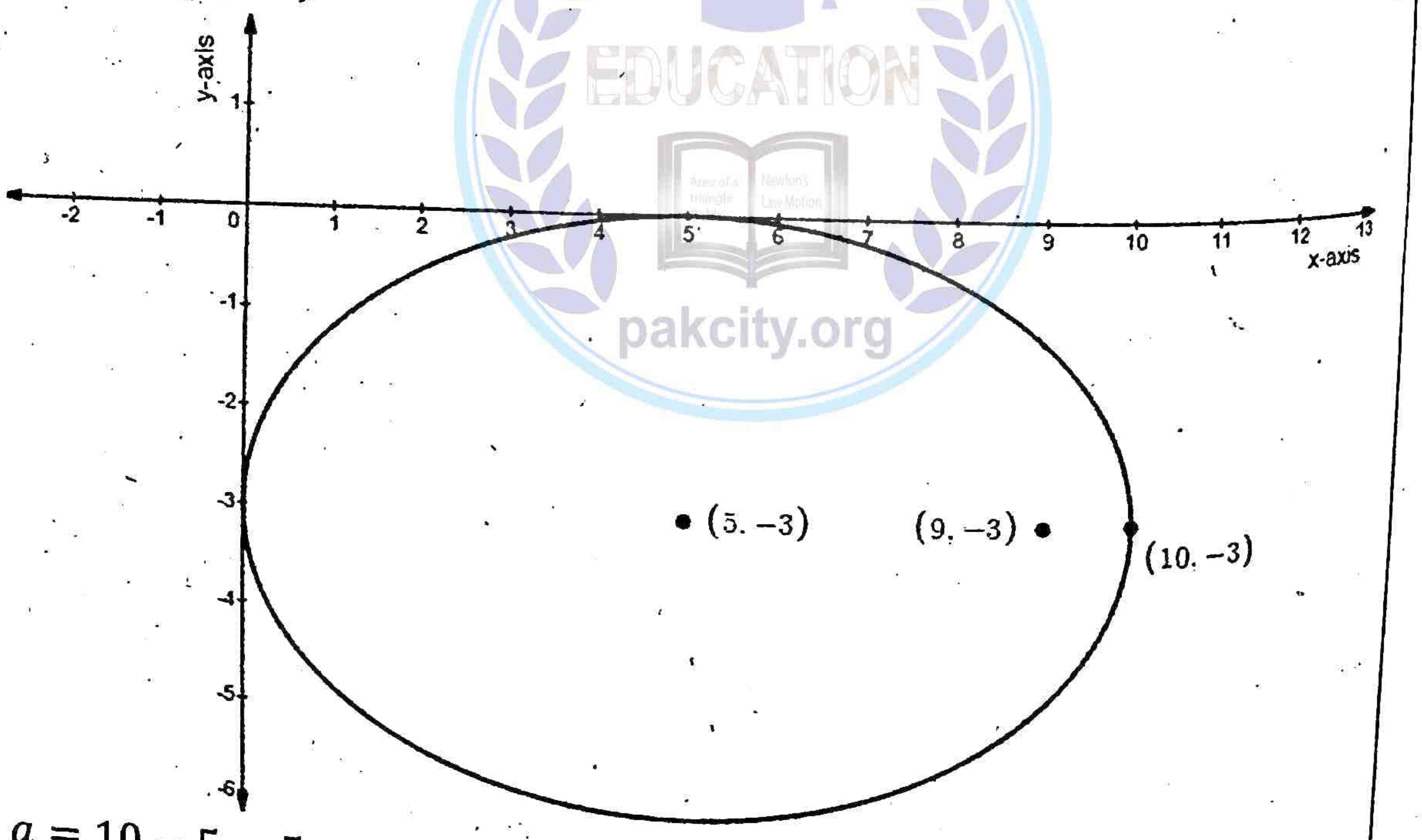
Q.5 Find equation of ellipse with centre at $(5,-3)$, one vertex at $(10,-3)$ and one focus at $(9,-3)$.

Solution:

Centre $(h,k) = (5,-3)$

Vertex $= (10,-3)$

Focus $= (9,-3)$



$a = 10 - 5 = 5$

$c = 9 -$

$\because c^2 =$

$4^2 = 5$

$b^2 = 2$

$b^2 = 9$

Major a

$\frac{(x-h)}{a^2}$

$\frac{(x-5)}{25}$

$\frac{(x-5)}{25}$

Q.6 If ell

(i) distan

(ii) distar

(iii) dista

Solution:

$9x^2 + 13$

$\div$ by 117

$\frac{x^2}{13} + \frac{y^2}{9}$

Comparin

$\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

$a^2 = 13 =$

$b^2 = 9 \Rightarrow$

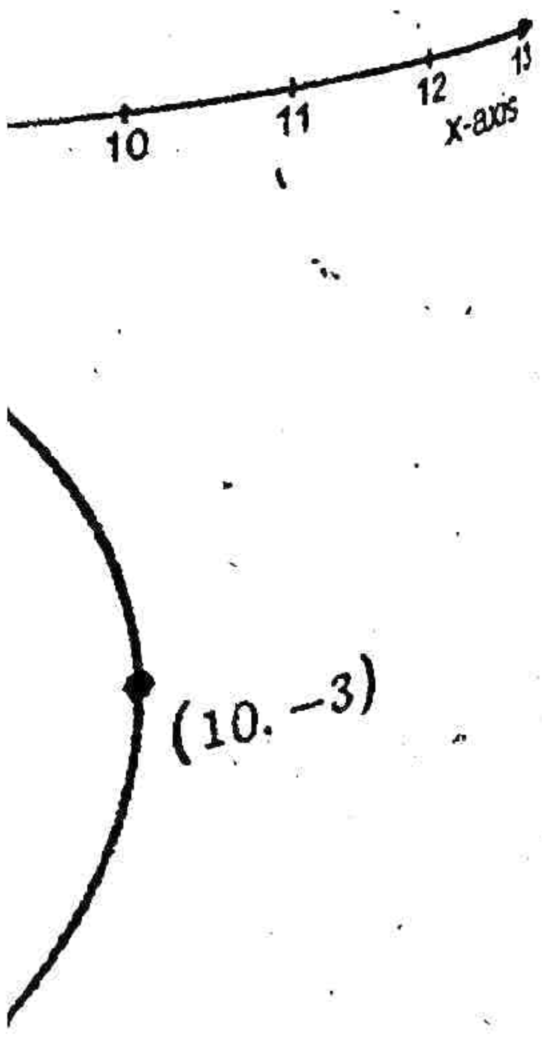
Centre = (

$c^2 = a^2 -$

$c = \sqrt{13 -}$

$^2 + 3y^2 = 6.$

vertex at $(10,-3)$



$c = 9 - 5 = 4$

$\because c^2 = a^2 - b^2$

$4^2 = 5^2 - b^2$

$b^2 = 25 - 16$

$b^2 = 9$

Major axis is parallel to x-axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-5)^2}{25} + \frac{\{y-(-3)\}^2}{9} = 1$$

$$\frac{(x-5)^2}{25} + \frac{(y+3)^2}{9} = 1$$

Q.6 If ellipse is $9x^2 + 13y^2 = 117$ then find:

(i) distance between foci

(ii) distance between vertices

(iii) distance between covertices

Solution:

$9x^2 + 13y^2 = 117$

$\div$ by 117

$$\frac{x^2}{13} + \frac{y^2}{9} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 13 \Rightarrow a = \sqrt{13}$

$b^2 = 9 \Rightarrow b = 3$

Centre $= (0,0)$

$c^2 = a^2 - b^2$

$c = \sqrt{13-9} = 2$

(i) distance between foci $= 2c = 2(2) = 4$

(ii) distance between vertices $= 2a = 2\sqrt{13}$

(iii) distance between covertices $= 2b = 2(3) = 6$

Q.7 Find eccentricity of ellipse if:

(i) axes are 32 and 24

(ii) latus rectum is equal to half of its major axis

Solution:

(i) axes are 32 and 24

Length of major axis $= 2a = 32$

$a = 16$

Length of minor axis $= 2b = 24$

$b = 12$

$\because b^2 = a^2(1 - e^2)$

$12^2 = 16^2(1 - e^2)$

$$\frac{144}{256} = 1 - e^2$$

$$e^2 = 1 - \frac{144}{256}$$

$$e^2 = \frac{7}{16}$$

$$e = \frac{\sqrt{7}}{4}$$

(ii) latus rectum is equal to half of its major axis

Length of major axis $= 2a$

Length of latus rectum $= \frac{2b^2}{a}$

Length of latus rectum $= \frac{1}{2}$[Length of major axis]

$$\frac{2b^2}{a} = \frac{1}{2}(2a)$$

$$\frac{2b^2}{a} = a$$

$2b^2 = a^2$

$\because b^2 = a^2(1 - e^2)$

$b^2 = 2b^2(1 - e^2)$

$1 = 2(1 - e^2)$

$1 = 2 - 2e^2$

$2e^2 = 2 - 1$

$2e^2 = 1$

$e^2 = \frac{1}{2}$

$e = \frac{1}{\sqrt{2}}$

Q.8 Find equation of the circle passing through focus of parabola $y^2 + 8x = 0$ and foci of ellipse $25x^2 + 16y^2 = 400$.

Solution:

$25x^2 + 16y^2 = 400$

÷ by 400

$\frac{25x^2}{400} + \frac{16y^2}{400} = \frac{400}{400}$

$\frac{x^2}{16} + \frac{y^2}{25} = 1$

Comparing with

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 16 \Rightarrow b = 4$

$c^2 = a^2 - b^2$

$c = \sqrt{25 - 16} = 3$

Major axis is along y-axis

Foci $(0, \pm c) = (0, \pm 3)$

Foci of ellipse are $F(0,3)$ and $F'(0,-3)$

$y^2 + 8x = 0$

$y^2 = -8x$

Comparing with $y^2 = 4ax$

$4a = -8$

$a = -2$

Principle axis is along x-axis

Focus of parabola $(a,0)=(-2,0)$

Points on circle are $(0,3),(0,-3)$ and $(-2,0)$

We have general equation of circle as

$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

$(0,3)$

$(1) \Rightarrow 0^2+3^2+2g(0)+2f(3)+c=0$

$0+9+0+6f+c=0$

$9+6f+c=0 \rightarrow (2)$

$(0,-3)$

$(1) \Rightarrow 0^2+(-3)^2+2g(0)+2f(-3)+c=0$

$0+9+0-6f+c=0$

$9-6f+c=0 \rightarrow (3)$

$(-2,0)$

$(1) \Rightarrow (-2)^2+0^2+2g(-2)+2f(0)+c=0$

$4+0-4g+0+c=0$

$4-4g+c=0 \rightarrow (3)$

By $(2)+(3)$

$18+2c=0$

$2c=-18$

$\boxed{c=-9}$

$(2) \Rightarrow 9+6f-9=0$

$6f=0$

$\boxed{f=0}$

$(3) \Rightarrow 4-4g-9=0$

$-5=4g$

$\boxed{g=-\frac{5}{4}}$

$(1) \Rightarrow x^2$

x^2+y^2-

x^2+y^2-

× by 2

$2x^2+2y$

Q.9 Find t

point $(4\sqrt{}$

Solution:

$25x^2+1$

÷ by 160

$\frac{25x^2}{1600}+\frac{1}{1}$

$\frac{x^2}{64}+\frac{y^2}{100}$

Comparin

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=$

$a^2=100$

$b^2=64$

$c^2=a^2-$

$c=\sqrt{10}$

Major axi

Foci $(0,\pm$

Foci of elli

Point = P

Length $\overline{PF}$

$(x_1,y_1)=$

$(x_2,y_2)=$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\frac{}{4}\right)x + 2(0)y - 9 = 0$

$x^2 + y^2 - \frac{5}{2}x + 0 - 9 = 0$

$x^2 + y^2 - \frac{5}{2}x - 9 = 0$

× by 2

$2x^2 + 2y^2 - 5x - 18 = 0$

Q.9 Find the length of, and the equations to, the focal radii drawn to a point $(4\sqrt{3}, 5)$ of the ellipse $25x^2 + 16y^2 = 1600$.

Solution:

$25x^2 + 16y^2 = 1600$

÷ by 1600

$\frac{25x^2}{1600} + \frac{16y^2}{1600} = \frac{1600}{1600}$

$\frac{x^2}{64} + \frac{y^2}{100} = 1$

Comparing with

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$a^2 = 100 \Rightarrow a = 10$

$b^2 = 64 \Rightarrow b = 8$

$c^2 = a^2 - b^2$

$c = \sqrt{100 - 64} = 6$

Major axis is along y-axis

Foci $(0, \pm c) = (0, \pm 6)$

Foci of ellipse are $F(0,6)$ and $F'(0,-6)$

Point $= P(4\sqrt{3}, 5)$

Length $\overline{PF}$

$(x_1, y_1) = F(0,6)$

$(x_2, y_2) = P(4\sqrt{3}, 5)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\overline{PF} = \sqrt{(4\sqrt{3} - 0)^2 + (5 - 6)^2}$$

$$\overline{PF} = \sqrt{(4\sqrt{3})^2 + (-1)^2}$$

$$\overline{PF} = \sqrt{48 + 1}$$

$$\overline{PF} = \sqrt{49}$$

$$\overline{PF} = 7 \text{ units}$$

Equation of $\overline{PF}$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{5 - 6}{4\sqrt{3} - 0}(x - 0)$$

$$y - 6 = \frac{-1}{4\sqrt{3}}x$$

$$4\sqrt{3}y - 24\sqrt{3} = -x$$

$$x + 4\sqrt{3}y - 24\sqrt{3} = 0$$

Length $\overline{PF'}$

$(x_1, y_1) = F'(0, -6)$

$(x_2, y_2) = P(4\sqrt{3}, 5)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\overline{PF} = \sqrt{(4\sqrt{3} - 0)^2 + (5 + 6)^2}$$

$$\overline{PF} = \sqrt{(4\sqrt{3})^2 + (11)^2}$$

$$\overline{PF} = \sqrt{48 + 121}$$

$$\overline{PF} = \sqrt{169}$$

$$\overline{PF} = 13 \text{ units}$$

Equation of $\overline{PF'}$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$y + 6 =$

$y + 6 =$

$4\sqrt{3}y +$

$11x - 4$

Q.10 Fin

to y-axis.

Solution:

Centre (

Points ar

Major ax

$\frac{(x-h)^2}{b^2}$

$\frac{(x-0)^2}{b^2}$

$\frac{x^2}{b^2} + \frac{(y}{}$

As $(2,1)$ i

$(1) \Rightarrow \frac{2^2}{b^2}$

$\frac{4}{b^2} + \frac{(0)}{a^2}$

$\frac{4}{b^2} + 0 =$

$\frac{4}{b^2} = 1$

$b^2 = 4$

As $(0,4)$ is

$(1) \Rightarrow \frac{0^2}{4}$

$0 + \frac{(3)^2}{a^2} =$

$$y+6=\frac{5+6}{4\sqrt{3}-0}(x-0)$$

$$y+6=\frac{11}{4\sqrt{3}}x$$

$$4\sqrt{3}y+24\sqrt{3}=11x$$

$$11x-4\sqrt{3}y-24\sqrt{3}=0$$

Q.10 Find equation of ellipse with centre at (0,1) and major axis parallel to y-axis. Also, it passes through (2,1) and (0,4).

Solution:

Centre $(h,k)=(0,1)$

Points are $(2,1)$ and $(0,4)$

Major axis parallel to y-axis

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-0)^2}{b^2}+\frac{(y-1)^2}{a^2}=1$$

$$\frac{x^2}{b^2}+\frac{(y-1)^2}{a^2}=1\rightarrow(1)$$

As $(2,1)$ is on (1)

$$(1)\Rightarrow\frac{2^2}{b^2}+\frac{(1-1)^2}{a^2}=1$$

$$\frac{4}{b^2}+\frac{(0)^2}{a^2}=1$$

$$\frac{4}{b^2}+0=1$$

$$\frac{4}{b^2}=1$$

$$b^2=4$$

As $(0,4)$ is on (1) and $b^2=4$

$$(1)\Rightarrow\frac{0^2}{4}+\frac{(4-1)^2}{a^2}=1$$

$$0+\frac{(3)^2}{a^2}=1$$

$$\frac{9}{a^2} = 1$$
$$a^2 = 9$$

$$(1) \Rightarrow \frac{x^2}{4} + \frac{(y-1)^2}{9} = 1$$

EXERCISE 9.4

The line $y = mx + c$ is tangent to

	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
Condition	$c^2 = a^2m^2 + b^2$	$c^2 = a^2 + b^2m^2$
Equation of tangent with slope m	$y = mx \pm \sqrt{a^2m^2 + b^2}$	
Equation of tangent at (x_1, y_1)	$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$	
Equation of normal at (x_1, y_1)	$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$	

Q.1 Find the condition when line $y = \sqrt{5}x + c$ is tangent to the ellipse $4x^2 + 9y^2 = 36$.

Solution:

$y = \sqrt{5}x + c$

Comparing with $y = mx + c$

$m = \sqrt{5}$ and $c = c$

$4x^2 + 9y^2 = 36$

÷ by 36

$$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$$
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 9$ and $b^2 = 4$

The Stude

$c^2 = a^2m^2$
$c^2 = (9)($
$c^2 = (9)(5$
$c^2 = 49$
$c = \pm 7$

Q.2 Show t

find point c

Solution:

$x = 2y + 4$

$\frac{x^2}{4} + \frac{y^2}{3} =$

Put value of

$(2) \Rightarrow \{2(y$

$4(y^2 + 4y$

$(y^2 + 4y +$

$3y^2 + 12y$

$4y^2 + 12y$
$4y^2 + 12y$
$B^2 - 4AC =$
$= 144 - 14$
$= 0$

Hence (1) to

$(2y)^2 + 2(2$
$(2y + 3)^2 =$
$2y + 3 = 0$
$y = -\frac{3}{2}$

$c^2 = a^2m^2 + b^2$
$c^2 = (9)(\sqrt{5})^2 + 4$
$c^2 = (9)(5) + 4$
$c^2 = 49$
$c = \pm 7$

Q.2 Show that the line $x = 2y + 4$ touches the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Also find point of contact.

Solution:

$x = 2y + 4 \rightarrow (1)$
$\frac{x^2}{4} + \frac{y^2}{3} = 1 \rightarrow (2)$

Put value of x from (1) in (2)

$(2) \Rightarrow \frac{\{2(y+2)\}^2}{4} + \frac{y^2}{3} = 1$

$\frac{4(y^2 + 4y + 4)}{4} + \frac{y^2}{3} = 1$

$(y^2 + 4y + 4) + \frac{y^2}{3} = 1$

$\frac{3y^2 + 12y + 12 + y^2}{3} = 1$

$4y^2 + 12y + 12 = 3$
$4y^2 + 12y + 9 = 0$
$B^2 - 4AC = 12^2 - 4(4)(9)$
$= 144 - 144$
$= 0$

Hence (1) touches (2)

$(2y)^2 + 2(2y)(3) + (3)^2 = 0$
$(2y + 3)^2 = 0$
$2y + 3 = 0$
$y = -\frac{3}{2}$

$(1) \Rightarrow x = 2\left(-\frac{3}{2}\right) + 4$

$x = -3 + 4$

$x = 1$

$\text{Point} = \left(1, -\frac{3}{2}\right)$

Q.3 Find the condition of tangency of line $y = mx + c$ to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Solution:

$y = mx + c \rightarrow (1)$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \rightarrow (2)$

Put value of y from (1) in (2)

$(2) \Rightarrow \frac{x^2}{b^2} + \frac{(mx + c)^2}{a^2} = 1$

$\frac{x^2}{b^2} + \frac{m^2x^2 + 2mcx + c^2}{a^2} = 1$

$\frac{a^2x^2 + b^2(m^2x^2 + 2mcx + c^2)}{a^2b^2} = 1$

$a^2x^2 + b^2m^2x^2 + 2b^2mcx + b^2c^2 = a^2b^2$

$x^2(a^2 + b^2m^2) + 2b^2mcx + b^2c^2 - a^2b^2 = 0$

$x^2(a^2 + b^2m^2) + 2b^2mcx + b^2(c^2 - a^2) = 0$

$B^2 - 4AC = 0$

$(2b^2mc)^2 - 4(a^2 + b^2m^2)\{b^2(c^2 - a^2)\} = 0$

$4b^4m^2c^2 - 4b^2(a^2 + b^2m^2)(c^2 - a^2) = 0$

÷ by $4b^2$

$b^2m^2c^2 - (a^2c^2 - a^4 + b^2m^2c^2 - a^2b^2m^2) = 0$

$b^2m^2c^2 - a^2c^2 + a^4 - b^2m^2c^2 + a^2b^2m^2 = 0$

$-a^2c^2 + a^4 + a^2b^2m^2 = 0$

$a^4 + a^2b^2m^2 = a^2c^2$

÷ by a^2

$c^2 = a^2 + b^2m^2$

(i) $\frac{x}{p}+$

Soluti

$\frac{x^2}{a^2}+$

(i) $\frac{x}{p}+$

$\frac{y}{q} = -$

$y = -$

Comp

$m =$

$c^2 =$

$q^2 =$

$q^2 =$

÷ by

$1 = \frac{a}{p}$

$1 = \frac{a}{p}$

$\frac{a^2}{p^2}+$

(ii) x c

$y \sin \alpha$

$y = -$

Compa

$m = -$

$\left(\frac{p}{\sin \alpha}\right)$

p^2

(i) $\frac{x}{p}+\frac{y}{q}=1$ (ii) $x\cos\alpha+y\sin\alpha=p$ (iii) $lx+my+n=0$

Solution:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

(i) $\frac{x}{p}+\frac{y}{q}=1$

$$\frac{y}{q}=-\frac{x}{p}+1$$

$$y=-\frac{q}{p}x+q$$

Comparing with $y=mx+c$

$$m=-\frac{q}{p},c=q$$

$$c^2=a^2m^2+b^2$$

$$q^2=a^2\left(-\frac{q}{p}\right)^2+b^2$$

$$q^2=a^2\left(\frac{q^2}{p^2}\right)+b^2$$

$\div$ by q^2

$$1=\frac{a^2q^2}{p^2q^2}+\frac{b^2}{q^2}$$

$$1=\frac{a^2}{p^2}+\frac{b^2}{q^2}$$

$$\frac{a^2}{p^2}+\frac{b^2}{q^2}=1$$

(ii) $x\cos\alpha+y\sin\alpha=p$

$$y\sin\alpha=-x\cos\alpha+p$$

$$y=-x\frac{\cos\alpha}{\sin\alpha}+\frac{p}{\sin\alpha}$$

Comparing with $y=mx+c$

$$m=-\frac{\cos\alpha}{\sin\alpha},c=\frac{p}{\sin\alpha}$$

$$\left(\frac{p}{\sin\alpha}\right)^2=a^2\left(-\frac{\cos\alpha}{\sin\alpha}\right)^2+b^2$$

$$\frac{p^2}{\sin^2\alpha}=a^2\left(\frac{\cos^2\alpha}{\sin^2\alpha}\right)+b^2$$

$$p^2 = \frac{a^2\cos^2\alpha}{\sin^2\alpha}(\sin^2\alpha) + b^2\sin^2\alpha$$

$$p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$$

(iii) $lx + my + n = 0$

$$my = -lx - n$$

$$y = -\frac{l}{m}x - \frac{n}{m}$$

Comparing with $y = Mx + C$

$$M = -\frac{l}{m}, C = -\frac{n}{m}$$

$$C^2 = a^2M^2 + b^2$$

$$\left(-\frac{n}{m}\right)^2 = a^2\left(-\frac{l}{m}\right)^2 + b^2$$

$$\frac{n^2}{m^2} = a^2\left(\frac{l^2}{m^2}\right) + b^2$$

$$n^2 = \frac{a^2l^2}{m^2}(m^2) + m^2b^2$$

$$n^2 = a^2l^2 + m^2b^2$$

Q.5 Find the equation of tangent to $\frac{x^2}{5} + \frac{y^2}{4} = 1$ with slope 3.

Solution:

$$m = 3$$

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 5$ and $b^2 = 4$

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$y = 3x \pm \sqrt{(5)(3)^2 + 4}$$

$$y = 3x \pm \sqrt{49}$$

$$y = 3x \pm 7$$

Q.6 Find the equation of tangent and normal to

(i) $9x^2 + 2$

(ii) $49x^2 +$

Solution:

(i) $9x^2 + 2$

÷ by 225

$\frac{x^2}{25} + \frac{y^2}{9} =$

Comparing

$\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

$a^2 = 25$ ar

$(x_1, y_1) =$

Equation

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2}$

$\frac{x(3)}{25} +$

$\frac{3x}{25} + \frac{12y}{5(9)}$

$\frac{3x}{25} + \frac{4y}{15} =$

Equation of

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1}$

$\frac{25x}{3} - \frac{9y}{\frac{12}{5}} =$

$\frac{25x}{3} - \frac{15y}{4}$

× by 12

$100x - 45y$

OR

Differentiate

(i) $9x^2 + 25y^2 = 225$ at $\left(3, \frac{12}{5}\right)$

(ii) $49x^2 + 64y^2 = 64 \times 49$ at $(8\cos\alpha, 7\sin\alpha)$

Solution:

(i) $9x^2 + 25y^2 = 225$

÷ by 225

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 25$ and $b^2 = 9$

$$(x_1, y_1) = \left(3, \frac{12}{5}\right)$$

Equation of tangent at (x_1, y_1)

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\frac{x(3)}{25} + \frac{y\left(\frac{12}{5}\right)}{9} = 1$$

$$\frac{3x}{25} + \frac{12y}{5(9)} = 1$$

$$\frac{3x}{25} + \frac{4y}{15} = 1$$

Equation of normal at (x_1, y_1)

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

$$\frac{25x}{3} - \frac{9y}{\frac{12}{5}} = 25 - 9$$

$$\frac{25x}{3} - \frac{15y}{4} = 16$$

× by 12

$$100x - 45y = 192$$

OR

Differentiate w.r.t "x"

$$\frac{d}{dx}(9x^2+25y^2)=\frac{d}{dx}(225)$$

$$18x+50yy'=0$$

$$50yy'=-18x$$

$$y'=-\frac{18x}{50y}$$

$$y'=-\frac{9x}{25y}$$

$$m\ at\ \left(3,\frac{12}{5}\right)=-\frac{9(3)}{25\left(\frac{12}{5}\right)}$$

$$m=-\frac{9(3)}{25\left(\frac{12}{5}\right)}=-\frac{9}{20}$$

$$(x_1,y_1)=\left(3,\frac{12}{5}\right)$$

Equation of tangent:

$$y-y_1=m(x-x_1)$$

$$y-\frac{12}{5}=-\frac{9}{20}(x-3)$$

$$\frac{20(5y-12)}{5}=-9(x-3)$$

$$4(5y-12)=-9x+27$$

$$9x+20y=48+27$$

$$9x+20y=75$$

$$\frac{3x}{25}+\frac{4y}{15}=1$$

Equation of normal:

$$y-y_1=-\frac{1}{m}(x-x_1)$$

$$y-\frac{12}{5}=\frac{20}{9}(x-3)$$

$$\frac{5y-12}{5}=\frac{20}{9}(x-3)$$

$$9(5y-12)=5(20)(x-3)$$

$$45y-108=100x-300$$

$$300-108=100x-45y$$

$$100x-45y=192$$

(ii) $49x^2$

÷ by 64

$\frac{x^2}{64}+\frac{y^2}{49}$

Compari

$\frac{x^2}{a^2}+\frac{y^2}{b^2}$

$a^2=64$

$(x_1,y_1)=$

Equatio

$\frac{xx_1}{a^2}+\frac{yy}{b^2}$

$\frac{x(8\cos\alpha}{64}$

$\frac{x\cos\alpha}{8}+$

× by 56

$7x\cos\alpha+$

Equation

$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}$

$\frac{64x}{8\cos\alpha}-\frac{}{7}$

$\frac{8x}{\cos\alpha}-\frac{7}{\sin}$

× by sin α c

$8x\sin\alpha-7$

Q.7 Find the

rectum with

Solution:

$\frac{x^2}{3}+\frac{y^2}{2}=1$

Comparing w

(ii) $49x^2 + 64y^2 = 64 \times 49$

÷ by 64×49

$$\frac{x^2}{64} + \frac{y^2}{49} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 64$ and $b^2 = 49$

$(x_1, y_1) = (8\cos\alpha, 7\sin\alpha)$

Equation of tangent at (x_1, y_1)

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\frac{x(8\cos\alpha)}{64} + \frac{y(7\sin\alpha)}{49} = 1$$

$$\frac{x\cos\alpha}{8} + \frac{y\sin\alpha}{7} = 1$$

× by 56

$7x\cos\alpha + 8y\sin\alpha = 56$

Equation of normal at (x_1, y_1)

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

$$\frac{64x}{8\cos\alpha} - \frac{49y}{7\sin\alpha} = 64 - 49$$

$$\frac{8x}{\cos\alpha} - \frac{7y}{\sin\alpha} = 15$$

× by $\sin\alpha\cos\alpha$

$8x\sin\alpha - 7y\cos\alpha = 15\sin\alpha\cos\alpha$

Q.7 Find the equation of tangent and normal at the end of the latus rectum with positive abscissa of the ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$.

Solution:

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

Comparing with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$a^2=3\Rightarrow a=\sqrt{3}$$

$$b^2=2\Rightarrow b=\sqrt{2}$$

$$c^2=a^2-b^2$$

$$c=\sqrt{3-2}=1$$

$$e=\frac{c}{a}=\frac{1}{\sqrt{3}}$$

End points of latus rectum are

$$\left(c,\frac{b^2}{a}\right)=\left(1,\frac{2}{\sqrt{3}}\right)\text{ and }\left(c,-\frac{b^2}{a}\right)=\left(1,-\frac{2}{\sqrt{3}}\right)$$

Equation of tangent at (x_1, y_1)

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

At $\left(1,\frac{2}{\sqrt{3}}\right)$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x(1)}{3}+\frac{y\left(\frac{2}{\sqrt{3}}\right)}{2}=1$$

$$\frac{x}{3}+\frac{2y}{2\sqrt{3}}=1$$

$$\frac{x}{3}+\frac{y}{\sqrt{3}}=1$$

$$\frac{x}{3}+\frac{y}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=1$$

$$\frac{x}{3}+\frac{\sqrt{3}y}{3}=1$$

$$x+\sqrt{3}y=3$$

At $\left(1,-\frac{2}{\sqrt{3}}\right)$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x(1)}{3}+\frac{y\left(-\frac{2}{\sqrt{3}}\right)}{2}$$

$$\frac{x}{3}-\frac{2y}{2\sqrt{3}}=1$$

$$\frac{x}{3}-\frac{y}{\sqrt{3}}=1$$

$$\frac{x}{3}-\frac{y}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=$$

$$\frac{x}{3}-\frac{\sqrt{3}y}{3}=1$$

$$x-\sqrt{3}y=3$$

Equation of no

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2$$

At $\left(1,\frac{2}{\sqrt{3}}\right)$

$$\frac{3x}{1}-\frac{2y}{\frac{2}{\sqrt{3}}}=3-$$

$$3x-\sqrt{3}y=1$$

At $\left(1,-\frac{2}{\sqrt{3}}\right)$

$$\frac{3x}{1}-\frac{2y}{-\frac{2}{\sqrt{3}}}=3-$$

$$3x+\sqrt{3}y=1$$

Q.8 Find the equati

where abscissa is 1.

Solution:

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x(1)}{3}+\frac{y\left(-\frac{2}{\sqrt{3}}\right)}{2}=1$$

$$\frac{x}{3}-\frac{2y}{2\sqrt{3}}=1$$

$$\frac{x}{3}-\frac{y}{\sqrt{3}}=1$$

$$\frac{x}{3}-\frac{y}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=1$$

$$\frac{x}{3}-\frac{\sqrt{3}y}{3}=1$$

$$x-\sqrt{3}y=3$$

Equation of normal at (x_1, y_1)

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

At $\left(1,\frac{2}{\sqrt{3}}\right)$

$$\frac{3x}{1}-\frac{2y}{\frac{2}{\sqrt{3}}}=3-2$$

$$3x-\sqrt{3}y=1$$

At $\left(1,-\frac{2}{\sqrt{3}}\right)$

$$\frac{3x}{1}-\frac{2y}{-\frac{2}{\sqrt{3}}}=3-2$$

$$3x+\sqrt{3}y=1$$

Q.8 Find the equation of tangent to the ellipse $\frac{x^2}{5}+\frac{9y^2}{20}=1$ at the points where abscissa is 1.

Solution:

$$\frac{x^2}{5}+\frac{9y^2}{20}=1 \rightarrow (1)$$

$$\frac{x^2}{5}+\frac{y^2}{\frac{20}{9}}=1$$

Comparing with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$a^2=5$$

$$b^2=\frac{20}{9}$$

$$x=1$$

$$(1)\Rightarrow \frac{(1)^2}{5}+\frac{9y^2}{20}=1$$

$$\frac{9y^2}{20}=1-\frac{1}{5}$$

$$\frac{9y^2}{20}=\frac{4}{5}$$

$$y^2=\frac{4}{5}\left(\frac{20}{9}\right)$$

$$y^2=\frac{16}{9}$$

$$y=\pm\frac{4}{3}$$

Points are $\left(1,\frac{4}{3}\right)$ and $\left(1,-\frac{4}{3}\right)$

Equation of tangent at (x_1,y_1)

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

At $\left(1,\frac{4}{3}\right)$

$$\frac{x(1)}{5}+\frac{y\left(\frac{4}{3}\right)}{\frac{20}{9}}=1$$

$$\frac{x}{5}+\frac{4y(9)}{3(20)}=1$$

$$\frac{x}{5}+\frac{3y}{5}=1$$

× by 5

$$x+3y=5$$

At $\left(1,-\frac{4}{3}\right)$

$$\frac{x(1)}{5}+\frac{y\left(-\frac{4}{3}\right)}{\frac{20}{9}}$$

$$\frac{x}{5}-\frac{4y(9)}{3(20)}=1$$

$$\frac{x}{5}-\frac{3y}{5}=1$$

× by 5

$$x-3y=5$$

Q.1 Find the equat
following conditio
(i) transverse and c
transverse axis is a
(ii) Hyperbola pass
along x-axis
(iii) transverse axis
$\sqrt{5}$.
(iv) transverse axis
eccentricity= $\frac{3}{2}$
(v) focus (5,0) and c
(vi) Eccentricity= 3
(vii) Eccentricity= 2

Solution:

(i) transverse and co
transverse axis is alo

$$\frac{x}{5}+\frac{3y}{5}=1$$

× by 5

$$x+3y=5$$

At $\left(1,-\frac{4}{3}\right)$

$$\frac{x(1)}{5}+\frac{y\left(-\frac{4}{3}\right)}{\frac{20}{9}}=1$$

$$\frac{x}{5}-\frac{4y(9)}{3(20)}=1$$

$$\frac{x}{5}-\frac{3y}{5}=1$$

× by 5

$$x-3y=5$$

EXERCISE 9.5

Q.1 Find the equation of hyperbola with centre at the origin satisfying the following conditions.

(i) transverse and conjugate axes are 16 and 12 respectively. Also, transverse axis is along y-axis

(ii) Hyperbola passes through $\left(\frac{3\sqrt{17}}{4},1\right)$ and $(3,0)$ with transverse axis along x-axis

(iii) transverse axis of length 8 units and along y-axis where eccentricity is $\sqrt{5}$.

(iv) transverse axis along x-axis with latus rectum $= 10$ units and eccentricity$=\frac{3}{2}$

(v) focus $(5,0)$ and directrix $x=2$

(vi) Eccentricity$=3$ and focus $=(8,0)$

(vii) Eccentricity$=2$ and vertex $=(0,4)$

Solution:

(i) transverse and conjugate axes are 16 and 12 respectively. Also, transverse axis is along y-axis

Length of transverse axis $= 2a = 16$

$a = 8$

Length of conjugate axis $= 2b = 12$

$b = 6$

Major axis is along y-axis

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{8^2} - \frac{x^2}{6^2} = 1$$

$$\frac{y^2}{64} - \frac{x^2}{36} = 1$$

(ii) Hyperbola passes through $\left(\frac{3\sqrt{17}}{4}, 1\right)$ and $(3,0)$ with transverse axis along x-axis

Major axis is along x-axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \rightarrow (1)$$

$$\left(\frac{3\sqrt{17}}{4}, 1\right)$$

$(3,0)$ is on (1)

$$(1) \Rightarrow \frac{(3)^2}{a^2} - \frac{0^2}{b^2} = 1$$

$$\frac{9}{a^2} - \frac{0}{b^2} = 1$$

$$\frac{9}{a^2} - 0 = 1$$

$$\frac{9}{a^2} = 1$$

$$a^2 = 9$$

$\left(\frac{3\sqrt{17}}{4}, 1\right)$ is on (1) and $a^2 = 9$

$$(1) \Rightarrow \frac{\left(\frac{3\sqrt{17}}{4}\right)^2}{9} - \frac{1^2}{b^2} = 1$$

The Studen

$$\frac{\frac{9(17)}{16}}{9} - \frac{1}{b^2}$$

$$\frac{9(17)}{16(9)} - \frac{1}{b^2}$$

$$\frac{17}{16} - \frac{1}{b^2} =$$

$$\frac{1}{b^2} = \frac{17}{16} -$$

$$\frac{1}{b^2} = \frac{1}{16}$$

$$b^2 = 16$$

$$(1) \Rightarrow \frac{x^2}{9} -$$

(iii) transver

$\sqrt{5}$.

Length of tr

$a = 4$

$e = \sqrt{5}$

$\because b^2 = a^2($

$b^2 = 4^2\left(\left(\sqrt{}\right.\right.$

$b^2 = 16(5 -$

$b^2 = 64$

Major axis is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{16} - \frac{x^2}{64} = 1$$

(iv) transverse

eccentricity=

Length of latus

$$\frac{\frac{\;}{16}}{9} - \frac{1}{b^2} = 1$$

$$\frac{9(17)}{16(9)} - \frac{1}{b^2} = 1$$

$$\frac{17}{16} - \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = \frac{17}{16} - 1$$

$$\frac{1}{b^2} = \frac{1}{16}$$

$$b^2 = 16$$

rse axis

$$(1) \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

(iii) transverse axis of length 8 units and along y-axis where eccentricity is $\sqrt{5}$.

Length of transverse axis $= 2a = 8$

$a = 4$

$e = \sqrt{5}$

$$\because b^2 = a^2(e^2 - 1)$$

$$b^2 = 4^2\left(\left(\sqrt{5}\right)^2 - 1\right)$$

$$b^2 = 16(5 - 1)$$

$$b^2 = 64$$

Major axis is along y-axis

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{16} - \frac{x^2}{64} = 1$$

(iv) transverse axis along x-axis with latus rectum $= 10$ units and eccentricity$= \frac{3}{2}$

Length of latus rectum $= \frac{2b^2}{a} = 10$

$\frac{b^2}{a} = 5$

$b^2 = 5a \to (1)$

$e = \frac{3}{2}$

$\because b^2 = a^2(e^2 - 1)$

$5a = a^2\left(\left(\frac{3}{2}\right)^2 - 1\right)$

$5 = a\left(\frac{9}{4} - 1\right)$

$5 = a\left(\frac{5}{4}\right)$

$a = 4$

$a^2 = 16$

$(1) \Rightarrow b^2 = 5(4)$

$b^2 = 20$

Major axis is along x-axis

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{16} - \frac{y^2}{20} = 1$

(v) focus (5,0) and directrix $x = 2$

$x = 2$

$\frac{a}{e} = 2$

$a = 2e \to (1)$

Focus $(ae, 0) = (5,0)$

$(1) \Rightarrow ae = 5$

$(2e)e = 5$

$e^2 = \frac{5}{2}$

$(1) \Rightarrow a^2 = 4e^2$

$a^2 = 4\left(\frac{5}{2}\right.$

$a^2 = 10$

$\because b^2 = a^2$

$b^2 = 10\left(\right.$

$b^2 = 10\left(\right.$

$b^2 = 15$

Transvers

$\frac{x^2}{a^2} - \frac{y^2}{b^2} =$

$\frac{x^2}{10} - \frac{y^2}{15} =$

(vi) Eccent

$e = 3$

Focus $(ae,$

$ae = 8$

$a(3) = 8$

$a = \frac{8}{3}$

$\because b^2 = a^2($

$b^2 = \left(\frac{8}{3}\right)^2$

$b^2 = \frac{64}{9}(9$

$b^2 = \frac{64}{9}(8$

$b^2 = \frac{512}{9}$

Transverse a

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a^2 = 4\left(\frac{5}{2}\right)$

$a^2 = 10$

$\because b^2 = a^2(e^2 - 1)$

$b^2 = 10\left(\frac{5}{2} - 1\right)$

$b^2 = 10\left(\frac{3}{2}\right)$

$b^2 = 15$

Transverse axis is along x-axis

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{10} - \frac{y^2}{15} = 1$

(vi) Eccentricity= 3 and focus = (8,0)

$e = 3$

Focus $(ae, 0) = (8,0)$

$ae = 8$

$a(3) = 8$

$a = \frac{8}{3}$

$\because b^2 = a^2(e^2 - 1)$

$b^2 = \left(\frac{8}{3}\right)^2 (3^2 - 1)$

$b^2 = \frac{64}{9}(9 - 1)$

$b^2 = \frac{64}{9}(8)$

$b^2 = \frac{512}{9}$

Transverse axis is along x-axis

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{\frac{64}{9}}-\frac{y^2}{\frac{512}{9}}=1$$

$$\frac{9x^2}{64}-\frac{9y^2}{512}=1$$

(vii) Eccentricity$=2$ and vertex $=(0,4)$

$e=2$

Vertex $(0,a)=(0,4)$

$a=4$

$\because b^2=a^2(e^2-1)$

$b^2=4^2(2^2-1)$

$b^2=16(3)$

$b^2=48$

Transverse axis is along y-axis

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

$$\frac{y^2}{4^2}-\frac{x^2}{48}=1$$

$$\frac{y^2}{16}-\frac{x^2}{48}=1$$

Q.2 Find equation of the hyperbola with centre $(1,3)$ and satisfying the following condition

(i) focus is $(2,3)$ and eccentricity is $\sqrt{3}$, whereas transverse axis is parallel to x-axis

(ii) focus is $(1,5)$ and an equation of directrix is $y=2$ where transverse axis is parallel to y-axis

---

Solution:

(i) focus is $(2,3)$ and eccentricity is $\sqrt{3}$, whereas transverse axis is parallel to x-axis

Center $(h,k)=(1,3)$ and $e=\sqrt{3}$

Focus $(c+h,k)=(2,3)$

$c+h=2$

$c+1=2$

$c=2-1$

$ae=1$

$a=\frac{1}{e}=\frac{1}{\sqrt{}}$

$b^2=a^2(e$

$b^2=a^2e^2$

$b^2=1^2-$

$b^2=1-\frac{1}{3}$

$b^2=\frac{2}{3}$

Transverse

$$\frac{(x-h)^2}{a^2}-$$

$$\frac{(x-1)^2}{\frac{1}{3}}-$$

$3(x-1)^2$

× by 2

$6(x-1)^2$

(ii) focus is (

axis is paral

Center $(h,k$

$h=1,k=$

$y=2$

$k-\frac{a}{e}=2$

$3-\frac{a}{e}=2$

$3-2=\frac{a}{e}$

$\frac{a}{e}=1$

$c = 2 - 1 = 1$

$ae = 1$

$a = \frac{1}{e} = \frac{1}{\sqrt{3}}$

$b^2 = a^2(e^2 - 1)$

$b^2 = a^2e^2 - a^2$

$b^2 = 1^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$b^2 = 1 - \frac{1}{3}$

$b^2 = \frac{2}{3}$

Transverse axis is parallel to x-axis

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-1)^2}{\frac{1}{3}} - \frac{(y-3)^2}{\frac{2}{3}} = 1$

$3(x-1)^2 - \frac{3}{2}(y-3)^2 = 1$

× by 2

$6(x-1)^2 - 3(y-3)^2 = 2$

(ii) focus is $(1,5)$ and an equation of directrix is $y = 2$ where transverse axis is parallel to y-axis

Center $(h,k) = (1,3)$

$h = 1, k = 3$

$y = 2$

$k - \frac{a}{e} = 2$

$3 - \frac{a}{e} = 2$

$3 - 2 = \frac{a}{e}$

$\frac{a}{e} = 1$

isfying the

xis is parallel

transverse

xis is parallel

$a = e \to (1)$

Focus $(h, c+k) = (1,5)$

$c + k = 5$

$c = 5 - 3$

$c = 2$

$ae = 2$

$(e)e = 2$

$e^2 = 2$

$(1) \Rightarrow a = e$

$a^2 = e^2$

$a^2 = 2$

$c^2 = a^2 + b^2$

$4 = 2 + b^2$

$b^2 = 2$

Transverse axis is parallel to y-axis

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-3)^2}{2} - \frac{(x-1)^2}{2} = 1$$

Q.3 Find eccentricity, foci, vertices and latus rectum of each of the following. Also, draw graph.

(i) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (ii) $\frac{y^2}{5} - \frac{x^2}{4} = 1$ (iii) $9x^2 - y^2 + 1 = 0$

(iv) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Solution:

(i) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Comparing with

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a^2 = 9 \Rightarrow a =$

$b^2 = 16 \Rightarrow b =$

Centre = $(0,0)$

$c^2 = a^2 + b^2$

$c = \sqrt{9+16} =$

$e = \frac{c}{a} = \frac{5}{3}$

Transverse axis

Vertices $(\pm a, 0$

Foci $(\pm c, 0) =$

Length of latus

Equation of dire

$x = \pm \frac{a}{e}$

$x = \pm \frac{3}{\frac{5}{3}}$

$x = \pm \frac{9}{5}$

$a^2 = 9 \Rightarrow a = 3$

$b^2 = 16 \Rightarrow b = 4$

Centre = $(0,0)$

$c^2 = a^2 + b^2$

$c = \sqrt{9 + 16} = 5$

$e = \frac{c}{a} = \frac{5}{3}$

Transverse axis is along x-axis

Vertices $(\pm a, 0) = (\pm 3,0)$

Foci $(\pm c, 0) = (\pm 5,0)$

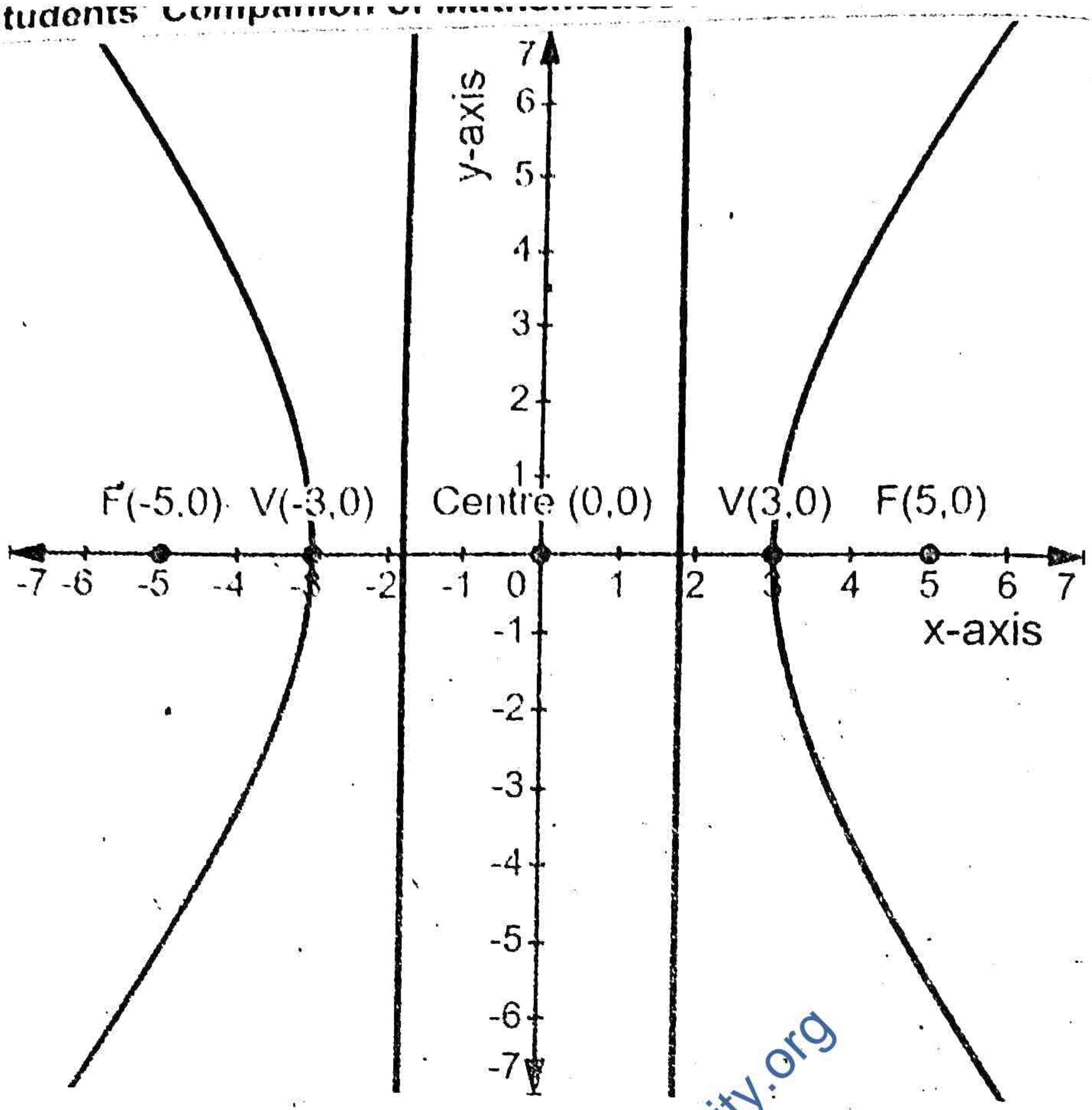
Length of latus rectum: $\frac{2b^2}{a} = \frac{2(16)}{3} = \frac{32}{3}$

Equation of directrices:

$x = \pm\frac{a}{e}$

$x = \pm\frac{3}{\frac{5}{3}}$

$x = \pm\frac{9}{5}$



(ii) $\frac{y^2}{5} - \frac{x^2}{4} = 1$

Comparing with

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$a^2 = 5 \Rightarrow a = \sqrt{5}$

$b^2 = 4 \Rightarrow b = 2$

Centre $= (0,0)$

$c^2 = a^2 + b^2$

$c = \sqrt{5+4} = 3$

$e = \frac{c}{a} = \frac{3}{\sqrt{5}}$

Transverse axis is along y-axis

Vertices $(0, \pm a) = (0, \pm\sqrt{5})$

Foci $(0, \pm c) = (0, \pm 3)$

Length of latus rectum: $\frac{2b^2}{a} = \frac{2(4)}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

Equatio

$y = \pm\frac{a}{e}$

$y = \pm\frac{\sqrt{5}}{\frac{3}{\sqrt{5}}}$

$y = \pm\frac{5}{3}$

(iii) $9x^2 -$

$1 = y^2 -$

$\frac{y^2}{1} - \frac{x^2}{\frac{1}{9}} =$

Comparing

$\frac{y^2}{a^2} - \frac{x^2}{b^2} =$

$a^2 = 1 \Rightarrow$

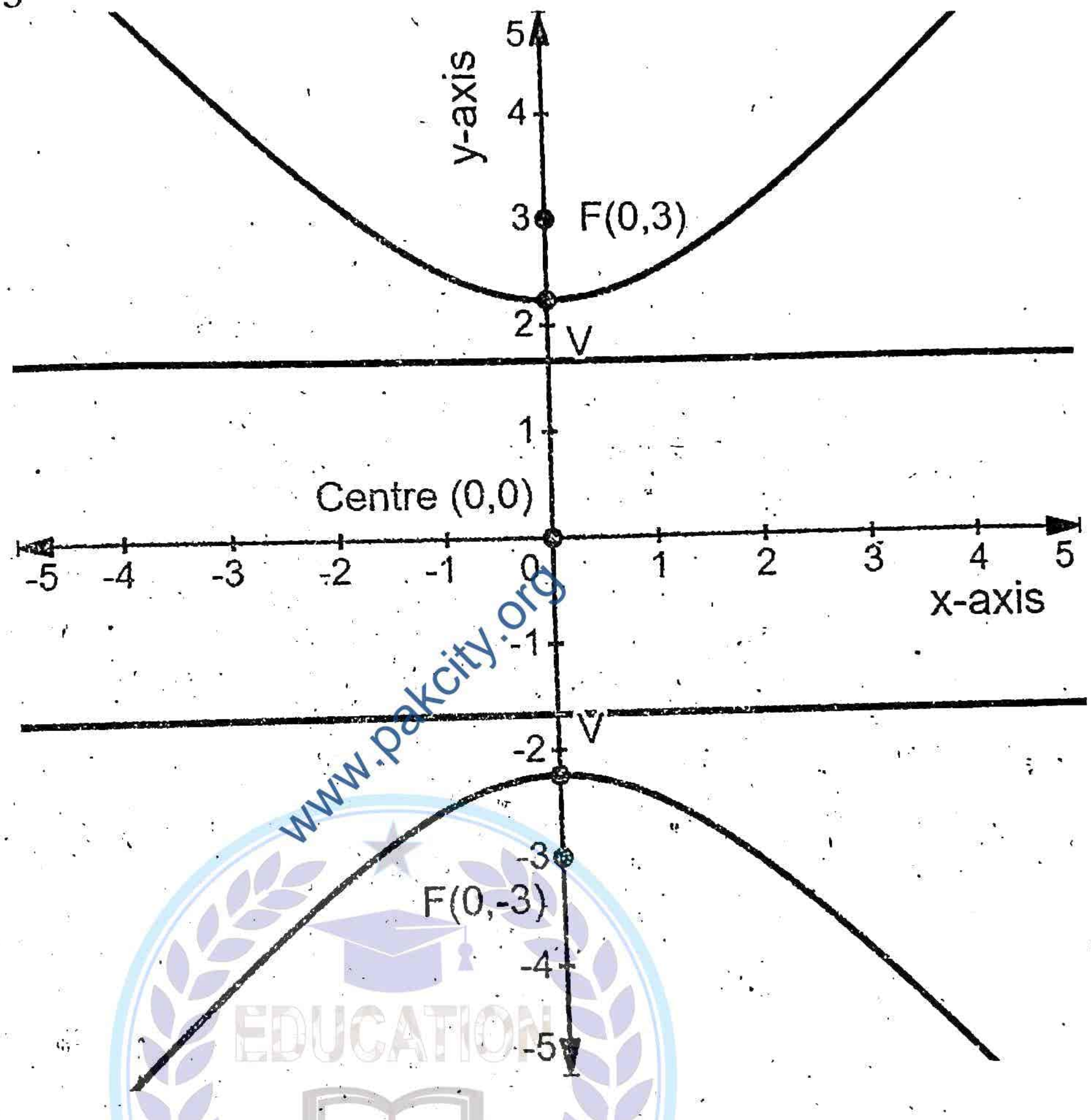
$b^2 = \frac{1}{9} \Rightarrow$

Equation of directrices:

$$y=\pm\frac{a}{e}$$

$$y=\pm\frac{\frac{\sqrt{5}}{3}}{\sqrt{5}}$$

$$y=\pm\frac{5}{3}$$



(iii) $9x^2-y^2+1=0$

$1=y^2-9x^2$

$$\frac{y^2}{1}-\frac{x^2}{\frac{1}{9}}=1$$

Comparing with

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

$a^2=1\Rightarrow a=1$

$b^2=\frac{1}{9}\Rightarrow b=\frac{1}{3}$

Centre $= (0,0)$

$$c^2 = a^2 + b^2$$

$$c = \sqrt{1 + \frac{1}{9}} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$$

$$e = \frac{c}{a} = \frac{\sqrt{10}}{3} \div 1 = \frac{\sqrt{10}}{3}$$

Transverse axis is along y-axis

Vertices $(0, \pm a) = (0, \pm 1)$

Foci $(0, \pm c) = \left(0, \pm \frac{\sqrt{10}}{3}\right)$

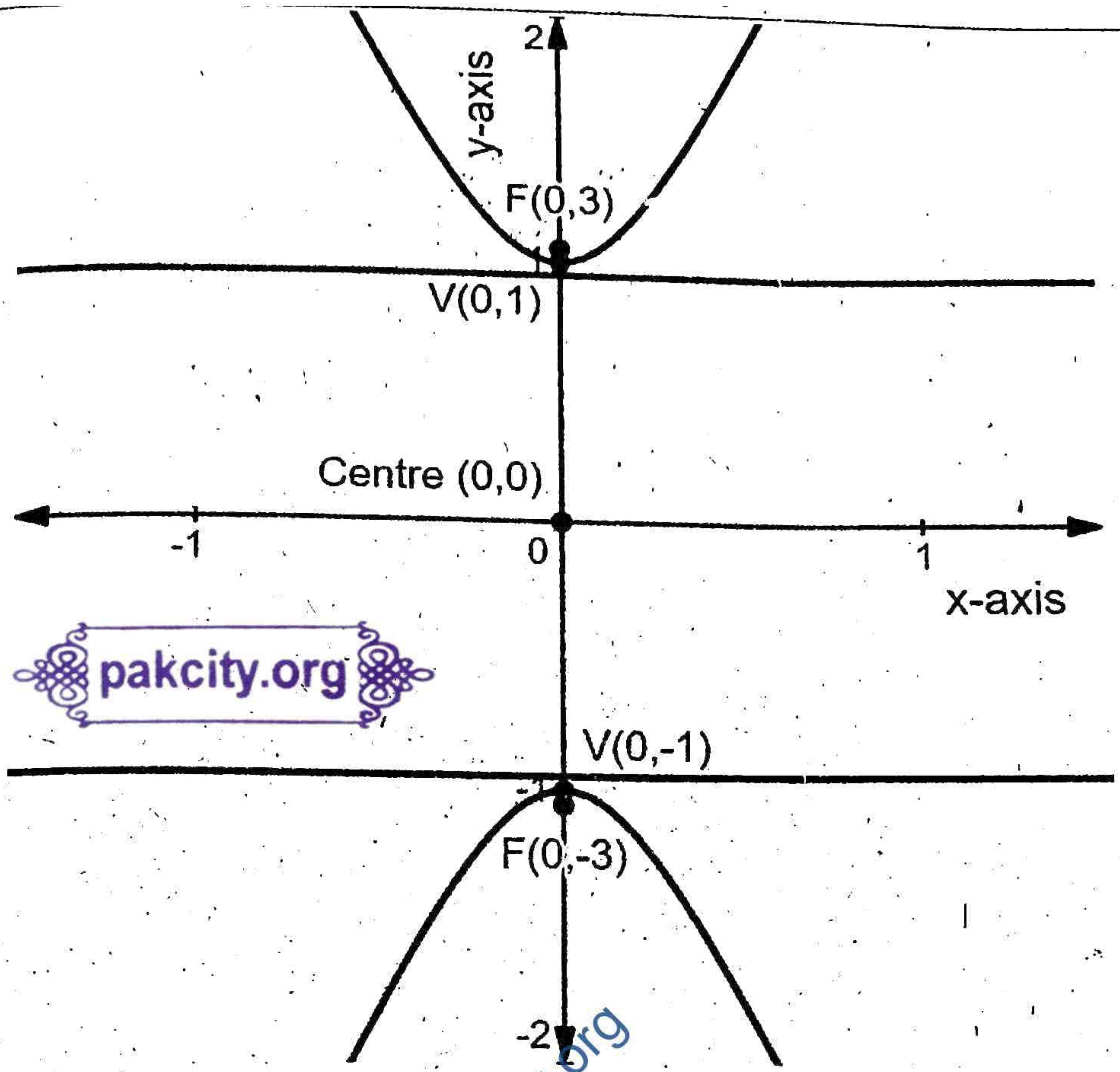
Length of latus rectum: $\frac{2b^2}{a} = \frac{2\left(\frac{1}{9}\right)}{1} = \frac{2}{9}$

Equation of directrices:

$$y = \pm \frac{a}{e}$$

$$y = \pm \frac{1}{\frac{\sqrt{10}}{3}}$$

$$y = \pm \frac{3}{\sqrt{10}}$$



(iv) $\frac{x^2}{4}-\frac{y^2}{9}=1$

Comparing with

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$a^2=4 \Rightarrow a=2$

$b^2=9 \Rightarrow b=3$

$c^2=a^2+b^2$

$c=\sqrt{4+9}=\sqrt{13}$

$e=\frac{c}{a}=\frac{\sqrt{13}}{2}$

Transverse axis is along x-axis

Vertices $(\pm a,0)=(\pm 2,0)$

Foci $(\pm c,0)=(\pm\sqrt{13},0)$

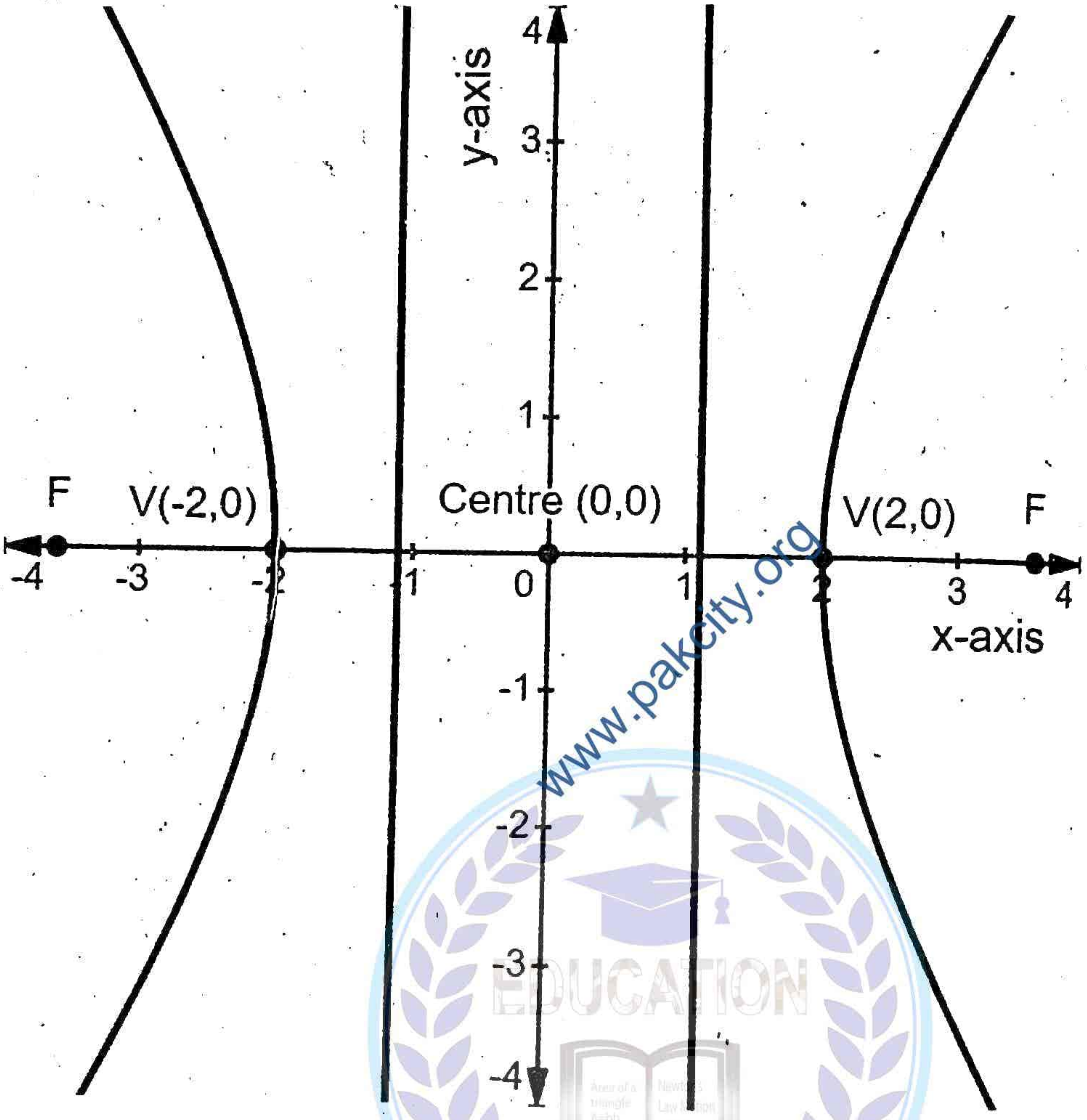
Length of latus rectum: $\frac{2b^2}{a}=\frac{2(9)}{2}=\sqrt{13}$

Equation of directrices:

$$x=\pm\frac{a}{e}$$

$$x=\pm\frac{2}{\frac{\sqrt{13}}{2}}$$

$$x=\pm\frac{4}{\sqrt{13}}$$



Q.4 Find centre, foci, eccentricity, vertices and equations of directrices. Also draw the graph

(i) $\frac{(x-5)^2}{9}-\frac{(y+3)^2}{16}=1$ (ii) $\frac{(y-4)^2}{36}-\frac{(x+5)^2}{64}=1$

(iii) $9x^2-4y^2+36x+8y-4=0$

(iv) $25x^2-150x-9y^2+72y+306=0$

Solution:

(i) $\frac{(x-5)^2}{9}-\frac{(y+3)^2}{16}=1$

$\frac{(x-5)^2}{9}-$

Comparing w

$\frac{(x-h)^2}{a^2}-$

$a^2=9\Rightarrow a$

$b^2=16\Rightarrow b$

Centre (h,k)

$c^2=a^2+b^2$

$c=\sqrt{9+16}$

$e=\frac{c}{a}=\frac{5}{3}$

Transverse axi

Vertices $(h\pm$

$=(8,-3)$ and

Foci $(h\pm c,k)$

$=(10,-3)$ an

Equation of di

$x=h\pm\frac{a}{e}$

$x=5\pm\frac{3}{\frac{5}{3}}$

$x=5\pm\frac{9}{5}$

$x=5+\frac{9}{5},x=$

$x=\frac{34}{5},x=\frac{16}{5}$

$$\frac{(x-5)^2}{9}-\frac{\{y-(-3)\}^2}{16}=1$$

Comparing with

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$a^2=9 \Rightarrow a=3$

$b^2=16 \Rightarrow b=4$

Centre $(h,k)=(5,-3)$

$c^2=a^2+b^2$

$c=\sqrt{9+16}=5$

$$e=\frac{c}{a}=\frac{5}{3}$$

Transverse axis is parallel to x-axis

Vertices $(h \pm a, k)=(5 \pm 3,-3)$

$=(8,-3)$ and $(2,-3)$

Foci $(h \pm c, k)=(5 \pm 5,-3)$

$=(10,-3)$ and $(0,-3)$

Equation of directrices:

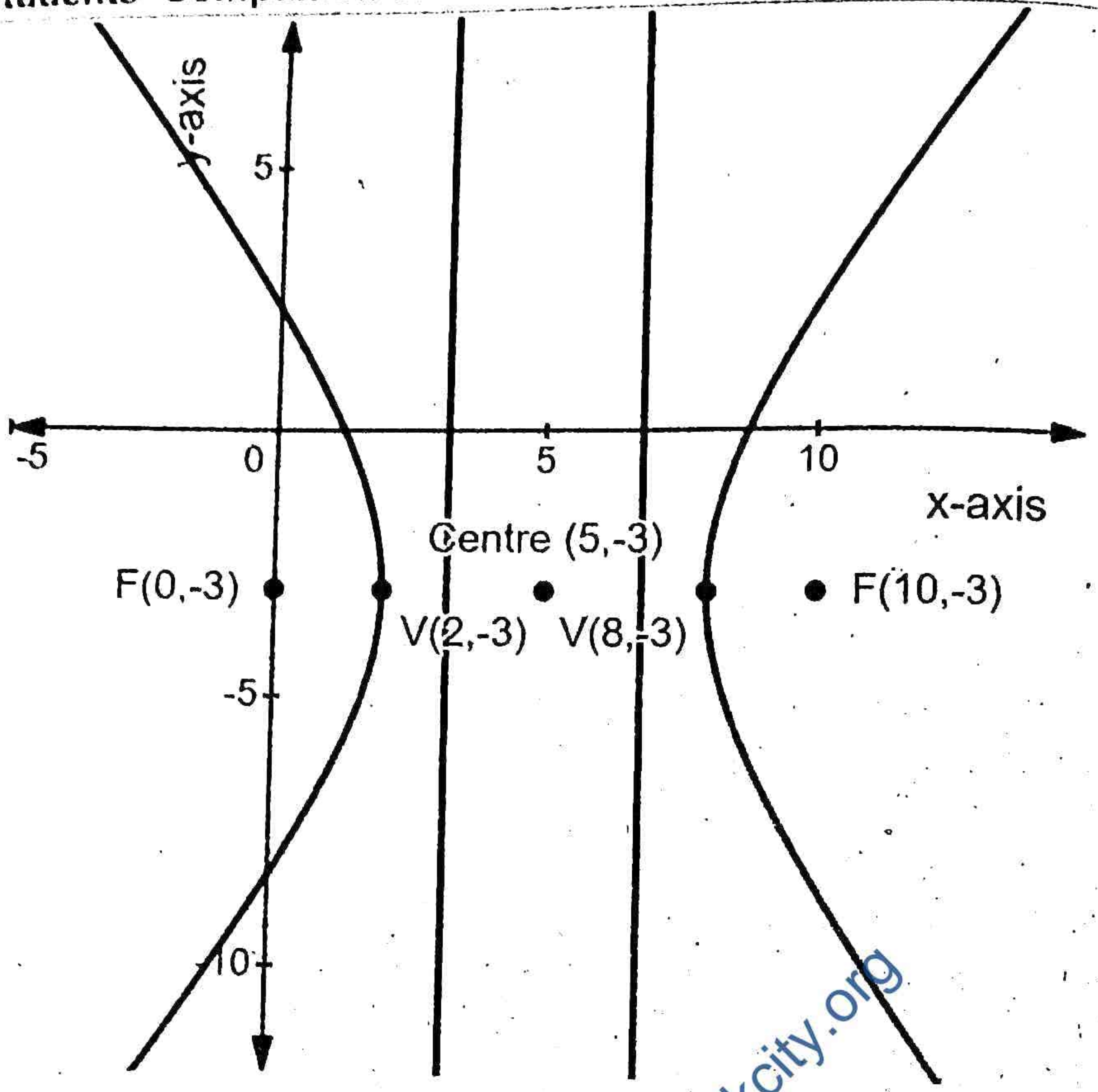
$$x=h \pm \frac{a}{e}$$

$$x=5 \pm \frac{3}{\frac{5}{3}}$$

$$x=5 \pm \frac{9}{5}$$

$$x=5+\frac{9}{5}, x=5-\frac{9}{5}$$

$$x=\frac{34}{5}, x=\frac{16}{5}$$



(ii) $\frac{(y-4)^2}{36}-\frac{(x+5)^2}{64}=1$

$$\frac{(y-4)^2}{36}-\frac{\{x-(-5)\}^2}{64}=1$$

Comparing with

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$a^2=36 \Rightarrow a=6$

$b^2=64 \Rightarrow b=8$

Centre $(h,k)=(-5,4)$

$c^2=a^2+b^2$

$c=\sqrt{36+64}=10$

$e=\frac{c}{a}=\frac{10}{6}=\frac{5}{3}$

Transverse axis is parallel to y-axis

Vertices $(h,k\pm a)=(-5,4\pm 6)$

$=(-5,10)$ and $(-$

Foci $(h,k\pm c)=($

$=(-5,14)$ and $(-$

Equation of direct

$y=k\pm\frac{a}{e}$

$y=4\pm\frac{6}{\frac{5}{3}}$

$y=4\pm\frac{18}{5}$

$y=4+\frac{18}{5}, y=$

$y=\frac{38}{5}, y=\frac{2}{5}$

(iii) $9x^2-4y^2+$

$9x^2+36x-4y^2$

$9\{x^2+4x\}-4\{y$

$9\{(x)^2+2(x)(2)$

$9(x+2)^2-4(y$

$= (-5,10)$ and $(-5,-2)$

Foci $(h, k \pm c) = (-5, 4 \pm 10)$

$= (-5,14)$ and $(-5,-6)$

Equation of directrices:

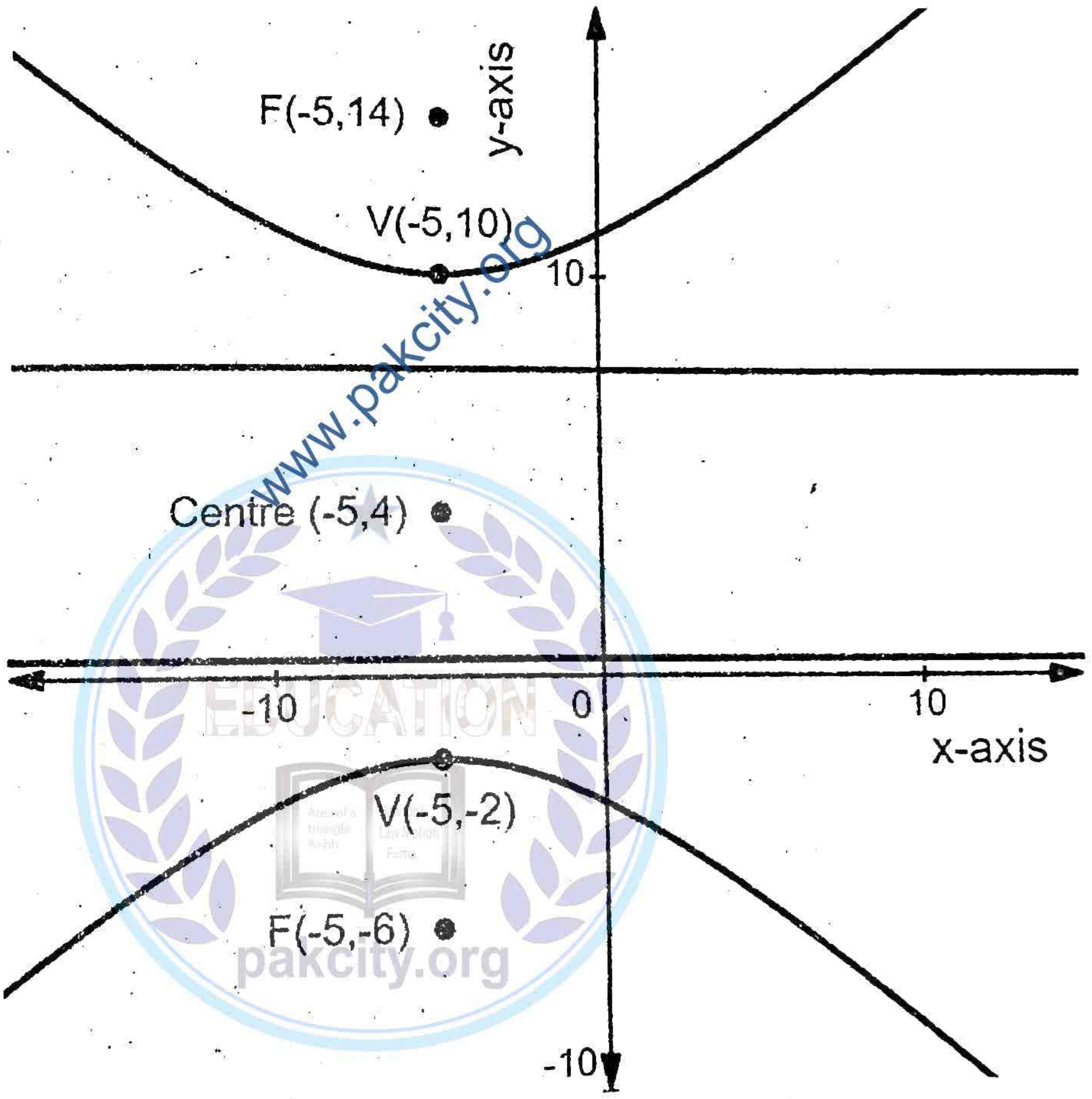
$y = k \pm \frac{a}{c}$

$y = 4 \pm \frac{6}{\frac{5}{3}}$

$y = 4 \pm \frac{18}{5}$

$y = 4 + \frac{18}{5}, y = 4 - \frac{18}{5}$

$y = \frac{38}{5}, y = \frac{2}{5}$



(iii) $9x^2 - 4y^2 + 36x + 8y - 4 = 0$

$9x^2 + 36x - 4y^2 + 8y = 4$

$9\{x^2 + 4x\} - 4\{y^2 - 2y\} = 4$

$9\{(x)^2 + 2(x)(2) + (2)^2\} - 4\{(y)^2 - 2(y)(1) + (1)^2\} = 4 + 36 - 4$

$9(x+2)^2 - 4(y-1)^2 = 36$

÷ by 36

$$\frac{9(x+2)^2}{36}-\frac{4(y-1)^2}{36}=\frac{36}{36}$$

$$\frac{(x+2)^2}{4}-\frac{(y-1)^2}{9}=1$$

$$\frac{\{x-(-2)\}^2}{4}-\frac{(y-1)^2}{9}=1$$

Comparing with

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$a^2=4 \Rightarrow a=2$

$b^2=9 \Rightarrow b=3$

Centre $(h,k)=(-2,1)$

$c^2=a^2+b^2$

$c=\sqrt{4+9}=\sqrt{13}$

$$e=\frac{c}{a}=\frac{\sqrt{13}}{2}$$

Transverse axis is parallel to x-axis

Vertices $(h \pm a, k)=(-2 \pm 2,1)$

$=(-4,1)$ and $(0,1)$

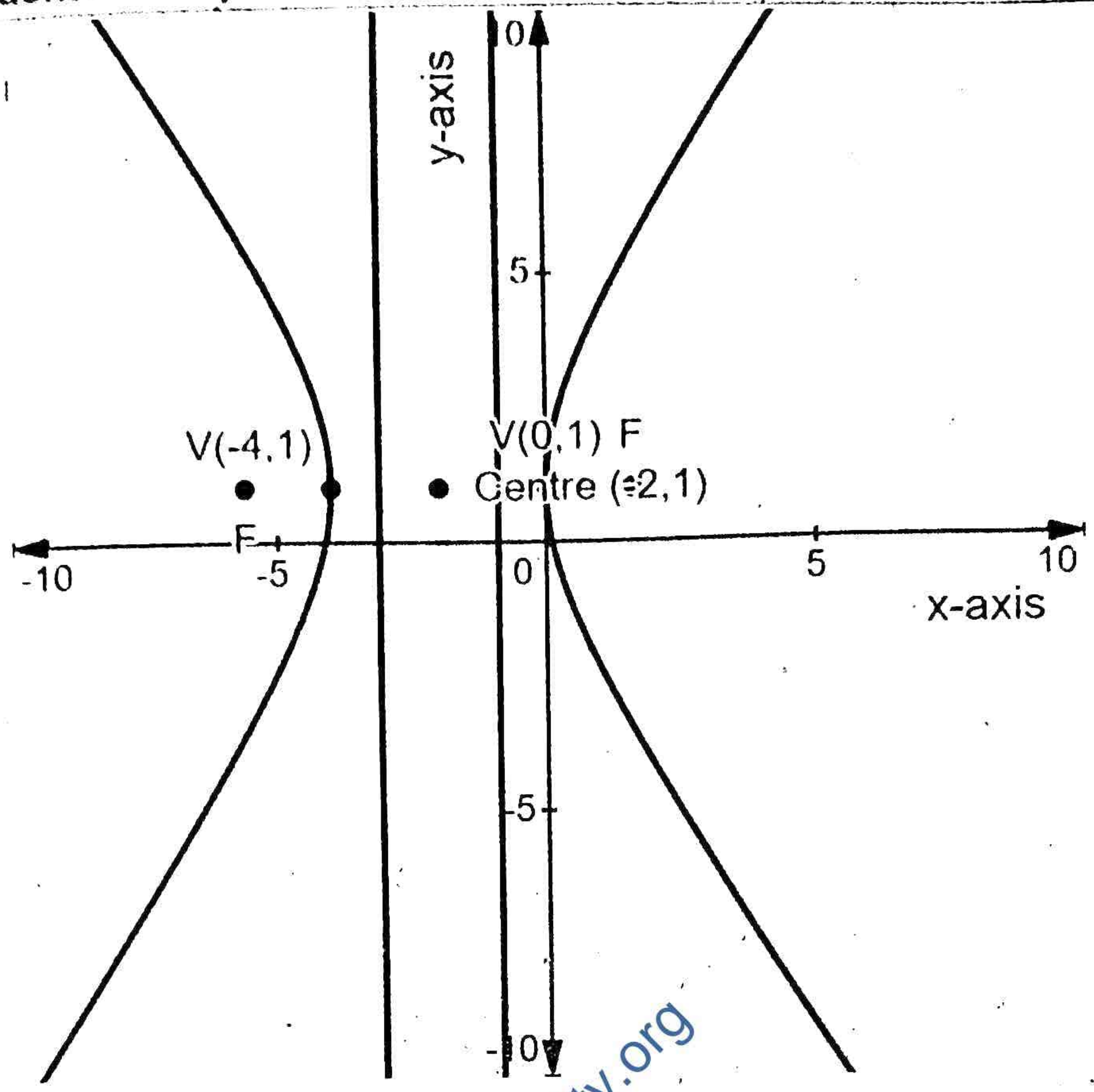
Foci $(h \pm c, k)=\left(-2 \pm \sqrt{13}, 1\right)$

Equation of directrices:

$$x=h \pm \frac{a}{e}$$

$$x=-2 \pm \frac{2}{\frac{\sqrt{13}}{2}}$$

$$x=-2 \pm \frac{4}{\sqrt{13}}$$



(iv) $25x^2 - 150x - 9y^2 + 72y + 306 = 0$

$25\{x^2 - 6x\} - 9\{y^2 - 8y\} = -306$

$25\{(x)^2 - 2(x)(3) + (3)^2\} - 9\{(y)^2 - 2(y)(4) + (4)^2\}$

$= -306 + 225 - 144$

$25(x-3)^2 - 9(y-4)^2 = -225$

× by -1

$-25(x-3)^2 + 9(y-4)^2 = 225$

$9(y-4)^2 - 25(x-3)^2 = 225$

÷ by 225

$$\frac{9(y-4)^2}{225} - \frac{25(x-3)^2}{225} = \frac{225}{225}$$

$$\frac{(y-4)^2}{25} - \frac{(x-3)^2}{9} = 1$$

Comparing with $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 9 \Rightarrow b = 3$

Centre $(h,k) = (3,4)$

$c^2 = a^2 + b^2$

$c = \sqrt{25+9} = \sqrt{34}$

$e = \frac{c}{a} = \frac{\sqrt{34}}{5}$

Transverse axis is parallel to y-axis

Vertices $(h, k \pm a) = (3,4 \pm 5)$

$= (3,9)$ and $(3,-1)$

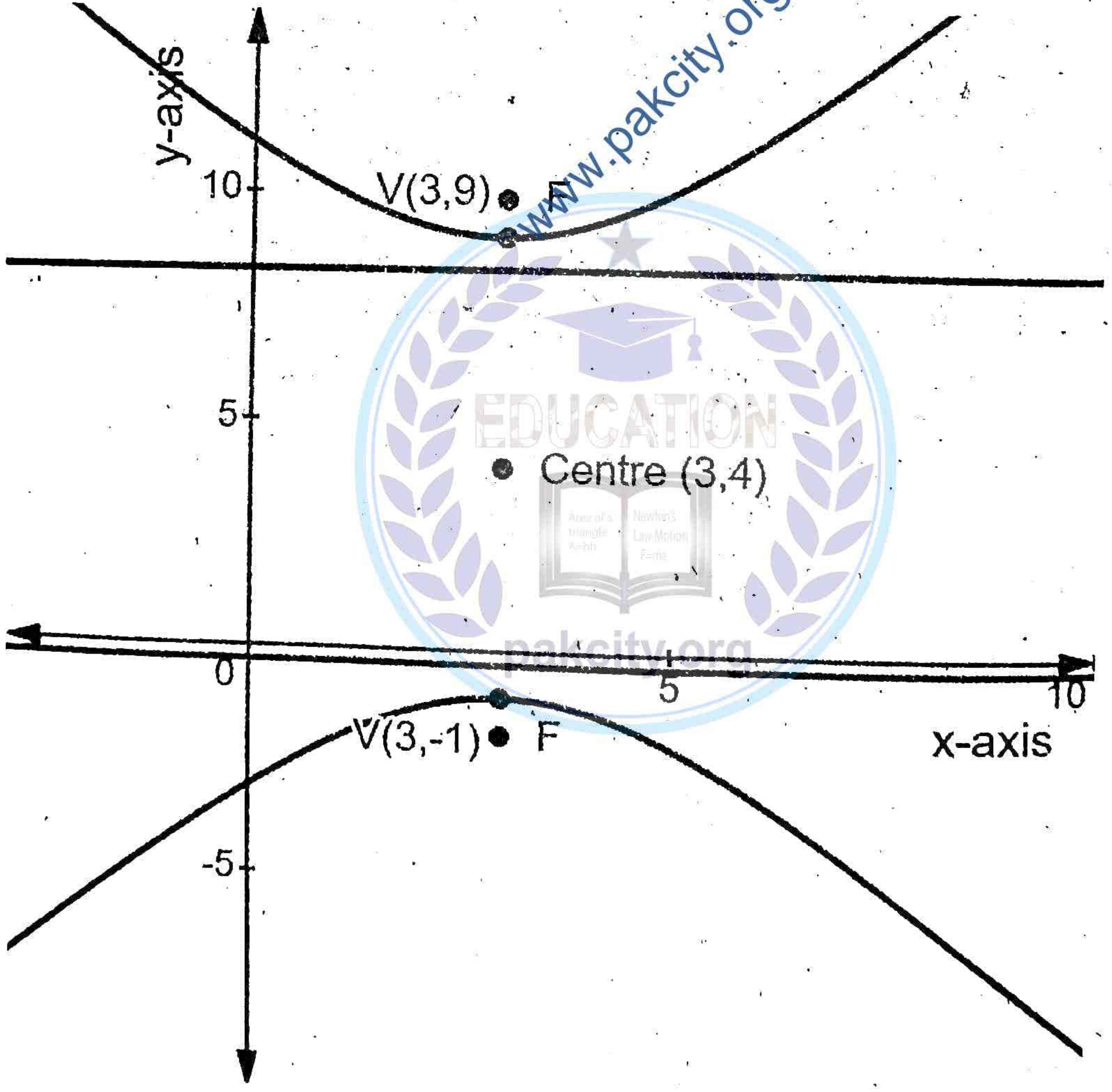
Foci $(h, k \pm c) = \left(3,4 \pm \sqrt{34}\right)$

Equation of directrices:

$y = k \pm \frac{a}{e}$

$y = 4 \pm \frac{5}{\frac{\sqrt{34}}{5}}$

$y = 4 \pm \frac{25}{\sqrt{34}}$



Q.5
vertices are
find equati

Solution:

Centre = (

vertices (±

$a = 4$

$\Rightarrow a^2 = 1$

And $b = 4$

$\Rightarrow b^2 = 1$

Transverse

$\frac{x^2}{a^2} - \frac{y^2}{b^2} =$

$\frac{x^2}{16} - \frac{y^2}{16} =$

$\frac{x^2 - y^2}{16} =$

$x^2 - y^2 =$

Conjugate

$\frac{y^2}{b^2} - \frac{x^2}{a^2} =$

$\frac{y^2}{16} - \frac{x^2}{16} =$

$\frac{y^2 - x^2}{16} =$

$y^2 - x^2 =$

Equation

$y^2 - x^2 =$

$y^2 = x^2$

$y = \pm x$

Q.5 Find equation of rectangular hyperbola with centre at origin whose vertices are $(\pm 4,0)$ and find equation of its conjugate hyperbola. Also, find equations of asymptotes of the rectangular hyperbola.

Solution:

Centre $= (0,0)$

vertices $(\pm a, 0) = (\pm 4,0)$

$a = 4$

$\Rightarrow a^2 = 16$

And $b = 4 \begin{bmatrix}\text{In rectangular hyperbola} \\ a = b\end{bmatrix}$

$\Rightarrow b^2 = 16$

Transverse axis is along x-axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

$$\frac{x^2 - y^2}{16} = 1$$

$$x^2 - y^2 = 16$$

Conjugate Hyperbola

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$\frac{y^2}{16} - \frac{x^2}{16} = 1$$

$$\frac{y^2 - x^2}{16} = 1$$

$$y^2 - x^2 = 16$$

Equation of asymptotes:

$y^2 - x^2 = 0$

$y^2 = x^2$

$y = \pm x$

Q.6 Find eccentricity of a hyperbola whose latus rectum is double the transverse axis.

Solution:

Length of transverse axis $= 2a$

Length of latus rectum $= \frac{2b^2}{a}$

Length of latus rectum $=$ 2[Length of major axis]

$$\frac{2b^2}{a} = 2(2a)$$
$$b^2 = 2a^2$$

$$\because b^2 = a^2(e^2 - 1)$$
$$2a^2 = a^2(e^2 - 1)$$
$$2 = e^2 - 1$$
$$e^2 = 2 + 1$$
$$e^2 = 3$$
$$e = \sqrt{3}$$

Q.7 Show that the eccentricities e_1 and e_2 of the two conjugate hyperbolas satisfy the relation $(e_1)^2 + (e_2)^2 = (e_1)^2(e_2)^2$.

Solution:

Let two conjugate hyperbolas are

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \rightarrow (1)$$
$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow (2)$$

Eccentricity of (1) $= e_1 = \frac{c}{a}$

Eccentricity of (2) $= e_2 = \frac{c}{b}$

$$(e_1)^2 + (e_2)^2 = (e_1)^2(e_2)^2$$

L.H.S $= (e_1)^2 + (e_2)^2$

$$= \left(\frac{c}{a}\right)^2 + \left(\frac{c}{b}\right)^2$$
$$= \frac{c^2}{a^2} + \frac{c^2}{b^2}$$
$$= \frac{c^2b^2 + c^2a^2}{a^2b^2}$$
$$= \frac{c^2(b^2 + a^2)}{a^2b^2}$$

$= \frac{c^2(c^2)}{a^2b^2}$ $[\because c$

$$= \left(\frac{c^2}{a^2}\right)\left(\frac{c^2}{b^2}\right)$$
$$= \left(\frac{c}{a}\right)^2\left(\frac{c}{b}\right)^2$$
$$= (e_1)^2(e_2)^2$$

= R.H.S

[illegible]

The line $y = m$

Condition
Equation of ta with slope m
Equation of ta (x_1, y_1)
Equation of n (x_1, y_1)

Q.1 For what va

10.

Solution:

$2x^2 - 5y^2 = 1$

÷ by 10

$\frac{2x^2}{10} - \frac{5y^2}{10} = \frac{1}{1}$

$$\frac{x^2}{5} - \frac{y^2}{2} = 1$$

Comparing with

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a^2 = 5$ and b^2

$$= \frac{c^2(b^2+a^2)}{a^2b^2}$$

$$= \frac{c^2(c^2)}{a^2b^2} \quad [\because c^2 = b^2 + a^2]$$

$$= \left(\frac{c^2}{a^2}\right)\left(\frac{c^2}{b^2}\right)$$

$$= \left(\frac{c}{a}\right)^2\left(\frac{c}{b}\right)^2$$

$$= (e_1)^2(e_2)^2$$

$$= \text{R.H.S}$$

EXERCISE 9.6

The line $y = mx + c$ is tangent to

	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Condition	$c^2 = a^2m^2 - b^2$	$c^2 = a^2 - b^2m^2$
Equation of tangent with slope m	$y = mx \pm \sqrt{a^2m^2 - b^2}$	
Equation of tangent at (x_1, y_1)	$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$	
Equation of normal at (x_1, y_1)	$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$	

Q.1 For what value of k, the line $y = 2kx$ will be tangent to $2x^2 - 5y^2 = 10$.

Solution:

$2x^2 - 5y^2 = 10$

÷ by 10

$$\frac{2x^2}{10} - \frac{5y^2}{10} = \frac{10}{10}$$

$$\frac{x^2}{5} - \frac{y^2}{2} = 1$$

Comparing with

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a^2 = 5$ and $b^2 = 2$

$y = 2kx + 0$

Comparing with $y = mx + c$

$m = 2k, c = 0$

$c^2 = a^2m^2 - b^2$

$0^2 = 5(2k)^2 - 2$

$2 = 5(4k^2)$

$k^2 = \frac{1}{10}$

$k = \pm\frac{1}{\sqrt{10}}$

Q.2 Find the condition when the line $y = mx + c$ is tangent to $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$

Solution:

$y = mx + c \rightarrow (1)$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \rightarrow (2)$

Put value of y from (1) in (2)

$(2) \Rightarrow \frac{(mx+c)^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{m^2x^2 + 2cmx + c^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{b^2(m^2x^2 + 2cmx + c^2) - a^2x^2}{a^2b^2} = 1$

$b^2m^2x^2 + 2b^2cmx + b^2c^2 - a^2x^2 = a^2b^2$

$x^2(b^2m^2 - a^2) + 2b^2cmx + b^2c^2 - a^2b^2 = 0$

$x^2(b^2m^2 - a^2) + 2b^2cmx + b^2(c^2 - a^2) = 0$

$B^2 - 4AC = 0$

$(2b^2cm)^2 - 4(b^2m^2 - a^2)\{b^2(c^2 - a^2)\} = 0$

$4b^4c^2m^2 - 4b^2(b^2m^2 - a^2)(c^2 - a^2) = 0$

÷ by $4b^2$

$b^2c^2m^2 - (b^2m^2c^2 - b^2m^2a^2 - a^2c^2 + a^4) = 0$

$b^2c^2m^2 - b^2m^2c^2 + b^2m^2a^2 + a^2c^2 - a^4 = 0$

a^2c^2

÷ by a^2

$c^2 = a^2$

Q.3 Find

slope is

Solution

$3x^2 - 4$

÷ by 12

$\frac{x^2}{4} - \frac{y^2}{3}$

Compari

$\frac{x^2}{a^2} - \frac{y^2}{b^2}$

$a^2 = 4$

Equatior

$y = mx$

$m = 3$

$y = 3x$

$y = 3x$

$y = 3x$

Q.4 Find

Solution:

$\frac{y^2}{a^2} - \frac{x^2}{b^2}$

Point = (

Differenti

$\frac{d}{dx}\left(\frac{y^2}{a^2} -\right.$

$\frac{2yy'}{a^2} - 2x$

$a^2c^2 = a^4 - b^2m^2a^2$

÷ by a^2

$c^2 = a^2 - b^2m^2$

Q.3 Find the equation of tangent to hyperbola $3x^2 - 4y^2 = 12$ when slope is 3.

Solution:

$3x^2 - 4y^2 = 12$

÷ by 12

$\frac{x^2}{4} - \frac{y^2}{3} = 1$

Comparing with

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a^2 = 4$ and $b^2 = 3$

Equation of tangent

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$m = 3$

$y = 3x \pm \sqrt{(4)(3)^2 - 3}$

$y = 3x \pm \sqrt{(4)(9) - 3}$

$y = 3x \pm \sqrt{33}$

Q.4 Find the equation of tangent and normal to $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ at (x_1, y_1).

Solution:

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \rightarrow (1)$

Point $= (x_1, y_1)$

Differentiate w.r.t "x"

$\frac{d}{dx}\left(\frac{y^2}{a^2} - \frac{x^2}{b^2}\right) = \frac{d}{dx}(1)$

$\frac{2yy'}{a^2} - \frac{2x}{b^2} = 0$

$$\frac{2yy'}{a^2} = \frac{2x}{b^2}$$

$$\frac{yy'}{a^2} = \frac{x}{b^2}$$

$$y' = \frac{a^2x}{b^2y}$$

$$m\ at\ (x_1, y_1) = \frac{a^2x_1}{b^2y_1}$$

Equation of tangent:

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{a^2x_1}{b^2y_1}(x - x_1)$$

Equation of normal:

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - y_1 = -\frac{b^2y_1}{a^2x_1}(x - x_1)$$

$$y - y_1 = \frac{b^2y_1}{a^2x_1}(x_1 - x)$$

Q.5 Find the equation of tangent and normal to $\frac{x^2}{5} - \frac{y^2}{7} = 1$ at $\left(2\sqrt{5}, \sqrt{7}\right)$.

Solution:

$$\frac{x^2}{5} - \frac{y^2}{7} = 1 \rightarrow (1)$$

Differentiate w.r.t "x"

$$\frac{d}{dx}\left(\frac{x^2}{5} - \frac{y^2}{7}\right) = \frac{d}{dx}(1)$$

$$\frac{2x}{5} - \frac{2yy'}{7} = 0$$

$$\frac{2yy'}{7} = \frac{2x}{5}$$

$$\frac{yy'}{7} = \frac{x}{5}$$

$$y' = \frac{7x}{5y}$$

$m\ at$

(x_1, y_1)

Equation

$y - y_1 =$

$y - \sqrt{7}$

$5\sqrt{7}(y$

$5\sqrt{7}y -$

$0 = 14$

$14\sqrt{5}x$

Equation

$y - y_1 =$

$y - \sqrt{7}$

$14\sqrt{5}(y$

$(14\sqrt{5}y$

$5\sqrt{7}x +$

$5\sqrt{7}x +$

$5\sqrt{7}x +$

Q.6 Find

translate

Solution

$(h, k) =$

$x = X +$

$y = Y +$

$$m\ at\ (2\sqrt{5},\sqrt{7}) = \frac{7(2\sqrt{5})}{5\sqrt{7}} = \frac{14\sqrt{5}}{5\sqrt{7}}$$

$$(x_1, y_1) = (2\sqrt{5}, \sqrt{7})$$

Equation of tangent:

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{7} = \frac{14\sqrt{5}}{5\sqrt{7}}(x - 2\sqrt{5})$$

$$5\sqrt{7}(y - \sqrt{7}) = 14\sqrt{5}(x - 2\sqrt{5})$$

$$5\sqrt{7}y - 35 = 14\sqrt{5}x - 140$$

$$0 = 14\sqrt{5}x - 5\sqrt{7}y + 35 - 140$$

$$14\sqrt{5}x - 5\sqrt{7}y - 105 = 0$$

Equation of normal:

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - \sqrt{7} = -\frac{5\sqrt{7}}{14\sqrt{5}}(x - 2\sqrt{5})$$

$$14\sqrt{5}(y - \sqrt{7}) = -5\sqrt{7}(x - 2\sqrt{5})$$

t $(2\sqrt{5},\sqrt{7})$

$$(14\sqrt{5}y - 14\sqrt{35}) = -5\sqrt{7}x + 10\sqrt{35}$$

$$5\sqrt{7}x + 14\sqrt{5}y - 14\sqrt{35} = 10\sqrt{35}$$

$$5\sqrt{7}x + 14\sqrt{5}y - 14\sqrt{35} - 10\sqrt{35} = 0$$

$$5\sqrt{7}x + 14\sqrt{5}y - 24\sqrt{35} = 0$$

Q.6 Find the transformed equation of $\frac{(x-6)^2}{25} + \frac{(y+7)^2}{16} = 1$ when axes are translated with new origin $(6, -7)$.

Solution:

$(h, k) = (6, -7)$

$x = X + h \Rightarrow x = X + 6$

$y = Y + k \Rightarrow y = Y + (-7) \Rightarrow y = Y - 7$

$$\frac{(x-6)^2}{25}+\frac{(y+7)^2}{16}=1$$

$$\frac{(X+6-6)^2}{25}+\frac{(Y-7+7)^2}{16}=1$$

$$\frac{X^2}{25}+\frac{Y^2}{16}=1$$

Q.7 If xy-axes are rotated through given angle θ then find the new coordinates of given point P.

(i) $(2,3), \theta = 60°$ (ii) $(6,7), \theta = 45°$

(iii) $(-4,6), \theta = 30°$

Solution:

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

$X = x\cos\theta + y\sin\theta \rightarrow (1)$

$Y = -x\sin\theta + y\cos\theta \rightarrow (2)$

(i) $(2,3), \theta = 60°$

$(x,y) = (2,3)$

$(1) \Rightarrow X = 2\cos 60° + 3\sin 60°$

$$X = 2\left(\frac{1}{2}\right)+3\left(\frac{\sqrt{3}}{2}\right)$$

$$X = \frac{2}{2}+\frac{3\sqrt{3}}{2}$$

$$X = \frac{2+3\sqrt{3}}{2}$$

$(2) \Rightarrow Y = -2\sin 60° + 3\cos 60°$

$$Y = -2\left(\frac{\sqrt{3}}{2}\right)+3\left(\frac{1}{2}\right)$$

$$Y = \frac{-2\sqrt{3}}{2}+\frac{3}{2}$$

$$Y = \frac{-2\sqrt{3}+3}{2}$$

$$\left(\frac{2+3\sqrt{3}}{2}, \frac{-2\sqrt{3}+3}{2}\right)$$

(ii) $(6,7), \theta =$

$(x,y) = (6,$

$(1) \Rightarrow X =$

$X = 6\left(\frac{1}{\sqrt{2}}\right)$

$X = \frac{6}{\sqrt{2}} +$

$X = \frac{6+7}{\sqrt{2}}$

$X = \frac{13}{\sqrt{2}}$

$(2) \Rightarrow Y =$

$Y = -6\left(\frac{1}{\sqrt{2}}\right.$

$Y = -\frac{6}{\sqrt{2}} +$

$Y = \frac{-6+7}{\sqrt{2}}$

$Y = \frac{1}{\sqrt{2}}$

Point $= \left(\frac{13}{\sqrt{2}}\right.$

(iii) $(-4,6),$

$(x,y) = (-4$

$(1) \Rightarrow X = -$

$X = -4\left(\frac{\sqrt{3}}{2}\right.$

$X = -2\sqrt{3} +$

$(2) \Rightarrow Y = -$

$Y = 4\left(\frac{1}{2}\right) +$

$Y = 2 + 3\sqrt{3}$

Point $= (-2$

(ii) $(6,7), \theta = 45°$

$(x,y) = (6,7)$

$(1) \Rightarrow X = 6\cos 45° + 7\sin 45°$

$$X = 6\left(\frac{1}{\sqrt{2}}\right) + 7\left(\frac{1}{\sqrt{2}}\right)$$

$$X = \frac{6}{\sqrt{2}} + \frac{7}{\sqrt{2}}$$

$$X = \frac{6+7}{\sqrt{2}}$$

$$X = \frac{13}{\sqrt{2}}$$

$(2) \Rightarrow Y = -6\sin 45° + 7\cos 45°$

$$Y = -6\left(\frac{1}{\sqrt{2}}\right) + 7\left(\frac{1}{\sqrt{2}}\right)$$

$$Y = -\frac{6}{\sqrt{2}} + \frac{7}{\sqrt{2}}$$

$$Y = \frac{-6+7}{\sqrt{2}}$$

$$Y = \frac{1}{\sqrt{2}}$$

$$\text{Point} = \left(\frac{13}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

(iii) $(-4,6), \theta = 30°$

$(x,y) = (-4,6)$

$(1) \Rightarrow X = -4\cos 30° + 6\sin 30°$

$$X = -4\left(\frac{\sqrt{3}}{2}\right) + 6\left(\frac{1}{2}\right)$$

$$X = -2\sqrt{3} + 3$$

$(2) \Rightarrow Y = -(-4)\sin 30° + 6\cos 30°$

$$Y = 4\left(\frac{1}{2}\right) + 6\left(\frac{\sqrt{3}}{2}\right)$$

$$Y = 2 + 3\sqrt{3}$$

$$\text{Point} = \left(-2\sqrt{3} + 3, 2 + 3\sqrt{3}\right)$$

Q.8 Find new origin O′ and new XY-axes with respect to xy-coordinate system if it is translated 6 units to the left, 5 units up and rotated $\frac{\pi}{6}$ radians anticlockwise.

Solution:

$(h,k) = (-6,5)$

$\theta = \frac{\pi}{6}$ radians

Slope of x-axis $= m = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(x_1,y_1) = (-6,5)$

Equation of x-axis

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{\sqrt{3}}(x - (-6))$

$\sqrt{3}y - 5\sqrt{3} = x + 6$

$0 = x - \sqrt{3}y + 5\sqrt{3} + 6$

Equation of y-axis

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 5 = -\sqrt{3}(x - (-6))$

$y - 5 = -\sqrt{3}(x + 6)$

$y - 5 = -\sqrt{3}x - 6\sqrt{3}$

$\sqrt{3}x + y + 6\sqrt{3} - 5 = 0$

Q.9 Find new coordinates of $P(4,5)$ if new origin is $(1,2)$ and XY-coordinate system is rotated with $\frac{\pi}{6}$ radians anticlockwise from xy-coordinate system.

Solution:

$(x,y) = P(4,5)$

$(h,k) = (1,2)$

$\theta = \frac{\pi}{6}$ radians

After translation

$(x,y) = (4-1, 5-2)$

$(x,y) = (3,3)$

	X
	c
x	si
y	

$X = x\cos\theta$

$Y = -x\sin$

$(1) \Rightarrow X =$

$X = 3\left(\frac{\sqrt{3}}{2}\right.$

$X = \frac{3\sqrt{3}+}{2}$

$(2) \Rightarrow Y =$

$Y = -3\left(\frac{1}{2}\right)$

$Y = \frac{-3+3}{2}$

OR

	X
$x-h$	co
$y-k$	sir

$X = (x-h)$

$Y = -(x-h$

$(x,y) = P(4$

$(h,k) = (1,2$

$\theta = \frac{\pi}{6}$ radians

$(1) \Rightarrow X = (4$

$X = 3\left(\frac{\sqrt{3}}{2}\right)+$

$X = \frac{3\sqrt{3}+3}{2}$

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

$$X = x\cos\theta + y\sin\theta \to (1)$$

$$Y = -x\sin\theta + y\cos\theta \to (2)$$

$$(1) \Rightarrow X = x\cos\theta + y\sin\theta$$

$$X = 3\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)$$

$$X = \frac{3\sqrt{3}+3}{2}$$

$$(2) \Rightarrow Y = -x\sin\theta + y\cos\theta$$

$$Y = -3\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)$$

$$Y = \frac{-3+3\sqrt{3}}{2}$$

OR

	X	Y
$x-h$	$\cos\theta$	$-\sin\theta$
$y-k$	$\sin\theta$	$\cos\theta$

$$X = (x-h)\cos\theta + (y-k)\sin\theta \to (1)$$

$$Y = -(x-h)\sin\theta + (y-k)\cos\theta \to (2)$$

$(x, y) = P(4,5)$
$(h, k) = (1,2)$
$\theta = \frac{\pi}{6}$ radians

$$(1) \Rightarrow X = (4-1)\cos\frac{\pi}{6} + (5-2)\sin\frac{\pi}{6}$$

$$X = 3\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)$$

$$X = \frac{3\sqrt{3}+3}{2}$$

nd XY-
rom xy-

$(2) \Rightarrow Y = -(4-1)\sin\frac{\pi}{6} + (5-2)\cos\frac{\pi}{6}$

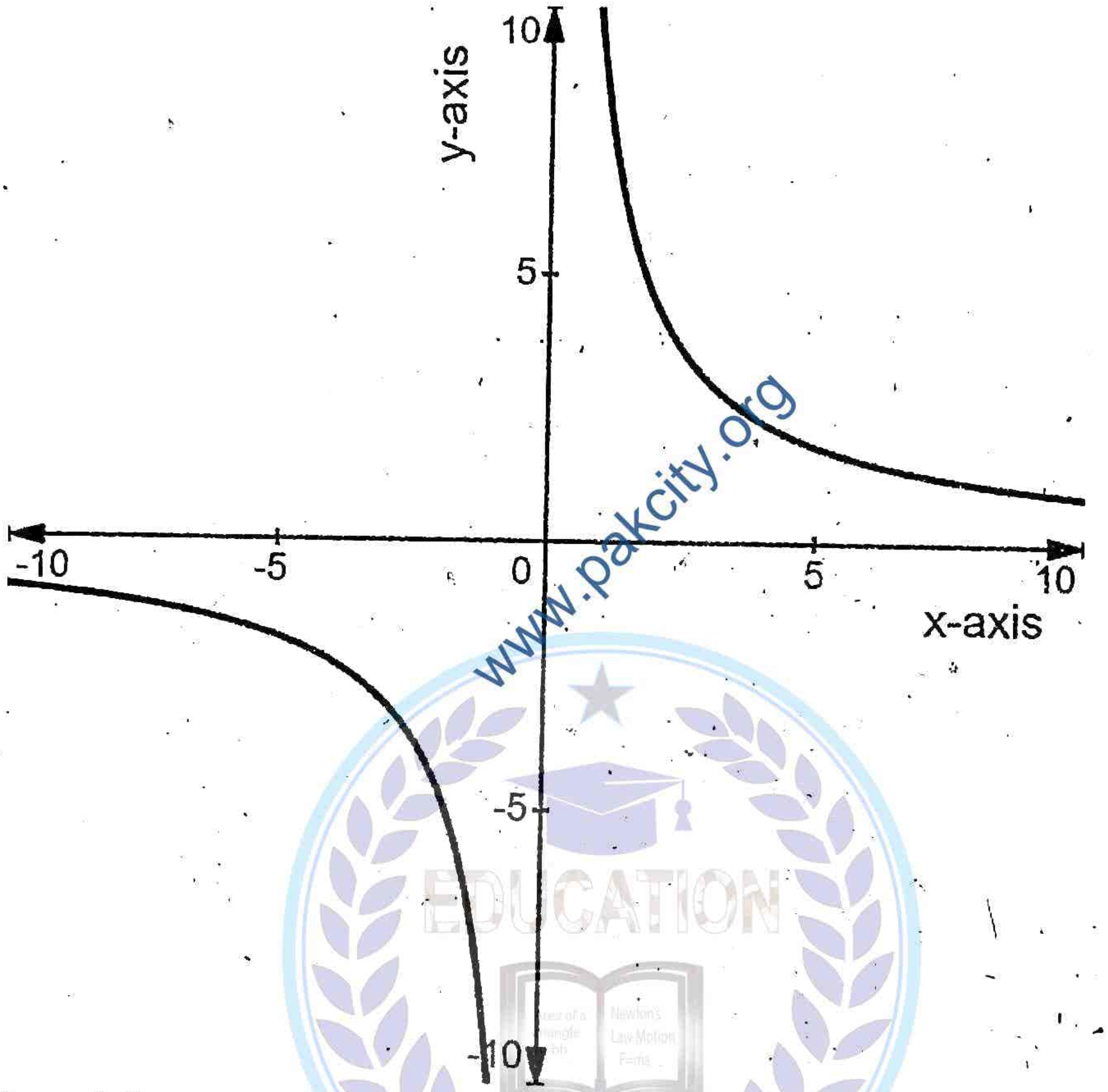
$Y = -3\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)$

$Y = \frac{-3+3\sqrt{3}}{2}$

Q.10 Identify and sketch the curve $xy = 9$.

Solution:

$xy = 9$



$xy - 9 = 0 \rightarrow (A)$

$Ax^2 + By^2 + Hxy + Gx + Fy + C = 0$

$A = B = G = F = 0$

$H = 1, c = -9$

$\tan 2\theta = \frac{H}{A-B}$

$\tan 2\theta = \frac{1}{0-0}$

$\tan 2\theta = \infty$

$2\theta = \tan^{-1}(\infty)$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

x

y

$X = x\,c$

$Y = -x$

$(1) \Rightarrow X$

$X = x\Big($

$X = \frac{x+}{\sqrt{}}$

$(2) \Rightarrow Y$

$Y = -x$

$Y = \frac{-x}{\sqrt{}}$

$(A) \Rightarrow \Big($

$\frac{y^2 - x^2}{2}$

÷ by 9

$\frac{y^2}{18} - \frac{x^2}{18}$

Comparin

$\frac{y^2}{a^2} - \frac{x^2}{b^2} =$

$a^2 = 18$ a

Centre =

Which is t

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

$$X = x\cos\theta + y\sin\theta \rightarrow (1)$$

$$Y = -x\sin\theta + y\cos\theta \rightarrow (2)$$

$$(1) \Rightarrow X = x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4}$$

$$X = x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$$

$$X = \frac{x+y}{\sqrt{2}}$$

$$(2) \Rightarrow Y = -x\sin\frac{\pi}{4} + y\cos\frac{\pi}{4}$$

$$Y = -x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$$

$$Y = \frac{-x+y}{\sqrt{2}}$$

$$(A) \Rightarrow \left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{y-x}{\sqrt{2}}\right) = 9$$

$$\frac{y^2 - x^2}{2} = 9$$

÷ by 9

$$\frac{y^2}{18} - \frac{x^2}{18} = 1$$

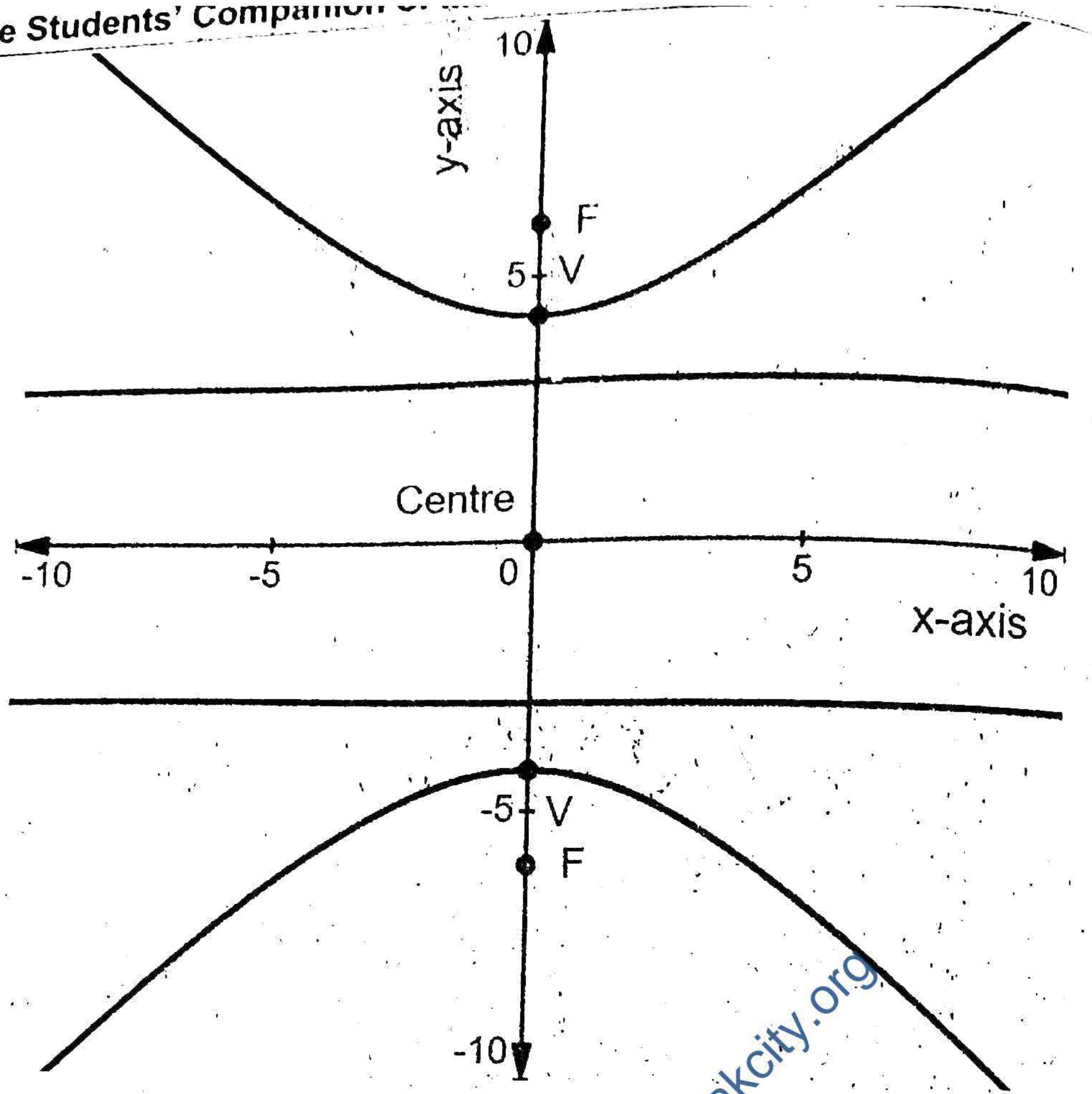
Comparing with

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$a^2 = 18$ and $b^2 = 18$

Centre $= (0,0)$

Which is the equation of hyperbola



Q.11 Through which angle the axes be rotated about origin so that the transformed equation of $9x^2 + 12xy + 4y^2 - x - y = 0$ does not contain the term involving XY.

Solution:

$$9x^2 + 12xy + 4y^2 - x - y = 0$$

$$Ax^2 + By^2 + Hxy + Gx + Fy + C = 0$$

$$A = 9, B = 4, H = 12, G = -1, F = -1, c = 0$$

$$\tan 2\theta = \frac{H}{A-B}$$

$$\tan 2\theta = \frac{12}{9-4}$$

$$\tan 2\theta = \frac{12}{5}$$

$$2\theta = \tan^{-1}\frac{12}{5}$$

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\frac{12}{5}}$$

1. Tick the correct o

(i) If the eccentricity

(a) parabola

(c) ✓ circle

(ii) The focus of par

(a) $(0,0)$

(c) $(-4,0)$

(iii) The latus rectur

(a) $-8, (3,-4)$

(c) $4, (-3,-4)$

(iv) The equation o

________.

(a) $6y = \frac{9}{2}(x+4)$

(c) $4y = \frac{9}{2}(x+6)$

(v) The latus rectun

(a) $\frac{5}{32}$

(c) $\frac{50}{4}$

(vi) The eccentricit

(a) $\sqrt{5}$

(c) 5

(vii) The centre of e

(a) $(\sqrt{10}, \sqrt{20})$

(c) ✓ $(-5,3)$

(viii) The equation

(a) $x = \pm\frac{4}{\sqrt{2}}$

(c) $x = \pm\frac{2}{\sqrt{5}}$

(ix) $ax^2 + by^2 + g$

numbers that repre

(a) a and b are non-

1. Tick the correct option.

(i) If the eccentricity is zero, then the conic is__________.

(a) parabola
(b) ellipse
(c) ✓ circle
(d) hyperbola

(ii) The focus of parabola $x^2 = -16y$ is __________.

(a) $(0,0)$
(b) $(4,0)$
(c) $(-4,0)$
(d) ✓ $(0,-4)$

(iii) The latus rectum and vertex of $(y-3)^2 = -8(x+4)$ is __________.

(a) $-8, (3,-4)$
(b) $8, (3,-4)$
(c) $4, (-3,-4)$
(d) ✓ $8, (-4,3)$

(iv) The equation of tangent at $(4,6)$ to the parabola $y^2 = 9x$ is __________.

(a) $6y = \frac{9}{2}(x+4)$
(b) $6y = 9(x-4)$
(c) $4y = \frac{9}{2}(x+6)$
(d) ✓ $3x - 4y + 12 = 0$

(v) The latus rectum of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is__________.

(a) $\frac{5}{32}$
(b) ✓ $\frac{32}{5}$
(c) $\frac{50}{4}$
(d) none

(vi) The eccentricity of the conic $\frac{x^2}{5} + \frac{y^2}{4} = 1$ is __________.

(a) $\sqrt{5}$
(b) ✓ $\frac{1}{\sqrt{5}}$
(c) 5
(d) none

(vii) The centre of ellipse $\frac{(x+5)^2}{10} + \frac{(y-3)^2}{20} = 1$ is __________.

(a) $(\sqrt{10}, \sqrt{20})$
(b) $(5,3)$
(c) ✓ $(-5,3)$
(d) none

(viii) The equation of directrix for the conic $\frac{x^2}{4} + \frac{y^2}{2} = 1$ is

(a) $x = \pm\frac{4}{\sqrt{2}}$
(b) $x = \pm\frac{\sqrt{5}}{4}$
(c) $x = \pm\frac{2}{\sqrt{5}}$
(d) ✓ $x = \pm 2\sqrt{2}$

(ix) $ax^2 + by^2 + gx + fy + c = 0$ where a, b, g, f and c are real numbers that represents hyperbola if

(a) a and b are non-zero and of same sign

(b) ✓ a and b are non-zero and of different sign

(c) either $a = 0$ or $b = 0$ (d) $a = b = 0$

(x) Auxiliary circle of ellipse $\frac{x^2}{6} + \frac{y^2}{5} = 1$ is ________.

(a) $x^2 + y^2 = 36$ (b) $x^2 + y^2 = 25$

(c) $x^2 + y^2 = 5$ (d) ✓ $x^2 + y^2 = 6$

(xi) The equation of directrix for $\frac{(x-h)^2}{p^2} + \frac{(y-k)^2}{q^2} = 1$ are ________ where $q > p$.

(a) $x = \pm\frac{p}{e}$ (b) $x - h = \pm\frac{q}{e}$

(c) ✓ $y - k = \pm\frac{q}{e}$ (d) $x - h = \pm\frac{p}{e}$

(xii) The vertices of hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$ are ________.

(a) $(\pm 5, 0)$ (b) $(0, \pm 5)$

(c) ✓ $(0, \pm 4)$ (d) $(\pm 4, 0)$

(xiii) Conjugate hyperbola to $\frac{x^2}{5} - \frac{y^2}{6} = 1$ is ________.

(a) $\frac{x^2}{5} - \frac{y^2}{6} = 1$ (b) ✓ $\frac{y^2}{6} - \frac{x^2}{5} = 1$

(c) $\frac{x^2}{6} - \frac{y^2}{5} = 1$ (d) none

(xiv) The eccentricity of rectangular hyperbola is ________.

(a) 1 (b) 2

(c) $\sqrt{3}$ (d) ✓ $\sqrt{2}$

(xv) The equation of tangent to $\frac{x^2}{6} + \frac{y^2}{5} = 1$ at $(\sqrt{6}, 0)$ is ________.

(a) $x = 6$ (b) ✓ $x = \sqrt{6}$

(c) $y = \sqrt{6}$ (d) none

(xvi) For what value of k, $y = k$ is tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is ________.

(a) ✓ ± 3 (b) ± 5

(c) $\pm\frac{7}{5}$ (d) none

(xvii) The equation of tangent to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ with slope 2 is ________.

(a) $y = 2x \pm \sqrt{23}$ (b) $y = 2x \pm \sqrt{41}$

(c) $x = 2y \pm \sqrt{23}$ (d) ✓ $y = 2x \pm \sqrt{55}$

(xviii) The equation of $xy = c^2$ represents

(a) parabola (b) ellipse

(c) ✓

(xix) If orig

(a) $(2,2)$

(c) $(4,4)$

(xx) If xy-c

abscissa is

(a) $x = \frac{x'}{2}$

(c) $x = \frac{x'}{\sqrt{3}}$

Q.2 Find th

(a) $\frac{(x-5)^2}{25} +$

Solution:

(a) $\frac{(x-5)^2}{25} +$

$\frac{(x-5)^2}{25} +$

Comparing

$\frac{(x-h)^2}{a^2} +$

$a^2 = 25 \Rightarrow$

$b^2 = 16 \Rightarrow$

Centre $(h,$

$c^2 = a^2 -$

$c = \sqrt{25 -}$

$e = \frac{c}{a} = \frac{3}{5}$

Major axis is

Vertices $(h$

$= (10, -3)$

Covertices (

(d) circle

(xix) If origin is shifted to (2,3) then coordinates of (5,6) are ________.

(a) (2,2)
(b) ✓ (3,3)
(c) (4,4)
(d) none

(xx) If xy-coordinate system is rotated at an angle of $\frac{\pi}{4}$ transformation for abscissa is

(a) $x = \frac{x'}{2} - \frac{y'}{\sqrt{2}}$
(b) ✓ $x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$
(c) $x = \frac{x'}{\sqrt{3}} - \frac{y'}{\sqrt{3}}$
(d) none

Q.2 Find the foci, vertices and directrices for the conic

(a) $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{16} = 1$ (b) $\frac{(x+4)^2}{9} - \frac{(y+7)^2}{16} = 1$

Solution:

(a) $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{16} = 1$

$$\frac{(x-5)^2}{25} + \frac{\{y-(-3)\}^2}{16} = 1$$

Comparing with

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 16 \Rightarrow b = 4$

Centre $(h,k) = (5,-3)$

$c^2 = a^2 - b^2$

$c = \sqrt{25-16} = 3$

$e = \frac{c}{a} = \frac{3}{5}$

Major axis is parallel to x-axis

Vertices $(h \pm a, k) = (5 \pm 5, -3)$
$= (10,-3)$ and $(0,-3)$

Covertices $(h, k \pm b) = (5, -3 \pm 4)$
$= (5,-7)$ and $(5,1)$

25
$= 6$
where
$\frac{q}{e}$
$\frac{p}{e}$
$= 1$
$+ \frac{y^2}{9} = 1$ is
s
$\sqrt{41}$
$\pm\sqrt{55}$

Foci $(h \pm c, k) = (5 \pm 3, -3)$

$= (8, -3)$ and $(2, -3)$

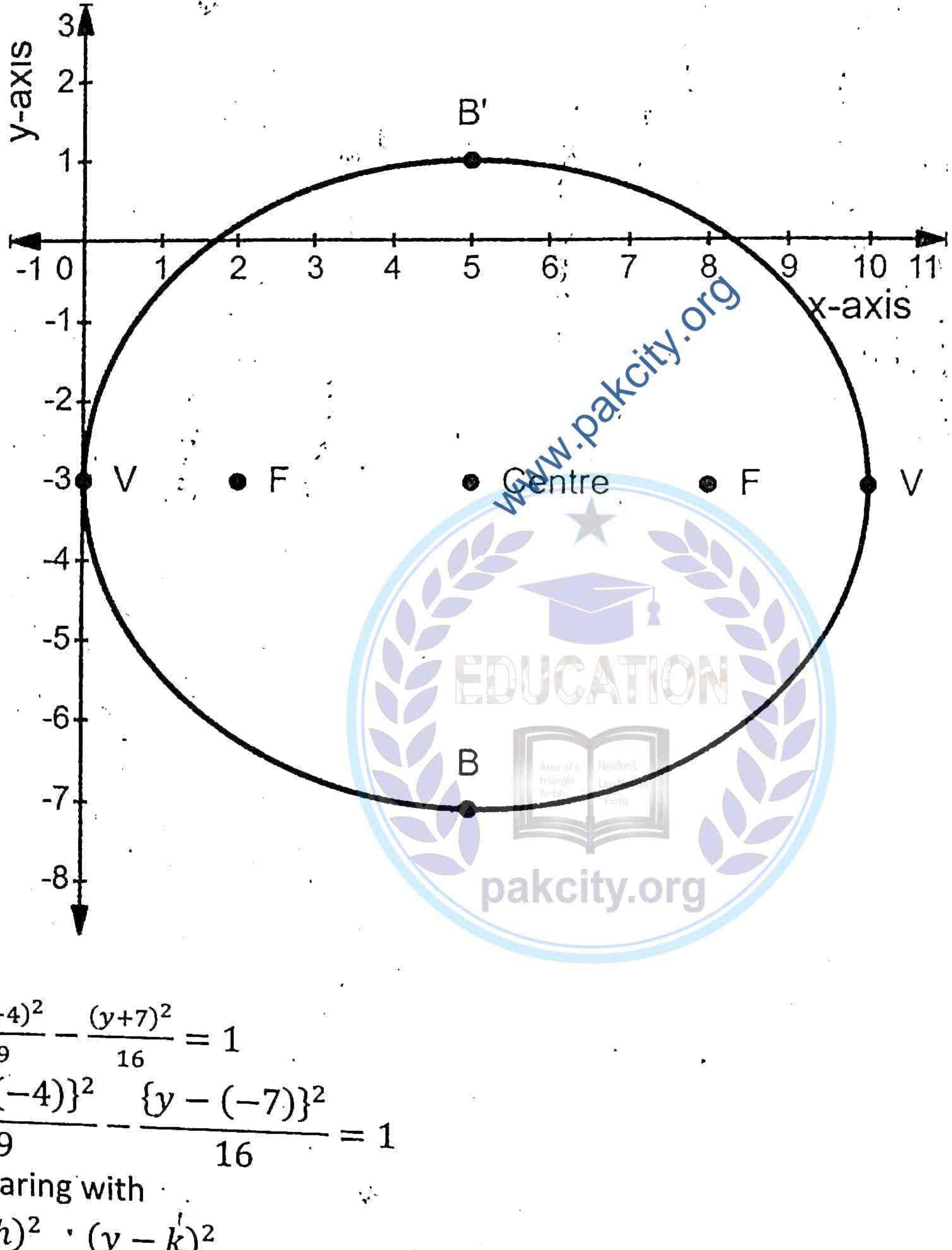
Equation of directrices:

$$x = h \pm \frac{a}{e}$$

$$x = 5 \pm \frac{5}{\frac{3}{5}}$$

$$x = 5 \pm \frac{25}{3}$$

$$x = \frac{40}{3} \text{ and } x = -\frac{10}{3}$$



(b) $\frac{(x+4)^2}{9} - \frac{(y+7)^2}{16} = 1$

$$\frac{\{x-(-4)\}^2}{9} - \frac{\{y-(-7)\}^2}{16} = 1$$

Comparing with

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 9$

$b^2 = 16 \Rightarrow$

Centre (h, k)

$c^2 = a^2 + b$

$c = \sqrt{9 + 16}$

$$e = \frac{c}{a} = \frac{5}{3}$$

Transverse a

Vertices $(h$

$= (-7, -7)$

Foci $(h \pm c,$

$= (-9, -7)$

Equation of

$$x = h \pm \frac{a}{e}$$

$$x = -4 \pm \frac{3}{\frac{5}{3}}$$

$$x = -4 \pm \frac{9}{5}$$

$$x = -4 + \frac{9}{5}$$

$$x = -\frac{11}{5}, x$$

$a^2 = 9 \Rightarrow a = 3$

$b^2 = 16 \Rightarrow b = 4$

Centre $(h, k) = (-4, -7)$

$c^2 = a^2 + b^2$

$c = \sqrt{9 + 16} = 5$

$e = \frac{c}{a} = \frac{5}{3}$

Transverse axis is parallel to x-axis

Vertices $(h \pm a, k) = (-4 \pm 3, -7)$

$= (-7, -7)$ and $(-1, -7)$

Foci $(h \pm c, k) = (-4 \pm 5, -7)$

$= (-9, -7)$ and $(1, -7)$

Equation of directrices:

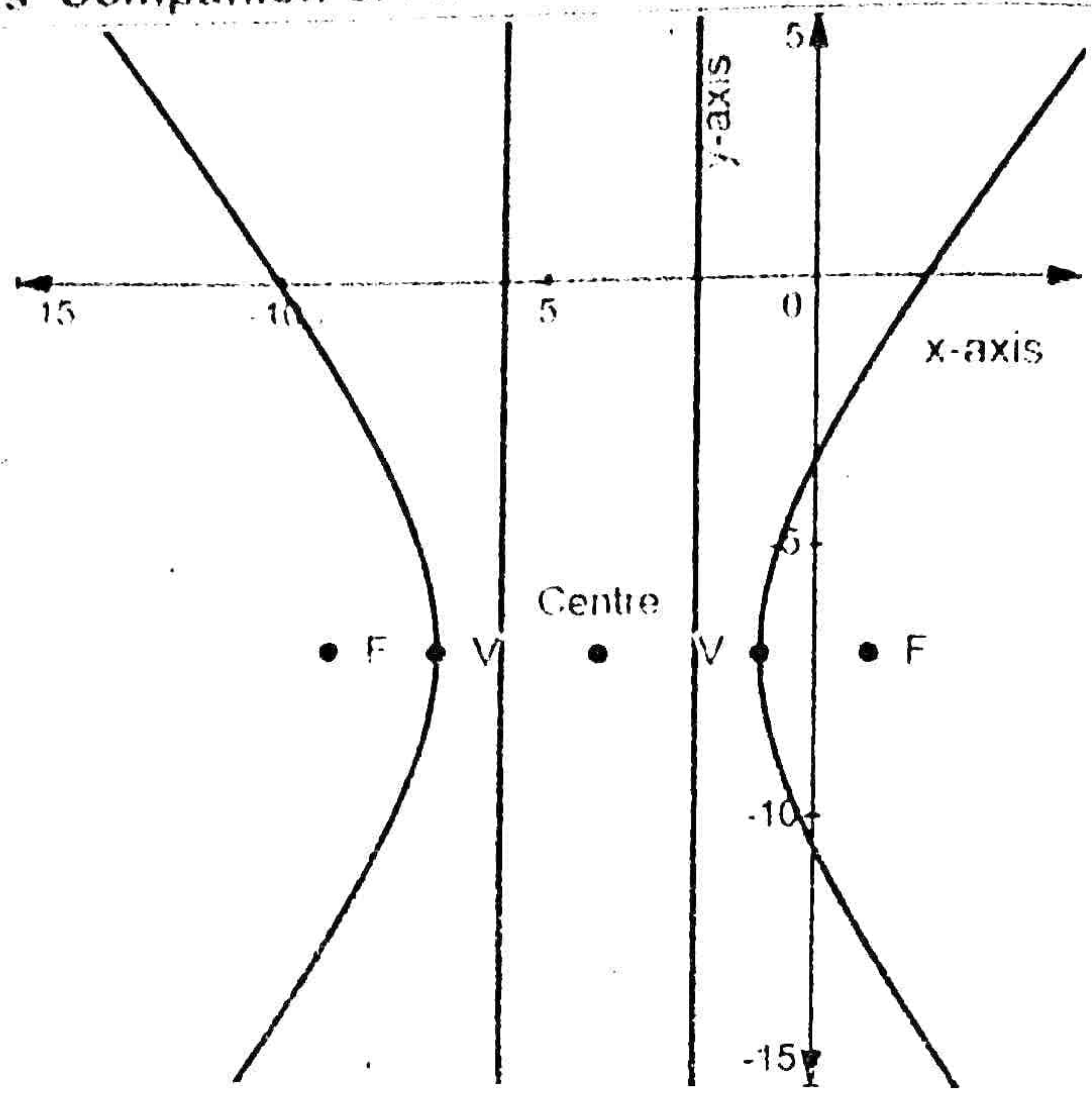
$x = h \pm \frac{a}{e}$

$x = -4 \pm \frac{3}{\frac{5}{3}}$

$x = -4 \pm \frac{9}{5}$

$x = -4 + \frac{9}{5}, x = -4 - \frac{9}{5}$

$x = -\frac{11}{5}, x = -\frac{29}{5}$



Q.3 Find the condition of tangency the line $y = x + c$ is tangent to the conic

(i) $y^2 = 10x$ (ii) $2x^2 + 3y^2 = 6$ (iii) $5x^2 - 7y^2 = 35$

Solution:

$y = x + c$

Comparing with $y = mx + c$

$m = 1$ and $c = c$

(i) $y^2 = 10x$

$y^2 = 4ax$

$4a = 10 \Rightarrow a = \frac{5}{2}$

$c = \frac{a}{m}$

$c = \frac{\frac{5}{2}}{1}$

$c = \frac{5}{2}$

(ii) $2x^2 + 3y^2 = 6$

÷ by 6

$\frac{2x^2}{6} + \frac{3y^2}{6} = \frac{6}{6}$

$\frac{x^2}{3} + \frac{y^2}{2} = 1$

Comparing with

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = 3$ and $b^2 = 2$

$c^2 = a^2m^2 + b^2$

$c^2 = (3)(1)^2 + 2$

$c^2 = 5$

$c = \pm\sqrt{5}$

(iii) $5x^2 - 7y^2 = 35$

÷ by 35

$\frac{5x^2}{35} - \frac{7y^2}{35} = \frac{35}{35}$

$\frac{x^2}{7} - \frac{y^2}{5} = 1$

Comparing with

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a^2 = 7$ and $b^2 = 5$

$c^2 = a^2m^2 - b^2$

$c^2 = (7)(1)^2 - 5$

$c^2 = 7 - 5$

$c^2 = 2$

$c = \pm\sqrt{2}$

Q.4 Find the transfo

is $(-5,3)$.

Solution:

$(h, k) = (-5,3)$

ıxis

angent to the

$^2 = 35$

$$\frac{2x^2}{6}+\frac{3y^2}{6}=\frac{6}{6}$$

$$\frac{x^2}{3}+\frac{y^2}{2}=1$$

Comparing with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$a^2 = 3$ and $b^2 = 2$

$$c^2 = a^2m^2 + b^2$$

$$c^2 = (3)(1)^2 + 2$$

$$c^2 = 5$$

$$c = \pm\sqrt{5}$$

(iii) $5x^2 - 7y^2 = 35$

÷ by 35

$$\frac{5x^2}{35}-\frac{7y^2}{35}=\frac{35}{35}$$

$$\frac{x^2}{7}-\frac{y^2}{5}=1$$

Comparing with

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$a^2 = 7$ and $b^2 = 5$

$$c^2 = a^2m^2 - b^2$$

$$c^2 = (7)(1)^2 - 5$$

$$c^2 = 7 - 5$$

$$c^2 = 2$$

$$c = \pm\sqrt{2}$$

Q.4 Find the transformed equation of $\frac{(x+5)^2}{7}-\frac{(y-3)^2}{5}=1$ when new origin is $(-5,3)$.

Solution:

$(h,k) = (-5,3)$

$x = X + h \Rightarrow x = X + (-5) \Rightarrow x = X$

$y = Y + k \Rightarrow y = Y + 3$

$$\frac{(x+5)^2}{7} - \frac{(y-3)^2}{5} = 1$$

$$\frac{(X-5+5)^2}{7} - \frac{(Y+3-3)^2}{5} = 1$$

$$\frac{X^2}{7} - \frac{Y^2}{5} = 1$$

Q.5 If xy-axes area rotated through angle θ, find coordinates of P if new coordinate is $(-2,7)$ and $\theta = 45°$.

Solution:

	X	Y
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

$x = X\cos\theta - Y\sin\theta \rightarrow (1)$

$y = X\sin\theta + Y\cos\theta \rightarrow (2)$

$(X, Y) = (-2,7)$ and $\theta = 45°$

$(1) \Rightarrow x = -2\cos 45° - 7\sin 45°$

$$x = -2\left(\frac{1}{\sqrt{2}}\right) - 7\left(\frac{1}{\sqrt{2}}\right)$$

$$x = -\frac{2}{\sqrt{2}} - \frac{7}{\sqrt{2}}$$

$$x = -\frac{9}{\sqrt{2}}$$

$(2) \Rightarrow y = -2\sin 45° + 7\cos 45°$

$$y = -2\left(\frac{1}{\sqrt{2}}\right) + 7\left(\frac{1}{\sqrt{2}}\right)$$

$$y = -\frac{2}{\sqrt{2}} + \frac{7}{\sqrt{2}}$$

$$y = \frac{5}{\sqrt{2}}$$

$$\left(-\frac{9}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$$