CHAPTER 08

CIRCLE

BASIC CONCEPTS AND FORMULAS

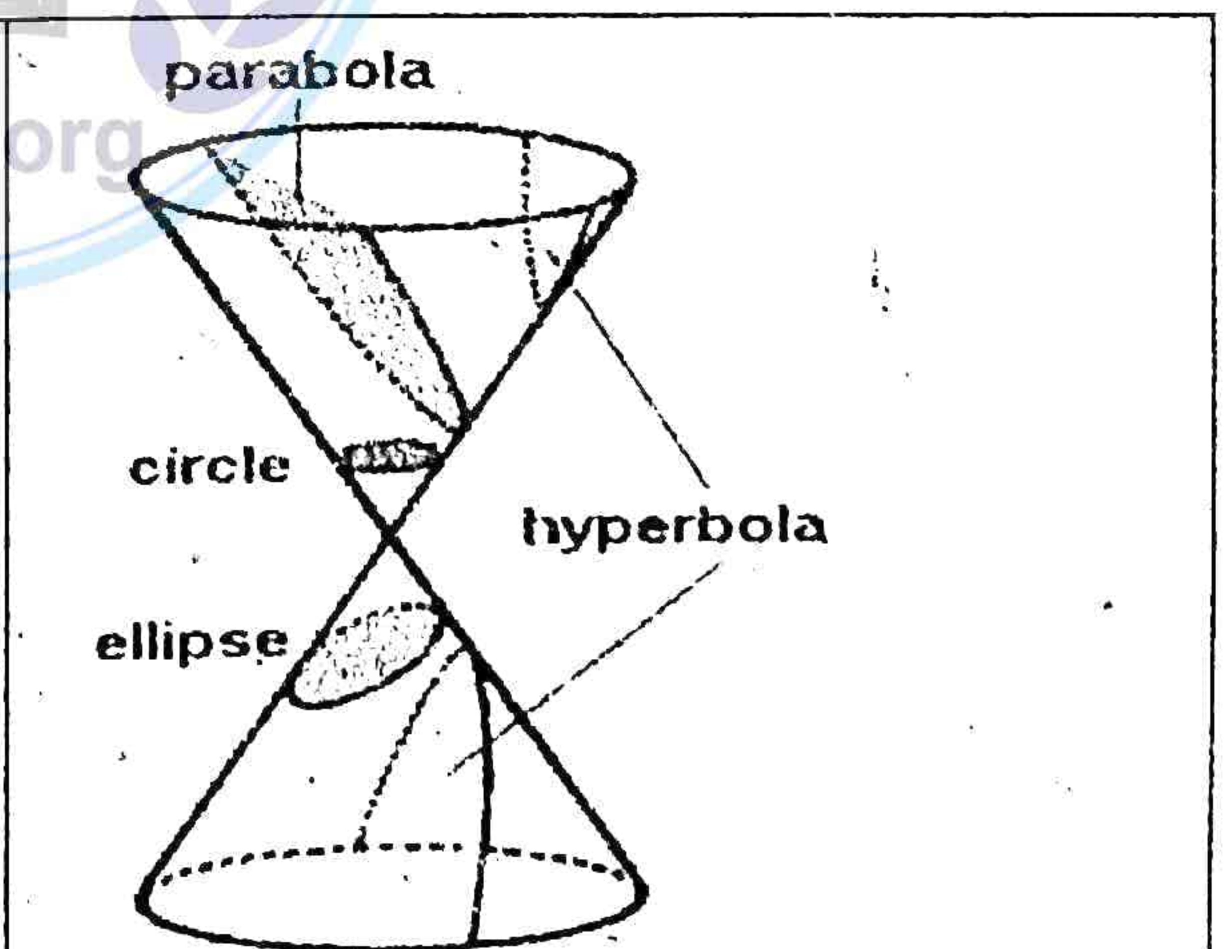
Conics and its family

A **conic** is the curve obtained as the intersection of a plane, called the *cutting plane*, with the surface of a double cone (a cone with two *nappes*). It is usually assumed that the cone is a right circular cone for the purpose of easy description, but this is not required; any double cone with some circular cross-section will suffice. Planes that pass through the vertex of the cone will intersect the cone in a point, a line or a pair of intersecting lines. These are called **degenerate conics** and some authors do not consider them to be conics at all. Unless otherwise stated, "conic" in this article will refer to a non-degenerate conic.

There are three types of conics: the ellipse, parabola, and hyperbola. The circle is a special kind of ellipse, although historically Apollonius considered as a fourth type. Ellipses arise when the intersection of the cone and plane is a closed curve. The circle is obtained when the cutting plane is parallel to the plane of the generating circle of the cone; for a right cone, this means the cutting plane is perpendicular to the axis. If the cutting plane is parallel to exactly one generating line of the cone, then the conic is unbounded and is called a *parabola*. In the remaining case, the figure is a *hyperbola*: the plane intersects *both* halves of the cone, producing two separate unbounded curves.

Applications:

Conic sections are important in astronomy: the orbits of two massive objects that interact according to Newton's law of universal gravitation are conic sections if their common center of mass is considered to be at rest. If they are bound together, they will both trace out ellipses; if they are moving apart, they will both follow parabolas or hyperbolas. See two-body problem.



The reflective properties of the conic sections are used in the design of searchlights, radio-telescopes and some optical telescopes. A searchlight uses a parabolic mirror as the reflector, with a bulb at the focus; and a similar construction is used for a parabolic microphone. The 4.2 meter Herschel optical telescope on La Palma, in the Canary islands, uses a primary parabolic mirror to reflect light towards a secondary hyperbolic mirror, which reflects it again to a focus behind the first mirror.

The General Equation for a Conic:

$$\mathbf{A}x^2 + \mathbf{B}xy + \mathbf{C}y^2 + \mathbf{D}x + \mathbf{E}y + \mathbf{F} = \mathbf{0}$$

For our course, we eliminate the term xy, as in the equation of a conic section it indicates there is a rotation of axes

$Ax^2 + Cy^2 + Dx + Ey + F = 0$	
Circle	A=C
Ellipse	• $A = 0$ and $C \neq 0$ • $A \neq 0$ and $C = 0$
Ellipse	$A \neq C$ and both positive
Hyperbola	From A or C, one is positive while other is negative $\lvert A\rvert = \lvert C\rvert$ or $\lvert A\rvert \neq \lvert C\rvert$
Circle	A=C

Eccentricity:

$$= \frac{distance\ from\ moving\ point\ to\ focus}{distance\ from\ moving\ point\ to\ directrix}$$

- e = 1, parabola
- 0 < e < 1, ellipse
- e > 1, hyperbola
- e = 0, circle

Circle:

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So the circle

center **(a,b)**

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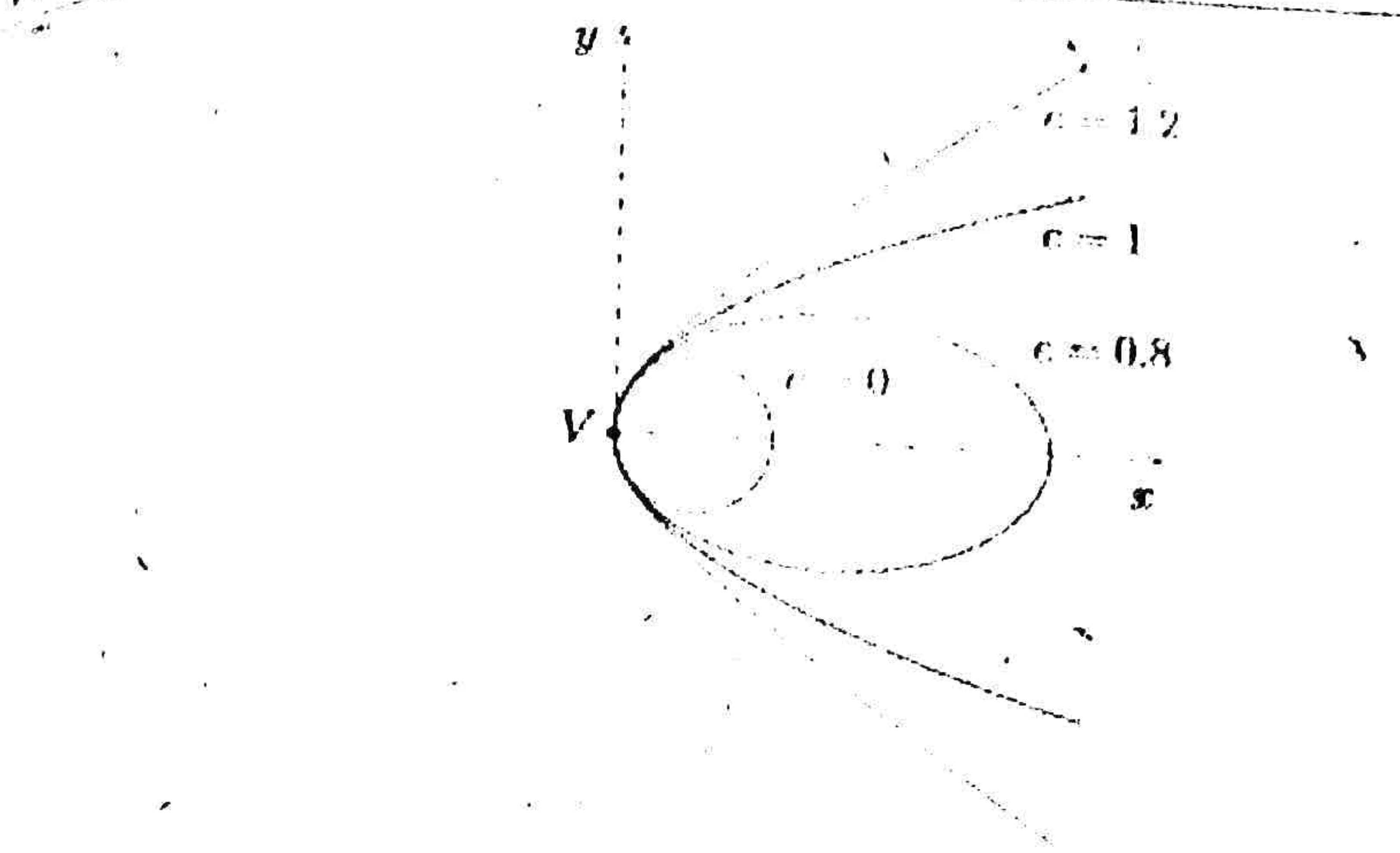
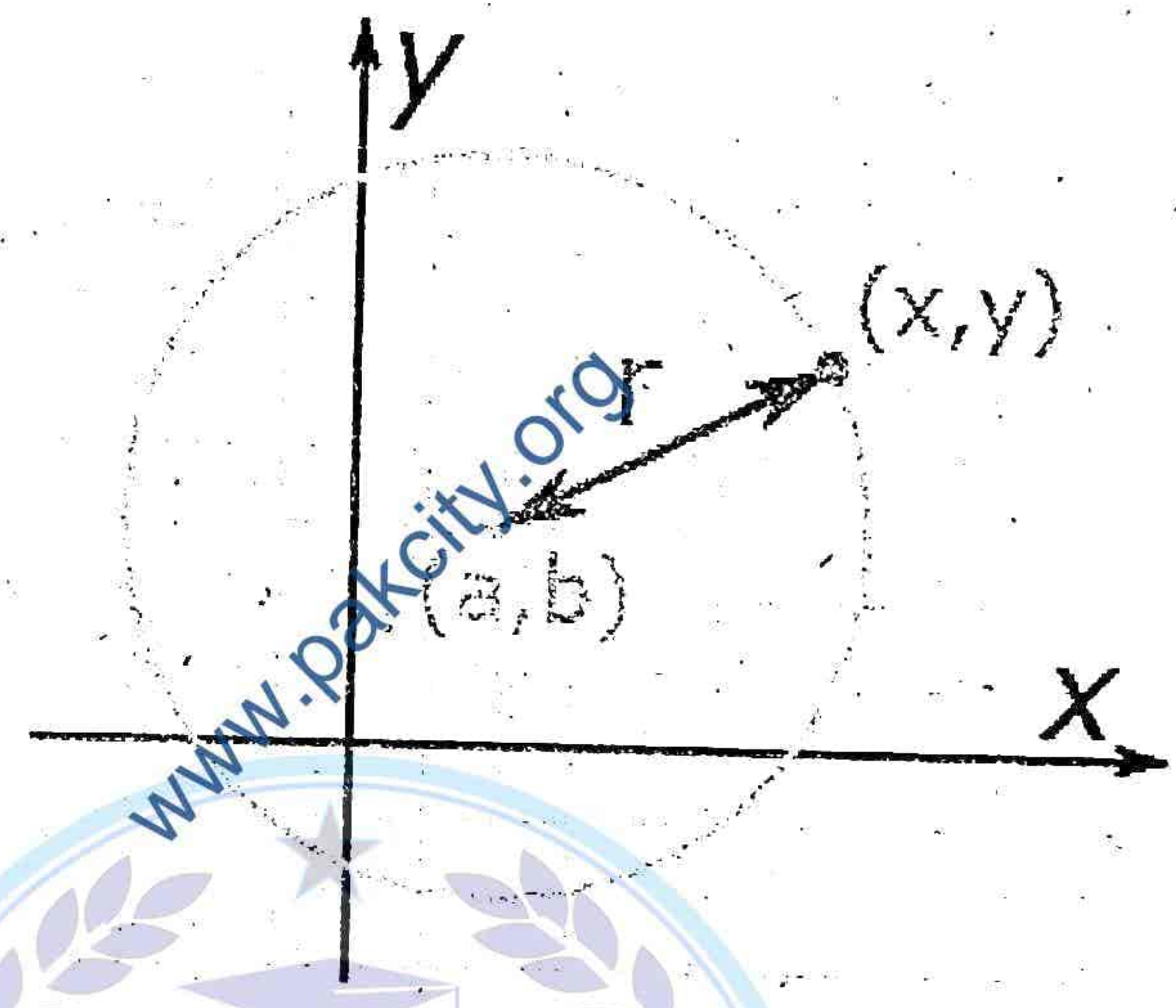


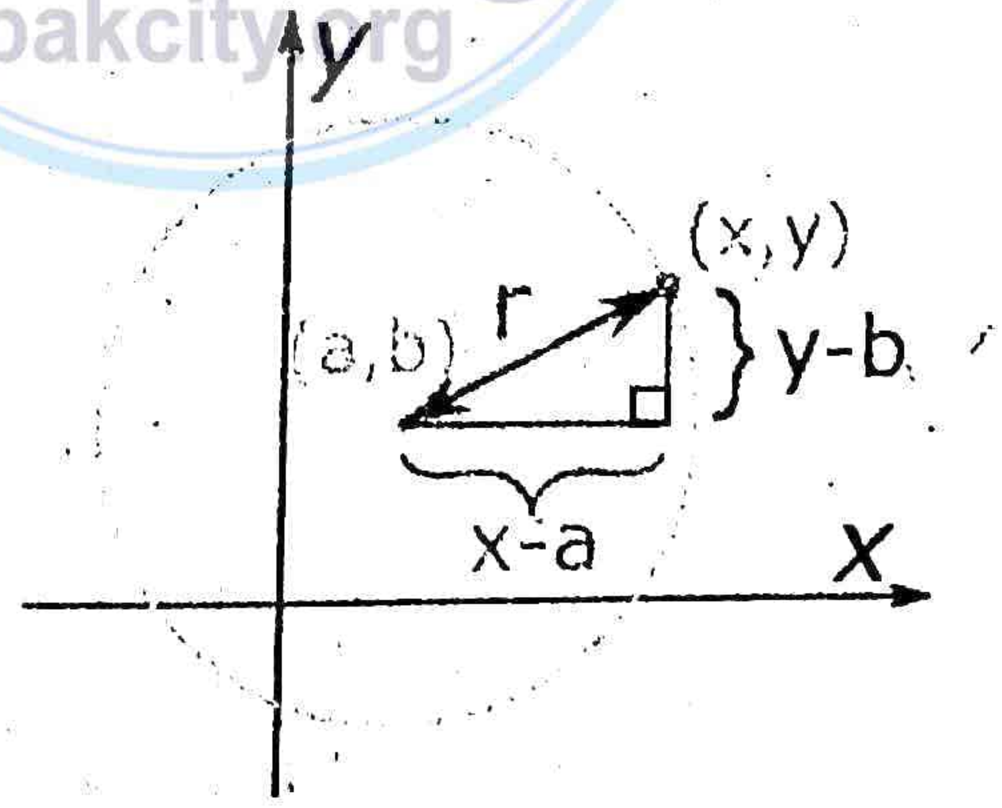
Circle:

Now let us put the center at **(a,b)**



So the circle is **all the points (x,y)** that are **"r"** away from the center **(a,b)**.

Now lets work out where the points are (using a right-angled triangle and Pythagoras):



$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(x-a)^2 + (y-b)^2 = r^2$

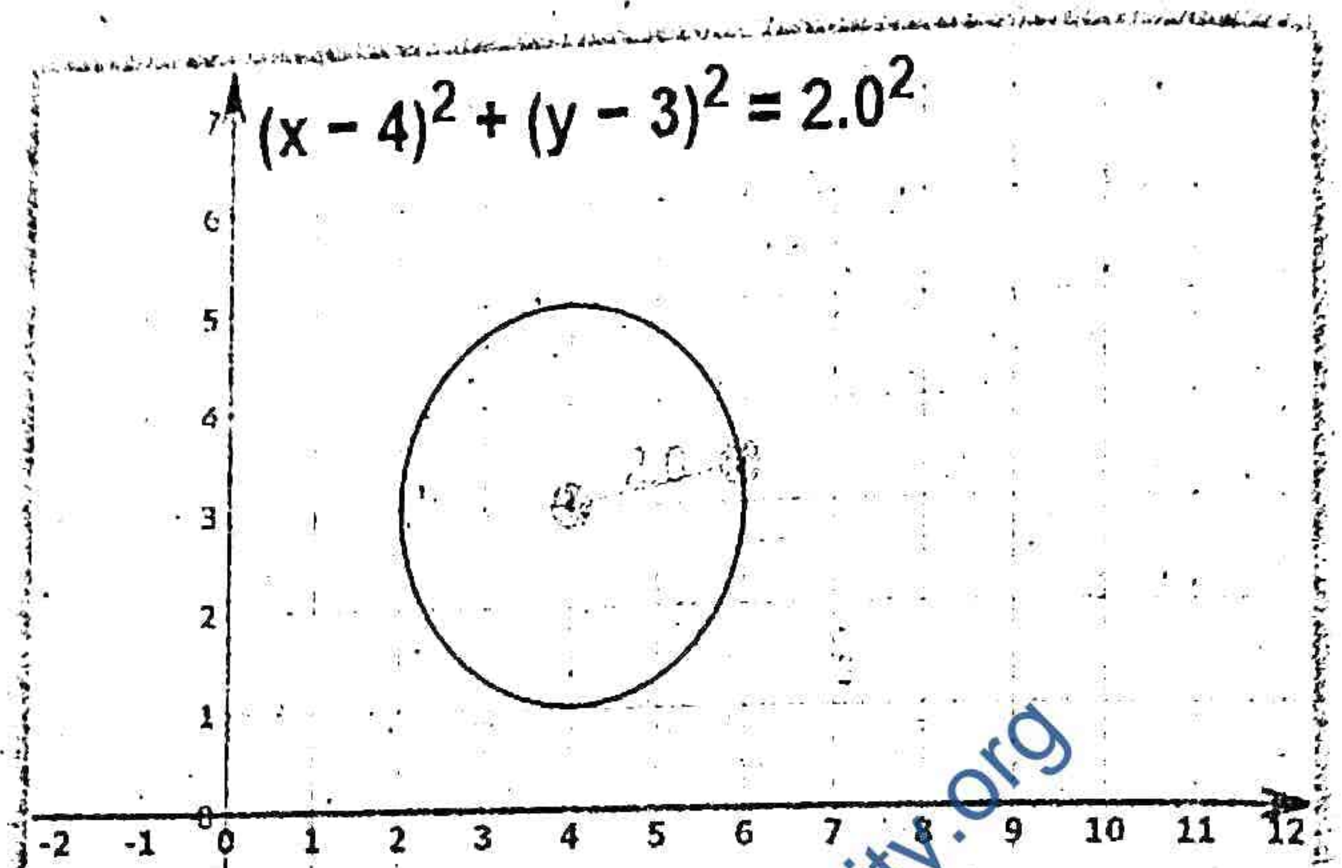
And that is the **"Standard Form"** for the equation of a circle.

Example#01

A circle with center at (3,4) and a radius of 6:

$(x-3)^2 + (y-4)^2 = 6^2$

Example#02



General equation of a circle

$x^2 + y^2 + 2gx + 2fy + C = 0$ ---(1)

$x^2 + 2gx + y^2 + 2fy = -C$

$(x^2 + 2gx + g^2) + (y^2 + 2fy + f^2)$

$= g^2 + f^2 - c$

$(x+g)^2 + (y+f)^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2$

Comparing with

$(x-a)^2 + (y-b)^2 = r^2$

$centre\ of\ (1) = (-g, -f)$

$r = \sqrt{g^2 + f^2 - c}$

Example

Find the equation of a circle passing through three non-collinear points (3,0), (0,4), (0,0)

Solution

$x^2 + y^2 + 2gx + 2fy + c = 0$ ----(1)

(0,0)

$(1) \Rightarrow 0^2 + 0^2$

$c = 0$

(0,4) and $c =$

$(1) \Rightarrow 0^2 + 4$

$f = -\frac{16}{8} \Rightarrow$

(3,0) and $c =$

$(1) \Rightarrow 3^2 +$

$g = -\frac{9}{6} \Rightarrow$

$(1) \Rightarrow x^2 +$

$x^2 + y^2 - 3$

Second Met

(3,0), (0,4),

$x^2 + y^2$

$3^2 + 0^2$

$0^2 + 4^2$

$0^2 + 0^2$

$\Rightarrow (x^2 +$

$+y$ 9 3 / 16 (/ 0 (

$\Rightarrow (x^2 +$

$+y(-48)$

$\Rightarrow x^2 +$

a circle

$(1) \Rightarrow 0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$

$\boxed{c = 0}$

$(0,4)$ and $c = 0$

$(1) \Rightarrow 0^2 + 4^2 + 2g(0) + 2f(4) + 0 = 0$

$f = -\frac{16}{8} \Rightarrow \boxed{f = -2}$

$(3,0)$ and $c = 0$

$(1) \Rightarrow 3^2 + 0^2 + 2g(3) + 2f(0) + 0 = 0$

$g = -\frac{9}{6} \Rightarrow \boxed{g = -\frac{3}{2}}$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2(-2)y + 0 = 0$

$x^2 + y^2 - 3x - 4y = 0$

Second Method

$(3,0), (0,4), (0,0)$

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 3^2+0^2 & 3 & 0 & 1 \\ 0^2+4^2 & 0 & 4 & 1 \\ 0^2+0^2 & 0 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x^2+y^2)\begin{vmatrix} 3 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix} - x\begin{vmatrix} 9 & 0 & 1 \\ 16 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$
$$+y\begin{vmatrix} 9 & 3 & 1 \\ 16 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 9 & 3 & 0 \\ 16 & 0 & 4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

ear points

$\Rightarrow (x^2+y^2)(12) - x(36)$
$+y(-48) - 0 = 0$

$\Rightarrow x^2 + y^2 - 3x - 4y = 0$

Third Method

$A(3,0), B(0,4), C(0,0)$

Let $O(a,b)$ be the centre of the circle

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|OA|^2=|OB|^2=|OC|^2$

$(a-3)^2+(b-0)^2=(a-0)^2+(b-4)^2$

$=(a-0)^2+(b-0)^2$

Eliminating x^2+y^2 from each side

$\Rightarrow -6a+9=-8b+16=0$

$-6a+9=0 \Rightarrow \boxed{a=\frac{3}{2}}$

$-8b+16=0 \Rightarrow \boxed{b=2}$

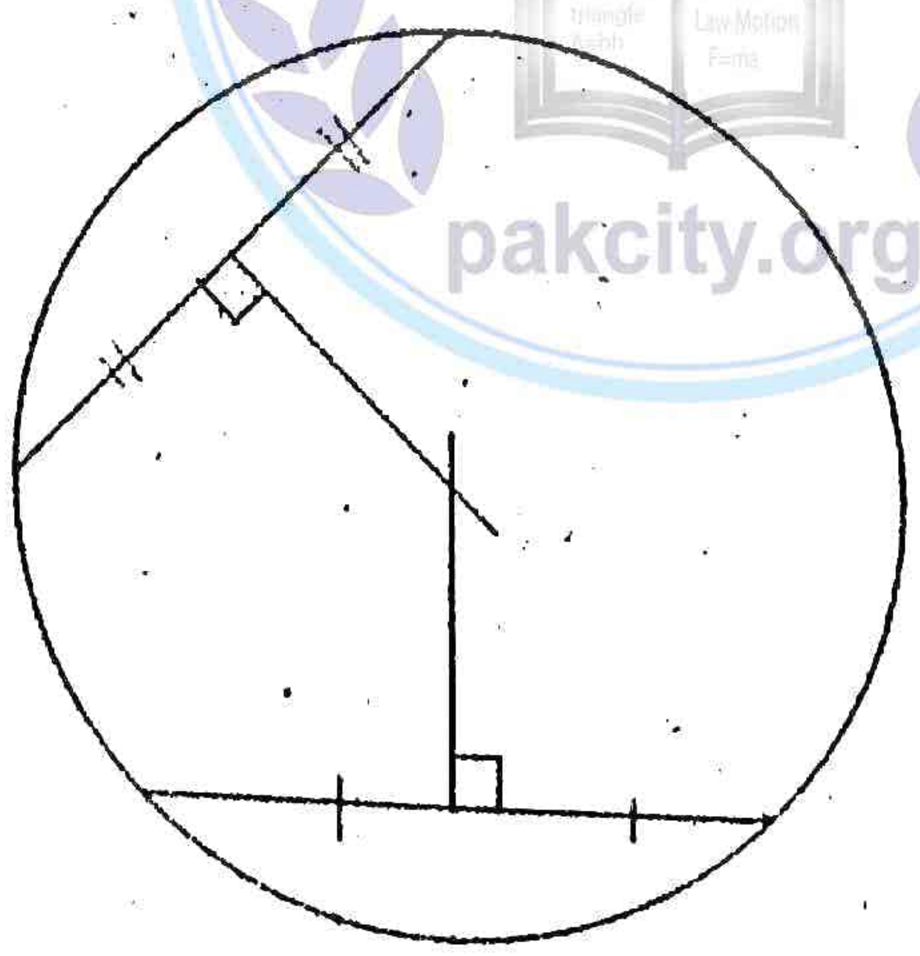
$$r=|OC|=\sqrt{2^2+\left(\frac{3}{2}\right)^2}=\frac{5}{2}$$

$$\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{25}{4}$$

Fourth Method

$P(3,0), Q(0,4), R(0,0)$

"The perpendicular bisectors of two chords meet at the centre"



Let L1 and L2 be the perpendicular bisectors of PQ and QR respectively.

Mid-point of P

Mid-point of (

Slope of PQ =

Slope of QR =

Since L1 ⊥ PQ

Gradient of L

Gradient of L

$L1: y-2=$

$L1: y-2=($

$\boxed{y=2}$

$L1 \Rightarrow 2-2$

$\boxed{x=\frac{3}{2}}$

$r=\sqrt{2^2+(}$

$\left(x-\frac{3}{2}\right)^2+($

Position of a

x_1

x_1

x_1

Tangents to th

Mid-point of PQ= $\left(\frac{3+0}{2},\frac{0+4}{2}\right)=\left(\frac{3}{2},2\right)$

Mid-point of QR= $\left(\frac{0+0}{2},\frac{0+4}{2}\right)=(0,2)$

Slope of PQ $=\frac{4-0}{0-3}=-\frac{4}{3}$

Slope of QR $=\frac{4-0}{0-0}=\frac{4}{0}$

Since L1 ⊥ PQ, and L2 ⊥ QR,

Gradient of L1 $=\frac{3}{4}$

Gradient of L2 $=0$

$$L1: y-2=\frac{3}{4}\left(x-\frac{3}{2}\right)$$

$$L1: y-2=0(x-0)$$

$$\boxed{y=2}$$

$$L1 \Rightarrow 2-2=\frac{3}{4}\left(x-\frac{3}{2}\right) \quad [\because y=2]$$

$$\boxed{x=\frac{3}{2}}$$

$$r=\sqrt{2^2+\left(\frac{3}{2}\right)^2}=\frac{5}{2}$$

$$\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{25}{4}$$

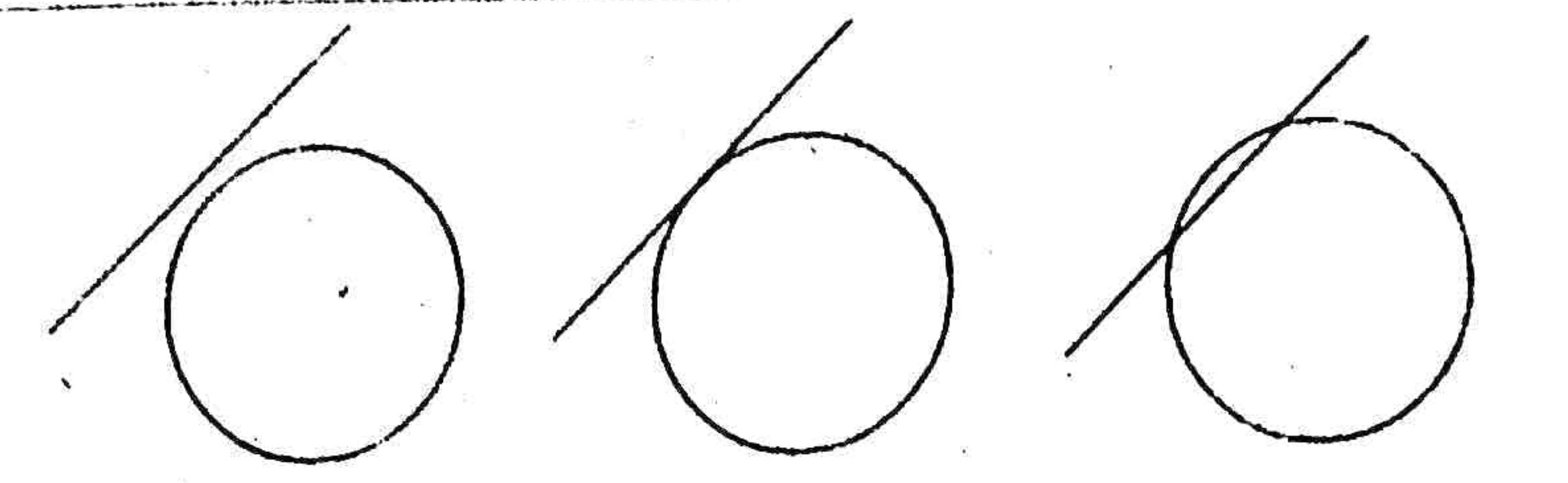
centre"

Position of a point with reference to circle

$x^2+y^2+2gx+2fy+c=0$ Then nature of point (x_1,y_1)	
$x_1^2+y_1^2+2gx_1+2fy_1+c>0$	Above
$x_1^2+y_1^2+2gx_1+2fy_1+c<0$	Below
$x_1^2+y_1^2+2gx_1+2fy_1+c=0$	On the circle

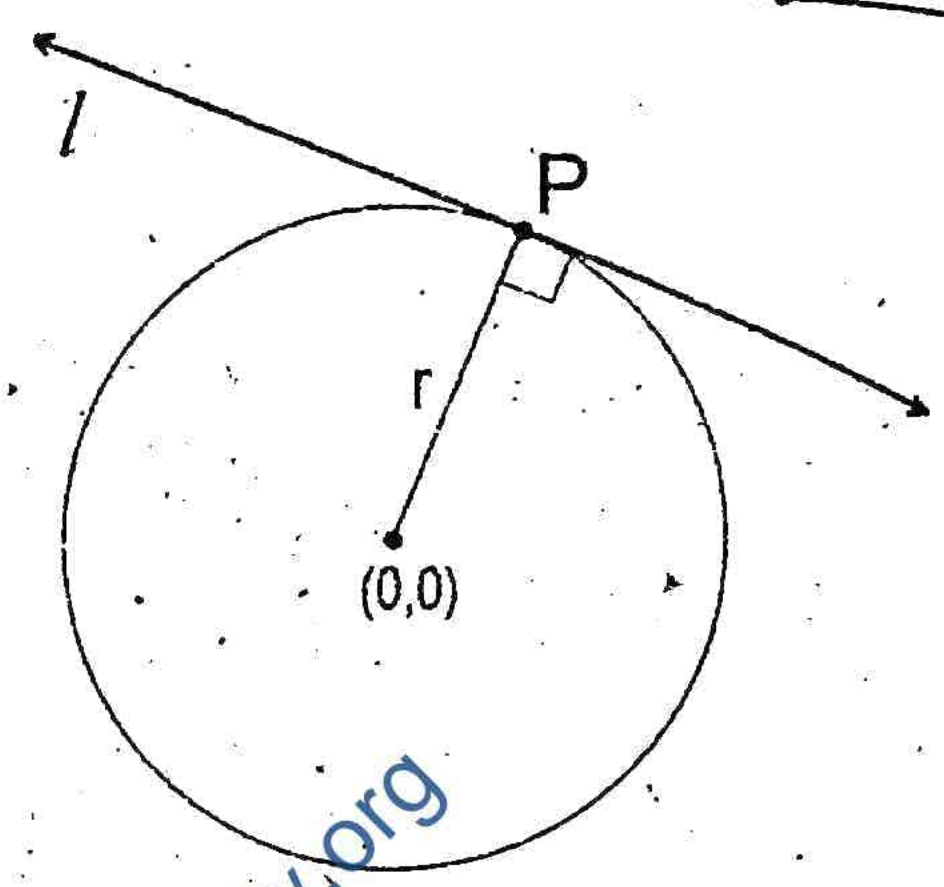
R

Tangents to the circle



Line outside the circle $B^2-4AC<0$ | Line touches the circle $B^2-4AC=0$ | Line through the circle $B^2-4AC>0$

Condition of tangency of $y=mx+c$ to the circle $x^2+y^2=r^2$



$y=mx+c$

$x^2+y^2=r^2$

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$r=\frac{|m(0)-0+c|}{\sqrt{m^2+(-1)^2}}$$

Squaring both sides

$c^2=r^2(1+m^2)$

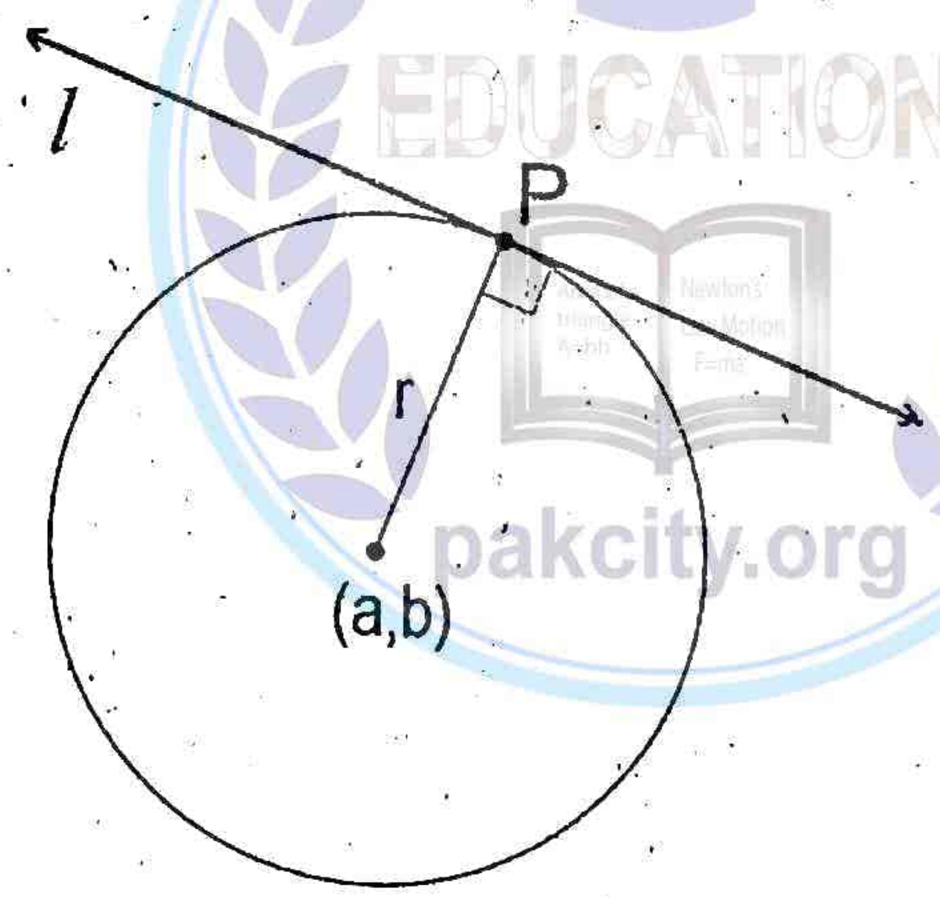
Is the condition of tangency

Condition of tangency of $y=mx+c$ to circle

$(x-a)^2+(y-b)^2=r^2$

$y=mx+c$

$(x-a)^2+(y-b)^2=r^2$



$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$r=\frac{|m(a)-b+c|}{\sqrt{m^2+(-1)^2}}$$

Squaring both sides

The Students'

$r^2(1+m^2)$ =

Is the conditi

Length of ta

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(x_1,y_1) to th

x^2+y^2+2,

$=\sqrt{x_1{}^2+y_1}$

Q.1 Describe

cone to prod

(i) circle

(v) a degene

Solution:

(i) cutting pla

not contain v

(ii) cutting pl

one nappe.

(iii) cutting pl

(iv) plane int

(v) cutting pl

OR

(i) Circle: If th

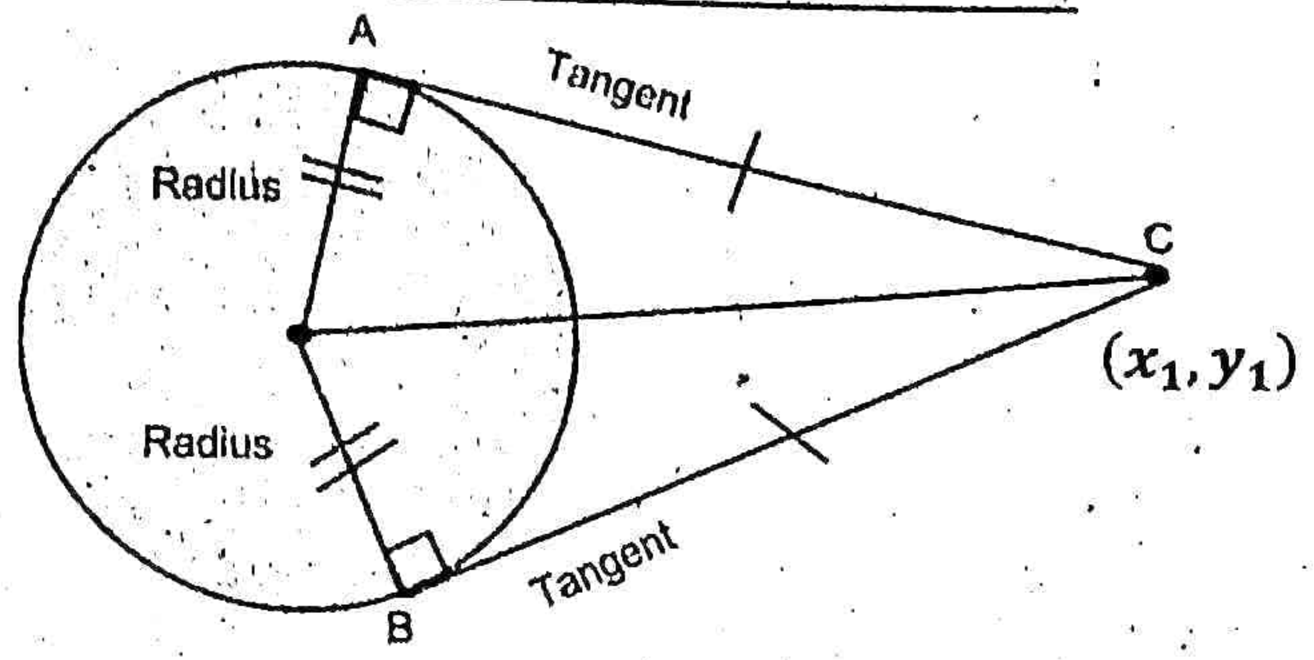
all the points

the apex of th

$r^2(1+m^2)=(am-b+c)^2$

Is the condition of tangency

Length of tangent from the point (x_1, y_1)



Length of tangent from the point (x_1, y_1) to the circle $x^2+y^2+2gx+2fy+c=0$ is

$=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

EXERCISE 8.1

Q.1 Describe the condition under which a plane cuts right circular cone to produce.

(i) circle (ii) parabola (iii) ellipse (iv) hyperbola

(v) a degenerate conic

Solution:

(i) cutting plane is perpendicular to the axis of the cone and does not contain vertex.

(ii) cutting plane is parallel to a generator of the cone and cuts only one nappe.

(iii) cutting plane is slightly tilted and cuts only one nappe.

(iv) plane intersects both nappes but does not contain the vertex.

(v) cutting plane passes through the vertex of the cone.

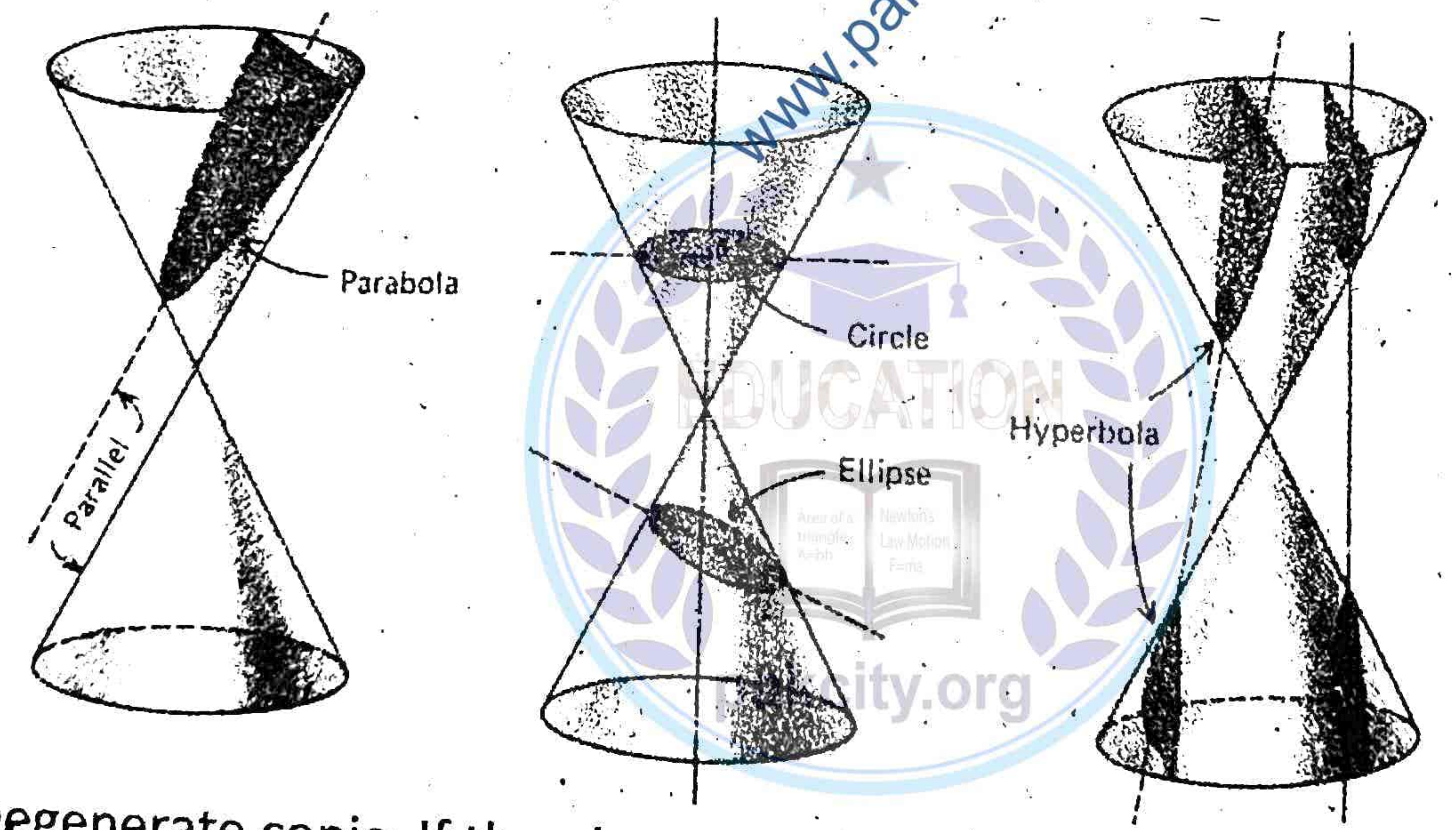
OR

(i) Circle: If the plane is perpendicular to the axis of the cone, then all the points on the intersection curve are the same distance from the apex of the cone. This makes the intersection curve a circle.

(ii) Parabola: If the plane is parallel to one of the generators of the cone, then the intersection curve is the set of all points that are the same distance away from the apex of the cone and the point of intersection of the plane and the generator. This makes the intersection curve like a parabola.

(iii) Ellipse: If the plane intersects the cone but is not perpendicular to the axis of the cone, then the intersection curve is the set of all points that are the same distance away from two points, called the foci of the ellipse. The distance between a point on the ellipse and each focus is called the focal length of the ellipse.

(iv) Hyperbola: If the plane intersects both nappes of the cone, then the intersection curve is the set of all points that are the same distance away from two points, called the foci of the hyperbola. However, the distance between a point on the hyperbola and each focus is different and is called the focal length of the hyperbola.



(v) Degenerate conic: If the plane intersects the cone in a line, a point, or two intersecting lines, then the intersection curve is a degenerate conic. This can happen if the plane passes through the apex of the cone, or if the plane is parallel to the base of the cone.

The Stude

Single

Q.2 Find

(i) centre

(ii) centr

(iii) (2, −

(iv) centr

(v) centr

(vi) centr

Solution:

Centre =

(x −

(i) centre

Centre =

$(x-0)^2$

x^2+y^2

x^2+y^2

(ii) centre

Centre =

$(x+5)^2$

x^2+10x

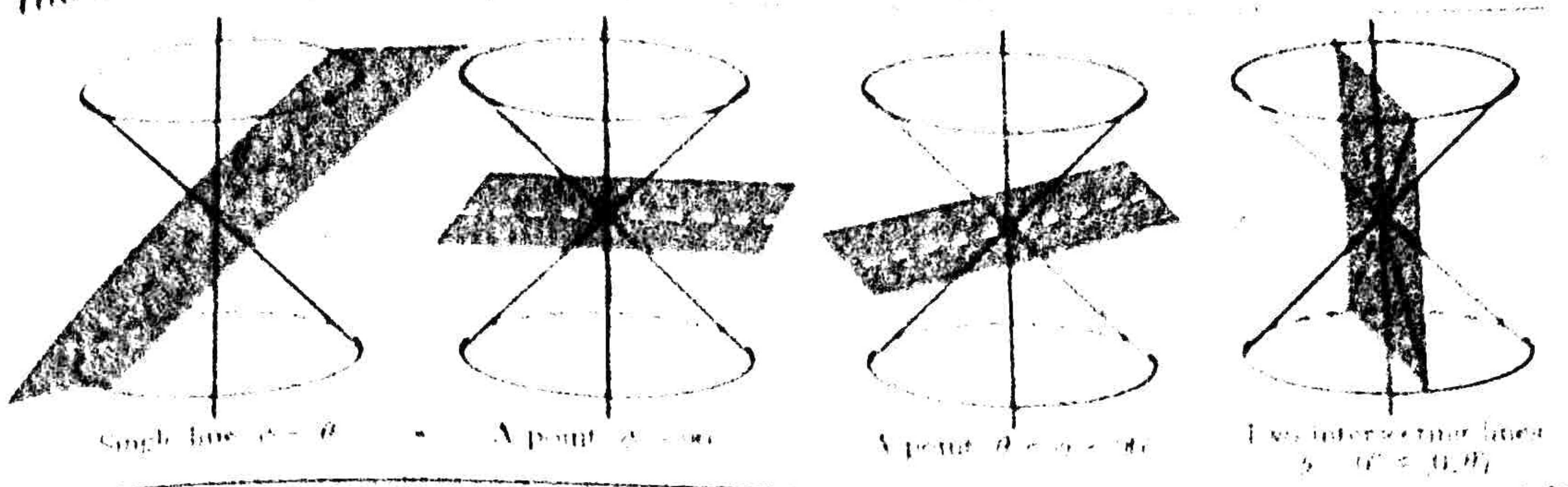
x^2+y^2

(iii) (2, −

$(x_1, y_1) =$

$(x_2, y_2) =$

Midpoint



Q.2 Find the equation of the circle if:

(i) centre is at origin and radius $5\sqrt{2}$ units.

(ii) centre is $(-5,7)$ and radius 6 units.

(iii) $(2,-3)$ and $(-4,7)$ are end points of its diameter.

(iv) centre is at origin and contains a point $(5,6)$.

(v) centre is at $(2,3)$ and contains a point $(5,7)$.

(vi) centre is at (p,q) and radius $\sqrt{p^2+q^2}$ units.

Solution:

Centre $= (a,b)$; Radius $= r$ units $(x-a)^2+(y-b)^2=r^2$

(i) centre is at origin and radius $5\sqrt{2}$ units.

Centre $= (0,0)$; Radius $= 5\sqrt{2}$ units

$$(x-0)^2+(y-0)^2=\left(5\sqrt{2}\right)^2$$

$$x^2+y^2=25(2)$$

$$x^2+y^2=50$$

(ii) centre is $(-5,7)$ and radius 6 units.

Centre $= (-5,7)$; Radius $= 6$ units

$$(x+5)^2+(y-7)^2=6^2$$

$$x^2+10x+25+y^2-14y+49=36$$

$$x^2+y^2+10x-14y+38=0$$

(iii) $(2,-3)$ and $(-4,7)$ are end points of its diameter.

$$(x_1,y_1)=(2,-3)$$

$$(x_2,y_2)=(-4,7)$$

$$\text{Midpoint}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Midpoint $= \left(\frac{2-4}{2}, \frac{-3+7}{2}\right) = (-1,2)$

$r = \frac{\sqrt{(-4-2)^2+(7+3)^2}}{2} = \frac{2\sqrt{34}}{2} = \sqrt{34}$

Centre $= (-1,2)$; Radius $= \sqrt{34}$ units

$(x-a)^2+(y-b)^2 = r^2$

$(x+1)^2+(y-2)^2 = \left(\sqrt{34}\right)^2$

$x^2+2x+1+y^2-4y+4 = 34$

$x^2+y^2+2x-4y-29 = 0$

OR

Let $P(x,y)$ be the point on circle

Angle inscribed in a semi-circle is right angle

$m_1m_2 = -1$

$\frac{y+3}{x-2} \times \frac{y-7}{x+4} = -1$

$\frac{y^2-4y-21}{x^2+2x-8} = -1$

$y^2-4y-21 = -x^2-2x+8$

$x^2+y^2+2x-4y-29 = 0$

OR

$x^2+y^2+2gx+2fy+c = 0 \rightarrow (1)$

Midpoint $= \left(\frac{2-4}{2}, \frac{-3+7}{2}\right) = (-1,2)$

$(-g,-f) = (-1,2)$

$g = 1, f = -2$

Put $(2,-3)$ in (1)

$(1) \Rightarrow 2^2+(-3)^2+2(1)(2)+2(-2)(-3)+c = 0$

$c = -29$

$(1) \Rightarrow x^2+y^2+2(1)x+2(-2)y-29 = 0$

$x^2+y^2+2x-4y-29 = 0$

(iv) centre is at origin and contains a point $(5,6)$.

Centre $= (-g,-f) = (0,0)$

$x^2+y^2+2gx+2fy+c = 0 \rightarrow (1)$

The Students' Com

As $(5,6)$ is on (1)

5^2+6^2+0+0+

$c = -61$

$(1) \Rightarrow x^2+y^2+$

$x^2+y^2-61 = 0$

(v) centre is at (2,

Centre $= (-g,-$

$g = -2, f = -3$

x^2+y^2+2gx+

As $(5,7)$ is on (1)

$5^2+7^2+2(-2)$

$c = -12$

$(1) \Rightarrow x^2+y^2+$

x^2+y^2-4x-

(vi) centre is at (

Centre $= (p,q)$;

$(x-a)^2+(y-$

$(x-p)^2+(y-$

$x^2-2px+p^2+$

x^2+y^2-2px-

Q.3 Find the cent

draw circles.

(i) x^2+y^2-25

(ii) $(x+3)^2+(y$

(iii) x^2+y^2-6x

(iv) x^2+y^2-8x

(v) $5x^2+5y^2+$

Solution:

(i) x^2+y^2-25

$x^2+y^2 = 5^2$

Comparing with (

Centre $= (0,0)$;

As $(5,6)$ is on (1)

$5^2+6^2+0+0+c=0$

$c=-61$

$(1) \Rightarrow x^2+y^2+0+0-61=0$

$x^2+y^2-61=0$

(v) centre is at $(2,3)$ and contains a point $(5,7)$.

$\text{Centre}=(-g,-f)=(2,3)$

$g=-2, f=-3$

$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

As $(5,7)$ is on (1)

$5^2+7^2+2(-2)(5)+2(-3)(7)+c=0$

$c=-12$

$(1) \Rightarrow x^2+y^2+2(-2)x+2(-3)y-12=0$

$x^2+y^2-4x-6y-12=0$

(vi) centre is at (p,q) and radius $\sqrt{p^2+q^2}$ units.

$\text{Centre}=(p,q)$; $\text{Radius}=\sqrt{p^2+q^2}$ units

$(x-a)^2+(y-b)^2=r^2$

$(x-p)^2+(y-q)^2=\left(\sqrt{p^2+q^2}\right)^2$

$x^2-2px+p^2+y^2-2qy+q^2=p^2+q^2$

$x^2+y^2-2px-2qy=0$

Q.3 Find the centre and radius of each of the following circles. Also draw circles.

(i) $x^2+y^2-25=0$

(ii) $(x+3)^2+(y-5)^2=49$

(iii) $x^2+y^2-6x+8y+10=0$

(iv) $x^2+y^2-8x+9=0$

(v) $5x^2+5y^2+20x-15y+10=0$

Solution:

(i) $x^2+y^2-25=0$

$x^2+y^2=5^2$

Comparing with $(x-a)^2+(y-b)^2=r^2$

$\text{Centre}=(0,0)$; $\text{Radius}=5$ units

(ii) $(x+3)^2 + (y-5)^2 = 49$

$\{x-(-3)\}^2 + (y-5)^2 = 7^2$

Comparing with $(x-a)^2 + (y-b)^2 = r^2$

Centre $= (-3,5)$; Radius $= 7$ units

(iii) $x^2 + y^2 - 6x + 8y + 10 = 0$

$x^2 + y^2 + 2(-3)x + +2(4)y + 10 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

By comparing

$g = -3$ and $f = 4$

Centre $= (-g, -f) = (3, -4)$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-3)^2 + (4)^2 - 10} = \sqrt{15}$ units

(iv) $x^2 + y^2 - 8x + 9 = 0$

$x^2 + y^2 + 2(-4)x + 2(0)y + 9 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

By comparing

$g = -4$ and $f = 0$

Centre $= (-g, -f) = (4,0)$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(-4)^2 + (0)^2 - 9} = \sqrt{7}$ units

(v) $5x^2 + 5y^2 + 20x - 15y + 10 = 0$

$\div$ by 5

$x^2 + y^2 + 4x - 3y + 2 = 0$

$x^2 + y^2 + 2(2)x + 2\left(-\frac{3}{2}\right)y + 2 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

By comparing

$g = 2$ and $f = -\frac{3}{2}$

Centre $= (-g, -f) = \left(-2, \frac{3}{2}\right)$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(2)^2 + (\ldots}$

Q.4 Find the va

$2y^2 - 8x + 4y$

Solution:

$2x^2 + 2y^2 - 8$

$\div$ by 2

$x^2 + y^2 - 4x$

$x^2 + y^2 + 2($

$x^2 + y^2 + 2g$

By comparing

$g = -2$ and

Centre $= (-$

$r^2 = g^2 + f$

$10^2 = (-2)$

$\frac{3k}{2} = 4 + 1$

$\frac{3k}{2} = -95$

$k = -95\left(\frac{2}{3}\right.$

$k = -\frac{190}{3}$

Q.5 Find the

concentric w

$24 = 0$. Also

Solution:

$x^2 + y^2 - 6$

The concent

$x^2 + y^2 - 6$

$$r = \sqrt{(2)^2 + \left(\frac{3}{2}\right)^2 - 2} = \frac{\sqrt{17}}{2} \text{ units}$$

Q.4 Find the value of k if the radius of the following circle $2x^2 + 2y^2 - 8x + 4y + 3k = 0$ is 10 units.

Solution:

$$2x^2 + 2y^2 - 8x + 4y + 3k = 0$$

$\div$ by 2

$$x^2 + y^2 - 4x + 2y + \frac{3k}{2} = 0$$

$$x^2 + y^2 + 2(-2)x + 2(1)y + \frac{3k}{2} = 0$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

By comparing

$g = -2$ and $f = 1, c = \frac{3k}{2}$

Centre $= (-g, -f) = (2, -1)$

$$r^2 = g^2 + f^2 - c$$

$$10^2 = (-2)^2 + (-1)^2 - \frac{3k}{2}$$

$$\frac{3k}{2} = 4 + 1 - 100$$

$$\frac{3k}{2} = -95$$

$$k = -95\left(\frac{2}{3}\right)$$

$$\boxed{k = -\frac{190}{3}}$$

Q.5 Find the equation of the circle passing through $(-3, -4)$ and is concentric with the circle whose equation is $x^2 + y^2 - 6x + 8y - 24 = 0$. Also identify the outer circle.

Solution:

$$x^2 + y^2 - 6x + 8y - 24 = 0 \rightarrow (1)$$

The concentric circle to (1) will be

$$x^2 + y^2 - 6x + 8y + C = 0 \rightarrow (2)$$

It passes through $(-3, -4)$

$(2) \Rightarrow (-3)^2 + (-4)^2 - 6(-3) + 8(-4) + C = 0$

$C = -11$

$(2) \Rightarrow x^2 + y^2 - 6x + 8y - 11 = 0$

Is the required equation of circle

$x^2 + y^2 - 6x + 8y - 11 = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \Rightarrow g = -3$

$2f = 8 \Rightarrow f = 4$

$c = -11$

$r = \sqrt{g^2 + f^2 - c}$

$r_1 = \sqrt{(-3)^2 + (4)^2 - (-11)} = \sqrt{36} = 6$

$x^2 + y^2 - 6x + 8y - 24 = 0 \rightarrow (1)$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \Rightarrow g = -3$

$2f = 8 \Rightarrow f = 4$

$c = -24$

$r = \sqrt{g^2 + f^2 - c}$

$r_2 = \sqrt{(-3)^2 + (4)^2 - (-24)} = \sqrt{49} = 7$

$r_2 > r_1$

Hence given circle is outer circle.

Q.6 Show that the equation $x = a\cos\theta$ and $y = a\sin\theta$ represent a circle with centre at origin and radius equal to a.

Solution:

$x = a\cos\theta \rightarrow (1)$

$y = a\sin\theta \rightarrow (2)$

Squaring (1) and (1) and then adding

$x^2 + y^2 = (a\cos\theta)^2 + (a\sin\theta)^2$

$x^2 + y^2 = a^2\cos^2\theta + a^2\sin^2\theta$

$x^2 + y^2 = a^2(\cos^2\theta + \sin^2\theta)$

$x^2 + y^2 =$
$x^2 + y^2 =$
Comparin
Centre =
Q.7 Prove
(i) throug
(ii) with c
(iii) with
(iv) with
Solution
$x^2 + y^2$
(i) throu
Point =
$(1) \Rightarrow 0$
$0 + 0 +$
$(1) \Rightarrow x$
(ii) with
Centre(
$(1) \Rightarrow x$
$x^2 + y^2$
$x^2 + y^2$
(iii) with
Centre
$(1) \Rightarrow$
$x^2 + y$
$x^2 + y$
(iv) with
Centre
$(1) \Rightarrow$
$x^2 + y$
$x^2 + y$

$x^2 + y^2 = a^2(1)$

$x^2 + y^2 = a^2$

Comparing with $(x-a)^2 + (y-b)^2 = r^2$

Centre $= (0,0)$; Radius $= 1$ units

Q.7 Prove that the equation of a circle

(i) through the origin has no constant term

(ii) with centre on x-axis has no term in y.

(iii) with centre on y-axis has no term in x.

(iv) with centre at origin has no term in x and y.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(i) through the origin has no constant term

Point $= (0,0)$

$(1) \Rightarrow 0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$

$0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$

$(1) \Rightarrow x^2 + y^2 + 2gx + 2fy = 0$

(ii) with centre on x-axis has no term in y.

Centre$(-g,-f) = (-g,0)$

$(1) \Rightarrow x^2 + y^2 + 2gx + 2(0)y + c = 0$

$x^2 + y^2 + 2gx + 0 + c = 0$

$x^2 + y^2 + 2gx + c = 0$

(iii) with centre on y-axis has no term in x.

Centre$(-g,-f) = (0,-f)$

$(1) \Rightarrow x^2 + y^2 + 2(0)x + 2fy + c = 0$

$x^2 + y^2 + 0 + 2fy + c = 0$

$x^2 + y^2 + 2fy + c = 0$

(iv) with centre at origin has no term in x and y.

Centre$(-g,-f) = (0,0)$

$(1) \Rightarrow x^2 + y^2 + 2(0)x + 2(0)y + c = 0$

$x^2 + y^2 + 0 + 0 + c = 0$

$x^2 + y^2 + c = 0$

ɔresent

EXERCISE 8.2

Q.1 Find the equation of the circle through the given points.

(i) (0,0), (0,3), (−4,0) (ii) (0,10), (−10,0), (8,6)

(iii) (0,3), (2, −1), (1,0) (iv) (7, −3), (−7,5), (11,5)

(v) (1,1), (2, −1), (3, −2)

Solution:

(i) (0,0), (0,3), (−4,0)

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(0,0)

$(1) \Rightarrow 0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$

$0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$

(0,3) and $c = 0$

$(1) \Rightarrow 0^2 + 3^2 + 2g(0) + 2f(3) + 0 = 0$

$0 + 9 + 0 + 6f + 0 = 0$

$6f = -9 \Rightarrow f = -\frac{3}{2}$

(−4,0) and $c = 0$

$(1) \Rightarrow (-4)^2 + 0^2 + 2g(-4) + 2f(0) + 0 = 0$

$16 + 0 - 8g + 0 + 0 = 0$

$8g = 16 \Rightarrow g = 2$

$(1) \Rightarrow x^2 + y^2 + 2(2)x + 2\left(-\frac{3}{2}\right)y + 0 = 0$

$x^2 + y^2 + 4x - 3y + 0 = 0$

(ii) (0,10), (−10,0), (8,6)

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(0,10)

$(1) \Rightarrow 0^2 + (10)^2 + 2g(0) + 2f(10) + c = 0$

$20f + c = -100 \rightarrow (2)$

(−10,0)

$(1) \Rightarrow (-10)^2 + 0^2 + 2g(-10) + 2f(0) + c = 0$

$100 + 0 - 20g + 0 + c = 0$

$100 - 20g + c = 0$

The Students'

$-20g + c =$ ·

(8,6)

$(1) \Rightarrow (8)^2 +$

$16g + 12f +$

By (2) − (3)

$20f + c + 2$

$20f + 20g =$

$f = -g \rightarrow ($

By (2) − (4)

$20f + c - 1$

$8f - 16g =$

$f - 2g = 0$

$-g - 2g =$

$-3g = 0$

$\boxed{g = 0}$

$(5) \Rightarrow \boxed{f =$

$(3) \Rightarrow -20$

$\boxed{c = -100}$

$(1) \Rightarrow x^2$ -

$x^2 + y^2 -$

(iii) (0,3), (

$x^2 + y^2 +$

(0,3)

$(1) \Rightarrow 0^2$

$0 + 9 + 0$

$9 + 6f +$

(1,0)

$(1) \Rightarrow 1^2$

$1 + 0 + 2$

$1 + 2g +$

(2,−1)

$(1) \Rightarrow (2$

$5 + 4$...

ven points.
0), (8,6)
5), (11,5)

$20g + c = -100 \rightarrow (3)$

(8,6)

$(1) \Rightarrow (8)^2 + (6)^2 + 2g(8) + 2f(6) + c = 0$

$16g + 12f + c = -100 \rightarrow (4)$

By (2) − (3)

$20f + c + 20g - c = -100 + 100$

$20f + 20g = 0$

$f = -g \rightarrow (5)$

By (2) − (4)

$20f + c - 16g - 12f - c = -100 + 100$

$8f - 16g = 0$

$f - 2g = 0$

$-g - 2g = 0$

$-3g = 0$

$\boxed{g = 0}$

$(5) \Rightarrow \boxed{f = 0}$

$(3) \Rightarrow -20(0) + c = -100$

$\boxed{c = -100}$

$(1) \Rightarrow x^2 + y^2 + 2(0)x + 2(0)y - 100 = 0$

$x^2 + y^2 - 100 = 0$

(iii) (0,3), (2, −1), (1,0)

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(0,3)

$(1) \Rightarrow 0^2 + 9 + 2g(0) + 2f(3) + c = 0$

$0 + 9 + 0 + 6f + c = 0$

$9 + 6f + c = 0 \rightarrow (2)$

(1,0)

$(1) \Rightarrow 1^2 + 0^2 + 2g(1) + 2f(0) + c = 0$

$1 + 0 + 2g + 0 + c = 0$

$1 + 2g + c = 0 \rightarrow (3)$

(2, −1)

$(1) \Rightarrow (2)^2 + (-1)^2 + 2g(2) + 2f(-1) + c = 0$

$5 + 4g - 2f + c = 0 \rightarrow (4)$

By (2) − (3)

$9 + 6f + c - 1 - 2g - c = 0$

$8 + 6f - 2g = 0$

$4 + 3f - g = 0 \rightarrow (5)$

By (4) − (3)

$5 + 4g - 2f + c - 1 - 2g - c = 0$

$4 + 2g - 2f = 0$

$2 + g - f = 0$

Adding in (5)

$\Rightarrow 6 + 2f = 0 \Rightarrow f = -3$

$(5) \Rightarrow 4 + 3(-3) - g = 0$

$g = -5$

$(3) \Rightarrow c = -1 - 2(-5) \Rightarrow c = 9$

$(1) \Rightarrow x^2 + y^2 + 2(-5)x + 2(-3)y + 9 = 0$

$x^2 + y^2 - 10x - 6y + 9 = 0$

(iv) $(7,-3), (-7,5), (11,5)$

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(7,-3)$

$(1) \Rightarrow (7)^2 + (-3)^2 + 2g(7) + 2f(-3) + c = 0$

$58 + 14g - 6f + c = 0 \rightarrow (2)$

$(-7,5)$

$(1) \Rightarrow (-7)^2 + (5)^2 + 2g(-7) + 2f(5) + c = 0$

$74 - 14g + 10f + c = 0 \rightarrow (3)$

$(11,5)$

$(1) \Rightarrow (11)^2 + (5)^2 + 2g(11) + 2f(5) + c = 0$

$146 + 22g + 10f + c = 0 \rightarrow (4)$

By (3) − (2)

$74 - 14g + 10f + c - (58 + 14g - 6f + c) = 0$

$16 - 28g + 16f = 0$

$4 - 7g + 4f = 0 \rightarrow (5)$

By (4) − (2)

$146 + 22g$ ·

$88 + 8g + 1$

$22 + 2g +$

By (6) − (5

$9g + 18 =$

$(6) \Rightarrow 22 +$

$f = -\frac{9}{2}$

$(2) \Rightarrow 58$

$c = -57$

$(1) \Rightarrow x^2$

$x^2 + y^2 -$

(v) (1,1),

$x^2 + y^2$ ·

(1,1)

$(1) \Rightarrow (1$

$2 + 2g +$

$(2,-1)$

$(1) \Rightarrow ($

$5 + 4g -$

$(3,-2)$

$(1) \Rightarrow ($

$13 + 6g$

By (2) −

$2 + 2g$ ·

$-3 - 2$

By (4) −

By (4) − (2)

$146+22g+10f+c-(58+14g-6f+c)=0$

$88+8g+16f=0$

$22+2g+4f=0 \rightarrow (6)$

By (6) − (5)

$9g+18=0 \Rightarrow g=-2$

$(6) \Rightarrow 22+2(-2)+4f=0$

$f=-\frac{9}{2}$

$(2) \Rightarrow 58+14(-2)-6\left(-\frac{9}{2}\right)+c=0$

$c=-57$

$(1) \Rightarrow x^2+y^2+2(-2)x+2\left(\frac{9}{2}\right)y-57=0$

$x^2+y^2-4x-9y-57=0$

(v) $(1,1),(2,-1),(3,-2)$

$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

$(1,1)$

$(1) \Rightarrow (1)^2+(1)^2+2g(1)+2f(1)+c=0$

$2+2g+2f+c=0 \rightarrow (2)$

$(2,-1)$

$(1) \Rightarrow (2)^2+(-1)^2+2g(2)+2f(-1)+c=0$

$5+4g-2f+c=0 \rightarrow (3)$

$(3,-2)$

$(1) \Rightarrow (3)^2+(-2)^2+2g(3)+2f(-2)+c=0$

$13+6g-4f+c=0 \rightarrow (4)$

By (2) − (3)

$2+2g+2f+c-(5+4g-2f+c)=0$

$-3-2g+4f=0 \rightarrow (5)$

By (4) − (3)

$13 + 6g - 4f + c - (5 + 4g - 2f + c) = 0$

$8 + 2g - 2f = 0 \rightarrow (6)$

By (5) + (6)

$-3 - 2g + 4f + 8 + 2g - 2f = 0$

$2f + 5 = 0 \Rightarrow f = -\dfrac{5}{2}$

$(6) \Rightarrow 4 + g - f = 0$

$g = f - 4$

$g = -\dfrac{5}{2} - 4 \Rightarrow g = -\dfrac{13}{2}$

$(3) \Rightarrow 5 + 4\left(-\dfrac{13}{2}\right) - 2\left(-\dfrac{5}{2}\right) + c = 0$

$c = 16$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\dfrac{13}{2}\right)x + 2\left(-\dfrac{5}{2}\right)y + 16 = 0$

$x^2 + y^2 - 13x - 5y + 16 = 0$

Q.2 Find the equation of circle through the points (1,2), (2,3) and having centre on (i) x-axis (ii) y-axis

---

Solution:

A(1,2), B(2,3) are points on circle

(i) centre on x-axis

Let O$(a, 0)$ be the centre on x-axis

$|\overline{AO}| = |\overline{BO}| = \text{radius}$

$|\overline{AO}|^2 = |\overline{BO}|^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(a - 1)^2 + (0 - 2)^2 = (a - 2)^2 + (0 - 3)^2$

$(a^2 - 2a + 1) + 4 = (a^2 - 4a + 4) + 9$

$2a = 8$

$a = 4$

$r^2 = |\overline{AO}|^2 = (4 - 1)^2 + (0 - 2)^2 = 13$

$(x - a)^2 + (y - b)^2 = r^2$

$(x - 4)^2 + (y - 0)^2 = 13$

$(x^2 - 8x + 16) + y^2 = 13$

$x^2 + y^2 - 8$...

(ii) centre o...

Let $O(0, b)$

$|\overline{AO}| = |\overline{BO}$...

$|\overline{AO}|^2 = |\overline{B}$...

$d = \sqrt{(x_2 -}$...

$(0 - 1)^2 +$...

$1 + (b^2 -$...

$2b = 8$

$b = 4$

$r^2 = |\overline{AO}|$...

$(x - a)^2 +$...

$(x - 0)^2 +$...

$x^2 + (y^2 -$...

$x^2 + y^2 -$...

Q.3 Find t...

having cen...

Solution:

$x^2 + y^2 +$...

(3,1) and ...

Since $(-g$...

$-g - f -$...

(3,1)

$(1) \Rightarrow (3$...

$10 + 6g$ ...

(2,2)

$(1) \Rightarrow (2$...

$8 + 4g +$...

By (4) − (...

$8 + 4g +$...

$x^2 + y$ ...

(ii) centre on y-axis

Let $O(0, b)$ be the centre on x-axis

$|\overline{AO}| = |\overline{BO}| =$ radius

$|\overline{AO}|^2 = |\overline{BO}|^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(0-1)^2 + (b-2)^2 = (0-2)^2 + (b-3)^2$

$1 + (b^2 - 4b + 4) = 4 + (b^2 - 6b + 9)$

$2b = 8$

$b = 4$

$r^2 = |\overline{AO}|^2 = (0-1)^2 + (4-2)^2 = 5$

$(x-a)^2 + (y-b)^2 = r^2$

$(x-0)^2 + (y-4)^2 = 5$

$x^2 + (y^2 - 8y + 16) = 5$

$x^2 + y^2 - 8y + 11 = 0$

, (2,3) and

Q.3 Find the equation of circle through the points (3,1), (2,2) and having centre on the line $x + y - 3 = 0$.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(3,1) and (2,2)

Since $(-g, -f)$ is on $x + y - 3 = 0$

$-g - f - 3 = 0 \rightarrow (2)$

(3,1)

$(1) \Rightarrow (3)^2 + (1)^2 + 2g(3) + 2f(1) + c = 0$

$10 + 6g + 2f + c = 0 \rightarrow (3)$

(2,2)

$(1) \Rightarrow (2)^2 + (2)^2 + 2g(2) + 2f(2) + c = 0$

$8 + 4g + 4f + c = 0 \rightarrow (4)$

By $(4) - (3)$

$8 + 4g + 4f + c - (10 + 6g + 2f + c) = 0$

$2 - 2g + 2f = 0$

$\div$ by 2

$-1 - g + f = 0$

Adding (2)

$-g - f - 3 = 0$

$-2g - 4 = 0$

$2g = -4 \Rightarrow g = -2$

$(2) \Rightarrow -(-2) - 3 = f$

$f = -1$

$(4) \Rightarrow 8 + 4(-2) + 4(-1) + c = 0$

$c = 4$

$(1) \Rightarrow x^2 + y^2 + 2(-2)x + 2(-1)y + 4 = 0$

$x^2 + y^2 - 4x - 2y + 4 = 0$

Q.4 Find the equation of circle through the points $(0, -1)$, $(3,0)$ and the line $3x + y - 9 = 0$ is tangent to it at $(3,0)$.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(0, -1)$, $(3,0)$ and $(3,0)$

$3x + y - 9 = 0$ is tangent

$y = -3x + 9 \rightarrow (1)$

$y = mx + c$

$m_1 = -3$

$(x_1, y_1) = (-g, -f)$

$(x_2, y_2) = (3,0)$

$m_2 = \dfrac{0 + f}{3 + g} = \dfrac{f}{3 + g}$

$m_1 m_2 = -1$

$\left(\dfrac{f}{3 + g}\right)(-3) = -1$

$3f = 3 + g$

$g = 3f - 3 \rightarrow (2)$

$(0, -1)$

The ...

$(1) \Rightarrow (0)^2$

$1 + 0 - 2f$

$1 - 2f + c$

$(3,0)$

$(1) \Rightarrow (3)^2$

$9 + 6g +$

$9 + 6g +$

By $(3) - ($

$9 + 6g +$

$8 + 6g +$

$\div$ by 2

$4 + 3(3f$

$4 + 9f -$

$10f - 5$

$(2) \Rightarrow g$

$g = -\dfrac{3}{2}$

$(2) \Rightarrow c$

$c = 2\left(\dfrac{1}{2}\right.$

$(1) \Rightarrow x^2$

$x^2 + y^2$

Q.5 Find

and is ta

Solution

$x^2 + y^2$

$(0,0)$, an

$y - 1 =$

1), (3,0)

$(1) \Rightarrow (0)^2 + (-1)^2 + 2g(0) + 2f(-1) + c = 0$

$1 + 0 - 2f + c = 0$

$1 - 2f + c = 0 \rightarrow (2)$

$(3,0)$

$(1) \Rightarrow (3)^2 + (0)^2 + 2g(3) + 2f(0) + c = 0$

$9 + 6g + 0 + c = 0$

$9 + 6g + c = 0 \rightarrow (3)$

By $(3) - (2)$

$9 + 6g + c - (1 - 2f + c) = 0$

$8 + 6g + 2f = 0$

$\div$ by 2

$4 + 3(3f - 3) + f = 0$

$4 + 9f - 9 + f = 0$

$10f - 5 = 0 \Rightarrow f = \frac{1}{2}$

$(2) \Rightarrow g = 3\left(\frac{1}{2}\right) - 3$

$g = -\frac{3}{2}$

$(2) \Rightarrow c = 2f - 1$

$c = 2\left(\frac{1}{2}\right) - 1 = 0$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2\left(\frac{1}{2}\right)y + 0 = 0$

$x^2 + y^2 - 3x + y = 0$

Q.5 Find the equation of circle through origin with x-intercept 2 and is tangent to the line $y - 1 = 0$.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(0,0)$, and $(2,0)$

$y - 1 = 0$ is tangent to (1)

(0,0)

$(1) \Rightarrow (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$

$c = 0$

(2,0) and $c = 0$

$(1) \Rightarrow (2)^2 + (0)^2 + 2g(2) + 2f(0) + 0 = 0$

$4 + 4g + 0 = 0$

$4 + 4g = 0 \Rightarrow g = -1$

$(x_1, y_1) = (-g, -f)$

$y - 1 = 0$

$$r = \left|\frac{-f-1}{\sqrt{0^2+1^2}}\right|$$

$$\sqrt{g^2 + f^2 - c} = \left|\frac{-(f+1)}{1}\right|$$

$(-1)^2 + f^2 - 0 = (f+1)^2$

$1 + f^2 = f^2 + 2f + 1$

$f = 0$

$(1) \Rightarrow x^2 + y^2 + 2(-1)x + 2(0)y + 0 = 0$

$x^2 + y^2 - 2x + 0 = 0$

$x^2 + y^2 - 2x = 0$

Q.6 Find the equation of circle containing the points (1,2), (2,3) and having centre on $x - y + 1 = 0$.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(1,2) and (2,3)

Since $(-g, -f)$ is on $x - y + 1 = 0$

$-g - (-f) + 1 = 0$

$-g + f + 1 = 0 \rightarrow (2)$

(1,2)

$(1) \Rightarrow (1)^2 + (2)^2 + 2g(1) + 2f(2) + c = 0$

$5 + 2g + 4f + c = 0 \rightarrow (3)$

(2,3)

$(1) \Rightarrow (2)^2 + (3$

$13 + 4g + 6f +$

By (4) − (3)

$13 + 4g + 6f$

$8 + 2g + 2f =$

÷ by 2

$4 + g + f = 0$

Adding (2)

$\Rightarrow 2f + 5 = 0$

$(2) \Rightarrow g = -$

$g = -\frac{3}{2}$

$(3) \Rightarrow 5 + 2($

$c = 8$

$(1) \Rightarrow x^2 + y$

$x^2 + y^2 - 3x$

Q.7 Find the e

vertices (1, −

Solution:

(1, −2), (−5,

$x^2 + y^2 + 2g$

(1, −2)

$(1) \Rightarrow (1)^2 +$

$5 + 2g - 4f$

(−5,2)

$(1) \Rightarrow (-5)^2$

$(2,3)$

$(1) \Rightarrow (2)^2 + (3)^2 + 2g(2) + 2f(3) + c = 0$

$13 + 4g + 6f + c = 0 \rightarrow (4)$

By $(4) - (3)$

$13 + 4g + 6f + c - (5 + 2g + 4f + c) = 0$

$8 + 2g + 2f = 0$

$\div$ by 2

$4 + g + f = 0$

Adding (2)

$\Rightarrow 2f + 5 = 0 \Rightarrow f = -\dfrac{5}{2}$

$(2) \Rightarrow g = -\dfrac{5}{2} + 1$

$g = -\dfrac{3}{2}$

$(3) \Rightarrow 5 + 2\left(-\dfrac{3}{2}\right) + 4\left(-\dfrac{5}{2}\right) + c = 0$

$c = 8$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\dfrac{3}{2}\right)x + 2\left(-\dfrac{5}{2}\right)y + 8 = 0$

$x^2 + y^2 - 3x - 5y + 8 = 0$

Q.7 Find the equation of the circum-circle of the triangle with vertices $(1,-2), (-5,2)$ and $(3,4)$.

Solution:

$(1,-2), (-5,2)$ and $(3,4)$

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(1,-2)$

$(1) \Rightarrow (1)^2 + (-2)^2 + 2g(1) + 2f(-2) + c = 0$

$5 + 2g - 4f + c = 0 \rightarrow (2)$

$(-5,2)$

$(1) \Rightarrow (-5)^2 + (2)^2 + 2g(-5) + 2f(2) + c = 0$

$29 - 10g + 4f + c = 0 \rightarrow (3)$

$(3,4)$

$(1) \Rightarrow (3)^2 + (4)^2 + 2g(3) + 2f(4) + c = 0$

$25 + 6g + 8f + c = 0 \rightarrow (4)$

By $(4) - (2)$

$25 + 6g + 8f + c - (5 + 2g - 4f + c) = 0$

$20 + 4g + 12f = 0$

$\div$ by 4

$5 + g + 3f = 0$

$g = -5 - 3f \rightarrow (4)$

By $(3) - (2)$

$29 - 10g + 4f + c - (5 + 2g - 4f + c) = 0$

$24 - 12g + 8f = 0$

$\div$ by 4

$6 - 3g + 2f = 0$

$6 - 3(-5 - 3f) + 2f = 0$

$11f + 21 = 0$

$f = -\dfrac{21}{11}$

$(4) \Rightarrow g = -5 - 3\left(-\dfrac{21}{11}\right)$

$g = \dfrac{8}{11}$

$(3) \Rightarrow 29 - 10\left(\dfrac{8}{11}\right) + 4\left(-\dfrac{21}{11}\right) + c = 0$

$c = \dfrac{155}{11}$

$(1) \Rightarrow x^2 + y^2 + 2\left(\dfrac{8}{11}\right)x + 2\left(-\dfrac{21}{11}\right)y + \dfrac{155}{11} = 0$

$11x^2 + 11y^2 +$

Q.8 Find the equ

$(1,-2), (3,-4)$

Solution:

$x^2 + y^2 + 2gx$

As (1) touches

$(1,-2)$ and $(3,$

$(1,-2)$ and $c =$

$(1) \Rightarrow (1)^2 + ($

$5 + 2g - 4f +$

$(3,-4)$ and $c =$

$(1) \Rightarrow (3)^2 + ($

$25 + 6g - 8f$

Multiply (2) by

$10 + 4g - 8f$

$g^2 - 2g - 15$

$g^2 - 5g + 3g$

$g(g - 5) + 3($

$(g - 5)(g + 3$

$\boxed{g = -3, 5}$

$(2) \Rightarrow 5 + 2($

$4f = 8 \Rightarrow f =$

$(2) \Rightarrow 5 + 2($

$4f = 40 \Rightarrow f$

$f = 2, g = -3$

$(1) \Rightarrow x^2 + y$

$x^2 + y^2 - 6x$

$11x^2 + 11y^2 + 16x - 42y + 155 = 0$

Q.8 Find the equation of circle containing the points $(1, -2), (3, -4)$ and touching x-axis.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

As (1) touches x-axis so $c = g^2$

$(1, -2)$ and $(3, -4)$

$(1, -2)$ and $c = g^2$

$(1) \Rightarrow (1)^2 + (-2)^2 + 2g(1) + 2f(-2) + g^2 = 0$

$5 + 2g - 4f + g^2 = 0 \rightarrow (2)$

$(3, -4)$ and $c = g^2$

$(1) \Rightarrow (3)^2 + (-4)^2 + 2g(3) + 2f(-4) + g^2 = 0$

$25 + 6g - 8f + g^2 = 0 \rightarrow (3)$

Multiply (2) by 2 and subtract from (3)

$10 + 4g - 8f + 2g^2 - (25 + 6g - 8f + g^2) = 0$

$g^2 - 2g - 15 = 0$

$g^2 - 5g + 3g - 15 = 0$

$g(g - 5) + 3(g - 5) = 0$

$(g - 5)(g + 3) = 0$

$\boxed{g = -3, 5}$

$(2) \Rightarrow 5 + 2(-3) - 4f + (-3)^2 = 0$

$4f = 8 \Rightarrow f = 2, g = -3$

$(2) \Rightarrow 5 + 2(5) - 4f + (5)^2 = 0$

$4f = 40 \Rightarrow f = 10$

$f = 2, g = -3, c = (-3)^2 = 9$

$(1) \Rightarrow x^2 + y^2 + 2(-3)x + 2(2)y + 9 = 0$

$x^2 + y^2 - 6x + 4y + 9 = 0$

$f = 10, g = 5, c = (5)^2 = 25$

$(1) \Rightarrow x^2 + y^2 + 2(5)x + 2(10)y + 25 = 0$

$x^2 + y^2 + 10x + 20y + 25 = 0$

Q.9 Find the equation of circle containing the points $(6,0)$ and touching the line $x = y$ at $(4,4)$.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \to (1)$

$y = x$ is tangent to (1)

$m_1 = 1$

$(x_1, y_1) = (-g, -f)$

$(x_2, y_2) = (4,4)$

$m_2 = \dfrac{4+f}{4+g}$

$m_1 m_2 = -1$

$\left(\dfrac{4+f}{4+g}\right)(1) = -1$

$4 + f = -4 - g$

$g = -8 - f \to (2)$

$(6,0)$

$(1) \Rightarrow (6)^2 + (0)^2 + 2g(6) + 2f(0) + c = 0$

$36 + 12g + 0 + c = 0$

$36 + 12g + c = 0 \to (3)$

$(4,4)$

$(1) \Rightarrow (4)^2 + (4)^2 + 2g(4) + 2f(4) + c = 0$

$32 + 8g + 8f + c = 0 \to (4)$

By $(4) - (3)$

$32 + 8g + 8f + c - (36 + 12g + c) = 0$

$-4 - 4g + 8f = 0$

$\div$ by 4

$-1 - (-8 - f) + 2f = 0$

The Students'

$f = -\dfrac{7}{3}$

$(2) \Rightarrow g = -$

$(3) \Rightarrow 36 +$

$c = 32$

$(1) \Rightarrow x^2 +$

$3x^2 + 3y^2$

Q.10 Show t

(i) touching

(ii) touching

Solution:

(i) touching

Centre$(-g$

$(1) \Rightarrow x^2 +$

$x^2 + y^2 +$

$x^2 + y^2 +$

(ii) touchin

Centre$(-g$

$(1) \Rightarrow x^2$

$x^2 + y^2 +$

$x^2 + y^2 +$

Q.11 Find

intercepts

Solution:

$x^2 + y^2 +$

$(0,0), (6,0$

(6,0) and

$f = -\frac{7}{3}$

$(2) \Rightarrow g = -8 - \left(-\frac{7}{3}\right) = -\frac{17}{3}$

$(3) \Rightarrow 36 + 12\left(-\frac{17}{3}\right) + c = 0$

$c = 32$

$(1) \Rightarrow x^2 + y^2 + 2\left(-\frac{17}{3}\right)x + 2\left(-\frac{7}{3}\right)y + 32 = 0$

$3x^2 + 3y^2 - 34x - 14y + 96 = 0$

Q.10 Show that the equation of circle with centre $(-g, -f)$ and:
(i) touching x-axis is of the form $x^2 + y^2 + 2gx + g^2 = 0$
(ii) touching y-axis is of the form $x^2 + y^2 + 2fy + f^2 = 0$

Solution:

(i) touching x-axis is of the form $x^2 + y^2 + 2gx + c = 0$

Centre$(-g, -f) = (-g, 0)$

$(1) \Rightarrow x^2 + y^2 + 2gx + 2(0)y + c = 0$

$x^2 + y^2 + 2gx + 0 + c = 0$

$x^2 + y^2 + 2gx + c = 0$

(ii) touching y-axis is of the form $x^2 + y^2 + 2fy + c = 0$

Centre$(-g, -f) = (0, -f)$

$(1) \Rightarrow x^2 + y^2 + 2(0)x + 2fy + c = 0$

$x^2 + y^2 + 0 + 2fy + c = 0$

$x^2 + y^2 + 2fy + c = 0$

Q.11 Find the equation of circle through the origin and having intercepts 6 and 8.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(0,0), (6,0)$ and $(0,8)$

$(0,0)$

$(1) \Rightarrow (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$

$c = 0$

$(6,0)$ and $c = 0$

$(1) \Rightarrow (6)^2 + (0)^2 + 2g(6) + 2f(0) + 0 = 0$

$36 + 0 + 12g + 0 = 0$

$12g = -36$

$g = -3$

$(0,8)$ and $c = 0$

$(1) \Rightarrow (0)^2 + (8)^2 + 2g(0) + 2f(8) + 0 = 0$

$0 + 64 + 0 + 16f = 0$

$16f = -64 \Rightarrow f = -4$

$(1) \Rightarrow x^2 + y^2 + 2(-3)x + 2(-4)y + 0 = 0$

$x^2 + y^2 - 6x - 8y = 0$

Q.12 Find the equation of circle through the two points $(b, 0)$ and $(-b, 0)$ and whose radius is a unit.

Solution:

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

$(b, 0)$ and $(-b, 0)$

$(b, 0)$

$(1) \Rightarrow (b)^2 + (0)^2 + 2g(b) + 2f(0) + c = 0$

$b^2 + 0 + 2gb + 0 + c = 0$

$b^2 + 2gb + c = 0 \rightarrow (1)$

$(-b, 0)$

$(1) \Rightarrow (-b)^2 + (0)^2 + 2g(-b) + 2f(0) + c = 0$

$b^2 + 0 - 2gb + 0 + c = 0$

$b^2 - 2gb + c = 0 \rightarrow (2)$

By $(1) + (2)$

$2b^2 + 2c = 0$

$c = -b^2$

$(1) \Rightarrow b^2 + 2gb$ ...

$2gb = 0$

$g = 0$

$r^2 = g^2 + f^2$ ...

$r = a$ units

$a^2 = 0 + f^2 -$ ...

$a^2 = f^2 + b^2$

$f^2 = a^2 - b^2$

$f = \pm\sqrt{a^2 - b}$ ...

$(1) \Rightarrow x^2 + y^2$ ...

$x^2 + y^2 + 0 \pm$ ...

$x^2 + y^2 \pm 2\sqrt{a}$ ...

Q.13 Find the e...

(5,0) and (−5, ...

Solution:

As circle passes ...

and B(−5,0) a...

units then AB i...

midpoint of AB ...

circle.

$(x_1, y_1) = A(5,$ ...

$(x_2, y_2) = B(-$ ...

Midpoint $= \left(\frac{x}{-}\right.$ ...

$(a, b) = \left(\frac{-5+}{2}\right.$ ...

$(a, b) = (0,0)$

$(x - a)^2 + (y -$ ...

$c = -b^2$

$(1) \Rightarrow b^2 + 2gb - b^2 = 0$
$2gb = 0$
$g = 0$

$r^2 = g^2 + f^2 - c$
$r = a$ units
$a^2 = 0 + f^2 - (-b^2)$
$a^2 = f^2 + b^2$
$f^2 = a^2 - b^2$
$f = \pm\sqrt{a^2 - b^2}$

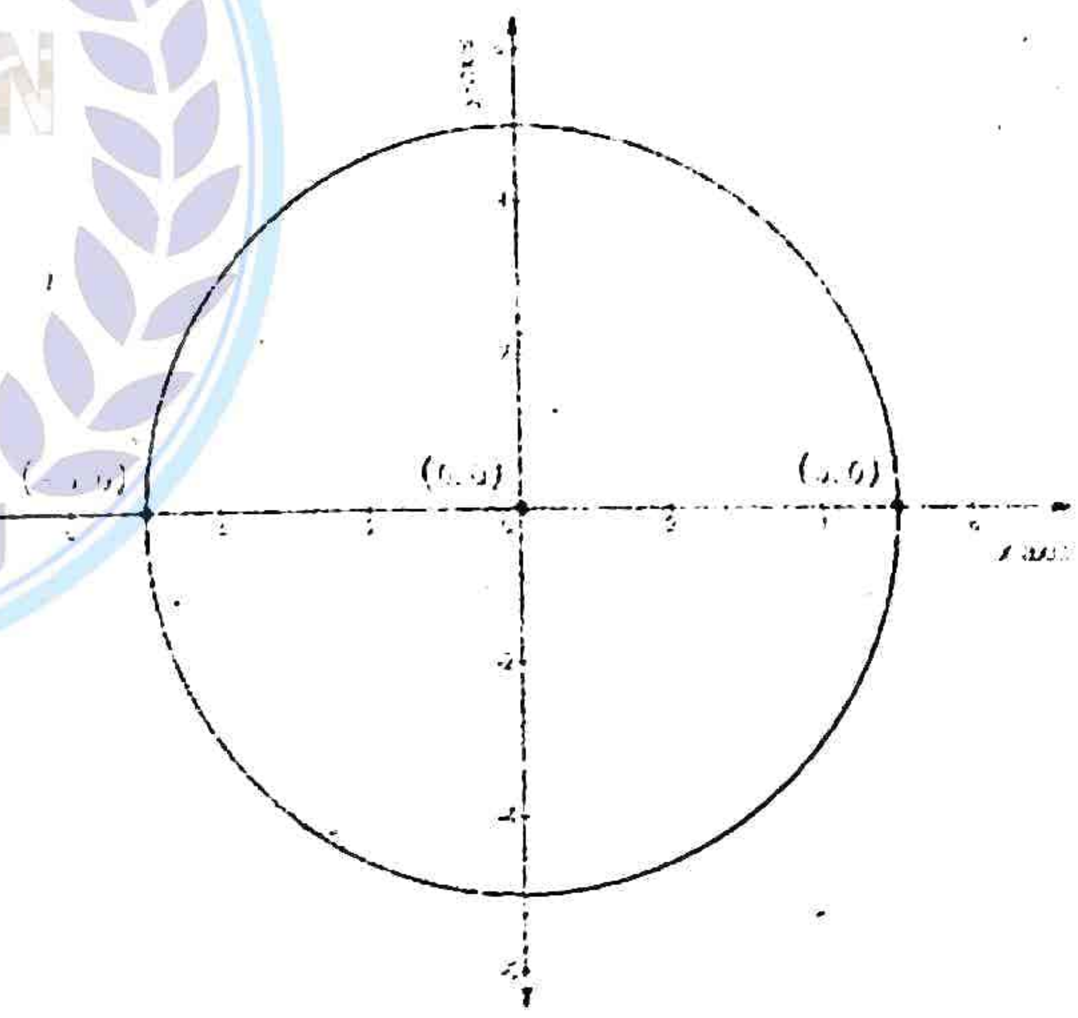
$(1) \Rightarrow x^2 + y^2 + 2(0)x + 2\left(\pm\sqrt{a^2 - b^2}\right)y - b^2 = 0$
$x^2 + y^2 + 0 \pm 2\sqrt{a^2 - b^2}y - b^2 = 0$
$x^2 + y^2 \pm 2\sqrt{a^2 - b^2}y - b^2 = 0$

$(b, 0)$ and

Q.13 Find the equation of circle which passes through the point $(5,0)$ and $(-5,0)$ and whose radius is 5 unit.

Solution:

As circle passes through $A(5,0)$ and $B(-5,0)$ and radius is 5 units then AB is diameter and midpoint of AB is centre of circle.



$(x_1, y_1) = A(5,0)$
$(x_2, y_2) = B(-5,0)$
$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$(a, b) = \left(\frac{-5 + 5}{2}, \frac{0 + 0}{2}\right)$
$(a, b) = (0,0)$
$(x - a)^2 + (y - b)^2 = r^2$

$(x-0)^2+(y-0)^2=5^2$

$x^2+y^2=25$

If question is

Q.13 Find the equation of circle which passes through the point (5,0) and (0, −5) and whose radius is 5 unit.

Solution:

$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

(5,0) and (0, −5)

(5,0)

$(1) \Rightarrow (5)^2+(0)^2+2g(5)+2f(0)+c=0$

$25+0+10g+0+c=0$

$25+10g+c=0 \rightarrow (2)$

(0, −5)

$(1) \Rightarrow (0)^2+(-5)^2+2g(0)+2f(-5)+c=0$

$25+0-10f+0+c=0$

$25-10f+c=0 \rightarrow (3)$

By (3) − (1)

$25-10f+c-(25+10g+c)=0$

$-10f-10g=0$

$f=-g$

$5^2=g^2+f^2-c$

$25-25+10f=2g^2$

$10f=2g^2$

$5f=f^2$

$f=0,5$

$g=0,-5$

$(1) \Rightarrow x^2+y^2+2(0)x+2(0)y-25=0$

$x^2+y^2-25=0$

$f=5$

$g=-5$

$(2) \Rightarrow 25+10(-$

$c=25$

$(1) \Rightarrow x^2+y^2+$

x^2+y^2-10x+

Q.1 Check wheth

neither to the cir

(i) $y=x+3$ (

Solution:

$x^2+y^2=25 \rightarrow$

(i) $y=x+3$

$(1) \Rightarrow x^2+(x$

$x^2+(x^2+2x$

$2x^2+2x-16$

÷ by 2

$x^2+x-8=0$

$B^2-4AC=(1$

$=33>0$

Line cuts circle

(ii) $y=\sqrt{3}x+$

$(1) \Rightarrow x^2+(\sqrt{}$

$x^2+(3x^2+2$

$4x^2+20\sqrt{3}x$

$B^2-4AC=($

$=400(3)-1$

$=0$

Line is tangent

he point

$f = 5$

$g = -5$

$(2) \Rightarrow 25 + 10(-5) + c = 0$

$c = 25$

$(1) \Rightarrow x^2 + y^2 + 2(-5)x + 2(5)y + 25 = 0$

$x^2 + y^2 - 10x + 10y + 25 = 0$

EXERCISE 8.3

Q.1 Check whether the following lines are tangent; secant or neither to the circle $x^2 + y^2 = 25$.

(i) $y = x + 3$ (ii) $y = \sqrt{3}x + 10$ (iii) $y = 2x + 15$

Solution:

$x^2 + y^2 = 25 \rightarrow (1)$

(i) $y = x + 3$

$(1) \Rightarrow x^2 + (x + 3)^2 = 25$

$x^2 + (x^2 + 2x + 9) = 25$

$2x^2 + 2x - 16 = 0$

÷ by 2

$x^2 + x - 8 = 0$

$B^2 - 4AC = (1)^2 - 4(1)(-8)$

$= 33 > 0$

Line cuts circle at two points i.e. secant line

(ii) $y = \sqrt{3}x + 10$

$(1) \Rightarrow x^2 + \left(\sqrt{3}x + 10\right)^2 = 25$

$x^2 + \left(3x^2 + 20\sqrt{3}x + 100\right) - 25 = 0$

$4x^2 + 20\sqrt{3}x + 75 = 0$

$B^2 - 4AC = \left(20\sqrt{3}\right)^2 - 4(4)(75)$

$= 400(3) - 1200$

$= 0$

Line is tangent to circle

(iii) $y = 2x + 15$

$(1) \Rightarrow x^2 + (2x + 15)^2 = 25$

$x^2 + (4x^2 + 60x + 225) = 25$

$5x^2 + 60x + 225 - 25 = 0$

$5x^2 + 60x + 200 = 0$

$\div$ by 5

$x^2 + 12x + 40 = 0$

$B^2 - 4AC = (12)^2 - 4(1)(40)$

$= 144 - 160$

$= -16 > 0$

Line does not touch circle i.e. neither tangent nor secant

Q.2 Find the condition of tangency and secancy of the line $y = 2x + k$ with the circle $x^2 + y^2 + 10x + 20y + c = 0$.

Solution:

$x^2 + y^2 + 10x + 20y + c = 0 \rightarrow (1)$

$y = 2x + k$

$(1) \Rightarrow x^2 + (2x + k)^2 + 10x + 20(2x + k) + c = 0$

$x^2 + (4x^2 + 4kx + k^2) + 10x + (40x + 20k) + c = 0$

$5x^2 + 4kx + 50x + k^2 + 20k + c = 0$

$5x^2 + 2x(2k + 25) + k^2 + 20k + c = 0 \rightarrow (1)$

$B^2 - 4AC = \{2(2k + 25)\}^2 - 4(5)(k^2 + 20k + c)$

$= 4(4k^2 + 100k + 625) - 4(5)(k^2 + 20k + c)$

$= 4\{(4k^2 + 100k + 625) - 5(k^2 + 20k + c)\}$

$= 4\{4k^2 + 100k + 625 - 5k^2 - 100k - 5c\}$

$= 4\{625 - k^2 - 5c\}$

(i) If line is tangent, then roots of (1) are equal

$B^2 - 4AC = 0$

$4\{625 - k^2 - 5c\} = 0$

$625 - k^2 - 5c = 0$

$k^2 = 625 - 5c$

(ii) If line is secant, then roots of (1) are real and different

$B^2 - 4AC > 0$

$4\{625 - k^2 - 5c\} > 0$

$625 - 5c > k^2$

$k^2 < 625 - 5c$

Q.3 Find the equation of tangent to $x^2 + y^2 = 36$ with the slope $\sqrt{3}$.

Solution:

$x^2 + y^2 = 36$

Comparing with $x^2 + y^2 = r^2$

$r^2 = 36$

Let $y = mx + c$ be the tangent to the circle

Slope of tangent line $= m = \sqrt{3}$

$c^2 = r^2(1 + m^2)$

$c^2 = 36\left(1 + \left(\sqrt{3}\right)^2\right)$

$c^2 = 36(4)$

$c = \pm 12$

$y = \sqrt{3}x \pm 12$

Q.4 Find the equation of tangent and normal:

(i) at $(1, -4)$ to the circle $x^2 + y^2 = 17$

(ii) at $(4,1)$ to the circle $x^2 + y^2 - 4x + 2y = 3$

Solution:

(i) at $(1, -4)$ to the circle $x^2 + y^2 = 17 \rightarrow (1)$

$\because (1, -4)$ lies on circle

Differentiate (1) w.r.t "x"

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(17)$$

$2x + 2yy' = 0$

$2yy' = -2x$

$$y' = -\frac{x}{y}$$

Slope of tangent line at $(1,-4) = -\frac{1}{-4} = \frac{1}{4}$

Slope of normal line at $(1,-4) = -4$

$(x_1, y_1) = (1,-4)$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - (-4) = \frac{1}{4}(x - 1)$

$4(y + 4) = x - 1$

$4y + 16 = x - 1$

$x - 4y = 17$

Equation of tangent

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - (-4) = -4(x - 1)$

$y + 4 = -4x + 4$

$4x + y = 0$

(ii) at (4,1) to the circle $x^2 + y^2 - 4x + 2y = 3$

$\because$ (4,1) lies on circle

Differentiate (1) w.r.t "x"

$\frac{d}{dx}(x^2 + y^2 - 4x + 2y) = \frac{d}{dx}(3)$

$2x + 2yy' - 4 + 2y' = 0$

$\div$ by 2

$y'(y + 1) = 2 - x$

$y' = \frac{2 - x}{y + 1}$

Slope of tangent at $(4,1) = \frac{2-4}{1+1} = -1$

Slope of normal at $(4,1) = 1$

$(x_1, y_1) = (4,1)$

Equati
$y - y_1 =$
$y - 1 =$
$y - 1 =$
$x + y =$

Equatio

$y - y_1$
$y - 1 =$
$x - y =$

Q.5 Fin
(i) from
(ii) fron

Solutio

$L = \sqrt{(}$

(i) from
$x^2 + y$
$(x_1, y_1$
$L = \sqrt{}$

(ii) fro
$x^2 + y$
$(x_1, y_1$
$L = \sqrt{}$

Q.6 Fi
to the

Solutic

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 4)$

$y - 1 = -x + 4$

$x + y = 5$

Equation of normal

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 1 = 1(x - 4)$

$x - y = 3$

Q.5 Find the length of tangent:

(i) from (6,1) to the circle $x^2 + y^2 = 4$

(ii) from (2,5) to the circle $x^2 + y^2 + 8x - 5y = 7$

Solution:

$L = \sqrt{(x_1)^2 + (y_1)^2 + 2gx_1 + 2fy_1 + c}$

(i) from (6,1) to the circle $x^2 + y^2 = 4$

$x^2 + y^2 - 4 = 0$

$(x_1, y_1) = (6,1)$

$L = \sqrt{(6)^2 + (1)^2 - 4} = \sqrt{33}$ units

(ii) from (2,5) to the circle $x^2 + y^2 + 8x - 5y = 7$

$x^2 + y^2 + 8x - 5y - 7 = 0$

$(x_1, y_1) = (2,5)$

$L = \sqrt{(2)^2 + (5)^2 + 8(2) - 5(5) - 7} = \sqrt{13}$ units

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Q.6 Find the condition that the line $y = mx + c$ may be tangnent to the circle $(x - h)^2 + (y - k)^2 = r^2$.

Solution:

$$x^2 + y^2 = r^2$$
$$y = mx + c$$
Condition: $c^2 = r^2(1 + m^2)$

$y = mx + c \rightarrow (1)$

$(x - h)^2 + (y - k)^2 = r^2 \rightarrow (2)$

Replace $x \rightarrow x + h$ and $y \rightarrow y + k$ in (1) and (2)

$(2) \Rightarrow (x + h - h)^2 + (y + k - k)^2 = r^2$

$x^2 + y^2 = r^2$

$(1) \Rightarrow y + k = m(x + h) + c$

$y = mx + mh + c - k$

Comparing with $y = Mx + C$

$C^2 = r^2(1 + M^2)$

$(mh + c - k)^2 = r^2(1 + m^2)$

Q.7 Show that circles $x^2 + y^2 - 6x - 6y + 10 = 0$ and $x^2 + y^2 = 2$ touch each other and find the point of contact.

Solution:

$x^2 + y^2 - 6x - 6y + 10 = 0 \rightarrow (1)$

$x^2 + y^2 = 2 \rightarrow (2)$

$(1) \Rightarrow x^2 - 6x + y^2 - 6y + 10 = 0$

$x^2 - 2(x)(3) + y^2 - 2(y)(3) = -10$

$x^2 - 2(x)(3) + 3^2 + y^2 - 2(y)(3) + 3^2 = -10 + 9 + 9$

$(x - 3)^2 + (y - 3)^2 = 8$

Comparing with $(x - a)^2 + (y - b)^2 = r^2$

Centre $(a, b) = C_1(3,3), r = \sqrt{8} = 2\sqrt{2}$

$(2) \Rightarrow x^2 + y^2 = 2$

Comparing with $(x - a)^2 + (y - b)^2 = r^2$

Centre $(a, b) = C_2(0,0), r = \sqrt{2}$

$|r_1 + r_2| = |2\sqrt{2} + \sqrt{2}| = |3\sqrt{2}| = 3\sqrt{2}$

$|C_1C_2|$ ...
$|C_1C_2| =$ ...

By (1) − ...
$-6x - 6$...
÷ by −6 ...
$x + y -$ ...
$y = 2 -$ ...

$(2) \Rightarrow x^2$ ...
$x^2 + (4$ ...
$2x^2 - 4$...
$x^2 - 2x$ ...
$(x - 1)^2$ ...
$x - 1 =$ ...
$x = 1$
$y = 2 -$ ...
$y = 1$
Point = ...

Q.8 Find ...
(i) at the ...
(ii) at th...
(iii) whic...
(iv) whic...

Solution

$x^2 + y^2$ ...
Differen...
$\frac{d}{dx}(x^2$ ...
$2x + 2y$ ...
$y' = m$ ...
(i) at the ...

$x^2 + y^2 =$

$|C_1C_2| = \sqrt{(3-0)^2 + (3-0)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{2(3)^2}$

$|C_1C_2| = 3\sqrt{2}$

By (1) – (2)

$-6x - 6y + 12 = 0$

$\div$ by -6

$x + y - 2 = 0$

$y = 2 - x \rightarrow (3)$

$(2) \Rightarrow x^2 + (2-x)^2 = 2$

$x^2 + (4 - 4x + x^2) = 2$

$2x^2 - 4x + 2 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

$y = 2 - 1 \rightarrow (3)$

$y = 1$

Point $= (1,1)$

Q.8 Find the equation of tangent(s) to the circle $x^2 + y^2 = 25$.

(i) at the point whose abscissa is 3.

(ii) at the point whose ordinate is -4.

(iii) which is parallel to $3x + 4y + 1 = 0$.

9 (iv) which is perpendicular to $3x + 4y + 1 = 0$.

Solution:

$x^2 + y^2 = 25 \rightarrow (1)$

Differentiate (1) w.r.t "x"

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$2x + 2yy' = 0$

$$y' = m = -\frac{x}{y}$$

(i) at the point whose abscissa is 3.

$(1) \Rightarrow 3^2 + y^2 = 25$

$y^2 = 16$

$y = \pm 4$

Points are (3,4) and (3, −4)

m at $(3,4) = -\frac{3}{4}$

m at $(3,-4) = -\frac{3}{-4} = \frac{3}{4}$

Equation of tangent

$(x_1, y_1) = (3,4)$

$y - 4 = -\frac{3}{4}(x - 3)$

$4y - 16 = -3x + 9$

$3x + 4y = 25$

$(x_1, y_1) = (3,-4)$

$y + 4 = \frac{3}{4}(x - 3)$

$4y + 16 = 3x - 9$

$3x - 4y = 25$

(ii) at the point whose ordinate is −4.

i.e. $y = -4$

$(1) \Rightarrow x^2 + (-4)^2 = 25$

$x^2 = 9$

$x = \pm 3$

Points are (3, −4) and (−3, −4)

m at $(3,-4) = -\frac{3}{-4} = \frac{3}{4}$

m at $(3,-4) = -\left(\frac{-3}{-4}\right) = -\frac{3}{4}$

Equation of tangent

$(x_1, y_1) = (3,-4)$

$y + 4 = \frac{3}{4}(x$

$4y + 16 = 3x$

$3x - 4y = 25$

$(x_1, y_1) = (-$

$y + 4 = -\frac{3}{4}($

$4y + 16 = -$

$3x + 4y + 2$

Q.9 Find the
through orig

Solution:

$x^2 + y^2 - 6$

The equation

$(1) \Rightarrow x^2 +$

$x^2 + m^2x^2$

$x^2(1 + m^2)$

$B^2 - 4AC =$

$\{-2(3 + m)$

$4(9 + 6m +$

÷ by 4

$9 + 6m + m$

$8m^2 - 6m$

$2m(4m - 3$

$m = 0, \frac{3}{4}$

$y = 0(x)$

$y = 0$

Or

$y = \frac{3}{4}x$

$4y = 3x$

$y + 4 = \frac{3}{4}(x - 3)$

$4y + 16 = 3x - 9$

$3x - 4y = 25$

$(x_1, y_1) = (-3, -4)$

$y + 4 = -\frac{3}{4}(x + 3)$

$4y + 16 = -3x - 9$

$3x + 4y + 25 = 0$

Q.9 Find the equations of tangents to $x^2 + y^2 - 6x - 2y + 9 = 0$ through origin. Find also their respective point of contact.

Solution:

$x^2 + y^2 - 6x - 2y + 9 = 0 \rightarrow (1)$

The equation of tangent through origin is $y = mx$

$(1) \Rightarrow x^2 + (mx)^2 - 6x - 2(mx) + 9 = 0$

$x^2 + m^2x^2 - 6x - 2mx + 9 = 0$

$x^2(1 + m^2) - 2x(3 + m) + 9 = 0$

$B^2 - 4AC = 0$

$\{-2(3 + m)\}^2 - 4(1 + m^2)(9) = 0$

$4(9 + 6m + m^2) - 4(1 + m^2)(9) = 0$

÷ by 4

$9 + 6m + m^2 - 9 - 9m^2 = 0$

$8m^2 - 6m = 0$

$2m(4m - 3) = 0$

$m = 0, \frac{3}{4}$

$y = 0(x)$

$y = 0$

Or

$y = \frac{3}{4}x$

$4y = 3x$

Solving $y = 0$ and (1)

$(1) \Rightarrow x^2 + (0)^2 - 6x - 2(0) + 9 = 0$

$x^2 + 0 - 6x - 0 + 9 = 0$

$x^2 - 6x + 9 = 0$

$(x)^2 - 2(x)(3) + (3)^2 = 0$

$(x - 3)^2 = 0$

$x - 3 = 0$

$x = 3$

Point $= (3,0)$

Solving $y = \frac{3}{4}x$ and (1)

$(1) \Rightarrow x^2 + \left(\frac{3}{4}x\right)^2 - 6x - 2\left(\frac{3}{4}x\right) + 9 = 0$

$x^2 + \frac{9x^2}{16} - 6x - \frac{3}{2}x + 9 = 0$

× by 16

$16x^2 + 9x^2 - 6(16)x - 3(8)x + 9(16) = 0$

$25x^2 - 120x + 144 = 0$

$(5x)^2 - 2(5x)(12) + (12)^2 = 0$

$(5x - 12)^2 = 0$

$5x - 12 = 0$

$5x = 12$

$x = 12$

$x = \frac{12}{5}$

$\boxed{y = \frac{3}{4}x}$

$y = \frac{3}{4}\left(\frac{12}{5}\right)$

$\boxed{y = \frac{9}{5}}$

The Studer

Point = (

Q.10 Shov

circle x^2 -

Solution:

$x^2 + y^2$ -

Comparir

$g = l, f$

Centre of

$ax + by$

$a(-l) +$

$-al - b$

$0 = 0$

Hence (

Q.11 Fin

the poin

$2gx + 2$

Solution

$y = mx$

$x^2 + y^2$

$x^2 + (m$

$x^2 + m$

$x^2(1 +$

Produc

$= \frac{c}{1 + m}$

$(1) \Rightarrow x$

$(2) \Rightarrow ($

$\text{Point} = \left(\frac{12}{5}, \frac{9}{5}\right)$

Q.10 Show that the line $ax + by + al + bm = 0$ is normal to the circle $x^2 + y^2 + 2lx + 2my + c = 0$ for all value of a and b.

Solution:

$x^2 + y^2 + 2lx + 2my + c = 0 \rightarrow (1)$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$

$g = l, f = m$

Centre of $(1) = (-l, -m)$

$ax + by + al + bm = 0 \rightarrow (2)$

$a(-l) + b(-m) + al + bm = 0$

$-al - bm + al + bm = 0$

$0 = 0$

Hence (2) is normal to (1).

Q.11 Find (i) the product of abscissa (ii) the product of ordinates of the points, where the line $y = mx$ meets the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Solution:

$y = mx \rightarrow (1)$

$x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (2)$

$x^2 + (mx)^2 + 2gx + 2f(mx) + c = 0$

$x^2 + m^2x^2 + 2gx + 2fmx + c = 0$

$x^2(1 + m^2) + 2x(g + fm) + c = 0$

Product of abscissa $= \frac{C}{A}$

$= \frac{c}{1 + m^2}$

$(1) \Rightarrow x = \frac{y}{m}$

$(2) \Rightarrow \left(\frac{y}{m}\right)^2 + y^2 + 2g\left(\frac{y}{m}\right) + 2fy + c = 0$

$$\frac{y^2}{m^2} + y^2 + \frac{2gy}{m} + 2fy + c = 0$$

$\times$ by m^2

$$y^2 + m^2y^2 + 2gym + 2fm^2y + m^2c = 0$$

$$y^2(1 + m^2) + 2ym(g + fm) + m^2c = 0$$

$$\text{Product of abscissa } = \frac{C}{A}$$

$$= \frac{m^2c}{1 + m^2}$$

Q.12 Prove that the line $y = x + k\sqrt{2}$ touches the circle $x^2 + y^2 = k^2$ and find its point of contact.

Solution:

$$x^2 + y^2 = k^2 \rightarrow (1)$$

$$x^2 + y^2 = r^2$$

$$y = x + k\sqrt{2} \rightarrow (2)$$

$$y = mx + c$$

$$m = 1, c = k\sqrt{2}$$

$$c^2 = r^2(1 + m^2)$$

$$\left(k\sqrt{2}\right)^2 = k^2(1 + 1^2)$$

$$2k^2 = 2k^2$$

Proved

$$(1) \Rightarrow x^2 + \left(x + k\sqrt{2}\right)^2 = k^2$$

$$x^2 + \left(x^2 + 2\sqrt{2}kx + 2k^2\right) - k^2 = 0$$

$$2x^2 + 2\sqrt{2}kx + k^2 = 0$$

$$\left(\sqrt{2}x\right)^2 + 2\left(\sqrt{2}x\right)(k) + (k)^2 = 0$$

$$\left(\sqrt{2}x + k\right)^2 = 0$$

$$\sqrt{2}x + k = 0$$

$$\sqrt{2}x = -k$$

The Student

$x = -\frac{k}{\sqrt{2}}$

$x = -\frac{k}{\sqrt{2}}($

$x = -\frac{\sqrt{2}k}{2}$

$(2) \Rightarrow y =$

$y = \frac{-\sqrt{2}k}{}$

$y = \frac{\sqrt{2}k}{2}$

Point = (

Q.13 Find
circle $x^2 +$

Solution:

$3x + 4y =$

$4y = c -$

$y = \frac{c - 3}{4}$

$x^2 + y^2 =$

$x^2 + \left(\frac{c -}{}\right.$

$x^2 + \frac{c^2 -}{}$

$\times$ by 16

$16x^2 + c^2$

$25x^2 - 6$

$25x^2 - 2$

$x = -\frac{k}{\sqrt{2}}$

$x = -\frac{k}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\boxed{x = -\frac{\sqrt{2}k}{2}}$

$(2) \Rightarrow y = -\frac{\sqrt{2}k}{2} + k\sqrt{2}$

$y = \frac{-\sqrt{2}k + 2\sqrt{2}k}{2}$

$\boxed{y = \frac{\sqrt{2}k}{2}}$

$\text{Point} = \left(-\frac{\sqrt{2}k}{2}, \frac{\sqrt{2}k}{2}\right)$

Q.13 Find the condition that the line $3x + 4y = c$ may touch the circle $x^2 + y^2 = 8x$.

Solution:

$3x + 4y = c$

$4y = c - 3x$

$y = \frac{c - 3x}{4}$

$x^2 + y^2 = 8x$

$x^2 + \left(\frac{c - 3x}{4}\right)^2 = 8x$

$x^2 + \frac{c^2 - 6xc + 9x^2}{16} = 8x$

× by 16

$16x^2 + c^2 - 6xc + 9x^2 = 128x$

$25x^2 - 6xc - 128x + c^2 = 0$

$25x^2 - 2x(3c + 64) + c^2 = 0$

$B^2 - 4AC = 0$

$\{-2(3c + 64)\}^2 - 4(25)(c^2) = 0$

$4(3c + 64)^2 = 4(25)(c^2)$

$(3c + 64)^2 = 25c^2$

$3c + 64 = \pm 5c$

$3c + 64 = 5c$

$c = 32$

And $3c + 64 = -5c$

$c = -8$

Q.14 Find whether the line $x + y = 2 + \sqrt{2}$ touches the circle $x^2 + y^2 - 2x - 2y - 1 = 0$.

Solution:

$x + y = 2 + \sqrt{2}$

$y = 2 + \sqrt{2} - x \rightarrow (1)$

$x^2 + y^2 - 2x - 2y - 1 = 0$

$x^2 + \left(2 + \sqrt{2} - x\right)^2 - 2x - 2\left(2 + \sqrt{2} - x\right) - 1 = 0$

$x^2 + \left(4 + 2 + x^2 + 4\sqrt{2} - 4x - 2\sqrt{2}x\right) - 2x - 4 - 2\sqrt{2} + 2x - 1 = 0$

$2x^2 - 4x - 2\sqrt{2}x + 2\sqrt{2} + 1 = 0$

$2x^2 - 2x\left(2 + \sqrt{2}\right) + 2\sqrt{2} + 1 = 0$

$B^2 - 4AC = \left\{-2\left(2 + \sqrt{2}\right)\right\}^2 - 4(2)\left(2\sqrt{2} + 1\right)$

$= 4\left(2 + \sqrt{2}\right)^2 - 8\left(2\sqrt{2} + 1\right)$

$= 4\left\{\left(4 + 4\sqrt{2} + 2\right) - 2\left(2\sqrt{2} + 1\right)\right\}$

$= 4\left\{4 + 4\sqrt{2} + 2 - 4\sqrt{2} - 2\right\}$

$= 4(4)$

$= 16 \neq 0$

Hence (1) is NOT tangent to (2)

The S... Mathematics XII

Q.15 Prove that the circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other, if $\frac{1}{f^2} + \frac{1}{g^2} = \frac{1}{c}$.

Solution:

$x^2 + y^2 + 2gx + c = 0 \rightarrow (1)$
$x^2 + y^2 + 2fy + c = 0 \rightarrow (2)$

By $(2) - (2)$

$x^2 + y^2 + 2fy + c - (x^2 + y^2 + 2gx + c) = 0$

$2fy - 2gx = 0$

$fy = gx \Rightarrow y = \frac{gx}{f}$

$(1) \Rightarrow x^2 + \left(\frac{gx}{f}\right)^2 + 2gx + c = 0$

$x^2 + \frac{g^2x^2}{f^2} + 2gx + c = 0$

$\times$ by f^2

$f^2x^2 + g^2x^2 + 2gf^2x + f^2c = 0$

$x^2(f^2 + g^2) + 2gf^2x + f^2c = 0$

$B^2 - 4AC = 0$

$(2gf^2)^2 - 4(f^2 + g^2)(f^2c) = 0$

$4g^2f^4 = 4f^2c(f^2 + g^2)$

$\div$ by $4f^2$

$g^2f^2 = c(f^2 + g^2)$

$\frac{1}{c} = \frac{f^2 + g^2}{g^2f^2}$

$\frac{1}{c} = \frac{f^2}{g^2f^2} + \frac{g^2}{g^2f^2}$

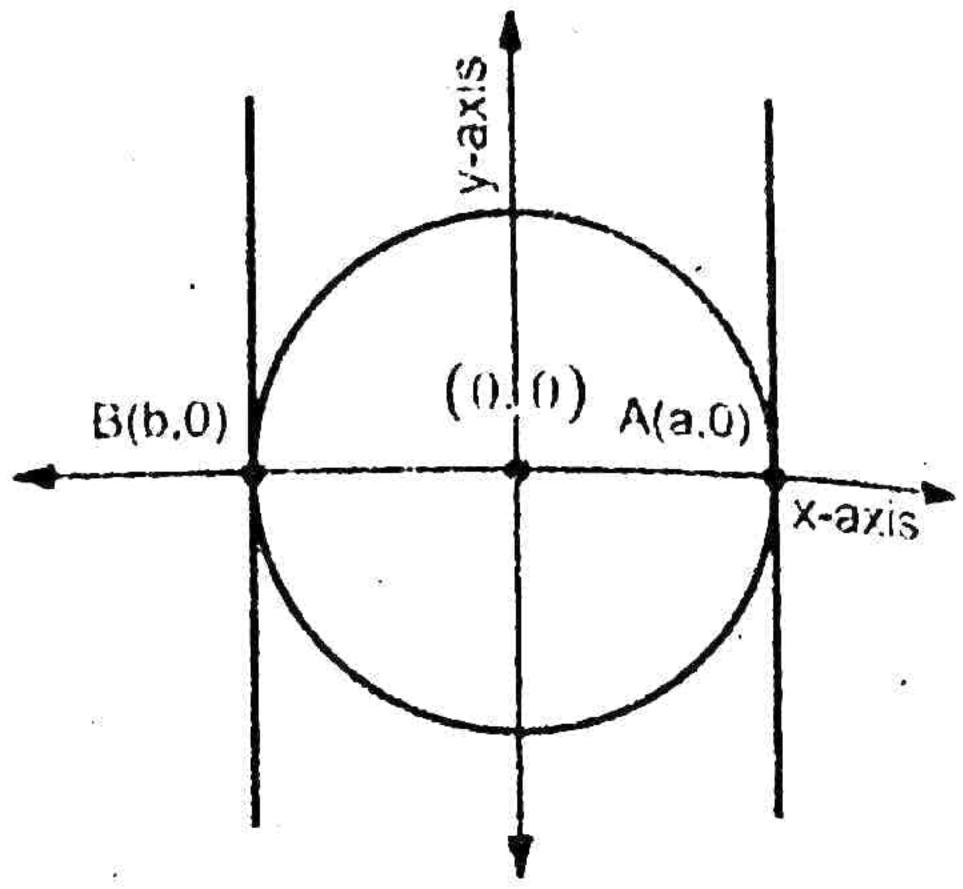
$\frac{1}{c} = \frac{1}{g^2} + \frac{1}{f^2}$

EXERCISE 8.4

Prove the following analytically.

Q.1 The tangents drawn at ends of a diameter of a circle are parallel.

Solution:



$x^2 + y^2 = r^2 \rightarrow (1)$

The equation of tangent to (1) is

$xx_1 + yy_1 = r^2 \rightarrow (2)$

Two points at end of diameter are

$A(a, 0)$ and $B(b, 0)$

Point $= (a, 0)$

$(2) \Rightarrow x(a) + y(0) = r^2$

$ax + 0 = r^2$

$ax = r^2$

$$\text{Slope} = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{a}{0} = \text{undefined}$$

Point $= (b, 0)$

$(2) \Rightarrow x(b) + y(0) = r^2$

$bx + 0 = r^2$

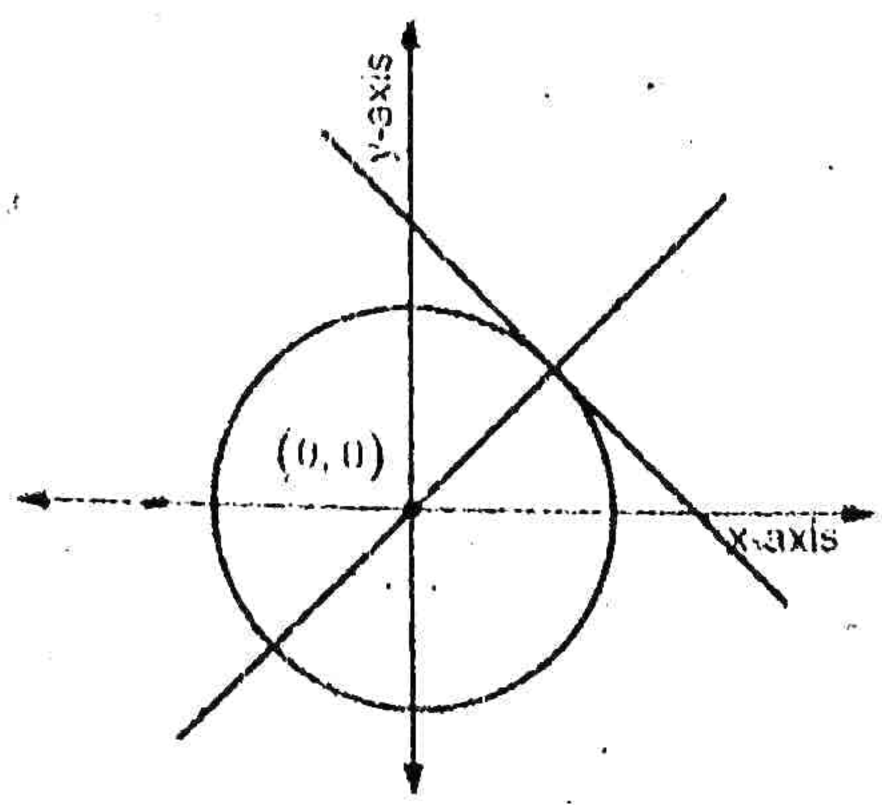
$bx = r^2$

$$\text{Slope} = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{b}{0} = \text{undefined}$$

Since both slopes are the same. Hence, they are parallel.

Q.2 A normal to a circle passes through the centre of circle.

Solution:



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Henre x^2 +

radius r uni

Point on ci

Equation of

Put circle (

$(1) \Rightarrow (0)$

$0 - 0 = 0$

$0 = 0$ Pro

Q.3 The mi

the circle c

Solution:

Let $A(2a,$

be the mid

$C = C\left(\frac{2a}{}\right.$

If O is the

$d = \sqrt{(x_2}$

$\overline{CA} = \sqrt{(2}$

$\overline{CB} = \sqrt{(a}$

Henre $x^2 + y^2 = r^2$ is equation of circle with centre at $(0,0)$ and radius r units

Point on circle $= (x_1, y_1)$

Equation of normal is $x_1 y - x y_1 = 0 \rightarrow (1)$

Put circle $(0,0)$ in (1)

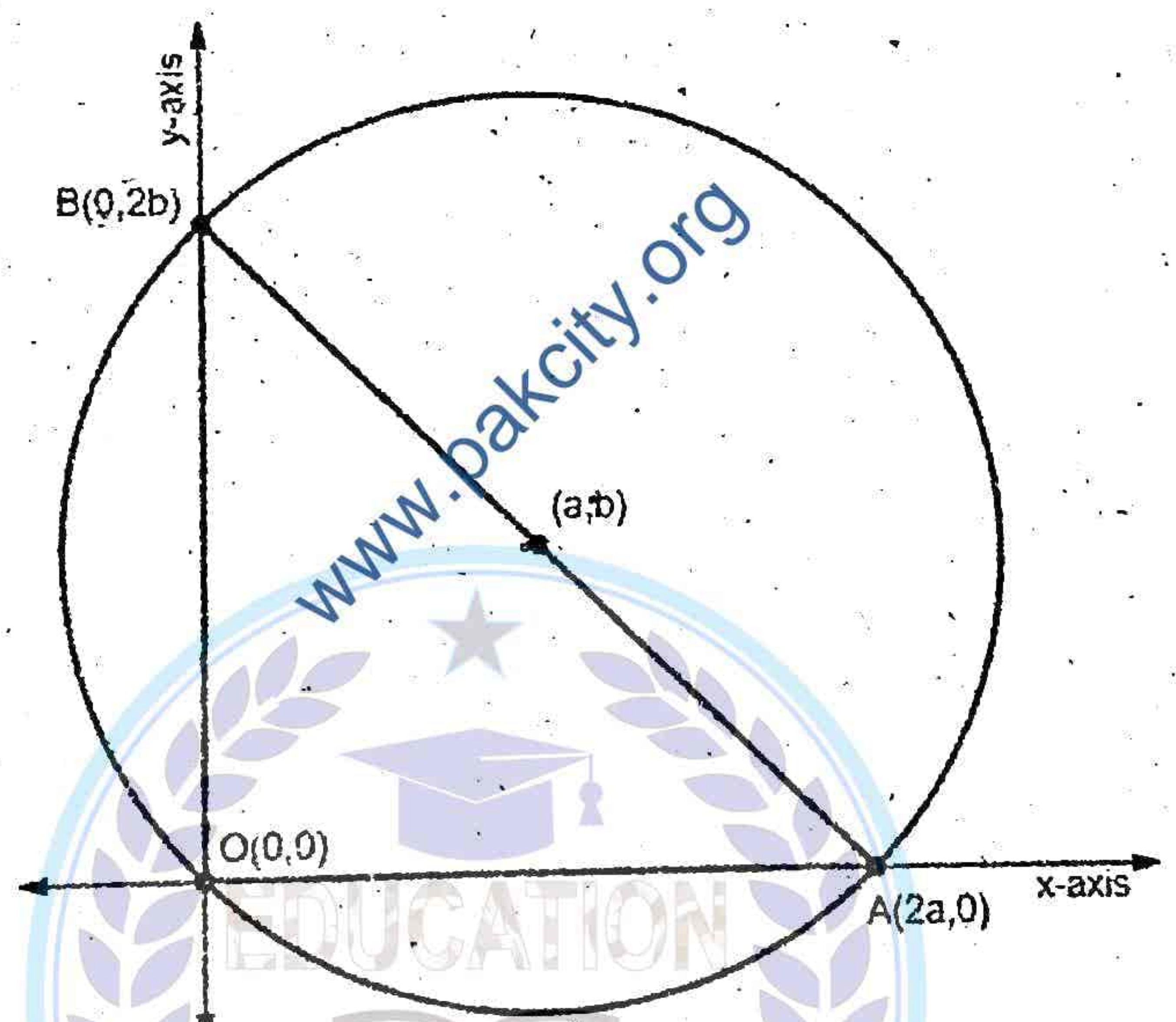
$(1) \Rightarrow (0)y - x(0) = 0$

$0 - 0 = 0$

$0 = 0$ Proved.

Q.3 The mid-point of hypotenuse of a right triangle is the centre of the circle circumscribing the triangle.

Solution:



Let $A(2a, 0), B(0,2b), O(0,0)$ be the vertices of right triangle and C be the midpoint of hypotenuse.

$$C = C\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right) = C(a, b)$$

If O is the centre of the circum-circle, then $\overline{CA} = \overline{CB} = \overline{OC}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\overline{CA} = \sqrt{(2a - a)^2 + (b - 0)^2} = \sqrt{(a)^2 + (b)^2} = \sqrt{a^2 + b^2}$$

$$\overline{CB} = \sqrt{(a - 0)^2 + (b - 2b)^2} = \sqrt{(a)^2 + (-b)^2} = \sqrt{a^2 + b^2}$$

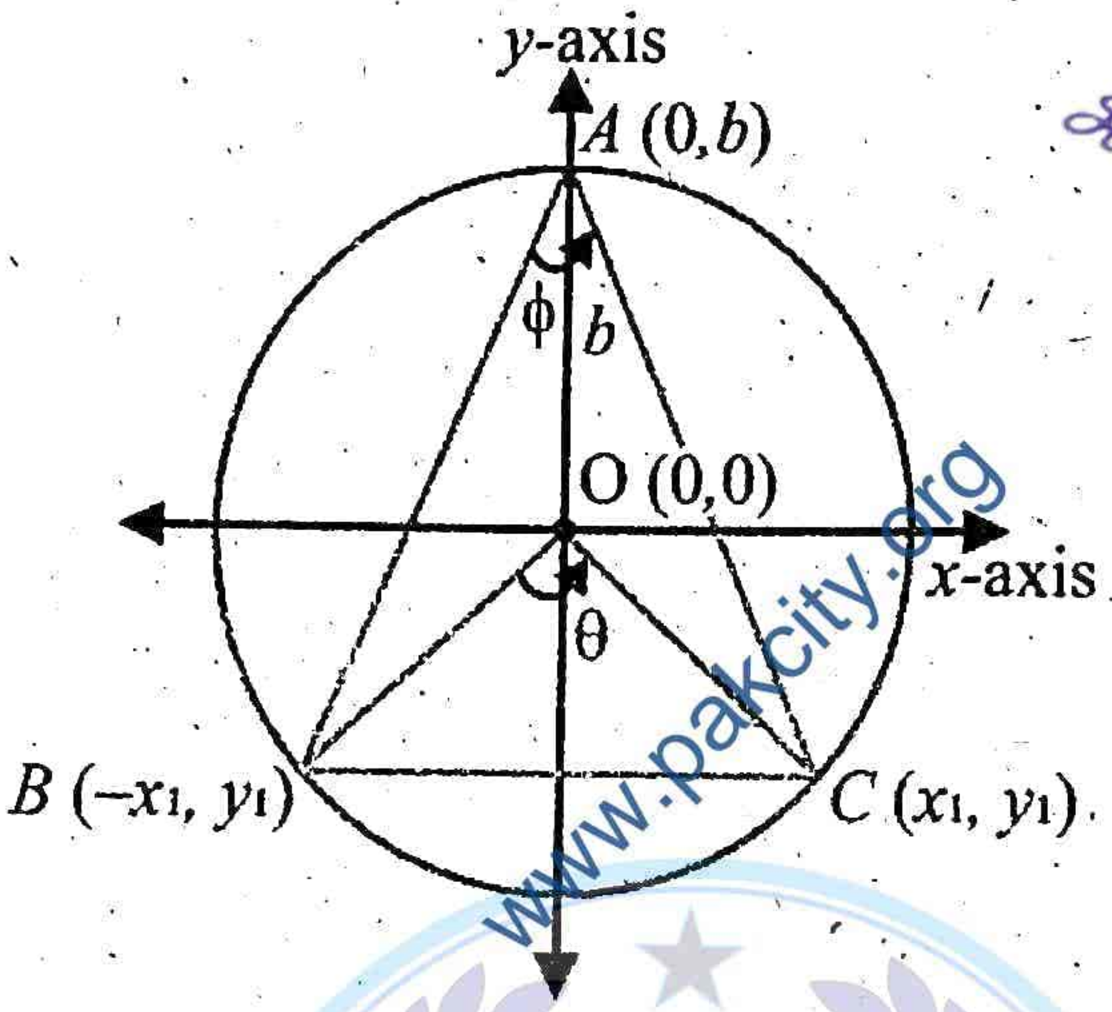
$\overline{OC} = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

Hence $\overline{CA} = \overline{CB} = \overline{OC} =$ radius of circle

Q.4 Measure of the central angle of major arc is double the measure of the inscribed angle of corresponding minor arc.

Solution:

Let $\widehat{BC}$ be a minor arc of circle $x^2 + y^2 = r^2$ such that its ends are $\mathrm{B}(-x_1, y_1)$ and $\mathrm{C}(x_1, y_1)$ whereas $\mathrm{A}(0, b)$ is any point of corresponding major arc on the given circle as shown.



Now, ∠BOC is the central angle of minor arc BC and ∠BAC is the angle subtended by then corresponding major arc BAC.

Here, Slope of $\overline{BO} = m_1 = \frac{y_1}{-x_1} = -\frac{y_1}{x_1}$

Slope of $\overline{CO} = m_2 = \frac{y_1}{x_1}$

Now, $\tan m\angle BOC = \frac{m_2 - m_1}{1 + m_1 m_2}$

i.e., $\tan\theta = \frac{\frac{y_1}{x_1} - \left(-\frac{y_1}{x_1}\right)}{1 + \left(\frac{y_1}{x_1}\right)\left(-\frac{y_1}{x_1}\right)}$

$$\tan\theta = \frac{\frac{y_1}{x_1} + \frac{y_1}{x_1}}{1 - \left(\frac{y_1}{x_1}\right)\left(\frac{y_1}{x_1}\right)}$$

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C is the

$$\tan\theta = \frac{\frac{2y_1}{x_1}}{1-\frac{(y_1)^2}{(x_1)^2}}$$

$$\tan\theta = \frac{\frac{2y_1}{x_1}}{\frac{(x_1)^2-(y_1)^2}{(x_1)^2}}$$

$$\tan\theta = \frac{2y_1}{x_1}\left\{\frac{(x_1)^2}{(x_1)^2-(y_1)^2}\right\}$$

$$\tan\theta = \frac{2y_1x_1}{(x_1)^2-(y_1)^2} \rightarrow (1)$$

We have

Slope of $\overline{AB} = m_3 = \frac{b-y_1}{x_1}$

And Slope of $\overline{AC} = m_4 = \frac{b-y_1}{-x_1} = -\frac{b-y_1}{x_1}$

Furthermore,

$x^2+y^2=r^2$

i.e., $(x_1)^2+(y_1)^2=b^2 \quad [\because r=b]$

$\Rightarrow (x_1)^2 = b^2-(y_1)^2 \rightarrow (2)$

Now,

$$\tan(\angle BAC) = \frac{m_4-m_3}{1+m_3m_4}$$

$$\tan\phi = \frac{-\frac{b-y_1}{x_1}-\frac{b-y_1}{x_1}}{1+\frac{b-y_1}{x_1}\left\{-\frac{b-y_1}{x_1}\right\}}$$

$$\tan\phi = \frac{-\frac{2(b-y_1)}{x_1}}{1-\left(\frac{b^2-2by_1+(y_1)^2}{(x_1)^2}\right)}$$

$$\tan\phi = \frac{-\frac{2(b-y_1)}{x_1}}{\frac{(x_1)^2-b^2+2by_1-(y_1)^2}{(x_1)^2}}$$

$$\tan\phi = \frac{2(b-y_1)}{x_1}\left\{\frac{(x_1)^2}{(x_1)^2-b^2+2by_1-(y_1)^2}\right\}$$

$$\tan\phi = \frac{-2x_1(b-y_1)}{(x_1)^2-b^2+2by_1-(y_1)^2}$$

$$\tan\phi = \frac{-2x_1(b-y_1)}{b^2-(y_1)^2-b^2+2by_1-(y_1)^2}$$

$$\tan\phi = \frac{-2x_1(b-y_1)}{-2(y_1)^2+2by_1}$$

$$\tan\phi = \frac{-x_1(b-y_1)}{y_1(b-y_1)}$$

$$\tan\phi = -\frac{x_1}{y_1} \rightarrow (3)$$

We know that

$$\tan 2\phi = \frac{2\tan\phi}{1-\tan^2\phi}$$

$$\tan 2\phi = \frac{2\left(-\frac{x_1}{y_1}\right)}{1-\left(-\frac{x_1}{y_1}\right)^2}$$

$$\tan 2\phi = \frac{-\frac{2x_1}{y_1}}{1-\frac{(x_1)^2}{(y_1)^2}}$$

$$\tan 2\phi = \frac{-\frac{2x_1}{y_1}}{\frac{(y_1)^2-(x_1)^2}{(y_1)^2}}$$

$$\tan 2\phi = -\frac{2x_1}{y_1}\left\{\frac{(y_1)^2}{(y_1)^2-(x_1)^2}\right\}$$

$$\tan 2\phi = -\frac{2x_1y_1}{(y_1)^2-(x_1)^2} \rightarrow (4)$$

From (1) and (4)

$$\tan\theta = \tan 2\phi$$

$$\theta = 2\phi$$

Area of Δ =

$= \frac{1}{2}\Big| \begin{matrix} 3-1 \\ 7-(- \end{matrix}$

$= \frac{1}{2}\Big| \begin{matrix} 2 & - \\ 10 & 1 \end{matrix}$

$= \frac{1}{2}(52)$

$= 26$ units

(ii) (2,3), (−

$(x_1, y_1) =$ (

Area of Δ =

$= \frac{1}{2}\Big| \begin{matrix} 2-(- \\ -1- \end{matrix}$

$= \frac{1}{2}\Big| \begin{matrix} 5 & 1 \\ -4 & 5 \end{matrix}$

$= \frac{1}{2}(29)$

$= \frac{29}{2}$ units

Q.3 Prove,

points are c

(i) (2,3), (5

(iii) (−1, −

Solution:

(i) (2,3), (5

Area of Δ =

$= \frac{1}{2}\{2\Big| \begin{matrix} 0 \\ 1 \end{matrix}$

$= \frac{1}{2}\{2(-1)$

$= 0$

Hence poin

(ii) (2,1), (4

Area of Δ =