CHAPTER 07

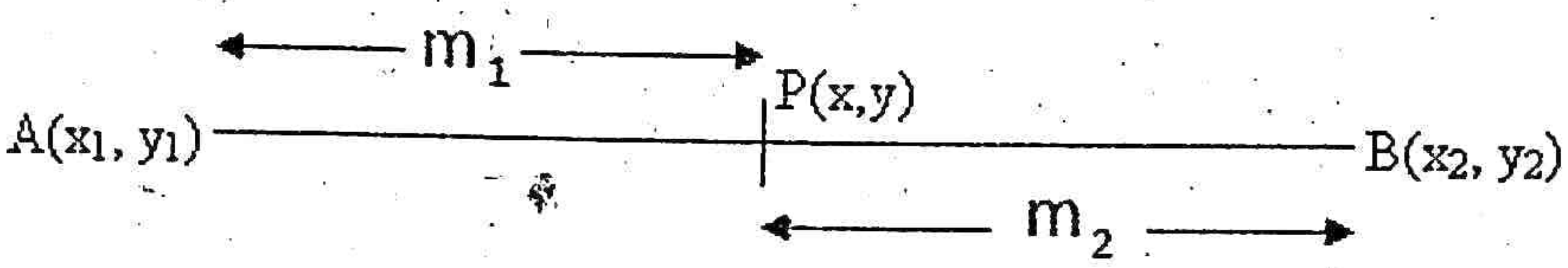
PLANE ANALYTIC GEOMETRY: STRAIGHT LINE

BASIC CONCEPTS AND FORMULAS

Distance Formula

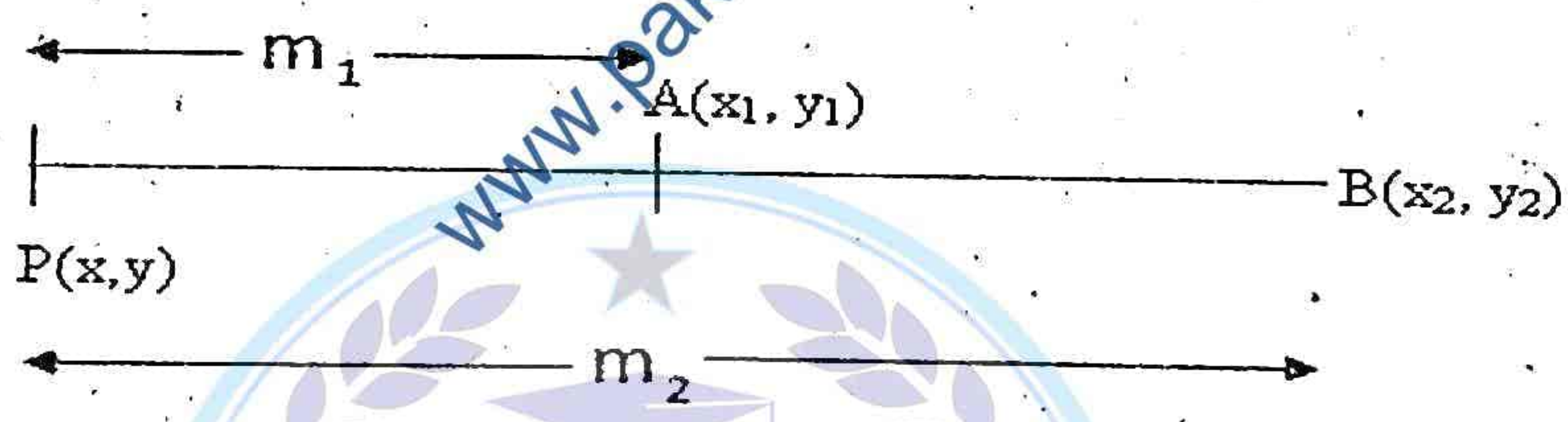
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Internal division



$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

External division



$$(x, y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

EQUATION OF STRAIGHT LINE

A line $ax + by + c = 0$ can be calculated in different forms

1. Point-slope form

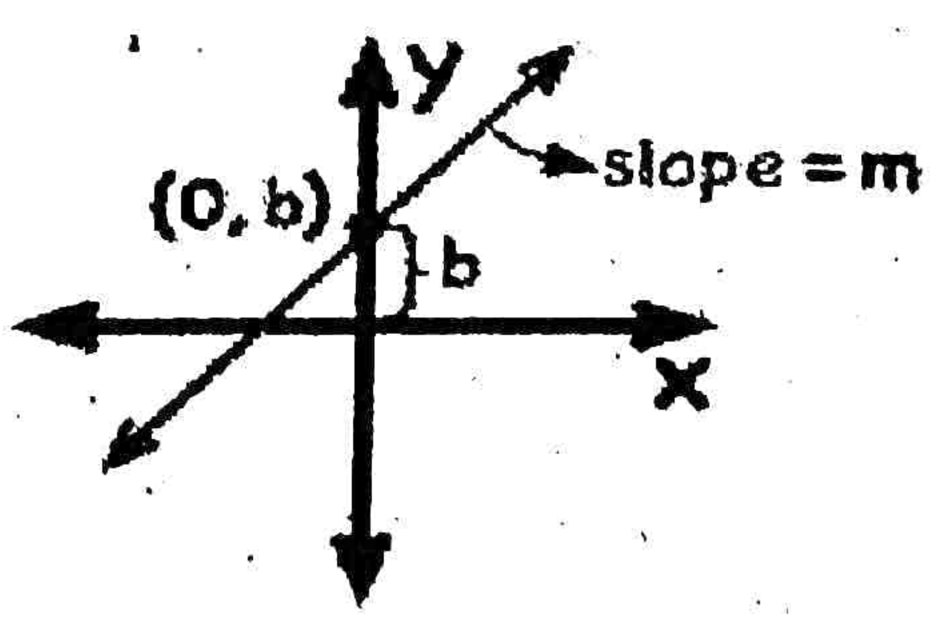
$$y - y_1 = m(x - x_1)$$

2. Two Point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

3. Slope-intercept form

$$y = mx + b$$



The Students' Comp...

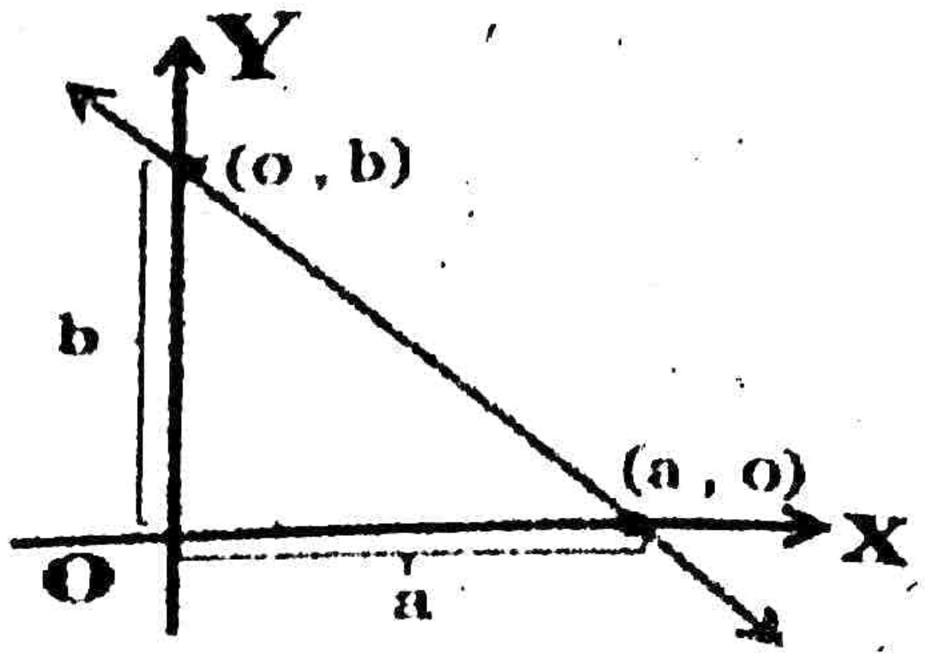
4. Two-intercept form

$$\frac{x}{a}+\frac{y}{b}=1$$

where

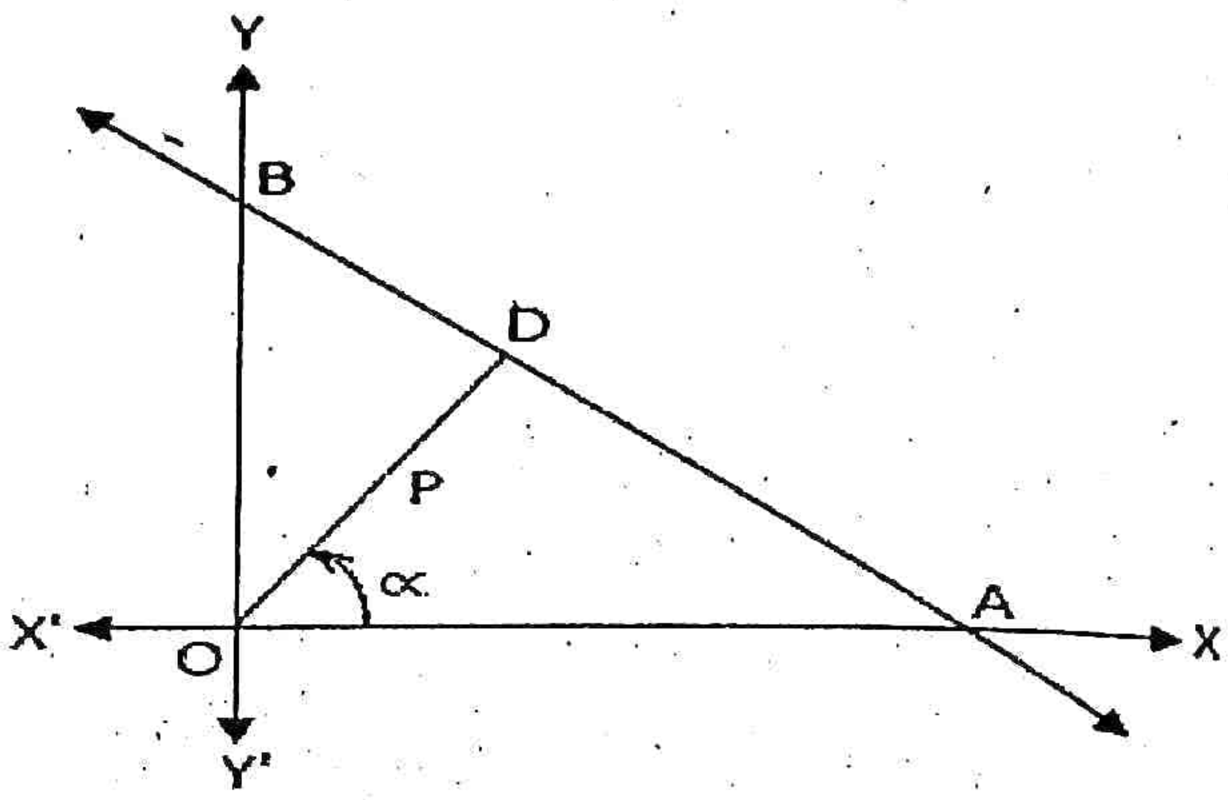
$a = x$-intercept

& $b = y$-intercept



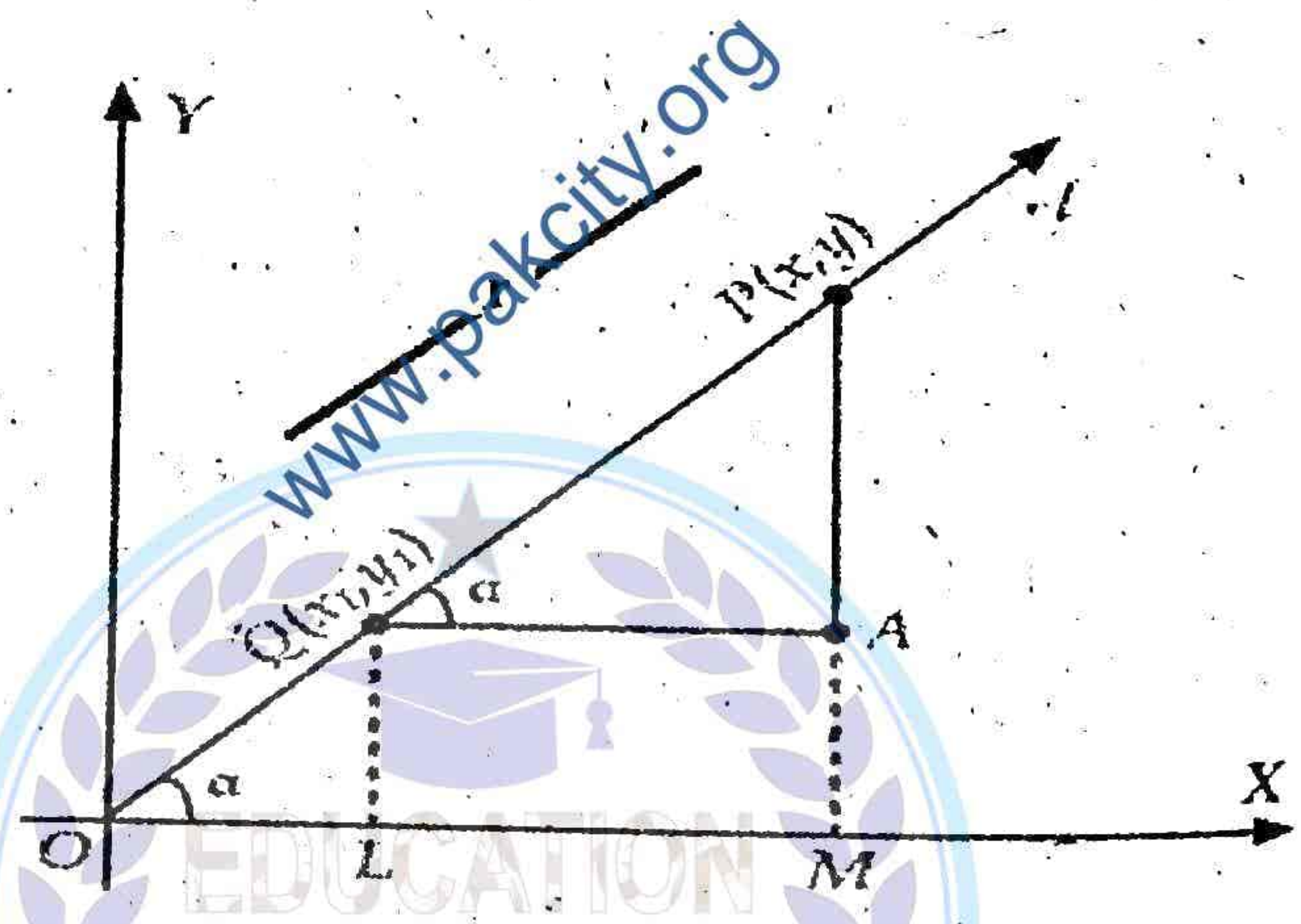
5. Normal or Perpendicular form

$$x\cos\alpha + y\sin\alpha = p$$



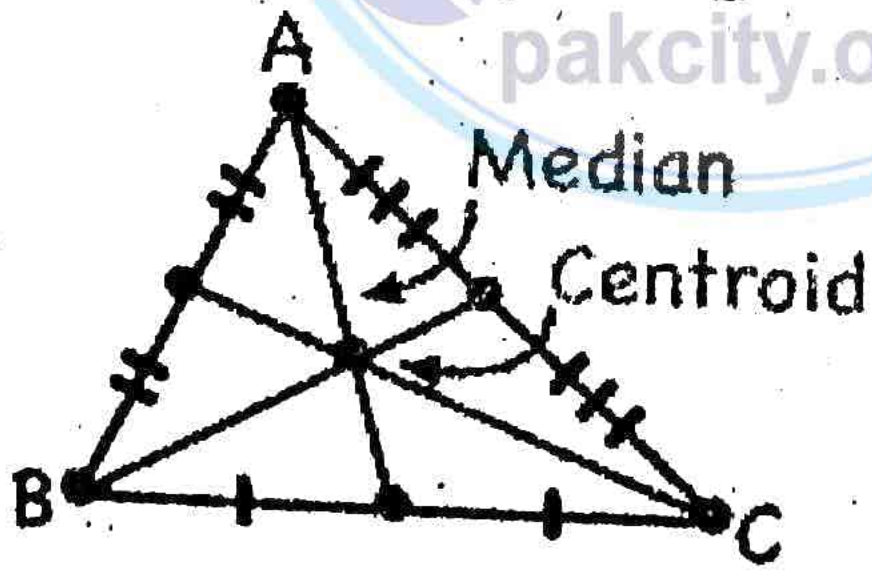
6. Symmetric form

$$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}=r$$



Median

Segment joining a vertex to the mid-point of opposite side is called a median.



Centroid

It is point of concurrency of medians; its ratio is 2: 1

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

The...

Altitude

Perpendicular fr...

Note: Point of c...

Perpendicular ...

A Line which pa... perpendicular o... segment.

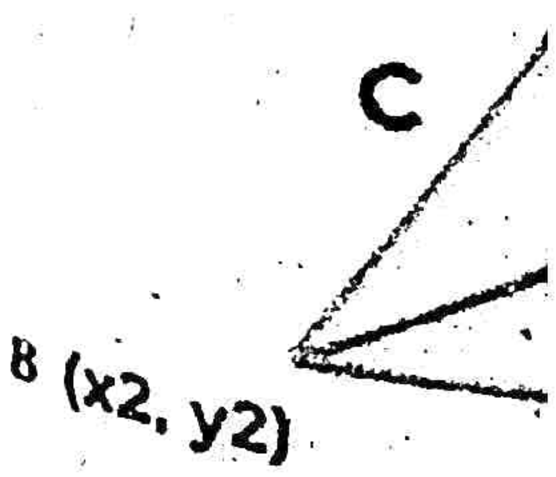
Note: Point of c...

Internal Bisect...

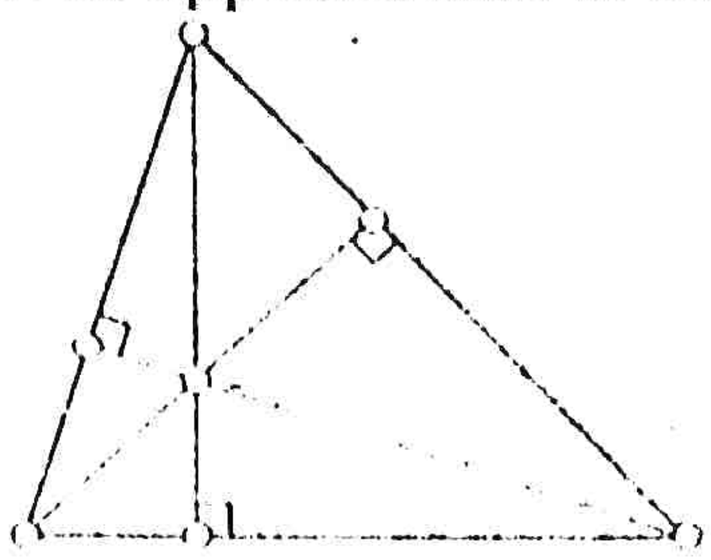
A Line that divi...

In-centre

Point of intersec...

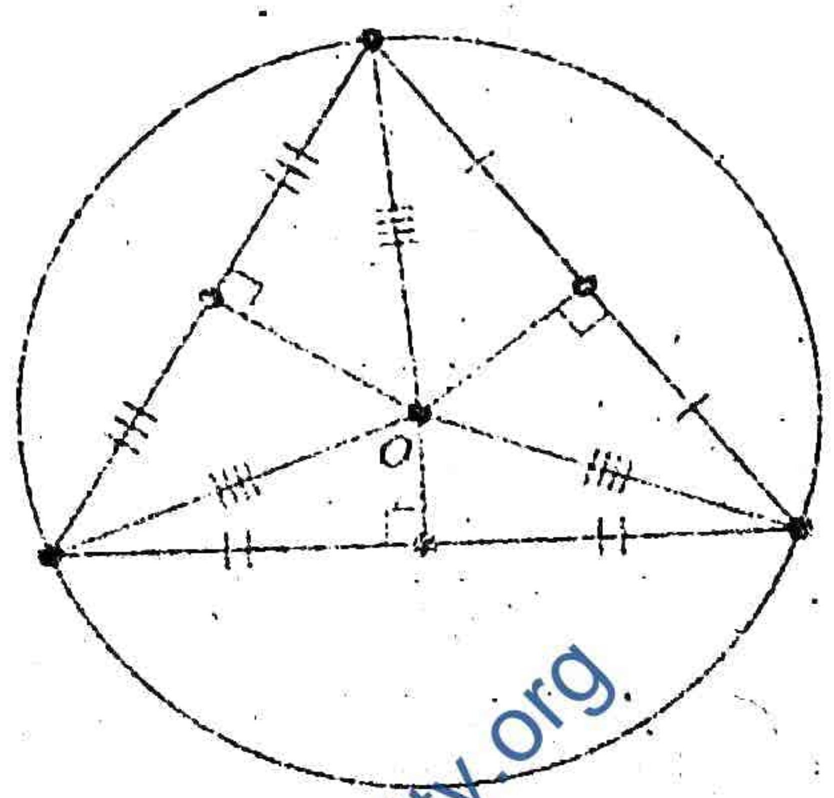


Altitude
Perpendicular from a vertex to opposite side is called altitude.



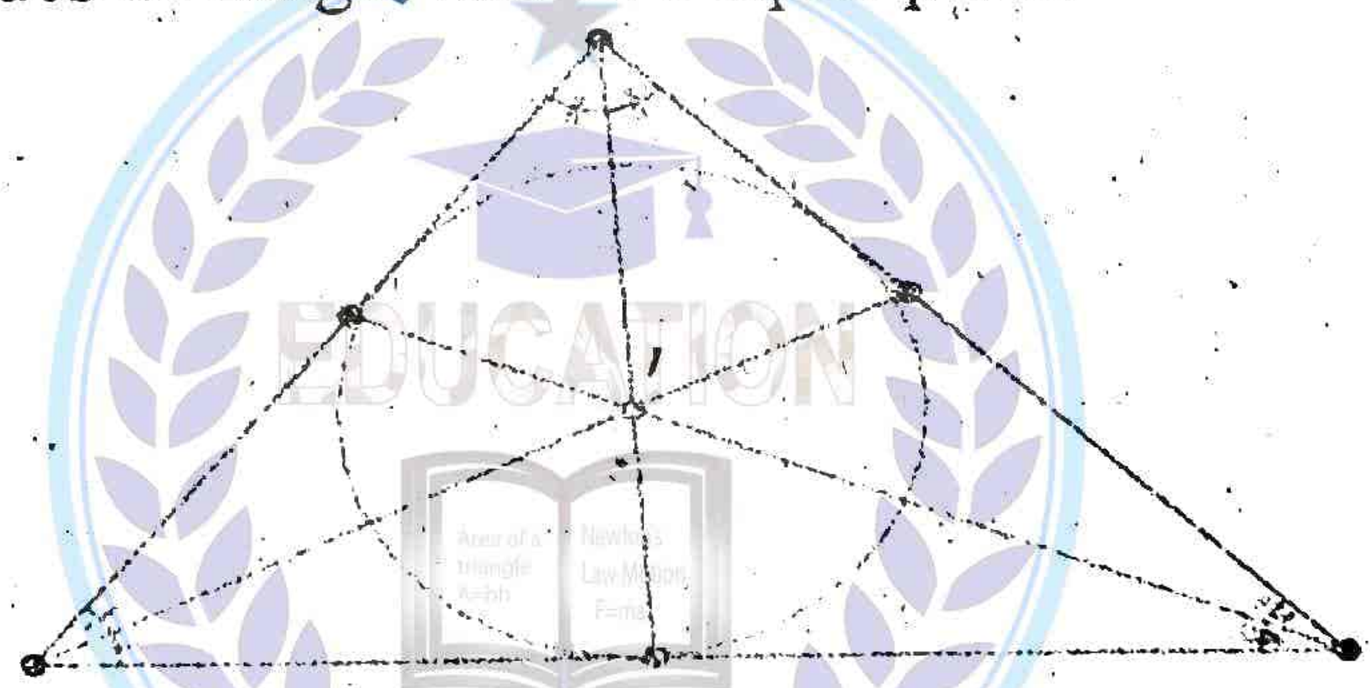
Note: Point of concurrency of altitudes is **ortho-centre.**

Perpendicular Bisector
A Line which passes through the mid-point of a segment and is perpendicular on the segment is called the perpendicular bisector of the segment.

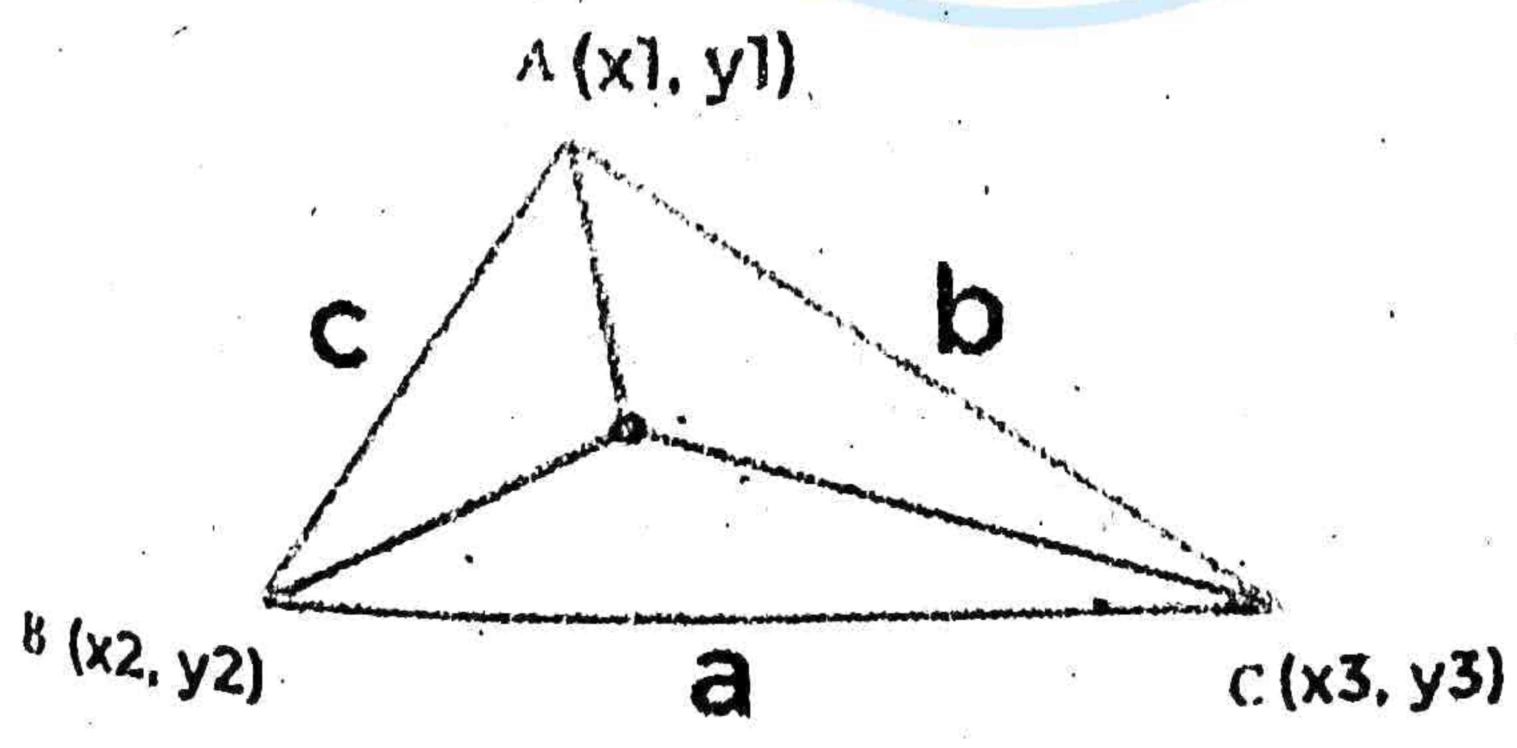


Note: Point of concurrency of perpendicular bisector is **circum-centre.**

Internal Bisector of angle
A Line that divides the angle into two equal parts.

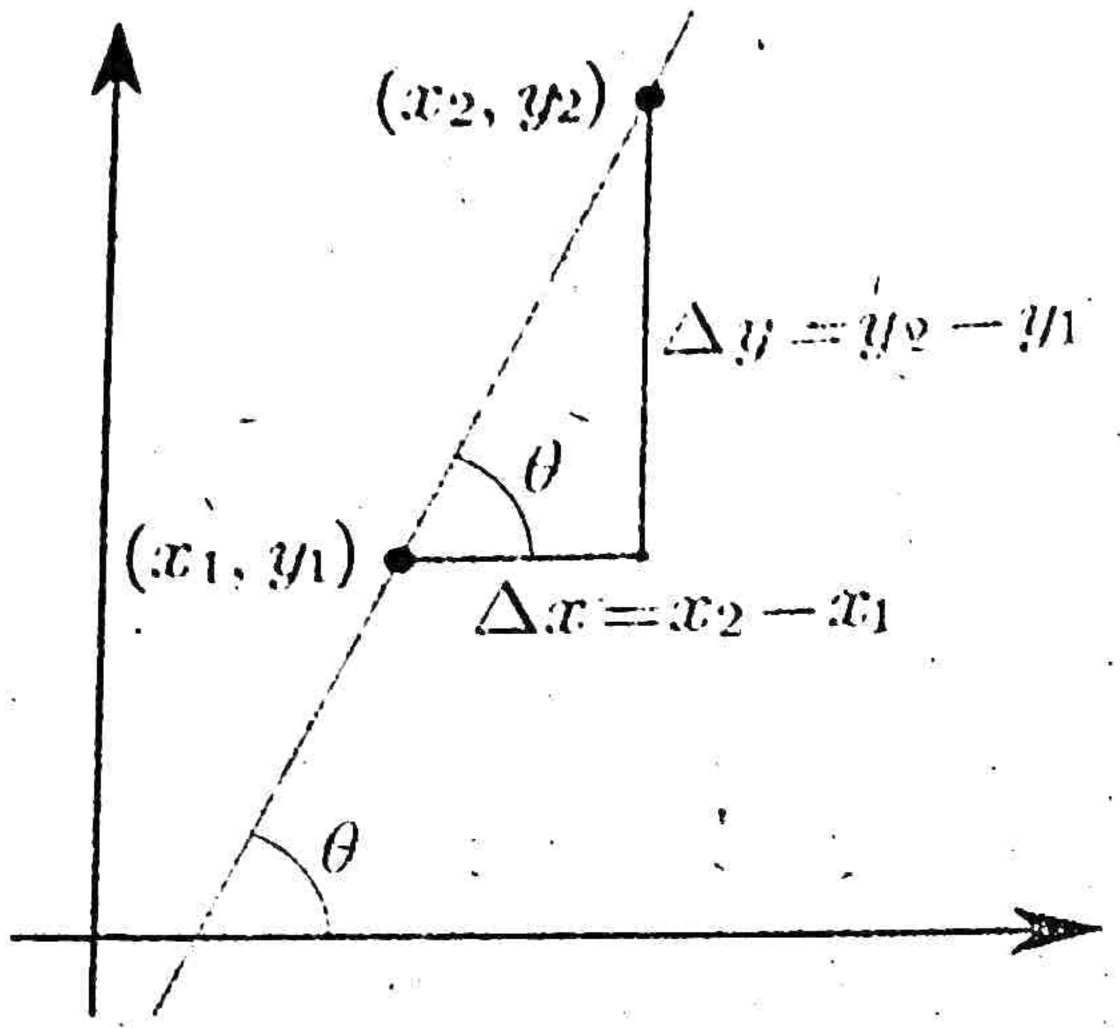


In-centre
Point of intersection of internal bisectors of angles



$$I(x, y) = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Slope of a line: Steepness of a line.



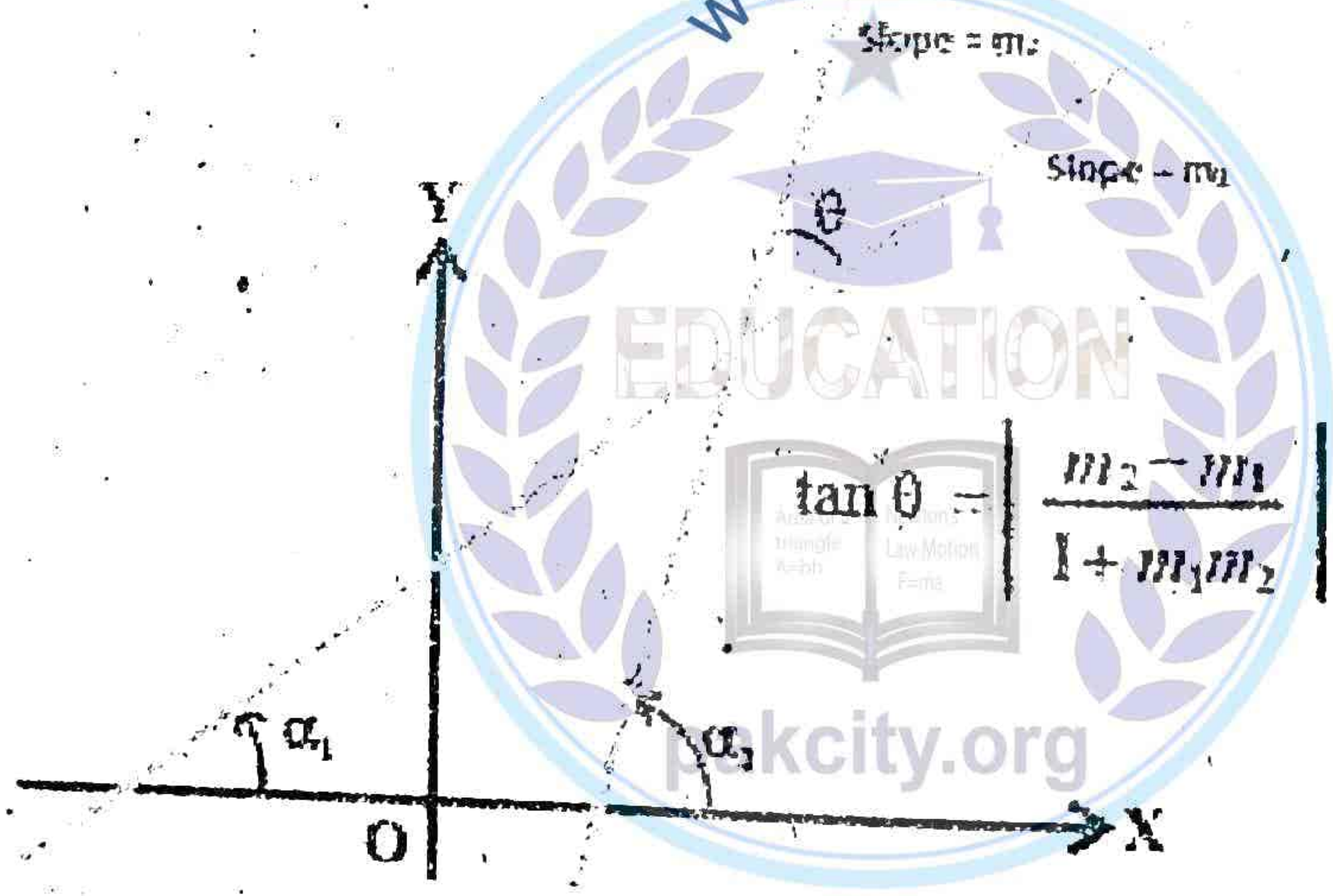
$$m = \frac{\text{rise or fall}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = tan\theta$$

$\theta =$ angle with positive $x -$ axis

$l: ax + by + c = 0$

$$m = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = -\frac{a}{b}$$

Angle between two lines:



$$\alpha_1 + (\pi - \alpha_2) + \theta = \pi$$

$$\theta = \alpha_2 - \alpha_1$$

$$\tan\theta = \tan(\alpha_2 - \alpha_1)$$

$$\tan\theta = \frac{tan\alpha_2 - tan\alpha_1}{1 + tan\alpha_2 tan\alpha_1}$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

Note: As two straig
calculated using fo
180^0.

$\tan\theta = \left|\frac{m_2 - m}{1 + m_2 m}\right.$

Angle from l_1 to l

$tan\theta = \frac{m_2 - m_1}{1 + m_2 m}$

Angle from l_2 to l

Angle from l_1 to l

$tan\theta = \frac{m_1 - m_2}{1 + m_2 m}$

Note: to find angl

Position of a poin

- $ax_1 + by_1 + c$
- $ax_1 + by_1 + c$
- $ax_1 + by_1 + c$

Note: For positior

Perpendicular dis

$l: ax + by + c =$

Point $= (x_1, y_1)$

$d = \left|\frac{ax_1 + by_1}{\sqrt{a^2 + b}}\right.$

Distance betwee

$ax + by + c_1 = ($

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Area of triangle

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\Delta = \frac{1}{2}\{x_1(y_2 - y_3$

- If points are c

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Area of Quadrilat

Note: As two straight lines intersect at two angles then one angle is calculated using formula while other can be obtained by subtracting from 180^0.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

Angle from l_1 to l_2

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

Angle from l_2 to l_1

Angle from l_1 to l_2

$$\tan\theta = \frac{m_1 - m_2}{1 + m_2 m_1}$$

Note: to find angles of triangle, take anti-clockwise direction in xy-plane.

Position of a point (x_1, y_1) w.r.t $ax + by + c = 0$

- $ax_1 + by_1 + c > 0$, above the line
- $ax_1 + by_1 + c < 0$, below the line
- $ax_1 + by_1 + c = 0$, on the line

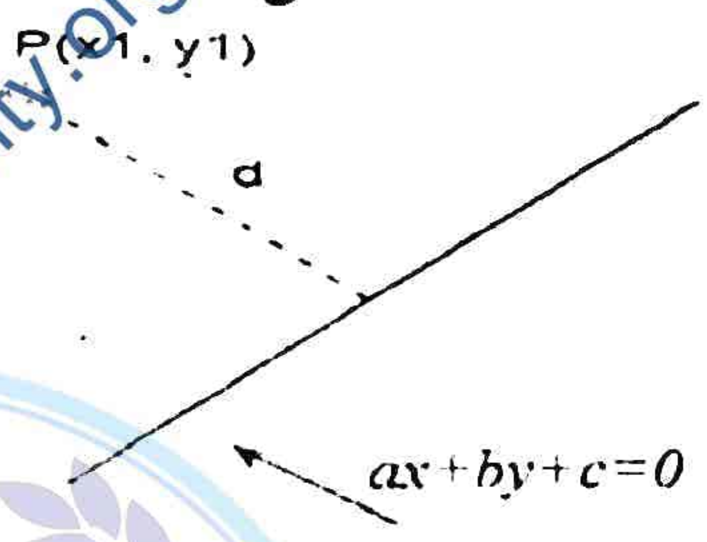
Note: For position of a point, coefficient of y must be positive.

Perpendicular distance of a point to a straight line

$l: ax + by + c = 0$

Point $= (x_1, y_1)$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$



Distance between parallel lines

$ax + by + c_1 = 0,\ ax + by + c_2 = 0$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Area of triangle

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_3)\}$$

- If points are collinear then

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Area of Quadrilateral

The Studen...

$$=\frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_4 & y_2-y_4 \end{vmatrix}$$

Concurrent lines

$ax+by+c=0$

$dx+ey+f=0$

$gx+hy+i=0$

Three lines are concurrent if $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=0$

2nd degree homogenous equations

$ax^2+2hxy+by^2=0$

Two lines are

$$y=\frac{-h\pm\sqrt{h^2-ab}}{b}x$$

$m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$

$tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$

θ = angle between homogenous lines

Nature of lines

- $\sqrt{h^2-ab}>0$, two real & different lines
- $\sqrt{h^2-ab}<0$, two imaginary lines
- $\sqrt{h^2-ab}=0$, two coincident/collinear lines

EXERCISE 7.1

Q.1 Find the distance between the following pairs of points:

(i) $A(-1,3)$ and $B(5,-5)$ (ii) $C(-1,0)$ and $D(0,-1)$

(iii) $E(1,-1)$ and $F(2,7)$ (iv) $G(-1,-4)$ and $H(5,-4)$

Solution:

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(i) $(x_1,y_1)=A(-1,3)$ and $(x_2,y_2)=B(5,-5)$

$\overline{AB}=\sqrt{(5+1)^2+(-5-3)^2}$

$\overline{AB}=\sqrt{(6)^2+(-8)^2}$

$\overline{AB}=\sqrt{36+64}$

$\overline{AB}=\sqrt{100}=10$ units

(ii) $(x_1,y_1)=C(-1,0)$ and $(x_2,y_2)=D(0,-1)$

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$\overline{CD}=\sqrt{(0+1)^2+}$...

$\overline{CD}=\sqrt{(1)^2+(-}$...

$\overline{CD}=\sqrt{1+1}$

$\overline{CD}=\sqrt{2}$ units

(iii) $(x_1,y_1)=E(1$...

$\overline{EF}=\sqrt{(2-1)^2+}$...

$\overline{EF}=\sqrt{(1)^2+(8}$...

$\overline{EF}=\sqrt{1+64}$

$\overline{EF}=\sqrt{65}$ units

(ii) $(x_1,y_1)=G(-$...

$\overline{CD}=\sqrt{(5+1)^2}$...

$\overline{CD}=\sqrt{(6)^2+(0}$...

$\overline{CD}=\sqrt{(6)^2}$

$\overline{CD}=6$ units

Q.2 Find the point...

Solution:

Let $P(0,y)$ be the...

$(x_1,y_1)=(5,2)$ a...

$d=\sqrt{(x_2-x_1)^2}$...

$(5\sqrt{2})^2=(0-5$...

$25(2)=(-5)^2+$...

$50=25+(y-2$...

$(y-2)^2=50-$...

$(y-2)^2=25$

$(y-2)=\pm5$

$y=\pm5+2$

$y=5+2$

$y=7$

Points on y-axis ar...

Q.3 Find the point...

Solution:

Let $P(x,0)$ be the...

$(x_1,y_1)=(-7,5)$

$\overline{CD} = \sqrt{(0+1)^2 + (-1-0)^2}$

$\overline{CD} = \sqrt{(1)^2 + (-1)^2}$

$\overline{CD} = \sqrt{1+1}$

$\overline{CD} = \sqrt{2}$ units

(iii) $(x_1, y_1) = E(1,-1)$ and $(x_2, y_2) = F(2,7)$

$\overline{EF} = \sqrt{(2-1)^2 + (7+1)^2}$

$\overline{EF} = \sqrt{(1)^2 + (8)^2}$

$\overline{EF} = \sqrt{1+64}$

$\overline{EF} = \sqrt{65}$ units

(ii) $(x_1, y_1) = G(-1,-4)$ and $(x_2, y_2) = H(5,-4)$

$\overline{CD} = \sqrt{(5+1)^2 + (-4+4)^2}$

$\overline{CD} = \sqrt{(6)^2 + (0)^2}$

$\overline{CD} = \sqrt{(6)^2}$

$\overline{CD} = 6$ units

Q.2 Find the point on y-axis which is $5\sqrt{2}$ units away from (5,2).

Solution:

Let $P(0, y)$ be the point on y-axis

$(x_1, y_1) = (5,2)$ and $(x_2, y_2) = P(0, y)$ and $d = 5\sqrt{2}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(5\sqrt{2})^2 = (0-5)^2 + (y-2)^2$

$25(2) = (-5)^2 + (y-2)^2$

$50 = 25 + (y-2)^2$

$(y-2)^2 = 50 - 25$

$(y-2)^2 = 25$

$(y-2) = \pm 5$

$y = \pm 5 + 2$

$y = 5 + 2$	$y = -5 + 2$
$y = 7$	$y = -3$

Points on y-axis are $(0,-3)$ and $(0,7)$

Q.3 Find the point on x-axis which is $\sqrt{41}$ units away from $(-7,5)$.

Solution:

Let $P(x, 0)$ be the point on x-axis

$(x_1, y_1) = (-7,5)$ and $(x_2, y_2) = P(x, 0)$ and $d = \sqrt{41}$

ints:

), -1)

H$(5,-4)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(\sqrt{41})^2 = (x + 7)^2 + (0 - 5)^2$

$41 = (x + 7)^2 + (-5)^2$

$41 = (x + 7)^2 + 25$

$(x + 7)^2 = 41 - 25$

$(x + 7)^2 = 16$

$x + 7 = \pm 4$

$x = \pm 4 - 7$

$x = 4 - 7$	$x = -4 - 7$
$x = -3$	$x = -11$

Points on x-axis are $(-3,0)$ and $(-11,0)$

Q.4 If ABC is triangle whose vertices are $A(-3,3)$, $B(2,6)$ and $C(3,0)$. Give the most specific name for ΔABC.

Solution:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(x_1, y_1) = A(-3,3)$ and $(x_2, y_2) = B(2,6)$

$\overline{AB} = \sqrt{(2 + 3)^2 + (6 - 3)^2}$

$\overline{AB} = \sqrt{(5)^2 + (3)^2}$

$\overline{AB} = \sqrt{34}$

$(x_1, y_1) = B(2,6)$ and $(x_2, y_2) = C(3,0)$

$\overline{BC} = \sqrt{(3 - 2)^2 + (0 - 6)^2}$

$\overline{BC} = \sqrt{(1)^2 + (-6)^2}$

$\overline{BC} = \sqrt{1 + 36}$

$\overline{BC} = \sqrt{37}$

$(x_1, y_1) = A(-3,3)$ and $(x_2, y_2) = C(3,0)$

$\overline{AC} = \sqrt{(3 + 3)^2 + (0 - 3)^2}$

$\overline{AC} = \sqrt{(6)^2 + (-3)^2}$

$\overline{AC} = \sqrt{36 + 9}$

$\overline{AC} = \sqrt{45} = 3\sqrt{5}$

$\overline{AB} \neq \overline{BC} \neq \overline{AC}$

$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$ [Pythagoras theorem]

$(\sqrt{45})^2 = (\sqrt{37})^2 + (\sqrt{34})^2$

$45 \neq 71$

It is Scalene trian

Q.5 If the points I and B(8,4), then

Solution:

$d = \sqrt{(x_2 - x_1)^2}$

$(x_1, y_1) = A(4,2)$

$|\overline{AP}| = \sqrt{(2 - 4)}$

$|\overline{AP}| = \sqrt{(-2)^2}$

$|\overline{AP}| = \sqrt{5}$

$(x_1, y_1) = A(4,2)$

$|\overline{AB}| = \sqrt{(8 - 4)}$

$|\overline{AB}| = \sqrt{(4)^2 +}$

$|\overline{AB}| = \sqrt{20} = 2$

$|\overline{AP}| = \frac{1}{2}|\overline{AB}|$

$\sqrt{5} = \frac{1}{2}(2\sqrt{5})$

$\sqrt{5} = \sqrt{5}$ Proved

Q.6 Find the coor
(i) M(−4,2) and
(iii) S(1,−2) and

Solution:

Midpoint $= \left(\frac{x_1}{}\right.$

(i) M(−4,2) and

Midpoint $= \left(\frac{-4}{2}\right.$

(ii) P(−1,−4) and

Midpoint $= \left(\frac{-1}{2}\right.$

(iii) S(1,−2) and

Midpoint $= \left(\frac{1 +}{2}\right.$

(iv) X(6,2) and Y(

It is Scalene triangle

Q.5 If the points P(2,1) lies on the line segment joining the points A(4,2) and B(8,4), then show that $|\overline{AP}| = \frac{1}{2}|\overline{AB}|$.

Solution:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(x_1, y_1) = A(4,2)$ and $(x_2, y_2) = P(2,1)$

$|\overline{AP}| = \sqrt{(2-4)^2 + (1-2)^2}$

$|\overline{AP}| = \sqrt{(-2)^2 + (-1)^2}$

$|\overline{AP}| = \sqrt{5}$

$(x_1, y_1) = A(4,2)$ and $(x_2, y_2) = B(8,4)$

$|\overline{AB}| = \sqrt{(8-4)^2 + (4-2)^2}$

$|\overline{AB}| = \sqrt{(4)^2 + (2)^2}$

$|\overline{AB}| = \sqrt{20} = 2\sqrt{5}$

$|\overline{AP}| = \frac{1}{2}|\overline{AB}|$

$\sqrt{5} = \frac{1}{2}(2\sqrt{5})$

$\sqrt{5} = \sqrt{5}$ Proved

Q.6 Find the coordinates of midpoint of the following points:

(i) M(−4,2) and N(−4,−2) (ii) P(−1,−4) and Q(5,−4)

(iii) S(1,−2) and T(2,−4) (iv) X(6,2) and Y(−2,−7)

Solution:

$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(i) M(−4,2) and N(−4,−2)

$\text{Midpoint} = \left(\frac{-4-4}{2}, \frac{2-2}{2}\right) = (-4,0)$

(ii) P(−1,−4) and Q(5,−4)

$\text{Midpoint} = \left(\frac{-1+5}{2}, \frac{-4-4}{2}\right) = (2,-4)$

(iii) S(1,−2) and T(2,−4)

$\text{Midpoint} = \left(\frac{1+2}{2}, \frac{-2-4}{2}\right) = \left(\frac{3}{2}, -3\right)$

(iv) X(6,2) and Y(−2,−7)

$\text{Midpoint} = \left(\frac{6-2}{2}, \frac{2-7}{2}\right) = \left(2, \frac{-5}{2}\right)$

Q.7 Find the point which divides the line segment joining $(4,-1)$ and $(4,3)$ in the ratio $3:1$ internally.

Solution:

$(x_1, y_1) = (4,-1)$ and $(x_2, y_2) = (4,3)$ and $m:n = 3:1$

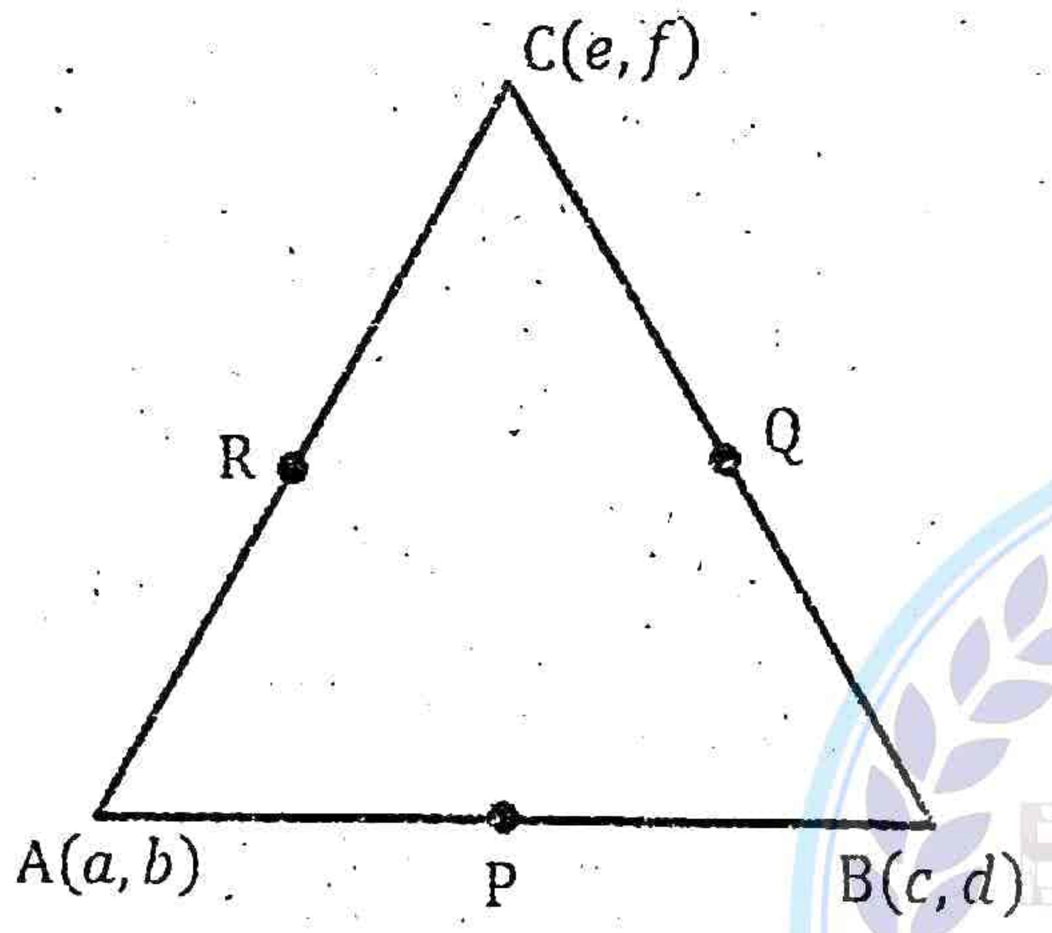
$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$(x, y) = \left(\frac{3(4)+1(4)}{3+1}, \frac{3(3)+1(-1)}{3+1}\right)$

$(x, y) = (4,2)$

Q.8 The points $P(-2,2)$, $Q(2,-1)$ and $R(-1,4)$ are midpoints of the sides of the triangle. Find the vertices.

Solution:



Let $A(a,b), B(c,d), C(e,f)$ be the vertices of triangle

P, Q, R are midpoints of $\overline{AB}, \overline{BC}, \overline{AC}$ respectively

$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$P(-2,2) = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

$\frac{a+c}{2} = -2 \Rightarrow a + c = -4 \rightarrow (1)$

$\frac{b+d}{2} = 2 \Rightarrow b + d = 4 \rightarrow (2)$

$Q(2,-1) = \left(\frac{c+e}{2}, \frac{d+f}{2}\right)$

$\frac{c+e}{2} = 2 \Rightarrow c + e = 4 \rightarrow (3)$

$\frac{d+f}{2} = -1 \Rightarrow$ …

$R(-1,4) = \left(\frac{a+}{2}\right.$ …

$\frac{a+e}{2} = -1 \Rightarrow$ …

$\frac{b+f}{2} = 4 \Rightarrow b$ …

By $(1) + (3) -$ …

$a + c + c + e -$ …

$a + c + c + e -$ …

$2c = 2 \Rightarrow \boxed{c =}$ …

$(1) \Rightarrow a = -4$ …

$a = -4 - 1 \Rightarrow$ …

$(2) \Rightarrow e = 4 -$ …

$e = 4 - 1 \Rightarrow \boxed{e}$ …

By $(2) + (4) -$ …

$b + d + d + f$ …

$b + d + d + f$ …

$2d = -6 \Rightarrow \boxed{d}$ …

$(2) \Rightarrow b = 4 -$ …

$b = 4 - (-3) =$ …

$(4) \Rightarrow f = -2$ …

$f = -2 - (-3)$ …

$A(-5,7), B(1,-$ …

**Q.9 $Z(4,5)$ and …
line segment ZX …**

Solution:

$(x_1, y_1) = Z(4,5$ …

Point of division …

Division is extern…

$(x, y) = \left(\frac{mx_2 -}{m -}\right.$ …

$Y = \left(\frac{(4)(7) -}{4 - 3}\right.$ …

$Y = (16, -19)$

−1) and

s of the sides

$\frac{d+f}{2}=-1 \Rightarrow d+f=-2 \rightarrow (4)$

$R(-1,4)=\left(\frac{a+e}{2},\frac{b+f}{2}\right)$

$\frac{a+e}{2}=-1 \Rightarrow a+e=-2 \rightarrow (5)$

$\frac{b+f}{2}=4 \Rightarrow b+f=8 \rightarrow (6)$

By (1) + (3) − (5)

$a+c+c+e-(a+e)=-4+4-(-2)$

$a+c+c+e-a-e=2$

$2c=2 \Rightarrow \boxed{c=1}$

$(1) \Rightarrow a=-4-c$

$a=-4-1 \Rightarrow \boxed{a=-5}$

$(2) \Rightarrow e=4-c$

$e=4-1 \Rightarrow \boxed{e=3}$

By (2) + (4) − (6)

$b+d+d+f-(b+f)=4-2-8$

$b+d+d+f-b-f=-6$

$2d=-6 \Rightarrow \boxed{d=-3}$

$(2) \Rightarrow b=4-d$

$b=4-(-3) \Rightarrow \boxed{b=7}$

$(4) \Rightarrow f=-2-d$

$f=-2-(-3) \Rightarrow \boxed{f=1}$

$A(-5,7), B(1,-3), C(3,1)$

Q.9 $Z(4,5)$ and $X(7,-1)$ are two given points and the point Y divides the line segment ZX externally in the ratio 4: 3. Find the coordinates of Y.

Solution:

$(x_1,y_1)=Z(4,5)$ and $(x_2,y_2)=X(7,-1)$ and $m:n=4:3$

Point of division = ?

Division is external

$(x,y)=\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$

$Y=\left(\frac{(4)(7)-3(4)}{4-3},\frac{(4)(-1)-3(5)}{4-3}\right)$

$Y=(16,-19)$

Q.10 If a point $P(k,7)$ divides the line segment joining $A(8,9)$ and $B(1,2)$ in a ratio $m:n$ then find ratio $m:n$ also find k.

Solution:

$(x_1,y_1) = A(8,9)$ and $(x_2,y_2) = B(1,2)$ and $m:n = m:1$

Point of division $= P(k,7)$

$$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(k,7) = \left(\frac{m(1)+1(8)}{m+1}, \frac{m(2)+1(9)}{m+1}\right)$$

$$(k,7) = \left(\frac{m+8}{m+1}, \frac{2m+9}{m+1}\right)$$

$$\frac{2m+9}{m+1} = 7$$

$$2m+9 = 7m+7$$

$$9-7 = 7m-2m$$

$$5m = 2 \Rightarrow \boxed{m = \frac{2}{5}}$$

$$k = \frac{m+8}{m+1}$$

$$k = \frac{\frac{2}{5}+8}{\frac{2}{5}+1} = \frac{\frac{42}{5}}{\frac{7}{5}} = \frac{42}{7} = 6$$

Q.11 $A(2,7)$ and $B(-4,-8)$ are coordinates of the line segment AB. There are two points that trisect the segment AB. Find the points of trisection.

Solution:

$(x_1,y_1) = A(2,7)$ and $(x_2,y_2) = B(-4,-8)$ and $m:n = 1:2$ or $2:1$

For $m:n = 1:2$

$$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(x,y) = \left(\frac{1(-4)+2(2)}{1+2}, \frac{1(-8)+2(7)}{1+2}\right)$$

$$(x,y) = (0,2)$$

For $m:n = 2:1$

$$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(x,y) = \left(\frac{2(-4)+1(2)}{2+1}, \frac{2(-8)+1(7)}{2+1}\right)$$

$$(x,y) = (-2,-3)$$

Q.12 The vertices P, Q and R of a triangle are $(2,1)$, $(5,2)$ and $(3,4)$ respectively. Find the coordinates of the circum-centre and find the radius of the circum-circle of the triangle.

Solution:

$P(2,1)$, $Q(5,2)$ and $R(3,4)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Let $O(x,y)$ be the coordinates of the circum center

$\overline{PO} = \overline{QO} = \overline{RO} =$ Circum radius

$(x_1, y_1) = P(2,1)$ and $(x_2, y_2) = O(x,y)$

$|\overline{PO}|^2 = (x-2)^2 + (y-1)^2$

$|\overline{PO}|^2 = (x^2 - 4x + 4) + (y^2 - 2y + 1)$

$|\overline{PO}|^2 = x^2 + y^2 - 4x - 2y + 5$

$(x_1, y_1) = Q(5,2)$ and $(x_2, y_2) = O(x,y)$

$|\overline{QO}|^2 = (x-5)^2 + (y-2)^2$

$|\overline{QO}|^2 = (x^2 - 10x + 25) + (y^2 - 4y + 4)$

$|\overline{QO}|^2 = x^2 + y^2 - 4y - 10x + 29$

$(x_1, y_1) = R(3,4)$ and $(x_2, y_2) = O(x,y)$

$|\overline{RO}|^2 = (x-3)^2 + (y-4)^2$

$|\overline{RO}|^2 = (x^2 - 6x + 9) + (y^2 - 8y + 16)$

$|\overline{RO}|^2 = x^2 + y^2 - 8y - 6x + 25$

Taking $|\overline{PO}|^2 = |\overline{QO}|^2$

$x^2 + y^2 - 4x - 2y + 5 = x^2 + y^2 - 4y - 10x + 29$

$10x - 4x + 4y - 2y = 29 - 5$

$6x + 2y = 24$

$3x + y = 12 \Rightarrow y = 12 - 3x \rightarrow (1)$

Taking $|\overline{PO}|^2 = |\overline{RO}|^2$

$x^2 + y^2 - 4x - 2y + 5 = x^2 + y^2 - 8y - 6x + 25$

$$6x - 4x + 8y - 2y = 25 - 5$$
$$2x + 6y = 20$$
$$x + 3y = 10$$
$$x + 3(12 - 3x) = 10$$
$$x + (36 - 9x) = 10$$
$$-8x = 10 - 36$$
$$8x = -26 \Rightarrow \boxed{x = \frac{13}{4}}$$
$$(1) \Rightarrow y = 12 - 3\left(\frac{13}{4}\right)$$
$$\boxed{y = \frac{9}{4}}$$

Circum radius $= \overline{PO} = \sqrt{(x-2)^2 + (y-1)^2}$

$$\overline{PO} = \sqrt{\left(\frac{13}{4} - 2\right)^2 + \left(\frac{9}{4} - 1\right)^2}$$
$$\overline{PO} = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{5}{4}\right)^2} = \sqrt{2\left(\frac{5}{4}\right)^2}$$
$$\overline{PO} = \frac{5\sqrt{2}}{4} \text{ units}$$

Q.13 $\overline{AB}$ is divided into 20 equal parts $P_1, P_2, P_3, \dots, P_{10}, \dots, P_{19}$. If A and B are (2,3) and (10,11) respectively, find the coordinates of P_{13}.

Solution:

$(x_1, y_1) = (2,3)$ and $(x_2, y_2) = (10,11)$ and $m:n = 13:7$

Point of division $= P_{13}$

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$
$$P_{13} = \left(\frac{13(10) + 7(2)}{13+7}, \frac{13(11) + 7(3)}{13+7}\right)$$
$$P_{13} = \left(\frac{36}{5}, \frac{41}{5}\right)$$

The Stu...

Q.14 If A, B are (3,4 ... units.

Solution:

Let $C(x, y$ ...

$(x_1, y_1) =$ ...

$d = \sqrt{(x_2}$ ...

$|\overline{AB}| = \sqrt{}$ ...

$|\overline{AB}| = \sqrt{}$ ...

$|\overline{AB}| = 5$

$\therefore$ B is the ...

$(x_1, y_1) =$ ...

Midpoint ...

$B(7,7) =$ ...

$\frac{3+x}{2} = 7$

$\frac{4+y}{2} = 7$

$C(x, y) =$ ...

Q.15 Find ... are respec...

(i) L(2,8), ...

Solution:

(i) L(2,8), ...

Q.14 If A, B and C are three collinear points and the coordinates of A and B are (3,4) and (7,7) respectively. Find the coordinates of C if $|\overline{AC}| = 10$ units.

Solution:

Let $C(x,y)$ be the required point

$(x_1, y_1) = A(3,4)$ and $(x_2, y_2) = B(7,7)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|\overline{AB}| = \sqrt{(7-3)^2 + (7-4)^2}$

$|\overline{AB}| = \sqrt{(4)^2 + (3)^2}$

$|\overline{AB}| = 5$ units

$\therefore$ B is the midpoint of $|\overline{AC}|$

$(x_1, y_1) = A(3,4)$ and $(x_2, y_2) = C(x,y)$ and midpoint $= B(7,7)$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$B(7,7) = \left(\frac{3 + x}{2}, \frac{4 + y}{2}\right)$$

$$\frac{3 + x}{2} = 7 \Rightarrow 3 + x = 14 \Rightarrow \boxed{x = 11}$$

$$\frac{4 + y}{2} = 7 \Rightarrow 4 + y = 14 \Rightarrow \boxed{y = 10}$$

$C(x,y) = (11,10)$

19. If A and B

P_{13}.

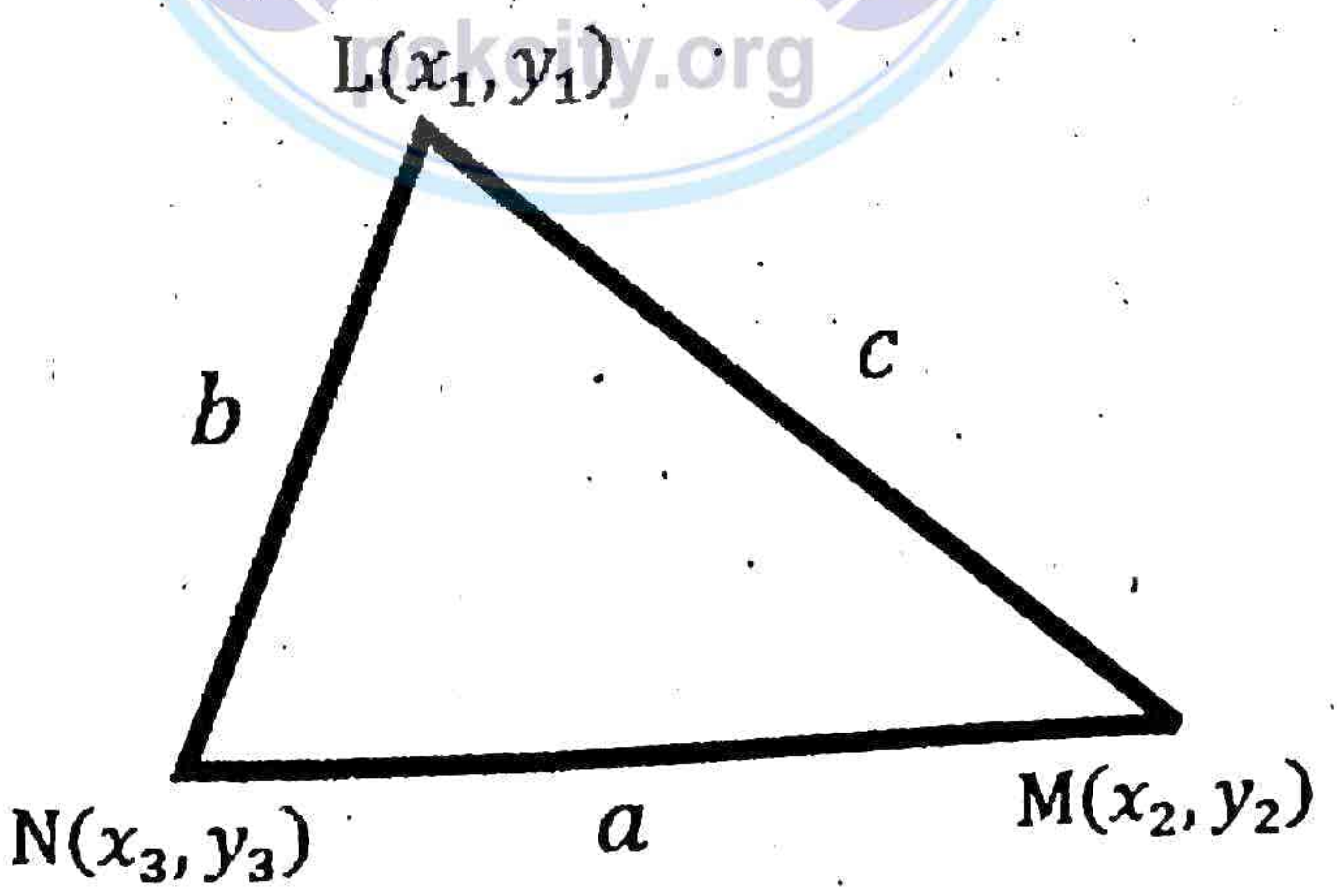
3(10,11)

Q.15 Find the coordinates of the incentre of triangle whose angular points are respectively:

(i) L(2,8), M(8,2) and N(9,9) (ii) P(−36,7), Q(20,7) and R(0,−8)

Solution:

(i) L(2,8), M(8,2) and N(9,9)



$(x_1, y_1) = L(2,8)$, $(x_2, y_2) = M(8,2)$ and $(x_3, y_3) = N(9,9)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$a = |\overline{NM}| = \sqrt{(9-8)^2 + (9-2)^2} = \sqrt{50} = 5\sqrt{2}$

$b = |\overline{LN}| = \sqrt{(9-2)^2 + (9-8)^2} = \sqrt{50} = 5\sqrt{2}$

$c = |\overline{LM}| = \sqrt{(8-2)^2 + (2-8)^2} = \sqrt{72} = 6\sqrt{2}$

$I(x,y) = \left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

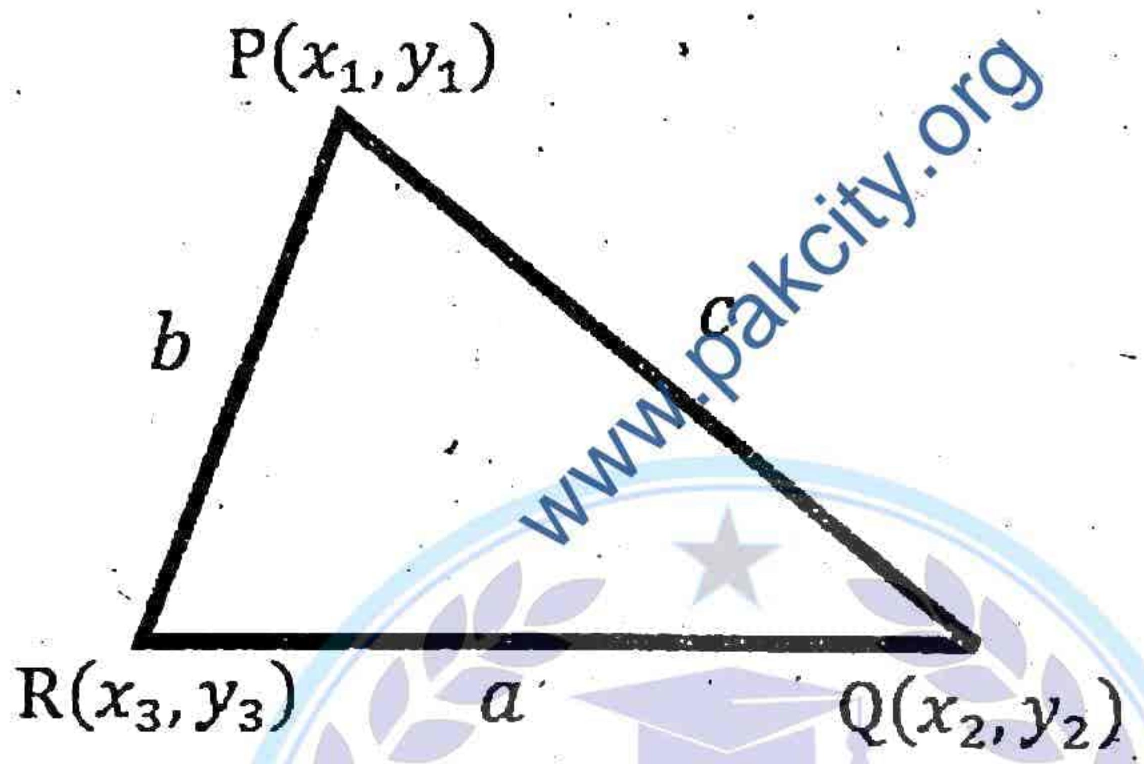
$I(x,y) =$

$\left(\dfrac{5\sqrt{2}(2) + 5\sqrt{2}(8) + 6\sqrt{2}(9)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \dfrac{5\sqrt{2}(8) + 5\sqrt{2}(2) + 6\sqrt{2}(9)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}\right)$

$I(x,y) = \left(\dfrac{10\sqrt{2} + 40\sqrt{2} + 54\sqrt{2}}{16\sqrt{2}}, \dfrac{40\sqrt{2} + 10\sqrt{2} + 54\sqrt{2}}{16\sqrt{2}}\right)$

$I(x,y) = \left(\dfrac{13\sqrt{2}}{2\sqrt{2}}, \dfrac{13\sqrt{2}}{2\sqrt{2}}\right) = \left(\dfrac{13}{2}, \dfrac{13}{2}\right)$

(ii) $P(-36,7), Q(20,7)$ and $R(0,-8)$



$(x_1, y_1) = P(-36,7), (x_2, y_2) = Q(20,7)$ and $(x_3, y_3) = R(0,-8)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$a = |\overline{RQ}| = \sqrt{(20-0)^2 + (7+8)^2} = \sqrt{625} = 25$

$b = |\overline{PR}| = \sqrt{(0+36)^2 + (-8-7)^2} = \sqrt{1529} = 39$

$c = |\overline{PQ}| = \sqrt{(20+36)^2 + (7-7)^2} = \sqrt{56^2} = 56$

$I(x,y) = \left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$I(x,y)$

$= \left(\dfrac{25(-36) + 39(20) + 56(0)}{25+39+56}, \dfrac{25(7) + 39(7) + 56(-8)}{25+39+56}\right)$

$= (-1,0)$

Q.16 The line segment joining $P(-8,10)$ and $Q(6,-4)$ is cut by x and y axes at A and B respectively. Find the ratio in which A and B divide $\overline{PQ}$.

Solution:

$(x_1, y_1) = P(-$

$(x,y) = \left(\dfrac{mx_2}{m}\right.$

$(x,y) = \left(\dfrac{m(6}{}\right.$

$(x,y) = \left(\dfrac{6m}{m+}\right.$

For X-axis: Poi

$\dfrac{-4m+10}{m+1} =$

$-4m + 10 =$

$4m = 10 \Rightarrow$

$\dfrac{5}{2} : 1 \Rightarrow 5:2$

For Y-axis: Poi

$\dfrac{6m-8}{m+1} = 0$

$6m - 8 = 0$

$6m = 8 \Rightarrow m$

$\dfrac{4}{3} : 1 \Rightarrow 4:3$

Q.17 Find the

points are:

(i) A(1,3), B(2

Solution:

Centroid G(x,

(i) A(1,3), B(2

$G(x,y) = \left(\dfrac{1}{}\right.$

$G(x,y) = \left(\dfrac{8}{3},\right.$

(ii) P(−2,5), Q

$G(x,y) = \left(\dfrac{-2}{}\right.$

Solution:

$(x_1, y_1) = P(-8,10)$ and $(x_2, y_2) = Q(6,-4)$ and $m:n = m:1$.

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$(x, y) = \left(\frac{m(6) + 1(-8)}{m+1}, \frac{m(-4) + 1(10)}{m+1}\right)$$

$$(x, y) = \left(\frac{6m - 8}{m+1}, \frac{-4m + 10}{m+1}\right)$$

For X-axis: Point of division $= (x, 0)$

$$\frac{-4m + 10}{m+1} = 0$$

$$-4m + 10 = 0$$

$$4m = 10 \Rightarrow \boxed{m = \frac{5}{2}}$$

$$\frac{5}{2}:1 \Rightarrow 5:2$$

For Y-axis: Point of division $= (0, y)$

$$\frac{6m - 8}{m+1} = 0$$

$$6m - 8 = 0$$

$$6m = 8 \Rightarrow \boxed{m = \frac{4}{3}}$$

$$\frac{4}{3}:1 \Rightarrow 4:3$$

Q.17 Find the coordinates of the centroid of a triangle whose angular points are:

(i) $A(1,3), B(2,7)$ and $C(5,6)$ (ii) $P(-2,5), Q(-7,1)$ and $R(-8,-4)$

Solution:

$$\text{Centroid } G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

(i) $A(1,3), B(2,7)$ and $C(5,6)$

$$G(x, y) = \left(\frac{1 + 2 + 5}{3}, \frac{3 + 7 + 6}{3}\right)$$

$$G(x, y) = \left(\frac{8}{3}, \frac{16}{3}\right)$$

(ii) $P(-2,5), Q(-7,1)$ and $R(-8,-4)$

$$G(x, y) = \left(\frac{-2 - 7 - 8}{3}, \frac{5 + 1 - 4}{3}\right)$$

$G(x,y) = \left(-\frac{17}{3}, \frac{2}{3}\right)$

Q.18 A straight line passes through the points (7,9) and (−1,1). Find a point on the line whose ordinate is 4.

Solution:

$(x_1, y_1) = (7,9)$ and $(x_2, y_2) = (-1,1)$ and $m:n = m:1$

Point of division $= (x, 4)$

$$(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$(x,4) = \left(\frac{m(-1) + 1(7)}{m+1}, \frac{m(1) + 1(9)}{m+1}\right)$$

$$(x,4) = \left(\frac{-m+7}{m+1}, \frac{m+9}{m+1}\right)$$

$$\frac{m+9}{m+1} = 4$$

$$m + 9 = 4m + 4$$

$$9 - 4 = 4m - m$$

$$3m = 5 \Rightarrow \boxed{m = \frac{5}{3}}$$

$$x = \frac{-m+7}{m+1}$$

$$x = \frac{-\frac{5}{3} + 7}{\frac{5}{3} + 1} = \frac{\frac{16}{3}}{\frac{8}{3}} = \frac{16}{8} = 2$$

Point $= (2,4)$

EXERCISE 7.2

Q.1 Find the slope of the line passing through the given pair of points.

(i) $A(3,7)$ and $B(2,9)$ (ii) $C(5,-2)$ and $D(3,6)$

(iii) $E(5,3)$ and $F(-2,3)$ (iv) $G(0,0)$ and $H(a,b)$

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

(i) $A(3,7)$ and $B(2,9)$

$(x_1, y_1) = A(3,7)$ and $(x_2, y_2) = B(2,9)$

$m = \frac{}{2-3}$

(ii) C(5, −2

$(x_1, y_1) =$

$m = \frac{6 - (}{3 -}$

(iii) E(5,3)

$(x_1, y_1) =$

$m = \frac{3 -}{-2 -}$

(iv) G(0,0

$(x_1, y_1) =$

$m = \frac{b -}{a -}$

Q.2 Find t

through tl

(i) A(2,1)

(iii) E(2,1

Solution:

For ⊥ lin

$m_1 m_2 =$

(i) A(2,1)

$(x_1, y_1) =$

$m = \frac{5 -}{4 -}$

Slope of p

(ii) C(−1,

$(x_1, y_1) =$

$m = \frac{5}{3 -}$

Slope of p

(iii) E(2,1

$(x_1, y_1) =$

$m = \frac{1 -}{-3 -}$

$m = \dfrac{9-7}{2-3} = -2$

(ii) $C(5,-2)$ and $D(3,6)$

$(x_1, y_1) = C(5,-2)$ and $(x_2, y_2) = D(3,6)$

$m = \dfrac{6-(-2)}{3-5} = -4$

(iii) $E(5,3)$ and $F(-2,3)$

$(x_1, y_1) = E(5,3)$ and $(x_2, y_2) = F(-2,3)$

$m = \dfrac{3-3}{-2-5} = 0$

(iv) $G(0,0)$ and $H(a,b)$

$(x_1, y_1) = G(0,0)$ and $(x_2, y_2) = H(a,b)$

$m = \dfrac{b-0}{a-0} = \dfrac{b}{a}$

Q.2 Find the slope of the perpendicular line when the given line passes through the following pair of points:

(i) $A(2,1)$ and $B(4,5)$ (ii) $C(-1,0)$ and $D(3,5)$

(iii) $E(2,1)$ and $F(-3,1)$ (iv) $G(-1,2)$ and $H(-1,-5)$

Solution:

For $\perp$ lines

$m_1 m_2 = -1 \Rightarrow m_2 = -\dfrac{1}{m_1}$

(i) $A(2,1)$ and $B(4,5)$

$(x_1, y_1) = A(2,1)$ and $(x_2, y_2) = B(4,5)$

$m = \dfrac{5-1}{4-2} = \dfrac{4}{2} = 2$

Slope of perpendicular line $= -\dfrac{1}{2}$

(ii) $C(-1,0)$ and $D(3,5)$

$(x_1, y_1) = C(-1,0)$ and $(x_2, y_2) = D(3,5)$

$m = \dfrac{5-0}{3-(-1)} = \dfrac{5}{4}$

Slope of perpendicular line $= -\dfrac{4}{5}$

(iii) $E(2,1)$ and $F(-3,1)$

$(x_1, y_1) = E(2,1)$ and $(x_2, y_2) = F(-3,1)$

$m = \dfrac{1-1}{-3-2} = 0$

Slope of perpendicular line $= \frac{1}{0}$ (undefined)

(iv) $G(-1,2)$ and $H(-1,-5)$

$(x_1, y_1) = G(-1,2)$ and $(x_2, y_2) = H(-1,-5)$

$$m = \frac{-5-2}{-1-(-1)} = \frac{-5-2}{-1+1} = -\frac{7}{0}$$

Slope of perpendicular line $= -\frac{0}{7} = 0$

Q.3 In each of the following the slope of the line is given, What is the slope of a line (a) parallel (b) perpendicular, to it?

(i) $\frac{2}{3}$ (ii) $-\frac{7}{2}$ (iii) -1 (iv) 4

Solution:

Slope of line	Parallel	Perpendicular
(i) $\frac{2}{3}$	$\frac{2}{3}$	$-\frac{3}{2}$
(ii) $-\frac{7}{2}$	$-\frac{7}{2}$	$\frac{2}{7}$
(iii) -1	-1	1
(iv) 4	4	$-\frac{1}{4}$

Q.4 Are the lines l_1 and l_2 through the given pair of points parallel, perpendicular or neither?

(i) $l_1: (1,2), (3,1)$ and $l_2: (0,-1), (2,0)$

(ii) $l_1: (0,3), (3,1)$ and $l_2: (-1,4), (-7,-5)$

(iii) $l_1: (2,-1), (5,-7)$ and $l_2: (0,0), (-1,2)$

(iv) $l_1: (1,0), (2,0)$ and $l_2: (5,-5), (-10,-5)$

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

(i) $l_1: (1,2), (3,1)$ and $l_2: (0,-1), (2,0)$

$(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (3,1)$

$$m_1 = \frac{1-2}{3-1} = -\frac{1}{2}$$

$(x_1, y_1) = (0,-1)$ and $(x_2, y_2) = (2,0)$

$$m_2 = \frac{0-(-1)}{2-0} = \frac{1}{2}$$

$m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so neither

(ii) $l_1: (0,3), (3,1)$ and $l_2: (-1,4), (-7,-5)$

$(x_1, y_1) = (0,3)$ and $(x_2, y_2) = (3,1)$

$$m_1 = \frac{1-3}{3-0} = -\frac{2}{3}$$

$(x_1, y_1) = (-1,4)$ and $(x_2, y_2) = (-7,-5)$

$$m_2 = \frac{-5-4}{-7-(-1)} = \frac{3}{2}$$

$m_1 m_2 = -1$ so perpendicular

(iii) $l_1: (2,-1), (5,-7)$ and $l_2: (0,0), (-1,2)$

$(x_1, y_1) = (2,-1)$ and $(x_2, y_2) = (5,-7)$

$$m_1 = \frac{-7-(-1)}{5-2} = -2$$

$(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (-1,2)$

$$m_2 = \frac{2-0}{-1-0} = -2$$

$m_1 = m_2$ so parallel

(iv) $l_1: (1,0), (2,0)$ and $l_2: (5,-5), (-10,-5)$

$(x_1, y_1) = (1,0)$ and $(x_2, y_2) = (2,0)$

$$m_1 = \frac{0-0}{2-1} = 0$$

$(x_1, y_1) = (5,-5)$ and $(x_2, y_2) = (-10,-5)$

$$m_2 = \frac{-5-(-5)}{-10-5} = 0$$

$m_1 = m_2$ so parallel

Q.5 The line through $(6,-4)$ and $(-3,2)$ is parallel to the line through $(2,1)$ and $(0, y)$. Find y.

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$(x_1, y_1) = (6,-4)$ and $(x_2, y_2) = (-3,2)$

$$m_1 = \frac{2-(-4)}{-3-6} = -\frac{2}{3}$$

$(x_1, y_1) = (2,1)$ and $(x_2, y_2) = (0, y)$

$$m_2 = \frac{y-1}{0-2} = \frac{y-1}{-2}$$

For parallel lines, $m_1 = m_2$

$$-\frac{y-1}{2} = -\frac{2}{3}$$

$\frac{y-1}{2} = \frac{2}{3}$

$3y - 3 = 4$

$3y = 4 + 3$

$3y = 7$

$\boxed{y = \frac{7}{3}}$

Q.6 The line through (2,5) and (−3, −2) is perpendicular to the line through (4, −1) and (x, 3). Find x.

Solution:

$(x_1, y_1) = (2,5)$ and $(x_2, y_2) = (-3, -2)$

$m_1 = \frac{-2-5}{-3-2} = \frac{7}{5}$

$(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (x, 3)$

$m_2 = \frac{3-(-1)}{x-4} = \frac{4}{x-4}$

For perpendicular lines, $m_1 m_2 = -1$

$\left(\frac{4}{x-4}\right)\left(\frac{7}{5}\right) = -1$

$28 = -5(x-4)$

$28 = -5x + 20$

$5x = 20 - 28$

$5x = -8$

$\boxed{x = -\frac{8}{5}}$

Q.7 Using slopes, prove that the (−1,4), (−3, −6) and (3, −2) are the vertices of right triangle.

Solution:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x_1, y_1) = A(-1,4)$ and $(x_2, y_2) = B(-3, -6)$

Slope of $\overline{AB} = \frac{-6-4}{-3-(-1)} = 5$

$(x_1, y_1) = A(-1,4)$ and $(x_2, y_2) = C(3, -2)$

Slope of $\overline{AC} = \frac{-2-4}{3-(-1)} = -\frac{3}{2}$

$(x_1, y_1) = B(-3, -6)$ and $(x_2, y_2) = C(3, -2)$

Slope of $\overline{BC}$ = -

(Slope of $\overline{AC}$)(

Since, $\overline{AC} \perp \overline{BC}$

Q.8 Using slope

(0, −1), (4, −3

Solution:

Let D(x, y) be

A(0, −1), B(4

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\because \overline{AB} \parallel \overline{DC}$

Slope of $\overline{AB}$ =

$\frac{-3-(-1)}{4-0} =$

$-\frac{1}{2} = \frac{3-y}{12-x}$

$-(12-x) =$

$-12 + x = 6$ -

$x = 18 - 2y$ -

$\because \overline{BC} \parallel \overline{AD}$

Slope of $\overline{BC}$ =

$\frac{3-(-3)}{12-4} = \frac{y}{}$

$\frac{3}{4} = \frac{y+1}{x}$

$3x = 4y + 4$

$3(18-2y) =$

$54 - 6y = 4y$

$54 - 4 = 6y +$

$50 = 10y \Rightarrow$ y

(1) $\Rightarrow x = 18$ -

$\boxed{x = 8}$

Slope of $\overline{BC} = \frac{-2-(-6)}{3-(-3)} = \frac{2}{3}$

$(\text{Slope of } \overline{AC})(\text{Slope of } \overline{BC}) = \left(-\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$

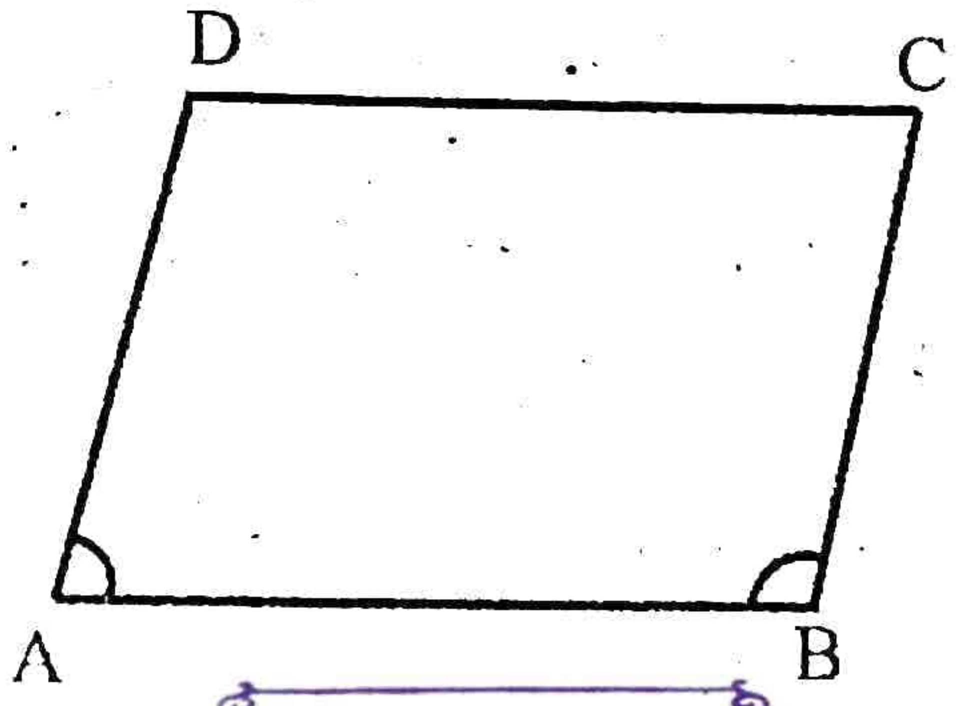
Since, $\overline{AC} \perp \overline{BC}$ then it is right angled triangle.

Q.8 Using slopes, find the fourth vertex of parallelogram if $(0,-1)$, $(4,-3)$ and , $(12,3)$ are its three consecutive vertices.

Solution:

Let $D(x,y)$ be the fourth vertex

$A(0,-1)$, $B(4,-3)$ and $C(12,3)$



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\because \overline{AB} \parallel \overline{DC}$

Slope of $\overline{AB}$ = Slope of $\overline{DC}$

$\frac{-3-(-1)}{4-0} = \frac{3-y}{12-x}$

$-\frac{1}{2} = \frac{3-y}{12-x}$

$-(12-x) = 2(3-y)$

$-12 + x = 6 - 2y$

$x = 18 - 2y \rightarrow (1)$

$\because \overline{BC} \parallel \overline{AD}$

Slope of $\overline{BC}$ = Slope of $\overline{AD}$

$\frac{3-(-3)}{12-4} = \frac{y-(-1)}{x-0}$

$\frac{3}{4} = \frac{y+1}{x}$

$3x = 4y + 4$

$3(18-2y) = 4y + 4$

$54 - 6y = 4y + 4$

$54 - 4 = 6y + 4y$

$50 = 10y \Rightarrow \boxed{y = 5}$

$(1) \Rightarrow x = 18 - 2(5)$

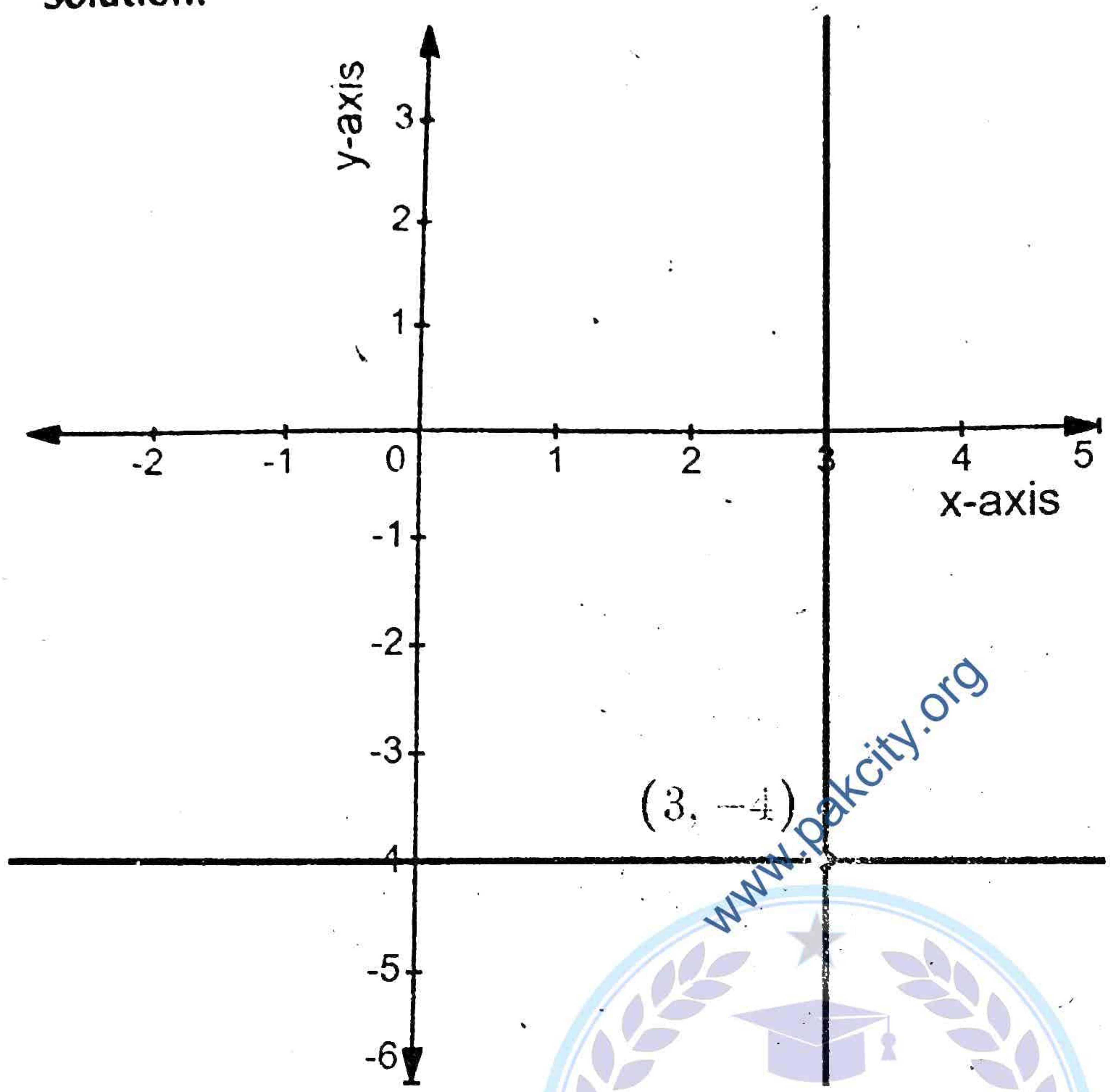
$\boxed{x = 8}$

The Students' Companion of Mathematics XII

EXERCISE 7.3

Q.1 Find the equation of the straight lines parallel to the coordinates axes and passing through the point $(3, -4)$.

Solution:



Point = $(3, -4)$

Equation of line parallel to x-axis: $y =$ constant

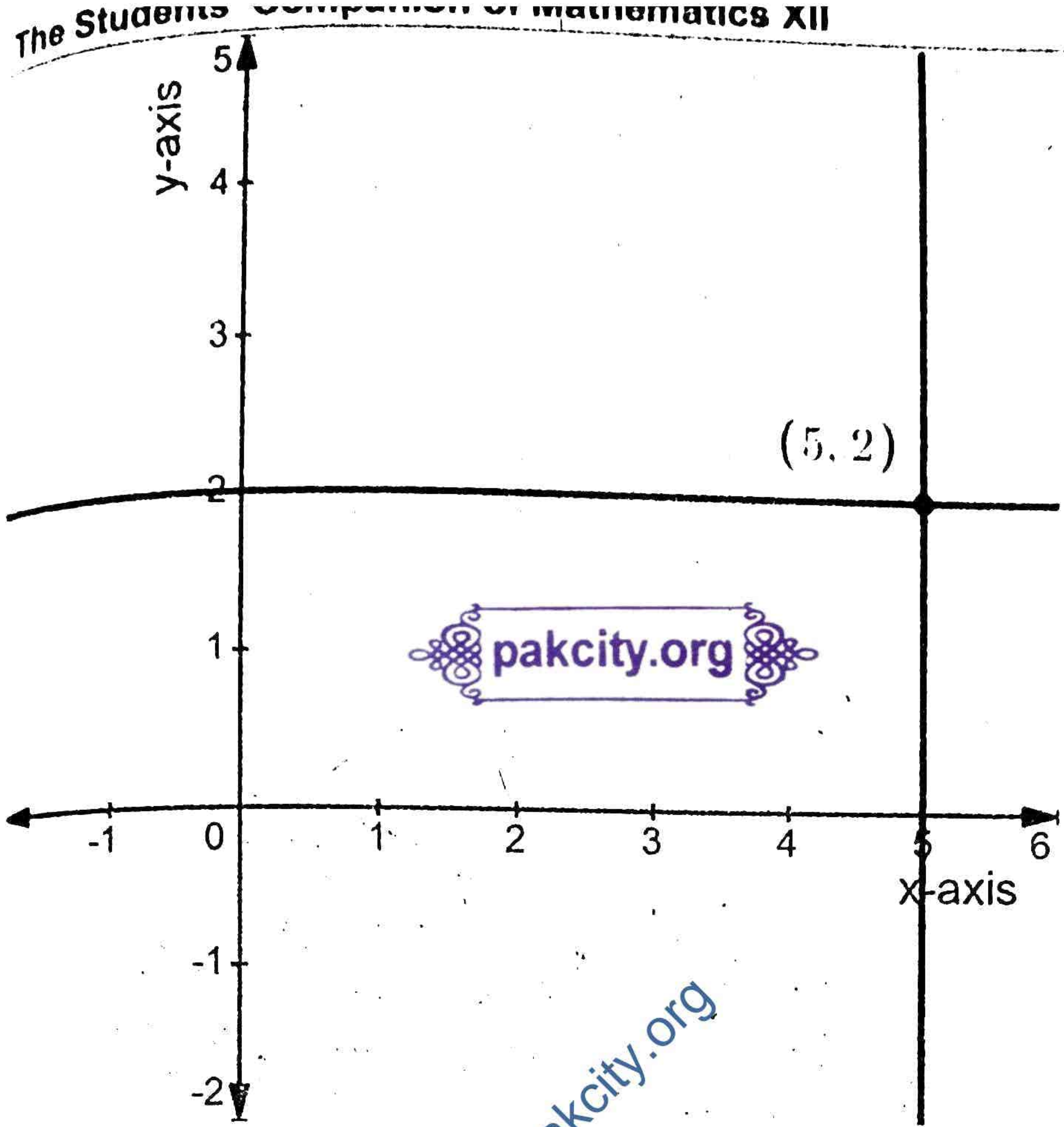
$\Rightarrow y = -4$

Equation of line parallel to y-axis: $x =$ constant

$\Rightarrow x = 3$

Q.2 Find the equation of the straight lines parallel to the coordinates axes and passing through the point $(5,2)$.

Solution:



Point = (5,2)

Equation of line parallel to x-axis: y = constant

$\Rightarrow y = 2$

Equation of line parallel to y-axis: x = constant

$\Rightarrow x = 5$

Q.3 Write the equation of the straight line parallel to x-axis which is at a distance of 5 units from above the x-axis.

Solution:

Equation of line parallel to x-axis: y = constant

$\Rightarrow y = 5$

Q.4 Find the equation of a line parallel to y-axis which is at a distance of 6 units of its left.

Solution:

Equation of line parallel to y-axis: x = constant

$\Rightarrow x = -6$

Q.6 Find the equation of the straight line determined by each of the following set of conditions.

(i) through $(5,-2)$ with the slope 4

(ii) through $(-1,-4)$ with the slope $-\frac{2}{3}$

(iii) through $\left(-\frac{1}{4},\frac{3}{4}\right)$ with the slope $\frac{2}{5}$

(iv) through $(0,b)$ with the slope m

(v) through the points $(7,-3)$ and $(-4,1)$

(vi) through the points $(5,-5)$ and $(-3,1)$

(vii) through the points $(a(t_1)^2, 2at_1)$ and $(a(t_2)^2, 2at_2)$

(viii) y-intercept $= 3$; slope $= 2$

(ix) y-intercept $= -2$; slope $= -\frac{2}{3}$

(x) y-intercept $= -5$; slope $= \frac{1}{2}$

(xi) y-intercept $= 0$; slope $= 0$

(xii) x-intercept $= 4$; y-intercept $= 3$

(xiii) x-intercept $= -2$; y-intercept $= 5$

(xiv) x-intercept $= -5$; y-intercept $= -1$

(xv) the line perpendicular from the origin to the line, $p = 3$ units and it makes an angle of $\alpha = 60°$ with x-axis.

(xvi) $p = \frac{3}{2}, \alpha = 150°$

Solution:

(i) through $(5,-2)$ with the slope 4

$(x_1,y_1) = (5,-2)$ and $m = 4$

$y - y_1 = m(x - x_1)$

$y - (-2) = 4(x - 5)$

$y + 2 = 4x - 20$

$0 = 4x - y - 22$

$\boxed{4x - y - 22 = 0}$

(ii) through $(-1,-4)$ with the slope $-\frac{2}{3}$

$(x_1,y_1) = (-1,-4)$ and $m = -\frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - (-4) = -\frac{2}{3}(x - (-1))$

$y + 4 = -\frac{2}{3}(x + 1)$

$3y + 12 = -2x - 2$

$2x + 3y + 14 = 0$

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(iii) through $(-$

$(x_1,y_1) = \left(-\frac{1}{4}\right.$

$y - y_1 = m(x -$

$y - \frac{3}{4} = \frac{2}{5}\left(x -\right.$

$\frac{4y-3}{4} = \frac{2}{5}\left(x\right.$

$\frac{4y-3}{4} = \frac{2}{5}\left(\frac{4x}{}\right.$

$5(4y - 3) = 8x$

$20y - 15 = 8x$

$0 = 8x - 20y +$

(iv) through $(0, b$

$(x_1,y_1) = (0,b)$

$y - y_1 = m(x -$

$y - b = m(x -$

$y - b = mx$

$y = mx + b$

(v) through the p

$(x_1,y_1) = (7,-3$

$y - y_1 = \frac{y_2 - y}{x_2 - x}$

$y - (-3) = \frac{1 -}{-4}$

$y + 3 = -\frac{4}{11}(x$

$11y + 33 = -4x$

$4x + 11y + 33 -$

$4x + 11y + 5 =$

(vi) through the p

$(x_1,y_1) = (5,-5)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$y - (-5) = \frac{1 - (}{-3 -}$

units and it

(iii) through $\left(-\frac{1}{4}, \frac{3}{4}\right)$ with the slope $\frac{2}{5}$

$(x_1, y_1) = \left(-\frac{1}{4}, \frac{3}{4}\right)$ and $m = \frac{2}{5}$

$y - y_1 = m(x - x_1)$

$y - \frac{3}{4} = \frac{2}{5}\left(x - \left(-\frac{1}{4}\right)\right)$

$\frac{4y - 3}{4} = \frac{2}{5}\left(x + \frac{1}{4}\right)$

$\frac{4y - 3}{4} = \frac{2}{5}\left(\frac{4x + 1}{4}\right)$

$5(4y - 3) = 8x + 2$

$20y - 15 = 8x + 2$

$0 = 8x - 20y + 15 + 2$

(iv) through $(0, b)$ with the slope m

$(x_1, y_1) = (0, b)$ and Slope $= m$

$y - y_1 = m(x - x_1)$

$y - b = m(x - 0)$

$y - b = mx$

$y = mx + b$

(v) through the points $(7, -3)$ and $(-4,1)$

$(x_1, y_1) = (7, -3)$ and $(x_2, y_2) = (-4,1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-3) = \frac{1 - (-3)}{-4 - 7}(x - 7)$

$y + 3 = -\frac{4}{11}(x - 7)$

$11y + 33 = -4x + 28$

$4x + 11y + 33 - 28 = 0$

$4x + 11y + 5 = 0$

(vi) through the points $(5, -5)$ and $(-3,1)$

$(x_1, y_1) = (5, -5)$ and $(x_2, y_2) = (-3,1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-5) = \frac{1 - (-5)}{-3 - 5}(x - 5)$

$y + 5 = -\frac{3}{4}(x - 5)$

$4y + 20 = -3x + 15$

$3x + 4y + 5 = 0$

(vii) through the points $(a(t_1)^2, 2at_1)$ and $(a(t_2)^2, 2at_2)$

$(x_1, y_1) = (a(t_1)^2, 2at_1)$ and $(x_2, y_2) = (a(t_2)^2, 2at_2)$

$y - 2at_1 = \frac{2at_2 - 2at_1}{a(t_2)^2 - a(t_1)^2}(x - a(t_1)^2)$

$y - 2at_1 = \frac{2a(t_2 - t_1)}{a(t_2 - t_1)(t_2 + t_1)}(x - a(t_1)^2)$

$y - 2at_1 = \frac{2}{(t_2 + t_1)}(x - a(t_1)^2)$

$(t_2 + t_1)(y - 2at_1) = 2(x - a(t_1)^2)$

$y(t_2 + t_1) - 2at_1(t_2 + t_1) = 2x - 2a(t_1)^2$

$y(t_2 + t_1) - 2at_1t_2 - 2a(t_1)^2 = 2x - 2a(t_1)^2$

$y(t_2 + t_1) - 2at_1t_2 = 2x$

$0 = 2x - y(t_2 + t_1) + 2at_1t_2$

$2x - y(t_2 + t_1) + 2at_1t_2 = 0$

(viii) y-intercept = 3; slope = 2

$y = mx + b$

$y = 3x + 2$

$0 = 3x - y + 2$

(ix) y-intercept = −2; slope = $-\frac{2}{3}$

$y = mx + b$

$y = -\frac{2}{3}x - 2$

× by 3

$3y = -2x - 6$

$2x + 3y + 6 = 0$

(x) y-intercept = −5; slope = $\frac{1}{2}$

$y = mx + b$

$y = \frac{1}{2}x - 5$

× by 2

$2y = x - 10$

$0 = x - 2y - 10$

(xi) y-intercept = 0; slope = 0

$y = mx + b$

$y = (0)x + 0$

$y = 0$

(xii) x-intercep

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4} + \frac{y}{3} = 1$

× by 12

$3x + 4y = 12$

(xiii) x-interce

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-2} + \frac{y}{5} = 1$

× by −10

$5x - 2y = -1$

(xiv) x-interce

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-5} + \frac{y}{-1} = 1$

× by −5

$x + 5y = -5$

(xv) the line pe

makes an angl

$x\cos\alpha + y\sin$

$x\cos 60° + y$

$x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right.$

× by 2

$x + \sqrt{3}y = 6$

(xvi) $p = \frac{3}{2}, \alpha =$

$x\cos\alpha + y\sin$

$x\cos 150° + y$

$x\left(-\frac{\sqrt{3}}{2}\right) + y($

$y = mx + b$

$y = (0)x + 0$

$y = 0$

(xii) x-intercept $= 4$; y-intercept $= 3$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{4} + \frac{y}{3} = 1$

$\times$ by 12

$3x + 4y = 12$

(xiii) x-intercept $= -2$; y-intercept $= 5$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-2} + \frac{y}{5} = 1$

$\times$ by -10

$5x - 2y = -10$

(xiv) x-intercept $= -5$; y-intercept $= -1$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-5} + \frac{y}{-1} = 1$

$\times$ by -5

$x + 5y = -5$

(xv) the line perpendicular from the origin to the line, $p = 3$ units and it makes an angle of $\alpha = 60°$ with x-axis.

$x\cos\alpha + y\sin\alpha = p$

$x\cos 60° + y\sin 60° = 3$

$x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 3$

$\times$ by 2

$x + \sqrt{3}y = 6$

(xvi) $p = \frac{3}{2}, \alpha = 150°$

$x\cos\alpha + y\sin\alpha = p$

$x\cos 150° + y\sin 150° = \frac{3}{2}$

$x\left(-\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = \frac{3}{2}$

× by 2

$-\sqrt{3}x + y = 3$

$\sqrt{3}x - y + 3 = 0$

Q.6 Reduce the equation $3x + 4y - 12 = 0$ to the
(i) slope-intercept form (ii) two-intercept form
(iii) normal or perpendicular form

Solution:

(i) slope-intercept form

$3x + 4y - 12 = 0$

$4y = -3x + 12$

$y = -\frac{3}{4}x + 3$

$y = mx + b$

(ii) two-intercept form

$3x + 4y - 12 = 0$

$3x + 4y = 12$

÷ by 12

$\frac{3x}{12} + \frac{4y}{12} = 1$

$\frac{x}{4} + \frac{y}{3} = 1$

$\frac{x}{a} + \frac{y}{b} = 1$

(iii) normal or perpendicular form

$3x + 4y - 12 = 0$

$3x + 4y = 12$

$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$

÷ by 5

$\frac{3x}{5} + \frac{4y}{5} = \frac{12}{5}$

$x\left(\frac{3}{5}\right) + y\left(\frac{4}{5}\right) = \frac{1}{5}$

$x \cos\alpha + y \sin\alpha = p$

Q.7 Find the equations of the sides of the triangle whose vertices are (1,4), (2, −3) and (−1, −2).

Solution:

A(1,4), B(2, −3) and C(−1, −2)

Equation of AB

$(x_1, y_1) = A(1,4)$ a

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}($

$y - 4 = \frac{-3-4}{2-1}(x$

$y - 4 = -7(x - 1$

$y - 4 = -7x + 7$

$7x + y - 4 - 7 =$

$7x + y - 11 = 0$

Equation of BC

$(x_1, y_1) = C(-1, -$

$y - (-2) = \frac{-3 -}{2 -}$

$y + 2 = -\frac{1}{3}(x +$

$3y + 6 = -x - 1$

$x + 3y + 6 + 1 =$

$x + 3y + 7 = 0$

Equation of AC

$(x_1, y_1) = A(1,4)$

$y - 4 = \frac{-2-4}{-1-1}($

$y - 4 = 3(x - 1)$

$y - 4 = 3x - 3$

$0 = 3x - y + 4 -$

$3x - y + 1 = 0$

Q.8 Find the equat
joining (−1,2) and

Solution:

Midpoint $= \left(\frac{x_1 +}{2}\right.$

$(x_1, y_1) = (-1,2)$

$(x_2, y_2) = (9,12)$

Midpoint $= \left(\frac{-1 +}{2}\right.$

$= (4,7)$

Equation of AB

$(x_1, y_1) = A(1,4)$ and $(x_2, y_2) = B(2,-3)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{-3 - 4}{2 - 1}(x - 1)$$

$$y - 4 = -7(x - 1)$$

$$y - 4 = -7x + 7$$

$$7x + y - 4 - 7 = 0$$

$$7x + y - 11 = 0$$

Equation of BC

$(x_1, y_1) = C(-1,-2)$ and $(x_2, y_2) = B(2,-3)$

$$y - (-2) = \frac{-3 - (-2)}{2 - (-1)}(x - (-1))$$

$$y + 2 = -\frac{1}{3}(x + 1)$$

$$3y + 6 = -x - 1$$

$$x + 3y + 6 + 1 = 0$$

$$x + 3y + 7 = 0$$

Equation of AC

$(x_1, y_1) = A(1,4)$ and $(x_2, y_2) = C(-1,-2)$

$$y - 4 = \frac{-2 - 4}{-1 - 1}(x - 1)$$

$$y - 4 = 3(x - 1)$$

$$y - 4 = 3x - 3$$

$$0 = 3x - y + 4 - 3$$

$$3x - y + 1 = 0$$

Q.8 Find the equation of the perpendicular bisector of the segment joining $(-1,2)$ and $(9,12)$.

Solution:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$(x_1, y_1) = (-1,2)$

$(x_2, y_2) = (9,12)$

$$\text{Midpoint} = \left(\frac{-1 + 9}{2}, \frac{2 + 12}{2}\right)$$

$$= (4,7)$$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \dfrac{12-2}{9+1} = \dfrac{10}{10} = 1$

$m_2 = -1$ [For perpendicular lines, $m_1 m_2 = -1$]

$y - y_1 = m(x - x_1)$

$y - 7 = -1(x - 4)$

$y - 7 = -x + 4$

$x + y - 7 - 4 = 0$

$x + y - 11 = 0$

Q.9 The x-intercept of a line is the reciprocal of its y-intercept and line passes through $(2, -1)$. Find its equation.

Solution:

Point $= (2, -1)$

x-intercept $= a$

y-intercept $= b$

According to question: $a = \frac{1}{b}$

$\dfrac{x}{a} + \dfrac{y}{b} = 1 \rightarrow (1)$

Put $(2, -1)$ and $a = \frac{1}{b}$ in (1)

$(1) \Rightarrow \dfrac{2}{\frac{1}{b}} + \dfrac{-1}{b} = 1$

$2b - \dfrac{1}{b} = 1$

$2b^2 - 1 = b$

$2b^2 - b - 1 = 0$

$2b^2 - 2b + b - 1 = 0$

$2b(b - 1) + (b - 1) = 0$

$(b - 1)(2b - 1) = 0$

$b = 1, \dfrac{1}{2}$

$b = 1 \Rightarrow a = 1 \rightarrow (A)$

$b = \dfrac{1}{2} \Rightarrow a = 2 \rightarrow (B)$

For (A), $(1) \Rightarrow$

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$\dfrac{x}{1} + \dfrac{y}{1} = 1$

$x + y - 1 = 0$

For (B), $(1) \Rightarrow$

$\dfrac{x}{2} + \dfrac{y}{\frac{1}{2}} = 1$

$\dfrac{x}{2} + 2y - 1 =$

$\times$ by 2

$x + 4y - 2 =$

Q.10 Find the sum of its inte

Solution:

Point $= (-2,$

x-intercept $=$

y-intercept $=$

According to q

$\dfrac{x}{a} + \dfrac{y}{b} = 1 \rightarrow ($

$bx + ay = ab$

Put $(-2, -4)$ a

$(1) \Rightarrow -2(3 -$

$-6 + 2a - 4a$

$a^2 - 3a - 6 -$

$a^2 - 5a - 6 =$

$a^2 - 6a + a -$

$a(a - 6) + 1($

$(a - 6)(a + 1)$

$\boxed{a = -1, 6}$

$a = -1 \Rightarrow b =$

$a = 6 \Rightarrow b = 3$

For (A), $(1) \Rightarrow$

$\dfrac{x}{-1} + \dfrac{y}{4} = 1$

$\times$ by -4

$4x - y = -4$

$4x - y + 4 = 0$

$\frac{x}{1}+\frac{y}{1}=1$

$x+y-1=0$

For (B), (1) $\Rightarrow$

$\frac{x}{2}+\frac{y}{\frac{1}{2}}=1$

$\frac{x}{2}+2y-1=0$

$\times$ by 2

$x+4y-2=0$

rcept and line

Q.10 Find the equation of the line which passes through $(-2,-4)$ and the sum of its intercepts equal to 3.

Solution:

Point $=(-2,-4)$

x-intercept $=a$

y-intercept $=b$

According to question: $a+b=3 \Rightarrow b=3-a$

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (1)$

$bx+ay=ab$

Put $(-2,-4)$ and $b=3-a$ in (1)

$(1) \Rightarrow -2(3-a)-4a=a(3-a)$

$-6+2a-4a=3a-a^2$

$a^2-3a-6-2a=0$

$a^2-5a-6=0$

$a^2-6a+a-6=0$

$a(a-6)+1(a-6)=0$

$(a-6)(a+1)=0$

$\boxed{a=-1,6}$

$a=-1 \Rightarrow b=3-(-1)=4 \rightarrow (A)$

$a=6 \Rightarrow b=3-6=-3 \rightarrow (B)$

For (A), (1) $\Rightarrow$

$\frac{x}{-1}+\frac{y}{4}=1$

$\times$ by -4

$4x-y=-4$

$4x-y+4=0$

For (B), $(1) \Rightarrow$

$$\frac{x}{6}+\frac{y}{-3}=1$$

$\times$ by 6

$$x-2y=6$$

Q.11 Find the equation of the line which passes through (5,6) and the y-intercept is twice that of the x-intercept.

Solution:

Point $=(5,6)$

x-intercept $=a$

y-intercept $=b$

According to question: $b=2a$

$$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (1)$$

Put (5,6) and $b=2a$ in (1)

$$(1) \Rightarrow \frac{5}{a}+\frac{6}{2a}=1$$

$$\frac{5}{a}+\frac{3}{a}=1$$

$$\frac{5+3}{a}=1$$

$$\boxed{a=8}$$

$$b=2(8)=16$$

$$(1) \Rightarrow \frac{x}{8}+\frac{y}{16}=1$$

$\times$ by 16

$$2x+y=16$$

Q.12 Find an equation of the line through $(11,-5)$ and parallel to a line having slope $\frac{3}{2}$.

Solution:

$$m_1=\frac{3}{2}$$

$$m_2=\frac{3}{2} \quad \begin{bmatrix}\text{For} \parallel \text{lines}\\ m_1=m_2\end{bmatrix}$$

$$(x_1,y_1)=(11,-5)$$

$$y-y_1=m(x-x_1)$$

$$y+5=\frac{3}{2}(x-11)$$

$2(y+5)=3(x$

$2y+10=3x-$

$0=3x-2y-$

$3x-2y-43=$

Q.13 Find an equ

the line having s

Solution:

$m_1=-\frac{3}{2}$

$m_2=\frac{2}{3}$ [For ... $m_1 m$

$(x_1,y_1)=(-4,$

$y-y_1=m(x$

$y+6=\frac{2}{3}(x+$

$3y+18=2x$

$2x-3y+8-$

$2x-3y-10$

Q.1 Determine

given straight l

(i) $3x+11y-$

(ii) $10x-12y$

(iii) $29x-17y$

Solution:

Note: for abov

(i) $3x+11y-$

$3x+11y-4$

Point $=(10,1$

$3(10)+11(1$

$-3<0$

(10,1) is below

Point $=(-4,6$

$3(-4)+11(6$

$10>0$

and the y-

l to a line

$2(y+5)=3(x-11)$

$2y+10=3x-33$

$0=3x-2y-10-33$

$3x-2y-43=0$

Q.13 Find an equation of the line through $(-4,-6)$ and perpendicular to the line having slope $-\frac{3}{2}$.

Solution:

$m_1=-\frac{3}{2}$

$m_2=\frac{2}{3}$ $\begin{bmatrix}\text{For } \perp \text{ lines}\\ m_1m_2=-1\end{bmatrix}$

$(x_1,y_1)=(-4,-6)$

$y-y_1=m(x-x_1)$

$y+6=\frac{2}{3}(x+4)$

$3y+18=2x+8$

$2x-3y+8-18=0$

$2x-3y-10=0$

EXERCISE 7.4

Q.1 Determine whether each of the specified points is above or below the given straight line:

(i) $3x+11y-44=0$, $(10,1)$, $(-4,6)$ and $(5,3)$

(ii) $10x-12y+17=0$, $(-20,-15)$, $(5,5)$ and $(100,84)$

(iii) $29x-17y+31=0$, $(0,2)$, $(-3,-3)$ and $(20,30)$

Solution:

Note: for above and below the line, coefficient of y must be positive.

(i) $3x+11y-44=0$, $(10,1)$, $(-4,6)$ and $(5,3)$

$3x+11y-44=0$

Point $=(10,1)$

$3(10)+11(1)-44=0$

$-3<0$

$(10,1)$ is below the line

Point $=(-4,6)$

$3(-4)+11(6)-44=0$

$10>0$

$(10,1)$ is above the line

Point $= (5,3)$

$3(5) + 11(3) - 44 = 0$

$4 > 0$

$(10,1)$ is above the line

(ii) $10x - 12y + 17 = 0, (-20,-15), (5,5)$ and $(100,84)$

$10x - 12y + 17 = 0$

$-10x + 12y - 17 = 0$

Point $= (-20,-15)$

$-10(-20) + 12(-15) - 17 = 0$

$3 > 0$

$(-20,-15)$ is above the line

Point $= (5,5)$

$-10(5) + 12(5) - 17 = 0$

$-7 < 0$

$(5,5)$ is below the line

Point $= (100,84)$

$-10(100) + 12(84) - 17 = 0$

$-9 < 0$

$(100,84)$ is below the line

(iii) $29x - 17y + 31 = 0, (0,2), (-3,-3)$ and $(20,30)$

$-29x + 17y - 31 = 0$

Point $= (0,2)$

$-29(0) + 17(2) - 31$

$3 > 0$

$(0,2)$ is above the line

Point $= (-3,-3)$

$-29(-3) + 17(-3) - 31$

$5 > 0$

$(-3,-3)$ is above the line

Point $= (20,30)$

$-29(20) + 17(30) - 31$

$-101 > 0$

$(20,30)$ is below the line

Q.2 In each of the following, find the perpendicular distance from the point to the line:

(i) $15x - 8y - 5$

(iii) $3x + 4y + 1$

(v) $5x + 12y -$

Solution:

$d = \left|\frac{ax_1 + by_1}{\sqrt{a^2 + b}}\right.$

(i) $15x - 8y -$

$(x_1, y_1) = (2,1$

$d = \left|\frac{15(2) -}{\sqrt{15^2 +}}\right.$

(ii) $3x - 4y +$

$d = \left|\frac{3(4) - 4}{\sqrt{3^2 +}}\right.$

(iii) $3x + 4y +$

$d = \left|\frac{3(3) + 4}{\sqrt{3^2}}\right.$

(iv) $2x - 7y +$

$d = \left|\frac{2(7) - 7}{\sqrt{2^2 +}}\right.$

(v) $5x + 12y -$

$d = \left|\frac{5(3) + 1}{\sqrt{5^2}}\right.$

Q.3 Find the d

(i) $5x - 12y +$

(ii) $x + y - 2$

(iii) $4x - 3y +$

Solution:

$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b}}\right.$

(i) $5x - 12y +$

$l_1: 5x - 12y +$

$a = 5, b = -1$

$d = \left|\frac{10 - (-}{\sqrt{5^2 + 1}}\right.$

(i) $15x - 8y - 5 = 0, (2,1)$ (ii) $3x - 4y + 5 = 0, (4, -3)$
(iii) $3x + 4y + 10 = 0, (3, -2)$ (iv) $2x - 7y + 1 = 0, (7,4)$
(v) $5x + 12y - 16 = 0, (3, -1)$

Solution:

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|.$$

(i) $15x - 8y - 5 = 0, (2,1)$

$(x_1, y_1) = (2,1)$ and $l: 15x - 8y - 5 = 0$

$$d = \left|\frac{15(2) - 8(1) - 5}{\sqrt{15^2 + (-8)^2}}\right| = \left|\frac{17}{\sqrt{289}}\right| = \left|\frac{17}{17}\right| = 1 \text{ units}$$

(ii) $3x - 4y + 5 = 0, (4, -3)$

$$d = \left|\frac{3(4) - 4(-3) + 5}{\sqrt{3^2 + (-4)^2}}\right| = \left|\frac{29}{\sqrt{25}}\right| = \left|\frac{29}{5}\right| = \frac{29}{5} \text{ units}$$

(iii) $3x + 4y + 10 = 0, (3, -2)$

$$d = \left|\frac{3(3) + 4(-2) + 10}{\sqrt{3^2 + (4)^2}}\right| = \left|\frac{11}{\sqrt{25}}\right| = \left|\frac{11}{5}\right| = \frac{11}{5} \text{ units}$$

(iv) $2x - 7y + 1 = 0, (7,4)$

$$d = \left|\frac{2(7) - 7(4) + 1}{\sqrt{2^2 + (-7)^2}}\right| = \left|\frac{-13}{\sqrt{53}}\right| = \frac{13}{\sqrt{53}} \text{ units}$$

(v) $5x + 12y - 16 = 0, (3, -1)$

$$d = \left|\frac{5(3) + 12(-1) - 16}{\sqrt{5^2 + 12^2}}\right| = \left|\frac{-13}{\sqrt{169}}\right| = \left|\frac{-13}{13}\right| = 1 \text{ units}$$

Q.3 Find the distance between the parallel lines:

(i) $5x - 12y + 10 = 0, 5x - 12y - 16 = 0$

(ii) $x + y - 2 = 0, 2x + 2y - 1 = 0$

(iii) $4x - 3y + 12 = 0, 4x - 3y - 12 = 0$

Solution:

$$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

(i) $5x - 12y + 10 = 0, 5x - 12y - 16 = 0$

$l_1: 5x - 12y + 10 = 0$ and $l_2: 5x - 12y - 16 = 0$

$a = 5, b = -12, c_1 = 10, c_1 = -16$

$$d = \left|\frac{10 - (-16)}{\sqrt{5^2 + 12^2}}\right|$$

$$d = \left|\frac{10+16}{\sqrt{169}}\right|$$

$$d = \left|\frac{26}{13}\right| = 2 \text{ units}$$

(ii) $x + y - 2 = 0, 2x + 2y - 1 = 0$

$l_1: x + y - 2 = 0$

and $l_2: 2x + 2y - 1 = 0$

$\div$ by 2

$$\Rightarrow x + y - \frac{1}{2} = 0$$

$$a = 1, b = 1, c_1 = -2, c_1 = -\frac{1}{2}$$

$$d = \left|\frac{-2-\left(-\frac{1}{2}\right)}{\sqrt{1^2+1^2}}\right|$$

$$d = \left|\frac{1}{\sqrt{2}}\left(-2+\frac{1}{2}\right)\right|$$

$$d = \left|-\frac{1}{\sqrt{2}}\left(\frac{3}{2}\right)\right|$$

$$d = \left|-\frac{3}{2\sqrt{2}}\right|$$

$$d = \frac{3}{2\sqrt{2}}$$

$$d = \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$d = \frac{3\sqrt{2}}{2(2)}$$

$$d = \frac{3\sqrt{2}}{4} \text{ units}$$

(iii) $4x - 3y + 12 = 0, 4x - 3y - 12 = 0$

$l_1: 4x - 3y + 12 = 0$ and $l_2: 4x - 3y - 12 = 0$

$$d = \left|\frac{12-(-12)}{\sqrt{4^2+(-3)^2}}\right|$$

$$d = \left|\frac{12+12}{\sqrt{25}}\right|$$

$$d = \left|\frac{24}{5}\right|$$

The Stud

$d = \frac{24}{5}$

Q.1 Wha

one of th

(−3,6)?

Solution

$m = \frac{y_2}{x_2}$

(x_1, y_1)

$m_1 = \frac{3}{2}$

(x_1, y_1)

$m_2 = \frac{6}{-}$

$\tan\theta =$

$\tan\theta =$

$\tan\theta =$

$\tan\theta =$

$\tan\theta =$

$\tan\theta =$

$\theta = \tan^{-}$

$\theta = 180$

Q.2 Find

$l_2: 2x +$

Solution:

$d = \frac{24}{5}$ units

EXERCISE 7.5

Q.1 What is the angle between two lines when they intersect at origin and one of the line passes through (2,3) and the other passes through (−3,6)?

Solution:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (2,3)$

$m_1 = \frac{3-0}{2-0} = \frac{3}{2}$

$(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (-3,6)$

$m_2 = \frac{6-0}{-3-0} = \frac{6}{-3} = -2$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-2 - \frac{3}{2}}{1 + (-2)\left(\frac{3}{2}\right)}\right|$

$\tan\theta = \left|\frac{-\frac{7}{2}}{1-3}\right|$

$\tan\theta = \left|\frac{-\frac{7}{2}}{-2}\right|$

$\tan\theta = \left|\frac{7}{2}\left(\frac{1}{2}\right)\right|$

$\tan\theta = \frac{7}{4}$

$\theta = \tan^{-1}\left(\frac{7}{4}\right) = 60.25°$

$\theta = 180° - 60.25° = 119.74°$

Q.2 Find the angle between the following lines: $l_1: 4x - 3y = 8$ and $l_2: 2x + 5y = 4$.

Solution:

$l_1: 4x - 3y = 8$ and $l_2: 2x + 5y = 4$

$m_1 = -\frac{4}{-3} = \frac{4}{3}$ and $m_2 = -\frac{2}{5}$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-\frac{2}{5} - \frac{4}{3}}{1 + \frac{4}{3}\left(-\frac{2}{5}\right)}\right|$

$\tan\theta = \left|\frac{-\frac{26}{15}}{\frac{7}{15}}\right|$

$\tan\theta = \left|\frac{-26}{7}\right|$

$\tan\theta = \frac{26}{7}$

$\theta = \tan^{-1}\left(\frac{26}{7}\right) = 74.93°$

Or $\theta = 180° - 74.93° = 105.06°$

Q.3 Find the acute angle between $l_1: y = 3x + 1$ and $l_2: y = -4x + 3$.

Solution:

$l_1: y = 3x + 1$ and $l_2: y = -4x + 3$

$m_1 = 3$ and $m_2 = -4$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-4 - 3}{1 + (3)(-4)}\right|$

$\tan\theta = \left|\frac{-7}{-11}\right|$

$\tan\theta = \frac{7}{11}$

$\theta = \tan^{-1}\frac{7}{11} = 32.47°$

Q.4 Find angle between two lines, one of which is the x-axis and the other line is $x - y + 4 = 0$.

Solution:

$\theta =$ Angle between the line and x-axis

$l: x - y + 4 = 0$

The Students' (

$m = -\frac{1}{-1} = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

Q.5 Find the an

$9 = 0.$

Solution:

$l_1: 2x - 3y + 7$

$m_1 = -\frac{2}{-3} =$

$\tan\theta = \left|\frac{m_2 -}{1 + m}\right.$

$\tan\theta = \left|\frac{-}{1 + (-}\right.$

$\tan\theta = \left|\frac{-\frac{29}{12}}{-\frac{2}{12}}\right|$

$\tan\theta = \left|\frac{29}{2}\right|$

$\tan\theta = \frac{29}{2}$

$\theta = \tan^{-1}\frac{29}{2} =$

Or $\theta = 180° -$

Q.6 Find the equ

45° with the lin

Solution:

$(x_1, y_1) = (3,2)$

$l_1: x - 2y = 3$

$m_1 = -\frac{1}{-2} = \frac{1}{2}$

$\theta =$ Angle betwe

$\tan\theta = \left|\frac{m_2 -}{1 + m_1}\right.$

$m = -\frac{1}{-1} = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

Q.5 Find the angle between the lines $2x - 3y + 7 = 0$ and $7x + 4y + 9 = 0$.

Solution:

$l_1: 2x - 3y + 7 = 0$ and $l_2: 7x + 4y + 9 = 0$

$m_1 = -\frac{2}{-3} = \frac{2}{3}$ and $m_2 = -\frac{7}{4}$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-\frac{7}{4} - \frac{2}{3}}{1 + \left(-\frac{7}{4}\right)\left(\frac{2}{3}\right)}\right|$

$\tan\theta = \left|\frac{-\frac{29}{12}}{-\frac{2}{12}}\right|$

$\tan\theta = \left|\frac{29}{2}\right|$

$\tan\theta = \frac{29}{2}$

$\theta = \tan^{-1}\frac{29}{2} = 86.05°$

Or $\theta = 180° - 86.05° = 93.94°$

Q.6 Find the equation of the line through point (3,2) and making angle 45° with the line $x - 2y = 3$.

Solution:

$(x_1, y_1) = (3,2)$

$l_1: x - 2y = 3$

$m_1 = -\frac{1}{-2} = \frac{1}{2}$

θ = Angle between the lines = 45°

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$x + 3.$

the other

$$\tan 45^\circ = \pm \frac{m_2 - \frac{1}{2}}{1 + \frac{1}{2}m_2}$$

$$1 = \frac{m_2 - \frac{1}{2}}{1 + \frac{1}{2}m_2}$$

$$1 = \frac{\frac{2m_2 - 1}{2}}{\frac{2 + m_2}{2}}$$

$$1 = \frac{2m_2 - 1}{2 + m_2}$$

$2 + m_2 = 2m_2 - 1$

$m_2 = 3$

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$

$0 = 3x - y - 7$

$$-1 = \frac{m_2 - \frac{1}{2}}{1 + \frac{1}{2}m_2}$$

$$-1 = \frac{\frac{2m_2 - 1}{2}}{\frac{2 + m_2}{2}}$$

$$-1 = \frac{2m_2 - 1}{2 + m_2}$$

$-2 - m_2 = 2m_2 - 1$

$3m_2 = -1$

$m_2 = -\frac{1}{3}$

$y - 2 = -\frac{1}{3}(x - 3)$

$3y - 6 = -x + 3$

$x + 3y - 9 = 0$

Q.7 Determine the measure of the acute angle between the straight line $x - y + 4 = 0$ and the straight line passing through the points (3,2) and (2,4).

Solution:

$l: x - y + 4 = 0$

Let

$m_1 =$ Slope of l

$m_1 = -\frac{1}{-1} = 1$

$(x_1, y_1) = (3,2)$ and $(x_2, y_2) = (2,4)$

$m_2 = \frac{4-2}{2-3} = -2$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-2-1}{1 + (-2)(1)}\right|$

The Students

$\tan\theta = \left|\frac{-3}{-1}\right|$

$\tan\theta = 3$

$\theta = \tan^{-1} 3 =$

Q.8 Find the e
intersection o
intersection.

Solution:

$l_1: 2x + 3y -$

The family of

$l_1 + kl_2 = 0$

$2x + 3y - 8$

$l_2: x - y + 1$

$l_1: 2x + 3y -$

$2(y - 1) + 3$

$2y - 2 + 3y$

$5y - 10 = 0$

$5y = 10$

$y = 2$

$(1) \Rightarrow x = 2$

$x = 1$

Point $= (1,2$

OR

$l_1: 2x + 3y -$

$\frac{x}{\begin{vmatrix} 3 & -8 \\ -1 & 1 \end{vmatrix}} =$

$\frac{x}{-5} = -\frac{y}{10} =$

$\frac{x}{-5} = -\frac{1}{5} \Rightarrow$

$-\frac{y}{10} = -\frac{1}{5} =$

$x = 1, y = 2$

Q.9 Find the e

(i) $2x + 3y +$

(ii) $x - 4y =$

$\tan\theta = \left|\frac{-3}{-1}\right|$

$\tan\theta = 3$

$\theta = \tan^{-1}3 = 71.56°$

Q.8 Find the equation of family of lines that pass through the point of intersection of $2x + 3y - 8 = 0$ and $x - y + 1 = 0$. Also find the point of intersection.

Solution:

$l_1: 2x + 3y - 8 = 0$ and $l_2: x - y + 1 = 0$

The family of line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$2x + 3y - 8 + k(x - y + 1) = 0$

$l_2: x - y + 1 = 0 \Rightarrow x = y - 1 \rightarrow (1)$

$l_1: 2x + 3y - 8 = 0$

$2(y - 1) + 3y - 8 = 0$

$2y - 2 + 3y - 8 = 0$

$5y - 10 = 0$

$5y = 10$

$y = 2$

$(1) \Rightarrow x = 2 - 1$

$x = 1$

Point $= (1,2)$

OR

$l_1: 2x + 3y - 8 = 0$ and $l_2: x - y + 1 = 0$

$$\frac{x}{\begin{vmatrix} 3 & -8 \\ -1 & 1 \end{vmatrix}} = -\frac{y}{\begin{vmatrix} 2 & -8 \\ 1 & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x}{-5} = -\frac{y}{10} = \frac{1}{-5}$$

$$\frac{x}{-5} = -\frac{1}{5} \Rightarrow x = 1$$

$$-\frac{y}{10} = -\frac{1}{5} \Rightarrow y = 2$$

$x = 1, y = 2$

Q.9 Find the equation of a line through the intersection of the lines:

(i) $2x + 3y + 1 = 0, 3x - 4y = 5$ and passing through the point $(2,1)$

(ii) $x - 4y = 3, x + 2y = 9$ and passing through origin.

straight line
ts (3,2) and

(iii) $3x + 2y = 8, 5x - 11y + 1 = 0$ and parallel to $6x + 13y = 25$.
(iv) $2x - 3y + 4 = 0, 3x + 3y - 5 = 0$ and parallel to y-axis.
(v) $5x - 6y = 1, 3x + 2y + 5 = 0$ and perpendicular to $5y - 3x = 11$.
(vi) $3x - 4y + 1 = 0, 5x + y - 1 = 0$ and cutting off equal intercepts from the axes.
(vii) $43x + 29y + 43 = 0, 23x + 8y + 6 = 0$ and having y-intercept -2.
(viii) $2x + 7y - 8 = 0, 3x + 2y + 5 = 0$ and making an angle of $45°$ with the line $2x + 3y - 7 = 0$.

Solution:

(i) $2x + 3y + 1 = 0, 3x - 4y = 5$ and passing through the point $(2,1)$

$l_1: 2x + 3y + 1 = 0$

$l_2: 3x - 4y - 5 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(2x + 3y + 1) + k(3x - 4y - 5) = 0 \rightarrow (1)$

It passes through $(2,1)$

$(1) \Rightarrow (2(2) + 3(1) + 1) + k(3(2) - 4(1) - 5) = 0$

$8 + k(-3) = 0$

$8 - 3k = 0 \Rightarrow k = \frac{8}{3}$

$(1) \Rightarrow (2x + 3y + 1) + \frac{8}{3}(3x - 4y - 5) = 0$

$3(2x + 3y + 1) + 8(3x - 4y - 5) = 0$

$6x + 9y + 3 + 24x - 32y - 40 = 0$

$30x - 23y - 37 = 0$

(ii) $x - 4y = 3, x + 2y = 9$ and passing through origin.

$l_1: x - 4y = 3$

$l_2: x + 2y = 9$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(x - 4y - 3) + k(x + 2y - 9) = 0 \rightarrow (1)$

It passes through $(0,0)$

$(1) \Rightarrow (0 - 4(0) - 3) + k(0 + 2(0) - 9) = 0$

$-3 - 9k = 0$

$9k = -3 \Rightarrow k = -\frac{1}{3}$

$(1) \Rightarrow (x -$

$3x - 12y -$

$2x - 14y =$

(iii) $3x + 2y$

$l_1: 3x + 2y$

$l_2: 5x - 11$

The line thr

$l_1 + kl_2 =$

$(3x + 2y -$

$3x + 2y -$

$x(3 + 5k)$

(1) is paral

$a_1b_2 - a_2b$

$6(2 - 11k$

$12 - 66k$

$-131k -$

$131k = -$

$k = -\frac{27}{131}$

$(1) \Rightarrow (3x$

$131(3x +$

$393x + 26$

$258x + 55$

(iv) $2x - 3$

$l_1: 2x - 3$

$l_2: 3x + 3$

The line th

$l_1 + kl_2 =$

$(2x - 3y$

$2x - 3y +$

$x(2 + 3k)$

(1) is paral

Slope of y-

$-\frac{2 + 3k}{-3 + 3k}$

y = 25.
- 3x = 11.
ntercepts
tercept −2.
of 45° with
int (2,1)

$(1) \Rightarrow (x - 4y - 3) - \frac{1}{3}(x + 2y - 9) = 0$

$3x - 12y - 9 - x - 2y + 9 = 0$

$2x - 14y = 0$

(iii) $3x + 2y = 8, 5x - 11y + 1 = 0$ and parallel to $6x + 13y = 25$.

$l_1: 3x + 2y - 8 = 0$

$l_2: 5x - 11y + 1 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(3x + 2y - 8) + k(5x - 11y + 1) = 0 \rightarrow (1)$

$3x + 2y - 8 + 5kx - 11ky + k = 0$

$x(3 + 5k) + y(2 - 11k) + k - 8 = 0$

(1) is parallel to $6x + 13y = 25$

$a_1b_2 - a_2b_1 = 0$

$6(2 - 11k) - 13(3 + 5k) = 0$

$12 - 66k - 65k - 39 = 0$

$-131k - 27 = 0$

$131k = -27$

$k = -\frac{27}{131}$

$(1) \Rightarrow (3x + 2y - 8) - \frac{27}{131}(5x - 11y + 1) = 0$

$131(3x + 2y - 8) - 27(5x - 11y + 1) = 0$

$393x + 262y - 1048 - 135x + 297y - 27 = 0$

$258x + 559y - 1075 = 0$

(iv) $2x - 3y + 4 = 0, 3x + 3y - 5 = 0$ and parallel to y-axis.

$l_1: 2x - 3y + 4 = 0$

$l_2: 3x + 3y - 5 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(2x - 3y + 4) + k(3x + 3y - 5) = 0 \rightarrow (1)$

$2x - 3y + 4 + 3kx + 3ky - 5k = 0$

$x(2 + 3k) + y(-3 + 3k) + 4 - 5k = 0$

(1) is parallel to y-axis

Slope of y-axis $= \frac{1}{0}$

$-\frac{2 + 3k}{-3 + 3k} = \frac{1}{0}$

$-3 + 3k = 0 \Rightarrow k = 1$

$(1) \Rightarrow (2x - 3y + 4) + 1(3x + 3y - 5) = 0$

$5x - 1 = 0$

(v) $5x - 6y = 1, 3x + 2y + 5 = 0$ and perpendicular to $5y - 3x = 11$.

$l_1: 5x - 6y - 1 = 0$

$l_2: 3x + 2y + 5 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(5x - 6y - 1) + k(3x + 2y + 5) = 0 \rightarrow (1)$

$5x - 6y - 1 + 3kx + 2ky + 5k = 0$

$x(5 + 3k) + y(-6 + 2k) - 1 + 5k = 0$

(1) is perpendicular to $-3x + 5y = 11$

$a_1a_2 + b_1b_2 = 0$

$-3(5 + 3k) + 5(-6 + 2k) = 0$

$-15 - 9k - 30 + 10k = 0$

$k - 45 = 0 \Rightarrow k = 45$

$(1) \Rightarrow (5x - 6y - 1) + 45(3x + 2y + 5) = 0$

$5x - 6y - 1 + 135x + 90y + 225 = 0$

$140x + 84y + 224 = 0$

(vi) $3x - 4y + 1 = 0, 5x + y - 1 = 0$ and cutting off equal intercepts from the axes.

$l_1: 3x - 4y + 1 = 0$

$l_2: 5x + y - 1 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(3x - 4y + 1) + k(5x + y - 1) = 0 \rightarrow (1)$

$3x - 4y + 1 + 5kx + ky - k = 0$

$x(3 + 5k) + y(-4 + k) = k - 1$

$\div$ by $(k - 1)$

$$\frac{x(3+5k)}{k-1} + \frac{y(-4+k)}{k-1} = 1$$

$$\frac{x}{\frac{k-1}{(3+5k)}} + \frac{y}{\frac{k-1}{(-4+k)}} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$a = \frac{k-1}{3+5k} \text{ and } b = \frac{k-1}{-4+k}$$

The St...

$\frac{k-1}{3+5k} = \frac{k ...}{-4 ...}$

$\frac{k-1}{3+5k} - \frac{k ...}{-4 ...}$

$(k-1)\{\frac{1}{3+ ...}$

$(k-1)\{\frac{-4}{(3+ ...}$

$(k-1)\{-4k ...$

$k = 1, -\frac{7}{4}$

$(1) \Rightarrow (3x - ...$

$8x - 3y = 0$

$(1) \Rightarrow (3x - ...$

$12x - 16y + ...$

$-23x - 23y ...$

(vii) $43x + 2 ...$

$l_1: 43x + 29 ...$

$l_2: 23x + 8y ...$

The line thro...

$l_1 + kl_2 = 0$

$(43x + 29y ...$

$43x + 29y + ...$

$x(43 + 23k) ...$

For y-interce...

$0(43 + 23k) ...$

$-58 - 16k ...$

$-10k - 15 ...$

$10k = -15 ...$

$(1) \Rightarrow (43x ...$

$2(43x + 29y ...$

$86x + 58y + ...$

$17x + 34y + ...$

(viii) $2x + 7y ...$

the line $2x + ...$

ular to $5y - 3x = 1$

$\frac{k-1}{3+5k} = \frac{k-1}{-4+k}$

$\frac{k-1}{3+5k} - \frac{k-1}{-4+k} = 0$

$(k-1)\left\{\frac{1}{3+5k} - \frac{1}{-4+k}\right\} = 0$

$(k-1)\left\{\frac{-4+k-3-5k}{(3+5k)(-4+k)}\right\} = 0$

$(k-1)\{-4k-7\} = 0$

$k = 1, -\frac{7}{4}$

$(1) \Rightarrow (3x-4y+1) + 1(5x+y-1) = 0$

$8x - 3y = 0$

$(1) \Rightarrow (3x-4y+1) - \frac{7}{4}(5x+y-1) = 0$

$12x - 16y + 4 - 35x - 7y + 7 = 0$

$-23x - 23y + 11 = 0$

(vii) $43x + 29y + 43 = 0, 23x + 8y + 6 = 0$ and having y-intercept -2.

$l_1: 43x + 29y + 43 = 0$

$l_2: 23x + 8y + 6 = 0$

equal intercepts

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(43x + 29y + 43) + k(23x + 8y + 6) = 0 \rightarrow (1)$

$43x + 29y + 43 + 23kx + 8ky + 6k = 0$

$x(43 + 23k) + y(29 + 8k) + 43 + 6k = 0$

For y-intercept: $x = 0$

$0(43 + 23k) + (-2)(29 + 8k) + 43 + 6k = 0$

$-58 - 16k + 43 + 6k = 0$

$-10k - 15 = 0$

$10k = -15 \Rightarrow k = -\frac{3}{2}$

$(1) \Rightarrow (43x + 29y + 43) - \frac{3}{2}(23x + 8y + 6) = 0$

$2(43x + 29y + 43) - 3(23x + 8y + 6) = 0$

$86x + 58y + 86 - 69x - 24y - 18 = 0$

$17x + 34y + 68 = 0$

(viii) $2x + 7y - 8 = 0, 3x + 2y + 5 = 0$ and making an angle of 45° with the line $2x + 3y - 7 = 0$.

$l_1: 2x + 7y - 8 = 0$

$l_2: 3x + 2y + 5 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(2x + 7y - 8) + k(3x + 2y + 5) = 0 \rightarrow (1)$

$2x + 7y - 8 + 3kx + 2ky + 5k = 0$

$x(2 + 3k) + y(7 + 2k) + 5k - 8 = 0$

$m_1 = -\dfrac{2 + 3k}{7 + 2k}$ and $m_2 = -\dfrac{2}{3}$

$\tan 45° = \pm\dfrac{m_1 - m_2}{1 + m_1 m_2}$

$1 = \dfrac{-\frac{2}{3} - m_2}{1 - \frac{2}{3}m_2}$

$1 = \dfrac{-2 - 3m_2}{3 - 2m_2}$

$3 - 2m_2 = -2 - 3m_2$

$m_2 = -5$

$-\dfrac{2 + 3k}{7 + 2k} = -5$

$2 + 3k = 5(7 + 2k)$

$2 + 3k = 35 + 10k$

$-33 = 7k$

$k = -\dfrac{33}{7}$

$1 = -\dfrac{-\frac{2}{3} - m_2}{1 - \frac{2}{3}m_2}$

$1 = -\dfrac{-2 - 3m_2}{3 - 2m_2}$

$3 - 2m_2 = 2 + 3m_2$

$1 = 5m_2$

$m_2 = \dfrac{1}{5}$

$-\dfrac{2 + 3k}{7 + 2k} = \dfrac{1}{5}$

$-5(2 + 3k) = 7 + 2k$

$-10 - 15k = 7 + 2k$

$-17 = 17k$

$k = -1$

$(1) \Rightarrow (2x + 7y - 8) - 1(3x + 2y + 5) = 0$

$2x + 7y - 8 - 3x - 2y - 5 = 0$

$-x + 5y - 13 = 0$

$(1) \Rightarrow (2x + 7y - 8) - \dfrac{33}{7}(3x + 2y + 5) = 0$

$7(2x + 7y - 8) - 33(3x + 2y + 5) = 0$

$14x + 49y - 56 - 99x - 66y - 165 = 0$

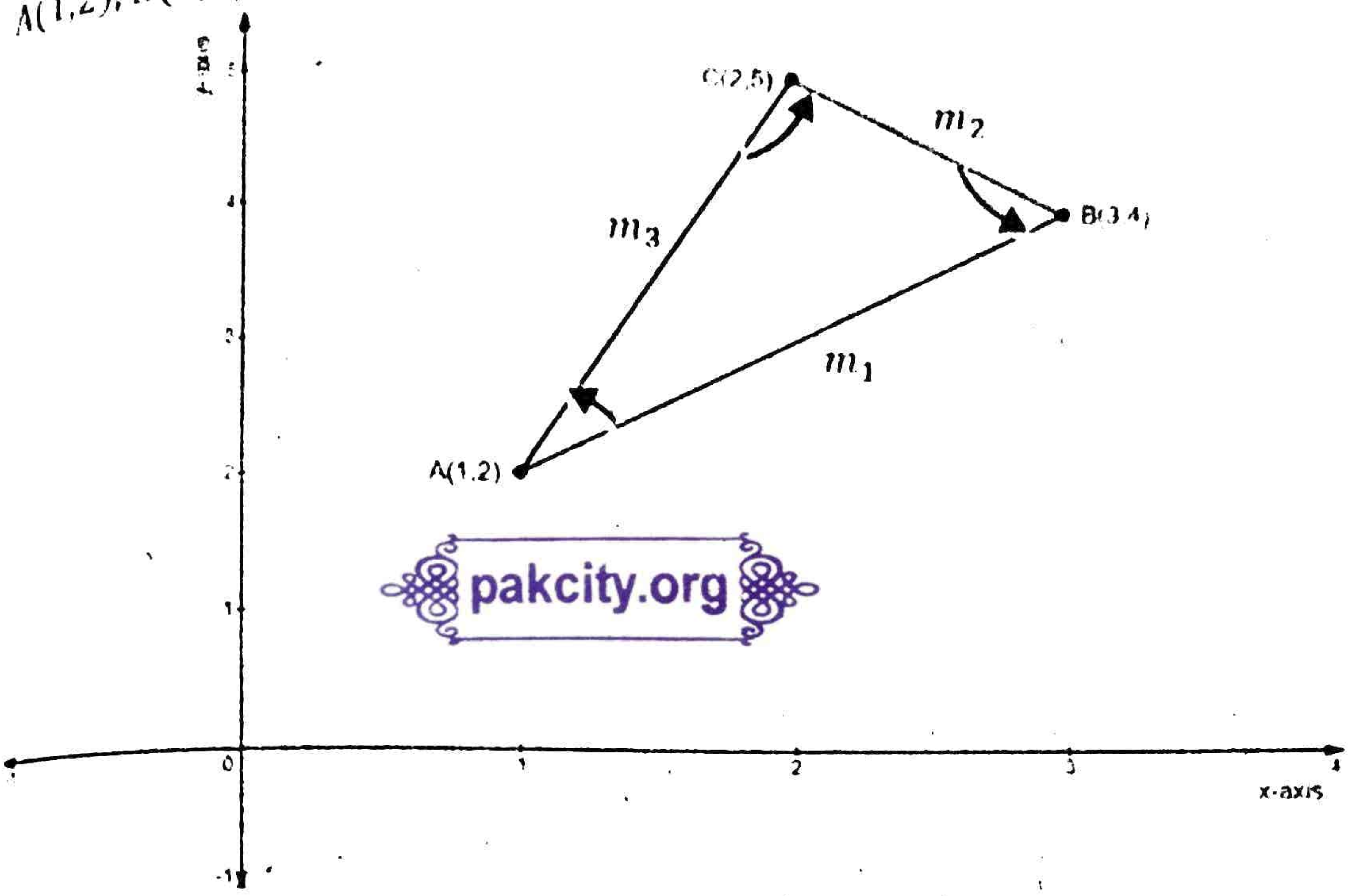
$-85x - 17y - 221 = 0$

$85x + 17y + 221 = 0$

Q.10 Find the angles of the triangle with the given vertices (1,2), (3,4) and (2,5).

Solution:

A(1,2), B(3,4) and C(2,5)



m_1 = Slope of $\overline{AB}$

m_2 = Slope of $\overline{BC}$

m_3 = Slope of $\overline{AC}$

$m_1 = \dfrac{4-2}{3-1} = 1$

$m_2 = \dfrac{5-4}{2-3} = -1$

$m_3 = \dfrac{5-2}{2-1} = 3$

$m_1 m_2 = (1)(-1) = -1$

$\overline{BC} \perp \overline{AB}$ i.e. $\angle B = 90°$

Other two angles are acute

$\tan\angle A = \dfrac{m_3 - m_1}{1 + m_1 m_3}$

$\tan\angle A = \dfrac{3-1}{1+(3)(1)} = \dfrac{1}{2}$

$\angle A = \tan^{-1}\dfrac{1}{2} = 26.56°$

$\tan\angle C = \dfrac{m_2 - m_3}{1 + m_2 m_3}$

..2), (3,4)

$\tan\angle C = \dfrac{-1-3}{1+(-1)(3)} = 2$

$\angle C = \tan^{-1} 2 = 63.43^\circ$

OR

$\angle A + \angle B + \angle C = 180^\circ$

$\angle C = 180^\circ - 26.56^\circ - 90^\circ = 63.43^\circ$

Q.11 What are the angle of the triangle with vertices $A(3,2)$, $B(4,5)$ and $C(-1,-1)$?

Solution:

$A(3,2)$, $B(4,5)$ and $C(-1,-1)$

$m_1 =$ Slope of $\overline{AB}$

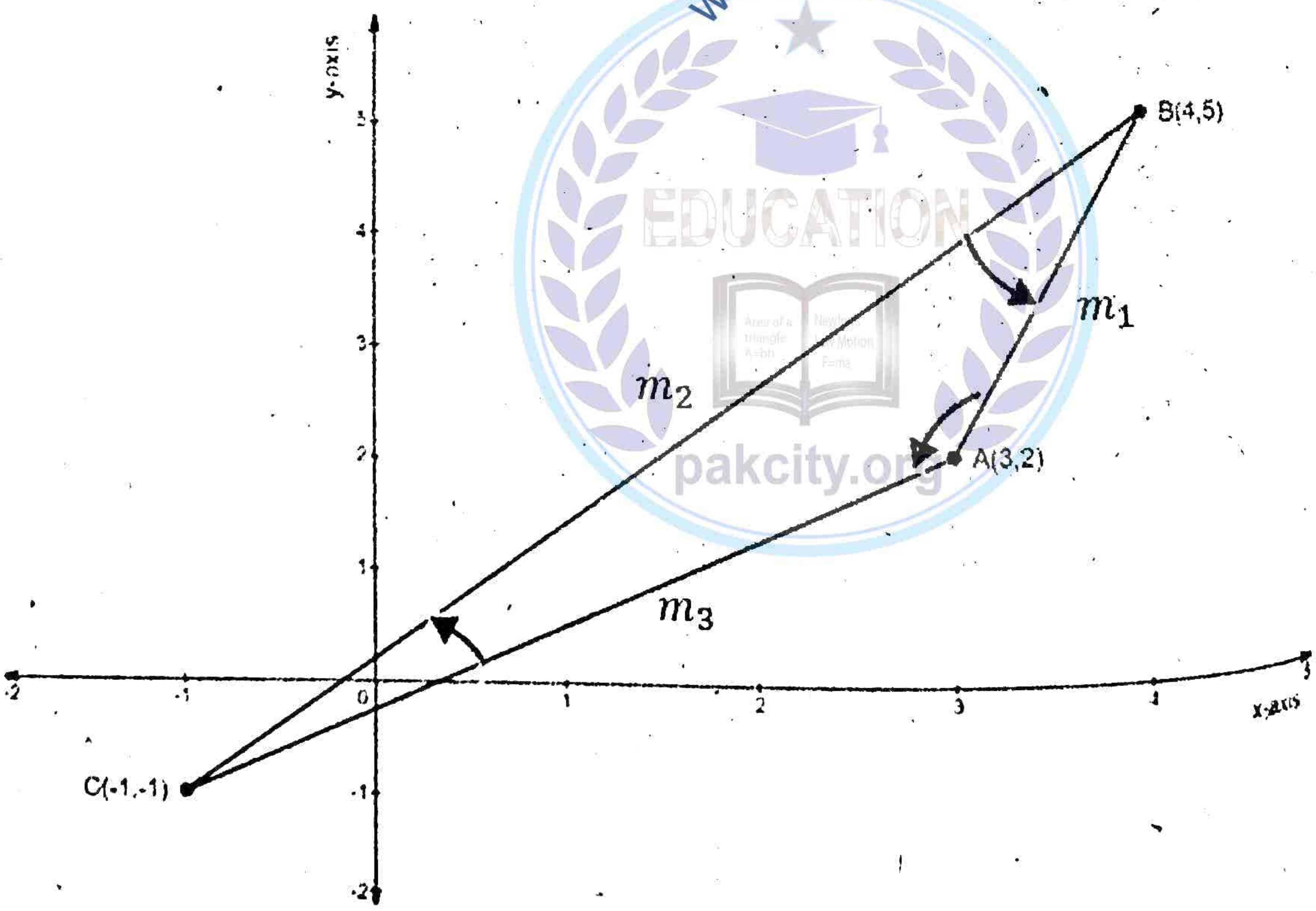
$m_2 =$ Slope of $\overline{BC}$

$m_3 =$ Slope of $\overline{AC}$

$m_1 = \dfrac{5-2}{4-3} = 3$

$m_2 = \dfrac{5+1}{4+1} = \dfrac{6}{5}$

$m_3 = \dfrac{2+1}{3+1} = \dfrac{3}{4}$



$\tan\angle A = \dfrac{m_3 - m_1}{1 + m_1 m_3}$

, B(4,5) and

$$m_1 = \frac{5-2}{4-3} = 3$$

$$m_2 = \frac{5+1}{4+1} = \frac{6}{5}$$

$$m_3 = \frac{2+1}{3+1} = \frac{3}{4}$$

$$\tan\angle A = \frac{\frac{3}{4}-3}{1+\left(\frac{3}{4}\right)(3)} = \frac{-\frac{9}{4}}{\frac{13}{4}} = -\frac{9}{13}$$

$$\angle A = \tan^{-1}\left(-\frac{9}{13}\right) = -34.69°$$

$$\text{or } \angle A = 180° - 34.69° = 145.31°$$

$$\tan\angle B = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\angle B = \frac{3-\frac{6}{5}}{1+3\left(\frac{6}{5}\right)} = \frac{\frac{9}{5}}{\frac{23}{5}} = \frac{9}{23}$$

$$\angle B = \tan^{-1}\left(\frac{9}{23}\right) = 21.37°$$

$$\tan\angle C = \frac{\frac{6}{5}-\frac{3}{4}}{1+\left(\frac{6}{5}\right)\left(\frac{3}{4}\right)} = \frac{\frac{9}{20}}{\frac{19}{10}} = \frac{9}{38}$$

$$\angle C = \tan^{-1}\left(\frac{9}{38}\right) = 13.32°$$

OR

$$\angle A + \angle B + \angle C = 180°$$

$$\angle C = 180° - 21.37° - 145.31° = 13.32°$$

Q.12 Find the angles of triangle where slopes of its sides are $3, \frac{1}{2}$ and -2.

Solution:

$m_1 = \text{Slope of } l_1 = 3$

$m_2\text{Slope of } l_2 = \frac{1}{2}$

$m_3 = \text{Slope of } l_3 = -2$

$$m_2 m_3 = \left(\frac{1}{2}\right)(-2)$$

$$m_2 m_3 = -1$$

$l_2 \perp l_3$ i.e. angle is 90°

Other both angles are acute

$$\tan\angle A = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{3 - \frac{1}{2}}{1 + (3)\left(\frac{1}{2}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{5}{2}}{\frac{5}{2}}\right|$$

$$\tan\theta = 1$$

$$\theta = 45^\circ$$

Third angle is $180^\circ - 90^\circ - 45^\circ = 45^\circ$

Q.1 If $A(2,5)$, $B(3,7)$ and $C(0,8)$ are the vertices of a triangle then find the

(i) equation of median through A

(ii) equation of altitude through B

(iii) equation of right bisector of side $\overline{AC}$

Solution:

$A(2,5)$, $B(3,7)$ and $C(0,8)$

(i) equation of median through A

Midpoint of $\overline{BC} = \left(\frac{3+0}{2}, \frac{7+8}{2}\right) = \left(\frac{3}{2}, \frac{15}{2}\right)$

$(x_1, y_1) = A(2,5)$

$(x_2, y_2) = \left(\frac{3}{2}, \frac{15}{2}\right)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 5 = \frac{\frac{15}{2} - 5}{\frac{3}{2} - 2}(x - 2)$$

$$y - 5 = \frac{\frac{5}{2}}{-\frac{1}{2}}(x - 2)$$

$$y - 5 = -5(x - 2)$$

$y - 5 = -5x +$

$5x + y - 5 - 1($

$5x + y - 15 =$

ii) equation of al

$(x_1, y_1) = A(2,$

$(x_2, y_2) = C(0,$

Slope of $\overline{AC} =$

Slope of altitud

$(x_1, y_1) = B(3,$

$y - y_1 = m(x$

$y - 7 = \frac{2}{3}(x -$

$3y - 21 = 2x$

$0 = 2x - 3y +$

(iii) equation o

$(x_1, y_1) = A($

$(x_2, y_2) = C($

Slope of $\overline{AC} =$

Slope of right

Midpoint of $\overline{A}$

$(x_1, y_1) = (1,$

$y - \frac{13}{2} = \frac{2}{3}($

$\frac{2y - 13}{2} = \frac{2}{3}$

$6y - 39 = 4$

$0 = 4x - 6y$

Q.2 Show tha

concurrency.

(i) $x - y = 6,$

(ii) $\frac{x}{a} + \frac{y}{b} = 1,$

(iii) $5x + y +$

Solution:

(i) $x - y = 6,$

$y - 5 = -5x + 10$

$5x + y - 5 - 10 = 0$

$5x + y - 15 = 0$

ii) equation of altitude through B

$(x_1, y_1) = A(2,5)$

$(x_2, y_2) = C(0,8)$

Slope of $\overline{AC} = \frac{8-5}{0-2} = -\frac{3}{2}$

Slope of altitude through B $= \frac{2}{3}$

$(x_1, y_1) = B(3,7)$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{3}(x - 3)$

$3y - 21 = 2x - 6$

$0 = 2x - 3y + 15$

(iii) equation of right bisector of side $\overline{AC}$

$(x_1, y_1) = A(2,5)$

$(x_2, y_2) = C(0,8)$

Slope of $\overline{AC} = \frac{8-5}{0-2} = -\frac{3}{2}$

Slope of right bisector of $\overline{AC} = \frac{2}{3}$

Midpoint of $\overline{AC} = \left(\frac{2+0}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

$(x_1, y_1) = \left(1, \frac{13}{2}\right)$

$y - \frac{13}{2} = \frac{2}{3}(x - 1)$

$\frac{2y - 13}{2} = \frac{2}{3}(x - 1)$

$6y - 39 = 4x - 4$

$0 = 4x - 6y + 35$

Q.2 Show that the following lines are concurrent. Also find their point of concurrency.

(i) $x - y = 6, 4y + 22 = 3x$ and $6x + 5y + 8 = 0$

(ii) $\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$ and $y = x$

(iii) $5x + y + 11 = 0, x + 7y + 9 = 0$ and $2x + y + 5 = 0$

Solution:

(i) $x - y = 6, 4y + 22 = 3x$ and $6x + 5y + 8 = 0$

$x-y-6=0, 0=3x-4y-22$ and $6x+5y+8=0$

$$\begin{vmatrix} 1 & -1 & -6 \\ 3 & -4 & -22 \\ 6 & 5 & 8 \end{vmatrix}$$

$$= 1\begin{vmatrix} -4 & -22 \\ 5 & 8 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -22 \\ 6 & 8 \end{vmatrix} - 6\begin{vmatrix} 3 & -4 \\ 6 & 5 \end{vmatrix}$$

$= 1(78) + 1(156) - 6(39)$

$= 0$

Hence, lines are concurrent

$l_1: x-y-6=0 \Rightarrow x = y+6$

$l_2 \Rightarrow 6(y+6)+5y+8=0$ [using l_1]

$6y+36+5y+8=0$

$11y+44=0$

$11y=-44$

$y=-4$

$l_1 \Rightarrow x=-4+6$

$x=2$

Point $=(2,-4)$

$l_1: x-y-6=0$ and $l_2: 6x+5y+8=0$

$$\frac{x}{\begin{vmatrix} -1 & -6 \\ 5 & 8 \end{vmatrix}} = -\frac{y}{\begin{vmatrix} 1 & -6 \\ 6 & 8 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -1 \\ 6 & 5 \end{vmatrix}}$$

$$\frac{x}{22} = -\frac{y}{44} = \frac{1}{11}$$

$$\frac{x}{22} = \frac{1}{11} \Rightarrow x = 2$$

$$-\frac{y}{44} = \frac{1}{11} \Rightarrow y = -4$$

(ii) $\frac{x}{a}+\frac{y}{b}=1, \frac{x}{b}+\frac{y}{a}=1$ and $y=x$

$bx+ay-ab=0, ax+by-ab=0$ and $x-y=0$

$$\begin{vmatrix} b & a & -ab \\ a & b & -ab \\ 1 & -1 & 0 \end{vmatrix}$$

$$= b\begin{vmatrix} b & -ab \\ -1 & 0 \end{vmatrix} - a\begin{vmatrix} a & -ab \\ 1 & 0 \end{vmatrix} - ab\begin{vmatrix} a & b \\ 1 & -1 \end{vmatrix}$$

$= b(-ab) - a(ab) - ab(-a-b)$

$= -ab^2 - a^2b + a^2b + ab^2$

$= 0$

Hence, lines ...

$l_1: y = x$

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{a}+\frac{x}{b}=1$

$x\left(\frac{1}{a}+\frac{1}{b}\right)=$

$x\left(\frac{b+a}{ab}\right)=$

$x=\frac{ab}{a+b}$

$l_1 \Rightarrow y = \frac{\ldots}{a}$

Point $= \left(\frac{\ldots}{a}\right.$

OR

$\frac{x}{a}+\frac{y}{b}=1, \frac{x}{b}$

$\frac{x}{\begin{vmatrix} a & -ab \\ b & -ab \end{vmatrix}} =$

$\frac{x}{ab^2-a^2b}$

$\frac{x}{ab(b-a)}$

$\frac{x}{ab(b-a)}$

$\frac{y}{ab(b-a)}$

$y=\frac{ab}{b+a}$

(iii) $5x+y$ ...

$\begin{vmatrix} 5 & 1 & 11 \\ 1 & 7 & 9 \\ 2 & 1 & 5 \end{vmatrix}$

$=5\begin{vmatrix} 7 & 9 \\ 1 & 5 \end{vmatrix}$

$=5(26)-$

Hence, lines are concurrent

$l_1: y = x$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{a} + \frac{x}{b} = 1$ [using l_1]

$x\left(\frac{1}{a} + \frac{1}{b}\right) = 1$

$x\left(\frac{b+a}{ab}\right) = 1$

$x = \frac{ab}{a+b}$

$l_1 \Rightarrow y = \frac{ab}{a+b}$

$\text{Point} = \left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

OR

$\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$ and $y = x$

$$\frac{x}{\begin{vmatrix} a & -ab \\ b & -ab \end{vmatrix}} = -\frac{y}{\begin{vmatrix} b & -ab \\ a & -ab \end{vmatrix}} = \frac{1}{\begin{vmatrix} b & a \\ a & b \end{vmatrix}}$$

$$\frac{x}{ab^2 - a^2b} = -\frac{y}{a^2b - ab^2} = \frac{1}{b^2 - a^2}$$

$$\frac{x}{ab(b-a)} = -\frac{y}{ab(a-b)} = \frac{1}{(b-a)(b+a)}$$

$$\frac{x}{ab(b-a)} = \frac{1}{(b-a)(b+a)} \Rightarrow x = \frac{ab}{b+a}$$

$$\frac{y}{ab(b-a)} = \frac{1}{(b-a)(b+a)}$$

$$y = \frac{ab}{b+a}$$

(iii) $5x + y + 11 = 0, x + 7y + 9 = 0$ and $2x + y + 5 = 0$

$$\begin{vmatrix} 5 & 1 & 11 \\ 1 & 7 & 9 \\ 2 & 1 & 5 \end{vmatrix}$$

$$= 5\begin{vmatrix} 7 & 9 \\ 1 & 5 \end{vmatrix} - 1\begin{vmatrix} 1 & 9 \\ 2 & 5 \end{vmatrix} + 11\begin{vmatrix} 1 & 7 \\ 2 & 1 \end{vmatrix}$$

$$= 5(26) - 1(-13) + 11(-13)$$

$= 0$

Hence, lines are concurrent

$l_1: 5x + y + 11 = 0 \Rightarrow y = -11 - 5x$

$2x + y + 5 = 0$

$2x + (-11 - 5x) + 5 = 0$ [using l_1]

$-3x - 6 = 0$

$3x = -6$

$x = -2$

$l_1 \Rightarrow y = -11 - 5(-2)$

$y = -11 + 10$

$y = -1$

Point $= (-2, -1)$

OR

$$\frac{x}{\begin{vmatrix}1 & 11\\ 7 & 9\end{vmatrix}} = -\frac{y}{\begin{vmatrix}5 & 11\\ 1 & 9\end{vmatrix}} = \frac{1}{\begin{vmatrix}5 & 1\\ 1 & 7\end{vmatrix}}$$

$$\frac{x}{-68} = -\frac{y}{34} = \frac{1}{34}$$

$$\frac{x}{-68} = \frac{1}{34} \Rightarrow x = -2$$

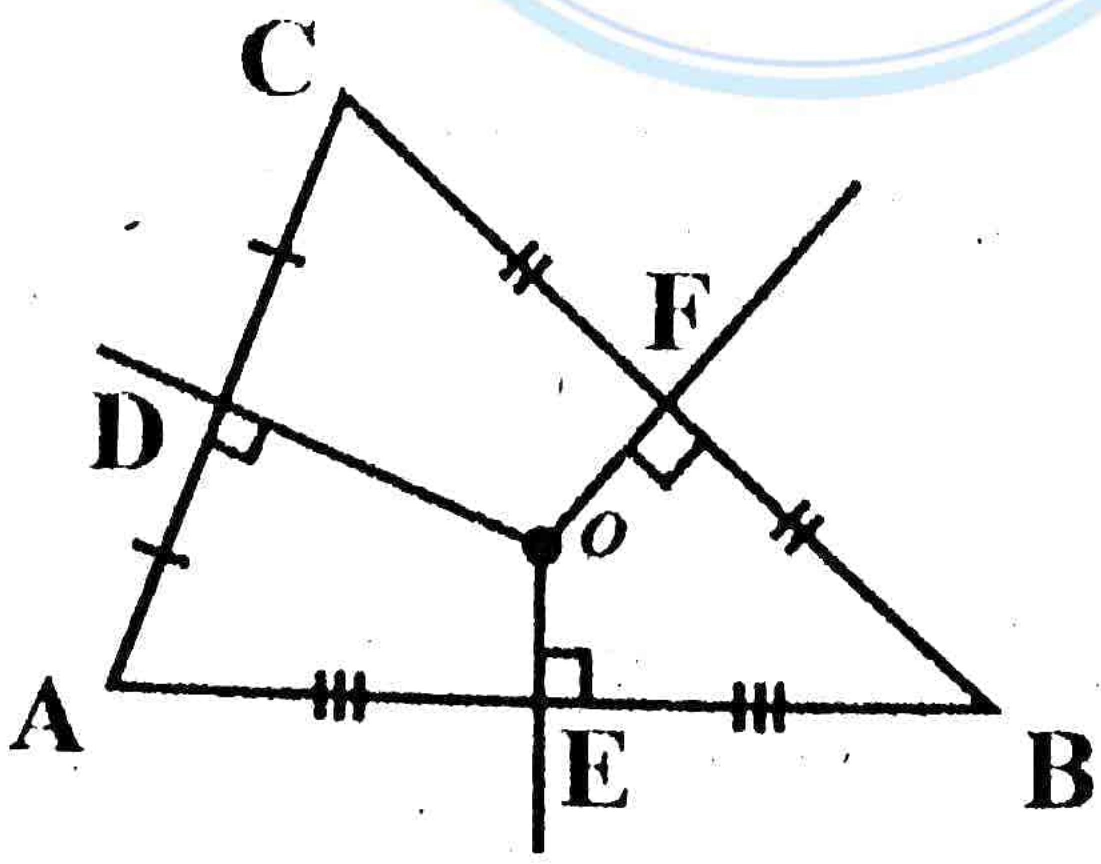
$$-\frac{y}{34} = \frac{1}{34} \Rightarrow y = -1$$

Q.3 If A(−1,5), B(2,3) and C(7,6) the vertices of triangle, then show right bisectors, medians and altitudes of the triangle are concurrent.

Solution:

A(−1,5), B(2,3) and C(7,6)

RIGHT BISECTORS ARE CONCURRENT



Midpoint $= \left(\frac{x_1}{\ldots}\right.$

$D = \left(\frac{-1+7}{2}, \frac{5}{\ldots}\right.$

$E = \left(\frac{-1+2}{2}, \frac{5}{\ldots}\right.$

$F = \left(\frac{2+7}{2}, \frac{3+}{2}\right.$

Slope of $\overline{AB} = \ldots$

Slope of $\overline{BC} = \frac{6}{7}\ldots$

Slope of $\overline{AC} = \frac{6}{7}\ldots$

Right bisector o

$y - y_1 = m(x - \ldots$

$(x_1, y_1) = E\left(\frac{1}{2}\right.$

$y - 4 = \frac{3}{2}\left(x - \ldots\right.$

$y - 4 = \frac{3}{2}\left(\frac{2x \ldots}{2}\right.$

$4y - 16 = 6x \ldots$

$0 = 6x - 4y + \ldots$

Right bisector o

$(x_1, y_1) = F\left(\frac{9}{2}\right.$

$y - \frac{9}{2} = -\frac{5}{3}\left(x \ldots\right.$

$\frac{2y - 9}{2} = -\frac{5}{3}\left(\ldots\right.$

$3(2y - 9) = - \ldots$

$6y - 27 = -1\ldots$

$10x + 6y - 72 \ldots$

Right bisector o

$(x_1, y_1) = D\left(3, \ldots\right.$

$y - \frac{11}{2} = -8(x \ldots$

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$D = \left(\frac{-1+7}{2}, \frac{5+6}{2}\right) = \left(3, \frac{11}{2}\right)$$

$$E = \left(\frac{-1+2}{2}, \frac{5+3}{2}\right) = \left(\frac{1}{2}, 4\right)$$

$$F = \left(\frac{2+7}{2}, \frac{3+6}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$$

$$\text{Slope of } \overline{AB} = \frac{5-3}{-1-2} = -\frac{2}{3}$$

$$\text{Slope of } \overline{BC} = \frac{6-3}{7-2} = \frac{3}{5}$$

$$\text{Slope of } \overline{AC} = \frac{6-5}{7+1} = \frac{1}{8}$$

<u>Right bisector of $\overline{AB}$</u>

$$y - y_1 = m(x - x_1)$$

$$(x_1, y_1) = E\left(\frac{1}{2}, 4\right)$$

$$y - 4 = \frac{3}{2}\left(x - \frac{1}{2}\right)$$

$$y - 4 = \frac{3}{2}\left(\frac{2x-1}{2}\right)$$

$$4y - 16 = 6x - 3$$

$$0 = 6x - 4y + 13$$

<u>Right bisector of $\overline{BC}$</u>

$$(x_1, y_1) = F\left(\frac{9}{2}, \frac{9}{2}\right)$$

$$y - \frac{9}{2} = -\frac{5}{3}\left(x - \frac{9}{2}\right)$$

$$\frac{2y-9}{2} = -\frac{5}{3}\left(\frac{2x-9}{2}\right)$$

$$3(2y-9) = -5(2x-9)$$

$$6y - 27 = -10x + 45$$

$$10x + 6y - 72 = 0$$

<u>Right bisector of $\overline{AC}$</u>

$$(x_1, y_1) = D\left(3, \frac{11}{2}\right)$$

$$y - \frac{11}{2} = -8(x - 3)$$

how right

$$\frac{2y-11}{2} = -8(x-3)$$

$$2y - 11 = -16x + 48$$

$$16x + 2y - 59 = 0$$

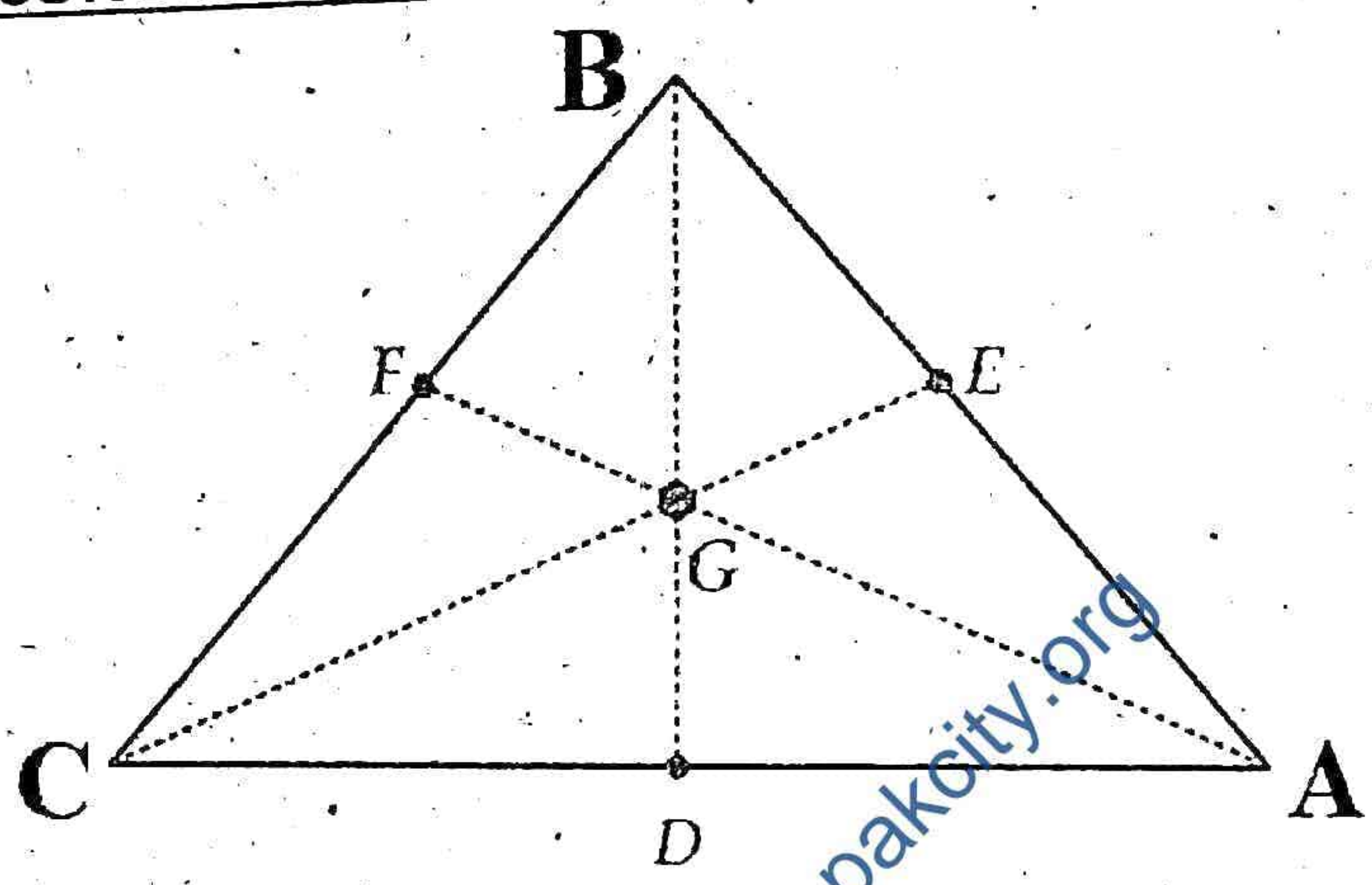
$$\begin{vmatrix} 6 & -4 & 13 \\ 10 & 6 & -72 \\ 16 & 2 & -59 \end{vmatrix}$$

$$= 6\begin{vmatrix} 6 & -72 \\ 2 & -59 \end{vmatrix} - (-4)\begin{vmatrix} 10 & -72 \\ 16 & -59 \end{vmatrix} + 13\begin{vmatrix} 10 & 6 \\ 16 & 2 \end{vmatrix}$$

$$= 6(-210) + 4(562) + 13(-76)$$

$$= 0$$

MEDIANS ARE CONCURRENT



$$D = \left(\frac{-1+7}{2}, \frac{5+6}{2}\right) = \left(3, \frac{11}{2}\right)$$

$$E = \left(\frac{-1+2}{2}, \frac{5+3}{2}\right) = \left(\frac{1}{2}, 4\right)$$

$$F = \left(\frac{2+7}{2}, \frac{3+6}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$$

Equation of median $\overline{AF}$

$$(x_1, y_1) = A(-1,5)$$

$$(x_2, y_2) = F\left(\frac{9}{2}, \frac{9}{2}\right)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 5 = \frac{\frac{9}{2} - 5}{\frac{9}{2} + 1}(x + 1)$$

$$y - 5 = -\frac{1}{11}(x + 1)$$

$$11y - 55 = -x - 1$$

$$x + 11y - 54 = 0$$

The Students' Comp

Equation of median $\overline{B}$

$$(x_1, y_1) = B(2,3)$$

$$(x_2, y_2) = D\left(3, \frac{11}{2}\right)$$

$$y - 3 = \frac{\frac{11}{2} - 3}{3 - 2}(x -$$

$$y - 3 = \frac{5}{2}(x - 2)$$

$$2y - 6 = 5x - 10$$

$$0 = 5x - 2y - 4$$

Equation of median

$$(x_1, y_1) = C(7,6)$$

$$(x_2, y_2) = E\left(\frac{1}{2}, 4\right)$$

$$y - 6 = \frac{4 - 6}{\frac{1}{2} - 7}(x -$$

$$y - 6 = \frac{4}{13}(x - 7)$$

$$13y - 78 = 4x - 2$$

$$0 = 4x - 13y + 50$$

$$\begin{vmatrix} 1 & 11 & -54 \\ 5 & -2 & -4 \\ 4 & -13 & 50 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & -4 \\ -13 & 50 \end{vmatrix} - 1$$

$$= 1(-152) - 11(2$$

$$= 0$$

ALTITUDES ARE CO

Slope of $\overline{AB} = \frac{5 -}{-1 -}$

Equation of median $\overline{BD}$

$(x_1, y_1) = B(2,3)$

$(x_2, y_2) = D\left(3, \frac{11}{2}\right)$

$y - 3 = \frac{\frac{11}{2} - 3}{3 - 2}(x - 2)$

$y - 3 = \frac{5}{2}(x - 2)$

$2y - 6 = 5x - 10$

$0 = 5x - 2y - 4$

Equation of median $\overline{CE}$

$(x_1, y_1) = C(7,6)$

$(x_2, y_2) = E\left(\frac{1}{2}, 4\right)$

$y - 6 = \frac{4 - 6}{\frac{1}{2} - 7}(x - 7)$

$y - 6 = \frac{4}{13}(x - 7)$

$13y - 78 = 4x - 28$

$0 = 4x - 13y + 50$

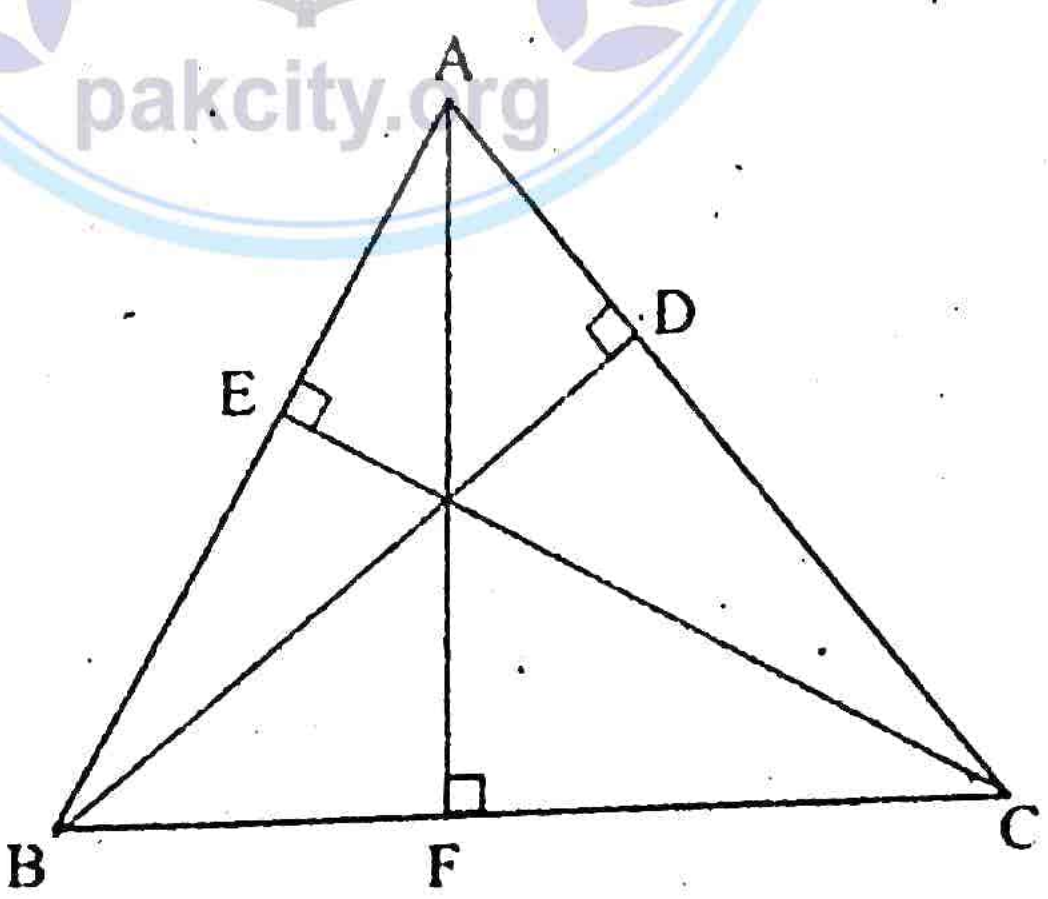
$$\begin{vmatrix} 1 & 11 & -54 \\ 5 & -2 & -4 \\ 4 & -13 & 50 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & -4 \\ -13 & 50 \end{vmatrix} - 11\begin{vmatrix} 5 & -4 \\ 4 & 50 \end{vmatrix} - 54\begin{vmatrix} 5 & -2 \\ 4 & -13 \end{vmatrix}$$

$= 1(-152) - 11(266) - 54(-57)$

$= 0$

ALTITUDES ARE CONCURRENT



Slope of $\overline{AB} = \frac{5 - 3}{-1 - 2} = -\frac{2}{3}$

Slope of $\overline{BC} = \dfrac{6-3}{7-2} = \dfrac{3}{5}$

Slope of $\overline{AC} = \dfrac{6-5}{7+1} = \dfrac{1}{8}$

Equation of altitude $\overline{AF}$

$(x_1, y_1) = A(-1,5)$

$y - y_1 = m(x - x_1)$

$y - 5 = -\dfrac{5}{3}(x+1)$

$3y - 15 = -5x - 5$

$5x + 3y - 10 = 0$

Equation of altitude $\overline{BD}$

$(x_1, y_1) = B(2,3)$

$y - 3 = -8(x-2)$

$y - 3 = -8x + 16$

$8x + y - 19 = 0$

Equation of altitude $\overline{CE}$

$(x_1, y_1) = C(7,6)$

$y - 6 = \dfrac{3}{2}(x-7)$

$2y - 12 = 3x - 21$

$-3x + 2y + 9 = 0$

$5x + 3y - 10 = 0$

$8x + y - 19 = 0$

$-3x + 2y + 9 = 0$

$$\begin{vmatrix} 5 & 3 & -10 \\ 8 & 1 & -19 \\ -3 & 2 & 9 \end{vmatrix}$$

$$= 5\begin{vmatrix} 1 & -19 \\ 2 & 9 \end{vmatrix} - 3\begin{vmatrix} 8 & -19 \\ -3 & 9 \end{vmatrix} - 10\begin{vmatrix} 8 & 1 \\ -3 & 2 \end{vmatrix}$$

$= 5(47) - 3(15) - 10(19)$

$= 0$

Q.1 Find t

(i) (11, −1

(iii) (−5, -

(iv) (−a, l

(v) (a cos

Solution:

(i) (11, −

Area of ∆

$= \frac{1}{2}\{11|$

$= \frac{1}{2}\{11($

= 57 un

(ii) (3,1)

Area of

$= \frac{1}{2}\{3|$

$= \frac{1}{2}\{3($

= 29 ur

(iii) (−5

Area of

$= \frac{1}{2}\{-5$

$= \frac{1}{2}\{-5$

$= \frac{105}{2}$

(iv) (−a

Area of

EXERCISE 7.7

Q.1 Find the area of the triangle whose vertices are:

(i) $(11,-12),(6,2)$ and $(-5,10)$ (ii) $(3,1),(-2,5)$ and $(-4,-5)$

(iii) $(-5,-2),(4,-6)$ and $(1,7)$

(iv) $(-a,b+c),(a,b-c)$ and $(a,-c)$

(v) $(a\cos\theta_1, b\sin\theta_1),(a\cos\theta_2, b\sin\theta_2)$ and $(a\cos\theta_3, b\sin\theta_3)$

Solution:

(i) $(11,-12),(6,2)$ and $(-5,10)$

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} 11 & -12 & 1 \\ 6 & 2 & 1 \\ -5 & 10 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{11\begin{vmatrix} 2 & 1 \\ 10 & 1 \end{vmatrix} - (-12)\begin{vmatrix} 6 & 1 \\ -5 & 1 \end{vmatrix} + 1\begin{vmatrix} 6 & 2 \\ -5 & 10 \end{vmatrix}\right\}$$

$$= \frac{1}{2}\{11(-8) + 12(11) + 1(70)\}$$

$= 57$ units

(ii) $(3,1),(-2,5)$ and $(-4,-5)$

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ -2 & 5 & 1 \\ -4 & -5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{3\begin{vmatrix} 5 & 1 \\ -5 & 1 \end{vmatrix} - 1\begin{vmatrix} -2 & 1 \\ -4 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 5 \\ -4 & -5 \end{vmatrix}\right\}$$

$$= \frac{1}{2}\{3(10) - 1(2) + (30)\}$$

$= 29$ units

(iii) $(-5,-2),(4,-6)$ and $(1,7)$

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} -5 & -2 & 1 \\ 4 & -6 & 1 \\ 1 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{-5\begin{vmatrix} -6 & 1 \\ 7 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -6 \\ 1 & 7 \end{vmatrix}\right\}$$

$$= \frac{1}{2}\{-5(-13) + 2(3) + (34)\}$$

$= \frac{105}{2}$ units

(iv) $(-a,b+c),(a,b-c)$ and $(a,-c)$

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} -a & b+c & 1 \\ a & b-c & 1 \\ a & -c & 1 \end{vmatrix}$$

Add R_1 in R_2 and R_3

$$=\frac{1}{2}\begin{vmatrix}-a & b+c & 1\\ 0 & 2b & 2\\ 0 & b & 2\end{vmatrix}$$

$$=\frac{1}{2}\left\{-a\begin{vmatrix}2b & 2\\ b & 2\end{vmatrix}-0+0\right\}$$

$$=\frac{1}{2}\{-a(4b-2b)\}$$

$$=\frac{1}{2}\{-a(2b)\}$$

$$=-ab$$

$$=ab \text{ units}$$

(v) $(a\cos\theta_1, b\sin\theta_1)$, $(a\cos\theta_2, b\sin\theta_2)$ and $(a\cos\theta_3, b\sin\theta_3)$

$$\text{Area of }\Delta=\frac{1}{2}\begin{vmatrix}a\cos\theta_1 & b\sin\theta_1 & 1\\ a\cos\theta_2 & b\sin\theta_2 & 1\\ a\cos\theta_3 & b\sin\theta_3 & 1\end{vmatrix}$$

By R_2-R_1 and R_3-R_1

$$=\frac{1}{2}\begin{vmatrix}a\cos\theta_1 & b\sin\theta_1 & 1\\ a\cos\theta_2-a\cos\theta_1 & b\sin\theta_2-b\sin\theta_1 & 0\\ a\cos\theta_3-a\cos\theta_1 & b\sin\theta_3-b\sin\theta_1 & 0\end{vmatrix}$$

$$=\frac{1}{2}\left\{1\begin{vmatrix}a\cos\theta_2-a\cos\theta_1 & b\sin\theta_2-b\sin\theta_1\\ a\cos\theta_3-a\cos\theta_1 & b\sin\theta_3-b\sin\theta_1\end{vmatrix}-0+0\right\}$$

$$=\frac{1}{2}\left\{\begin{array}{l}ab(\cos\theta_2-\cos\theta_1)(\sin\theta_3-\sin\theta_1)\\ -ab(\sin\theta_2-\sin\theta_1)(\cos\theta_3-\cos\theta_1)\end{array}\right\}$$

$$=\frac{ab}{2}\left\{\begin{array}{l}(\sin\theta_3\cos\theta_2-\sin\theta_3\cos\theta_1-\sin\theta_1\cos\theta_2+\sin\theta_1\cos\theta_1)\\ -(\cos\theta_3\sin\theta_2-\cos\theta_3\sin\theta_1-\cos\theta_1\sin\theta_2+\cos\theta_1\sin\theta_1)\end{array}\right\}$$

$$=\frac{ab}{2}\left\{\begin{array}{l}\sin\theta_3\cos\theta_2-\sin\theta_3\cos\theta_1-\sin\theta_1\cos\theta_2+\sin\theta_1\cos\theta_1\\ -\cos\theta_3\sin\theta_2+\cos\theta_3\sin\theta_1+\cos\theta_1\sin\theta_2-\cos\theta_1\sin\theta_1\end{array}\right\}$$

$$=\frac{ab}{2}\left\{\begin{array}{l}\sin\theta_3\cos\theta_2-\cos\theta_3\sin\theta_2+\sin\theta_1\cos\theta_3-\cos\theta_1\sin\theta_3\\ \qquad +\sin\theta_2\cos\theta_1-\cos\theta_2\sin\theta_1\end{array}\right\}$$

$$=\frac{ab}{2}\{\sin(\theta_2-\theta_1)+\sin(\theta_1-\theta_3)+\sin(\theta_3-\theta_2)\}\text{ units}$$

Q.2 Find the area of a quadrilateral whose consecutive vertices are:

(i) $(3,-3)$, $(7,5)$, $(1,2)$ and $(-3,4)$

(ii) $(2,3)$, $(-1,2)$, $(-3,2)$ and $(3,-3)$

Solution:

(i) $(3,-3)$, $(7,5)$, $(1,2)$ and $(-3,4)$

$(x_1,y_1)=(3,-3)$, $(x_2,y_2)=(7,5)$, $(x_3,y_3)=(1,2)$, $(x_4,y_4)=(-3,4)$

$$\text{Area of }\Delta=\frac{1}{2}\begin{vmatrix}x_1- & \\ x_2- & \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}3-1 & -3 \ldots\\ 7-(-3) & 5\ldots\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}2 & -5\\ 10 & 1\end{vmatrix}$$

$$=\frac{1}{2}(52)$$

$$=26 \text{ units}$$

(ii) $(2,3)$, $(-1,2)$, $(-\ldots$

$(x_1,y_1)=(2,3)$, $(x_2 \ldots$

$$\text{Area of }\Delta=\frac{1}{2}\begin{vmatrix}x_1- & \\ x_2- & \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}2-(-3) & 3\ldots\\ -1-3 & 2-\ldots\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}5 & 1\\ -4 & 5\end{vmatrix}$$

$$=\frac{1}{2}(29)$$

$$=\frac{29}{2}\text{ units}$$

Q.3 Prove, by the m...

points are collinear:

(i) $(2,3)$, $(5,0)$ and (...

(iii) $(-1,-1)$, $(5,7)$ ...

Solution:

(i) $(2,3)$, $(5,0)$ and (...

$$\text{Area of }\Delta=\frac{1}{2}\begin{vmatrix}2 & 3 & \\ 5 & 0 & \\ 4 & 1 & \end{vmatrix}$$

$$=\frac{1}{2}\left\{2\begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix}-3\begin{vmatrix}\ldots\end{vmatrix}\ldots\right.$$

$$=\frac{1}{2}\{2(-1)-3(1)\ldots$$

$$=0$$

Hence points are col...

(ii) $(2,1)$, $(4,-1)$ an...

$$\text{Area of }\Delta=\frac{1}{2}\begin{vmatrix}2 & \\ 4 & -\\ 1 & \end{vmatrix}$$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3-1 & -3-2 \\ 7-(-3) & 5-4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & -5 \\ 10 & 1 \end{vmatrix}$$

$$= \frac{1}{2}(52)$$

$= 26$ units

(ii) $(2,3), (-1,2), (-3,2)$ and $(3,-3)$

$(x_1, y_1) = (2,3), (x_2, y_2) = (-1,2), (x_3, y_3) = (-3,2), (x_4, y_4) = (3,-3)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2-(-3) & 3-2 \\ -1-3 & 2-(-3) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 5 & 1 \\ -4 & 5 \end{vmatrix}$$

$$= \frac{1}{2}(29)$$

$= \frac{29}{2}$ units

Q.3 Prove, by the method of the area of a triangle, that the following points are collinear:

(i) $(2,3), (5,0)$ and $(4,1)$ (ii) $(2,1), (4,-1)$ and $(1,2)$

(iii) $(-1,-1), (5,7)$ and $(8,11)$

Solution:

(i) $(2,3), (5,0)$ and $(4,1)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 5 & 0 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{2\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 5 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 0 \\ 4 & 1 \end{vmatrix}\right\}$$

$$= \frac{1}{2}\{2(-1) - 3(1) + (5)\}$$

$= 0$

Hence points are collinear

(ii) $(2,1), (4,-1)$ and $(1,2)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 4 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left\{2\begin{vmatrix} -1 & 1 \\ 2 & 1\end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 1 & 1\end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 1 & 2\end{vmatrix}\right\}$

$= \frac{1}{2}\{2(-3) - 1(3) + 1(9)\}$

$= 0$

Hence points are collinear

(iii) $(-1,-1), (5,7)$ and $(8,11)$

$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} -1 & -1 & 1 \\ 5 & 7 & 1 \\ 8 & 11 & 1\end{vmatrix}$

$= \frac{1}{2}\left\{-1\begin{vmatrix} 7 & 1 \\ 11 & 1\end{vmatrix} - (-1)\begin{vmatrix} 5 & 1 \\ 8 & 1\end{vmatrix} + 1\begin{vmatrix} 5 & 7 \\ 8 & 11\end{vmatrix}\right\}$

$= \frac{1}{2}\{-1(-4) + (-3) + (-1)\}$

$= 0$

Hence points are collinear

Q.4 Find the area of triangle formed by the lines:

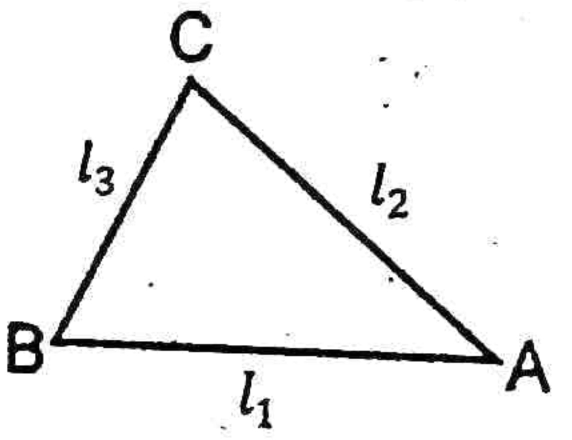
(i) $y = 0, y = 2x$ and $y = 6x + 5$

(ii) $y - x = 0, y + x = 0$ and $x - c = 0$

(iii) $y = 2x + 3, 2y + 3x = 3$ and $x + y + 2 = 0$

Solution:

(i) $l_1: y = 0, l_2: y = 2x$ and $l_3: y = 6x + 5$



<u>For vertex A:</u> Solving l_1 and l_2

$2x = 0 \Rightarrow x = 0$

$A(0,0)$

<u>For vertex B:</u> Solving l_1 and l_3

$6x + 5 = 0 \Rightarrow x = -\frac{5}{6}$

$B\left(-\frac{5}{6}, 0\right)$

<u>For vertex C:</u> Solving l_2 and l_3

$6x + 5 = 2x \Rightarrow 4x = -5 \Rightarrow x = -\frac{5}{4}$

$l_2 \Rightarrow y = 2\left(-\frac{5}{4}\right) = -\frac{5}{2}$

$C\left(-\frac{5}{4}, -\frac{5}{2}\right)$

$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 \\ -\frac{5}{6} & 0 \\ -\frac{5}{4} & - \end{vmatrix}$

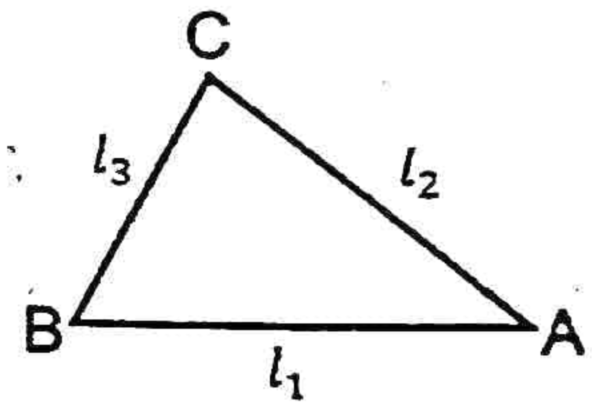
$= \frac{1}{2}\left\{0 - 0 + 1\begin{vmatrix} -\frac{5}{6} \\ -\frac{5}{4}\end{vmatrix}\right.$

$= \frac{1}{2}\left\{\left(\frac{25}{12} - 0\right)\right\}$

$= \frac{25}{24}$ units

(ii) $y - x = 0, y + x =$

$l_1: y = x, l_2: y = -x$ ar



<u>For vertex A:</u> Solving l_1

$x = -x \Rightarrow 2x = 0 \Rightarrow$

$\Rightarrow y = 0$

$A(0,0)$

<u>For vertex B:</u> Solving l_1

$l_1 \Rightarrow y = c$

$B(c, c)$

<u>For vertex C:</u> Solving l_2

$l_2 \Rightarrow y = -c$

$C(c, -c)$

$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 \\ c & c \\ c & -c\end{vmatrix}$

$= \frac{1}{2}\left\{0 - 0 + 1\begin{vmatrix} c & c \\ c & -c\end{vmatrix}\right.$

$l_2 \Rightarrow y = 2\left(-\frac{5}{4}\right) = -\frac{5}{2}$

$C\left(-\frac{5}{4}, -\frac{5}{2}\right)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -\frac{5}{6} & 0 & 1 \\ -\frac{5}{4} & -\frac{5}{2} & 1 \end{vmatrix}$$

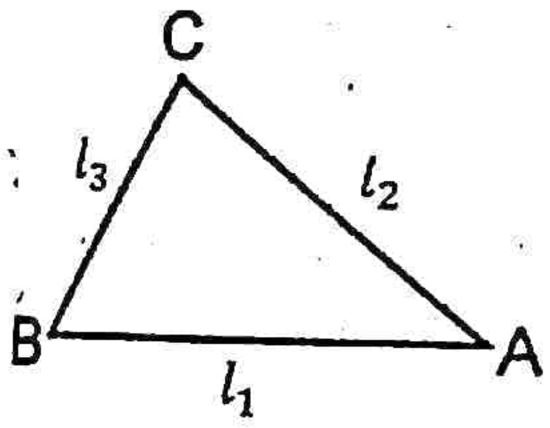
$$= \frac{1}{2}\left\{0 - 0 + 1\begin{vmatrix} -\frac{5}{6} & 0 \\ -\frac{5}{4} & -\frac{5}{2} \end{vmatrix}\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{25}{12} - 0\right)\right\}$$

$$= \frac{25}{24} \text{ units}$$

(ii) $y - x = 0, y + x = 0$ and $x - c = 0$

$l_1: y = x, l_2: y = -x$ and $l_3: x = c$



For vertex A: Solving l_1 and l_2

$x = -x \Rightarrow 2x = 0 \Rightarrow x = 0$

$\Rightarrow y = 0$

$A(0,0)$

For vertex B: Solving l_1 and l_3

$l_1 \Rightarrow y = c$

$B(c, c)$

For vertex C: Solving l_2 and l_3

$l_2 \Rightarrow y = -c$

$C(c, -c)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ c & c & 1 \\ c & -c & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{0 - 0 + 1\begin{vmatrix} c & c \\ c & -c \end{vmatrix}\right\}$$

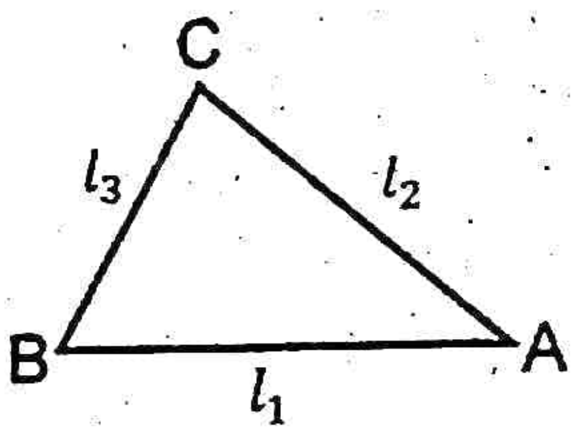
$= \frac{1}{2}\{(-c^2 - c^2)\}$

$= \frac{-2c^2}{2}$

$= c^2$ units

(iii) $y = 2x + 3, 2y + 3x = 3$ and $x + y + 2 = 0$

$l_1: y = 2x + 3, l_2: y = -\frac{3}{2}x + \frac{3}{2}$ and $y = -x - 2$



For vertex A: Solving l_1 and l_2

$2x + 3 = -\frac{3}{2}x + \frac{3}{2}$

$4x + 6 = -3x + 3$

$7x = -9 \Rightarrow x = -\frac{3}{7}$

$l_1 \Rightarrow y = 2\left(-\frac{3}{7}\right) + 3 = \frac{15}{7}$

$A\left(-\frac{3}{7}, \frac{15}{7}\right)$

For vertex B: Solving l_1 and l_3

$2x + 3 = -x - 2 \Rightarrow 3x = -5 \Rightarrow x = -\frac{5}{3}$

$l_1 \Rightarrow y = 2\left(-\frac{5}{3}\right) + 3 = -\frac{1}{3}$

$B\left(-\frac{5}{3}, -\frac{1}{3}\right)$

For vertex C: Solving l_2 and l_3

$-\frac{3}{2}x + \frac{3}{2} = -x - 2$

$-3x + 3 = -2x - 4$

$7 = x$

$l_1 \Rightarrow y = -7 - 2 = -9$

$B(7, -9)$

Area of $\Delta = \frac{1}{2}\begin{vmatrix} -\frac{3}{7} & \frac{15}{7} \\ -\frac{5}{3} & -\frac{1}{3} \\ 7 & -9 \end{vmatrix}$

By $R_2 - R_1$ and $R_3 - R_1$

$= \frac{1}{2}\begin{vmatrix} -\frac{3}{7} & \frac{15}{7} & 1 \\ -\frac{26}{21} & -\frac{52}{21} & 0 \\ \frac{52}{7} & -\frac{78}{7} & 0 \end{vmatrix}$

$= \frac{1}{2}\left\{1\begin{vmatrix} -\frac{26}{21} & -\frac{52}{21} \\ \frac{52}{7} & -\frac{78}{7} \end{vmatrix}\right.$

$= \frac{1}{2}\left\{\left(-\frac{26}{21}\right)\left(-\frac{78}{7}\right) - \right.$

$= \frac{1}{2}\left\{\left(\frac{676}{21}\right)\right\}$

$= \frac{676}{42}$ units

Q.1 Find the equation
the following equatio

(i) $x^2 - 5xy + 6y^2 =$

(iii) $9x^2 - 6xy + y^2 =$

(v) $7x^2 - 3xy + 5y^2$

Solution:

$y = mx \Rightarrow \frac{y}{x} = m$

(i) $x^2 - 5xy + 6y^2 =$

$\div$ by x^2

$1 - 5\left(\frac{y}{x}\right) + 6\left(\frac{y}{x}\right)^2 =$

$B(7,-9)$

$$\text{Area of }\Delta = \frac{1}{2}\begin{vmatrix} -\frac{3}{7} & \frac{15}{7} & 1 \\ -\frac{5}{3} & -\frac{1}{3} & 1 \\ 7 & -9 & 1 \end{vmatrix}$$

By $R_2 - R_1$ and $R_3 - R_1$

$$= \frac{1}{2}\begin{vmatrix} -\frac{3}{7} & \frac{15}{7} & 1 \\ -\frac{26}{21} & -\frac{52}{21} & 0 \\ \frac{52}{7} & -\frac{78}{7} & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left\{1\begin{vmatrix} -\frac{26}{21} & -\frac{52}{21} \\ \frac{52}{7} & -\frac{78}{7} \end{vmatrix} - 0 + 0\right\}$$

$$= \frac{1}{2}\left\{\left(-\frac{26}{21}\right)\left(-\frac{78}{7}\right) - \left(-\frac{52}{21}\right)\left(\frac{52}{7}\right)\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{676}{21}\right)\right\}$$

$$= \frac{676}{42} \text{ units}$$

EXERCISE 7.8

Q.1 Find the equations of the pair of lines represented jointly by each of the following equations. State nature of lines also trace the pair of lines.

(i) $x^2 - 5xy + 6y^2 = 0$

(ii) $4x^2 - xy - 5y^2 = 0$

(iii) $9x^2 - 6xy + y^2 = 0$

(iv) $10x^2 - 3xy - y^2 = 0$

(v) $7x^2 - 3xy + 5y^2 = 0$

Solution:

$y = mx \Rightarrow \frac{y}{x} = m$

(i) $x^2 - 5xy + 6y^2 = 0$

$\div$ by x^2

$$1 - 5\left(\frac{y}{x}\right) + 6\left(\frac{y}{x}\right)^2 = 0$$

$1-5m+6m^2=0$

$6m^2-5m+1=0$

$6m^2-2m-3m+1=0$

$2m(3m-1)-(3m-1)=0$

$(3m-1)(2m-1)=0$

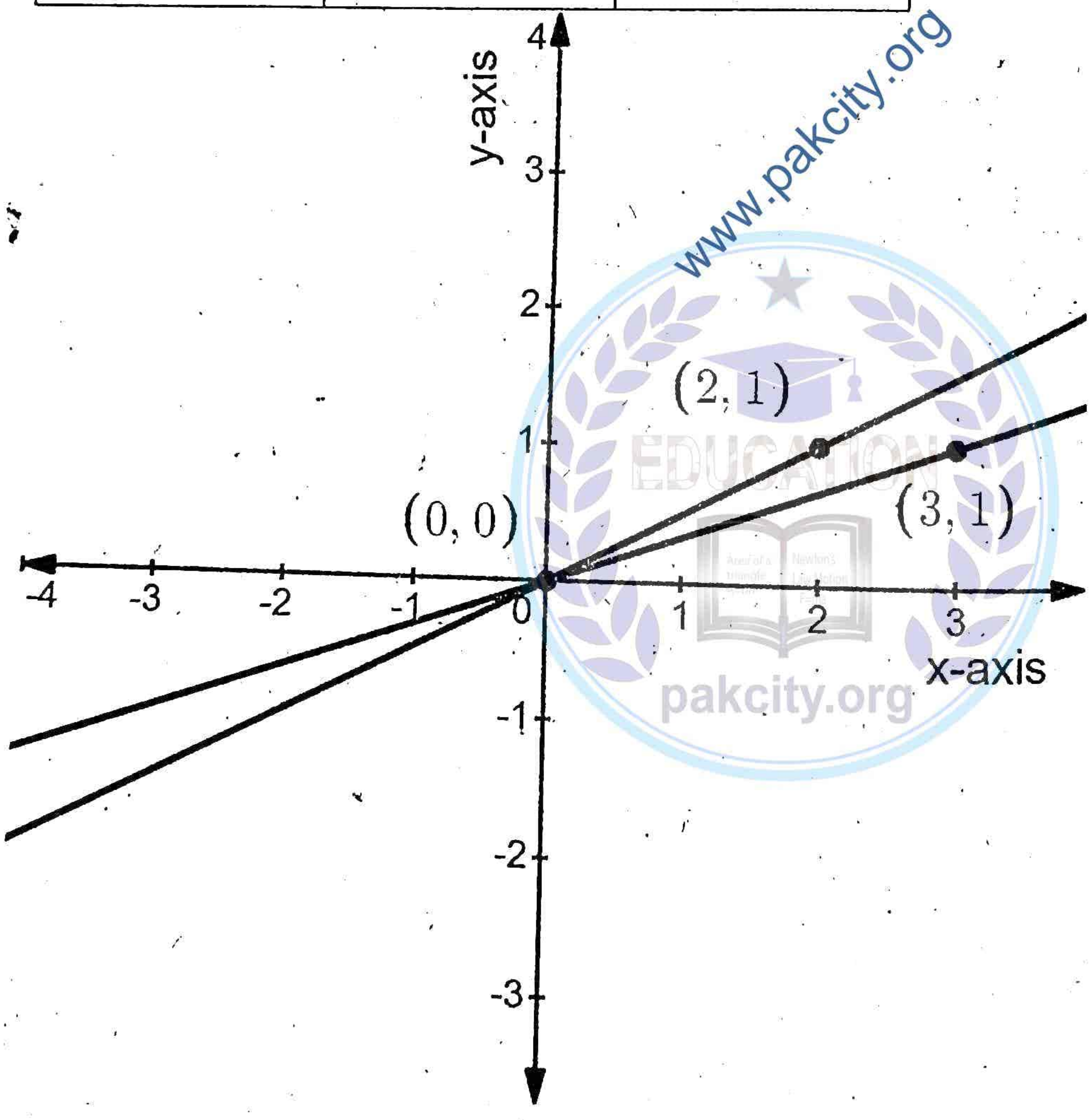
$3m-1=0 \Rightarrow m=\frac{1}{3}$

$2m-1=0 \Rightarrow m=\frac{1}{2}$

$y=\frac{1}{3}x$ and $y=\frac{1}{2}x$

Two real and different lines passing through origin.

Line	x	y
$y=\frac{1}{3}x$	0	0
	3	1
$y=\frac{1}{2}x$	0	0
	2	1



(ii) $4x^2-xy-5y^2=$

÷ by x^2

$4-\left(\frac{y}{x}\right)-5\left(\frac{y}{x}\right)^2=0$

$4-m-5m^2=0$

$5m^2+m-4=0$

$5m^2+5m-4m-4$

$5m(m+1)-4(m+$

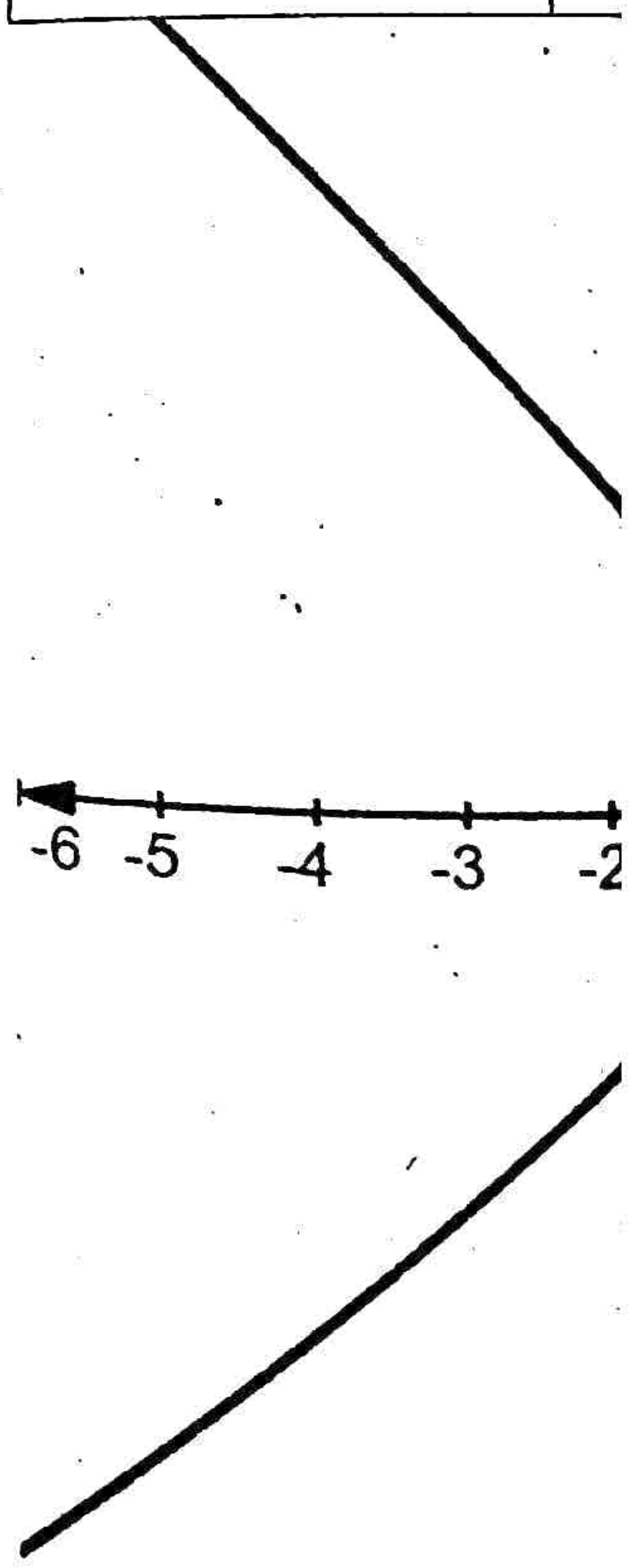
$(m+1)(5m-4)=$

$m=-1,\frac{4}{5}$

$y=-x$ and $y=\frac{4}{5}x$

Two real and differen

Line	x
$y=-x$	0
	5
$y=\frac{4}{5}x$	0
	5



(ii) $4x^2 - xy - 5y^2 = 0$

$\div$ by x^2

$4 - \left(\frac{y}{x}\right) - 5\left(\frac{y}{x}\right)^2 = 0$

$4 - m - 5m^2 = 0$

$5m^2 + m - 4 = 0$

$5m^2 + 5m - 4m - 4 = 0$

$5m(m+1) - 4(m+1) = 0$

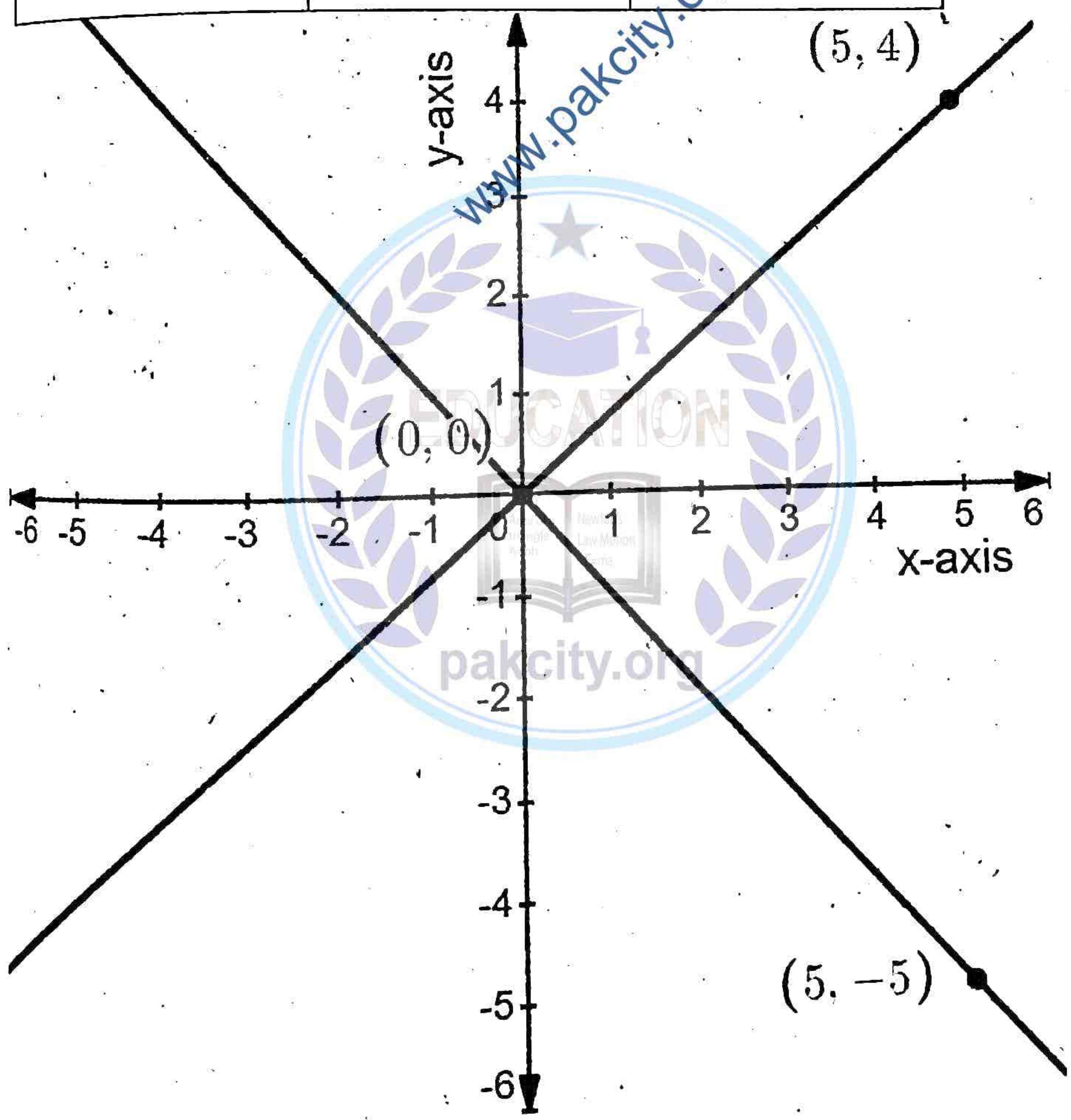
$(m+1)(5m-4) = 0$

$m = -1, \frac{4}{5}$

$y = -x$ and $y = \frac{4}{5}x$

Two real and different lines passing through origin.

Line	x	y
$y = -x$	0	0
	5	-5
$y = \frac{4}{5}x$	0	0
	5	4



(iii) $9x^2 - 6xy + y^2 = 0$

$\div$ by x^2

$9 - 6\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = 0$

$9 - 6m + m^2 = 0$

$m^2 - 6m + 9 = 0$

$(m)^2 - 2(m)(3) + (3)^2 = 0$

$(m-3)^2 = 0$

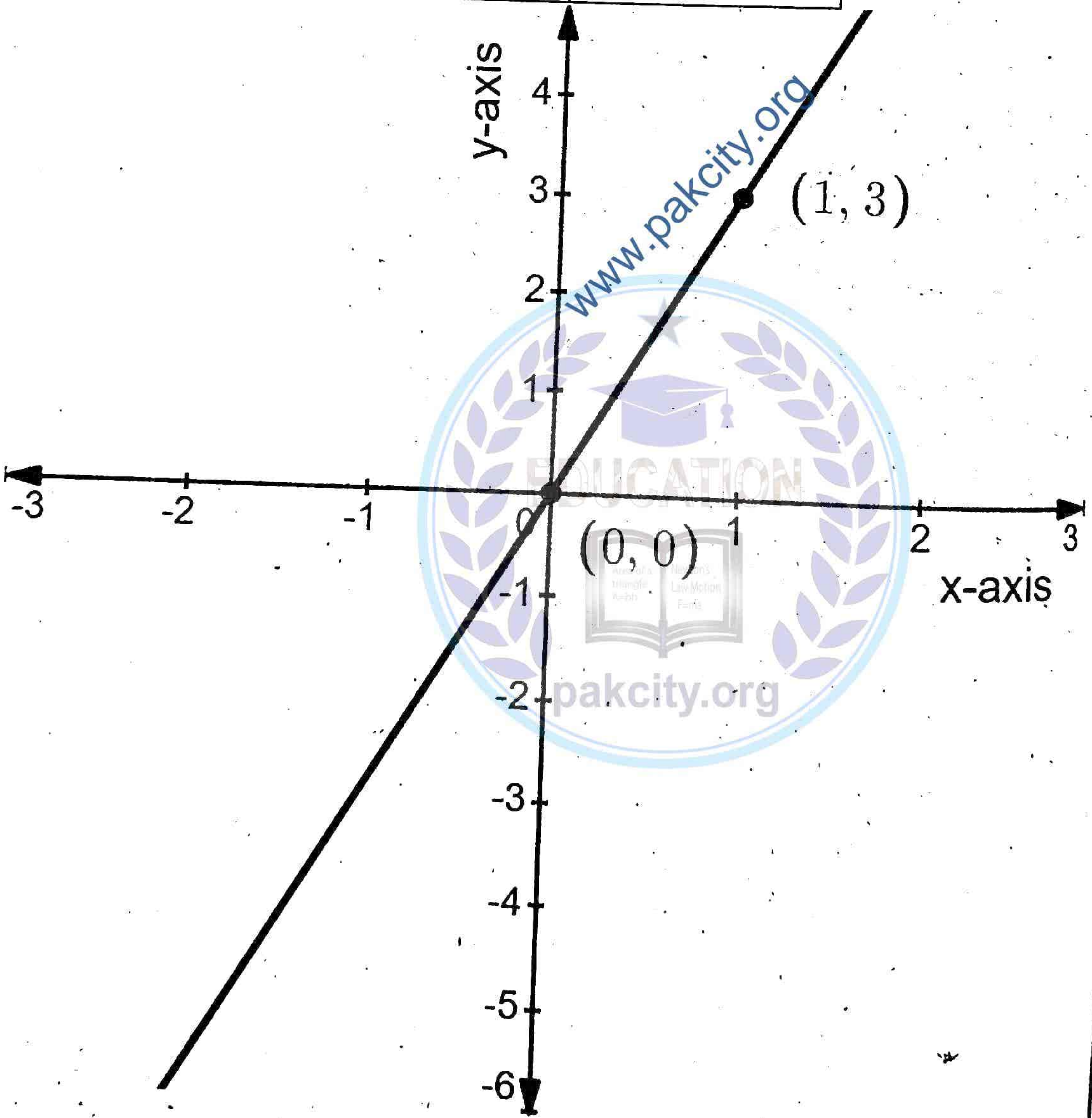
$(m-3)(m-3) = 0$

$\boxed{m = 3,3}$

$y = 3x$ and $y = 3x$

Two real and same lines passing through origin.

Line	x	y
$y = 3x$	0	0
	1	3



The Student

(iv) $10x^2 -$ …

$\div$ by x^2

$10 - 3\left(\frac{y}{x}\right)$ …

$10 - 3m -$ …

$m^2 + 3m -$ …

$m^2 + 5m -$ …

$m(m+5) -$ …

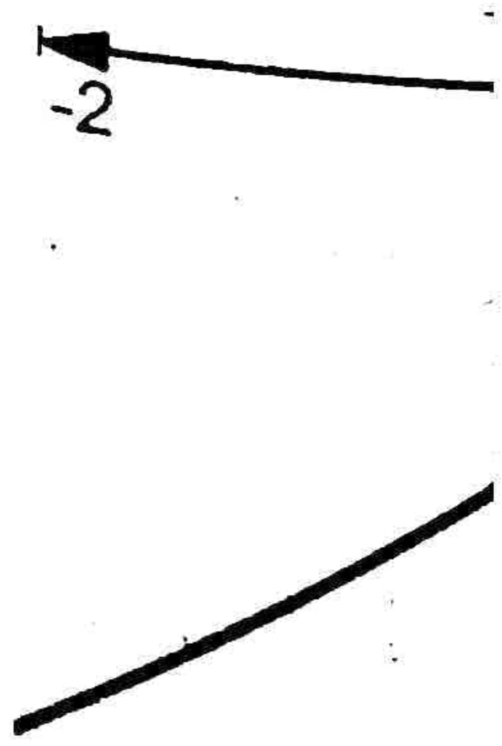
$(m+5)(m$ …

$m = 2, -5$

$y = 2x$ and …

Two real and …

Line	
$y = 2x$	
$y = -5x$	



(iv) $10x^2 - 3xy - y^2 = 0$

÷ by x^2

$$10 - 3\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2 = 0$$

$$10 - 3m - m^2 = 0$$

$$m^2 + 3m - 10 = 0$$

$$m^2 + 5m - 2m - 10 = 0$$

$$m(m + 5) - 2(m + 5) = 0$$

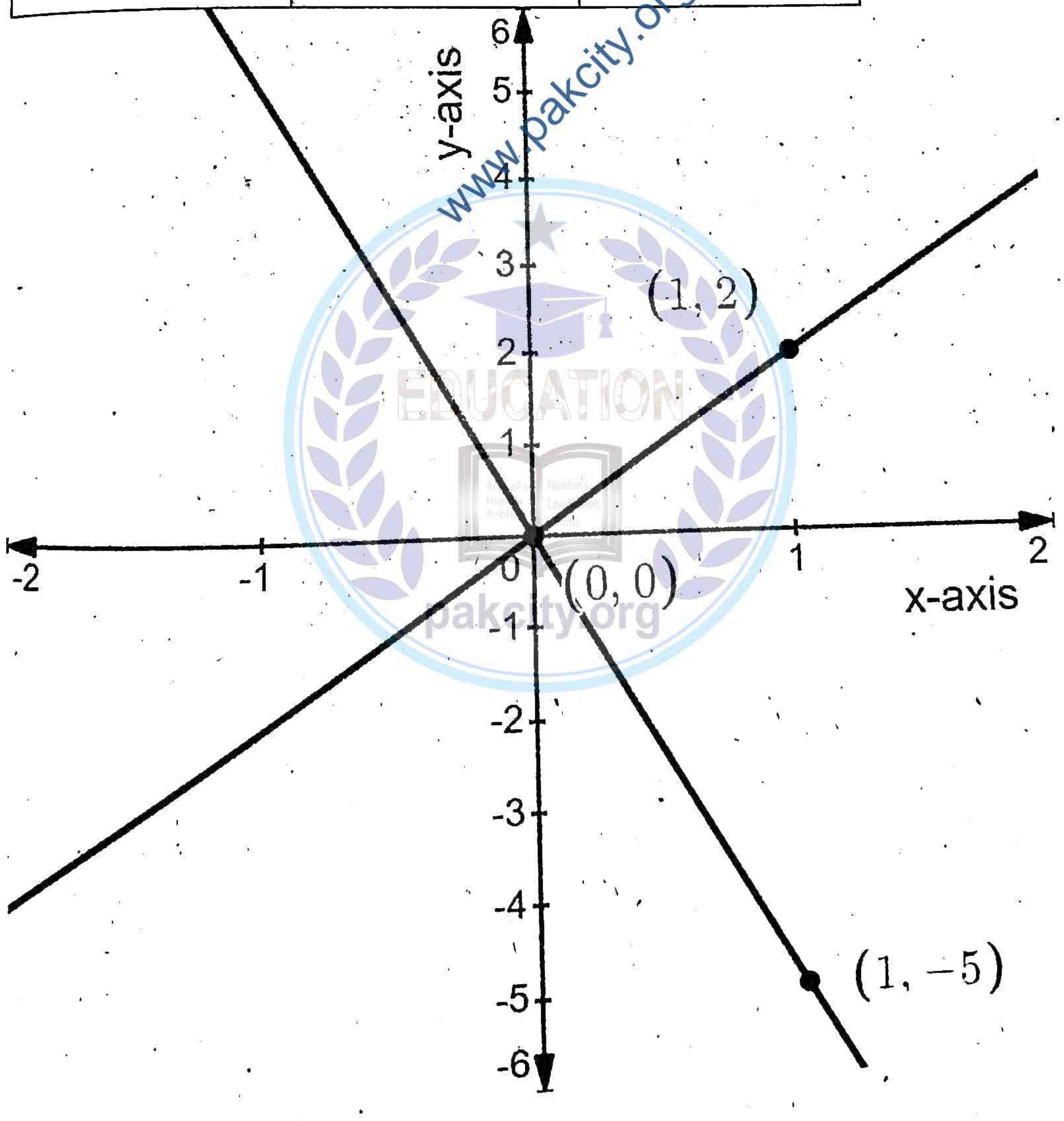
$$(m + 5)(m - 2) = 0$$

$$m = 2, -5$$

$y = 2x$ and $y = -5x$

Two real and different lines passing through origin.

Line	x	y
$y = 2x$	0	0
	1	2
$y = -5x$	0	0
	1	−5



(v) $7x^2 - 3xy + 5y^2 = 0$

$\div$ by x^2

$$7 - 3\left(\frac{y}{x}\right) + 5\left(\frac{y}{x}\right)^2 = 0$$

$$7 - 3m + 5m^2 = 0$$

$$5m^2 - 3m + 7 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(7)}}{2(5)}$$

$$m = \frac{3 \pm i\sqrt{131}}{10}$$

$$y = \frac{3 + i\sqrt{131}}{10}x \text{ and } y = \frac{3 - i\sqrt{131}}{10}x$$

Two imaginary lines.

Q.2 Find the combines equation of the pair of lines through the origin which are perpendicular to the lines represented by:

(i) $2x^2 - 5xy + y^2 = 0$ (ii) $6x^2 - 13xy + 6y^2 = 0$

Solution:

(i) $2x^2 - 5xy + y^2 = 0$

$$ax^2 + ahxy + by^2 = 0$$

$$a = 2, b = 1, 2h = -5$$

$$m_1 + m_2 = -\frac{2h}{b} = \frac{5}{1} = 5$$

$$m_1 m_2 = \frac{a}{b} = \frac{2}{1} = 2$$

The combined equation of perpendicular lines is

$$\left(y + \frac{1}{m_1}x\right)\left(y + \frac{1}{m_2}x\right) = 0$$

$$\left(\frac{m_1 y + x}{m_1}\right)\left(\frac{m_2 y + x}{m_2}\right) = 0$$

$$m_1 m_2 y^2 + xy(m_1 + m_2) + x^2 = 0$$

$$2y^2 + 5xy + x^2 = 0$$

(ii) $6x^2 - 13xy + 6y^2 = 0$

$$ax^2 + ahxy + by^2 = 0$$

$$a = 6, b = 6, 2h = -13$$

$m_1 + m_2 =$

$m_1 m_2 = \frac{a}{b}$

The combin

$\left(y + \frac{1}{m_1}x\right)$

$\left(\frac{m_1 y + x}{m_1}\right)$

$m_1 m_2 y^2 +$

$(1)y^2 + xy$

$6y^2 + 13x$

Q.3 Trace t

(i) $x^2 - 3x$

(iii) $6x^2 -$

Solution:

(i) $x^2 - 3x$

$x^2 - 2xy$ -

$x(x - 2y)$

$(x - 2y)($

$x - y = 0$

$x - 2y = ($

$m_1 + m_2 = \frac{13}{6}$

$m_1 m_2 = \frac{a}{b} = \frac{6}{6} = 1$

The combined equation of perpendicular lines is

$\left(y + \frac{1}{m_1}x\right)\left(y + \frac{1}{m_2}x\right) = 0$

$\left(\frac{m_1 y + x}{m_1}\right)\left(\frac{m_2 y + x}{m_2}\right) = 0$

$m_1 m_2 y^2 + xy(m_1 + m_2) + x^2 = 0$

$(1)y^2 + xy\left(\frac{13}{6}\right) + x^2 = 0$

$6y^2 + 13xy + 6x^2 = 0$

Q.3 Trace the pair of lines given by the following equations:

(i) $x^2 - 3xy + 2y^2 = 0$ (ii) $x^2 - 6xy + 9y^2 = 0$

(iii) $6x^2 - xy - y^2 = 0$ (iv) $8x^2 - 3xy - y^2 = 0$

Solution:

(i) $x^2 - 3xy + 2y^2 = 0$

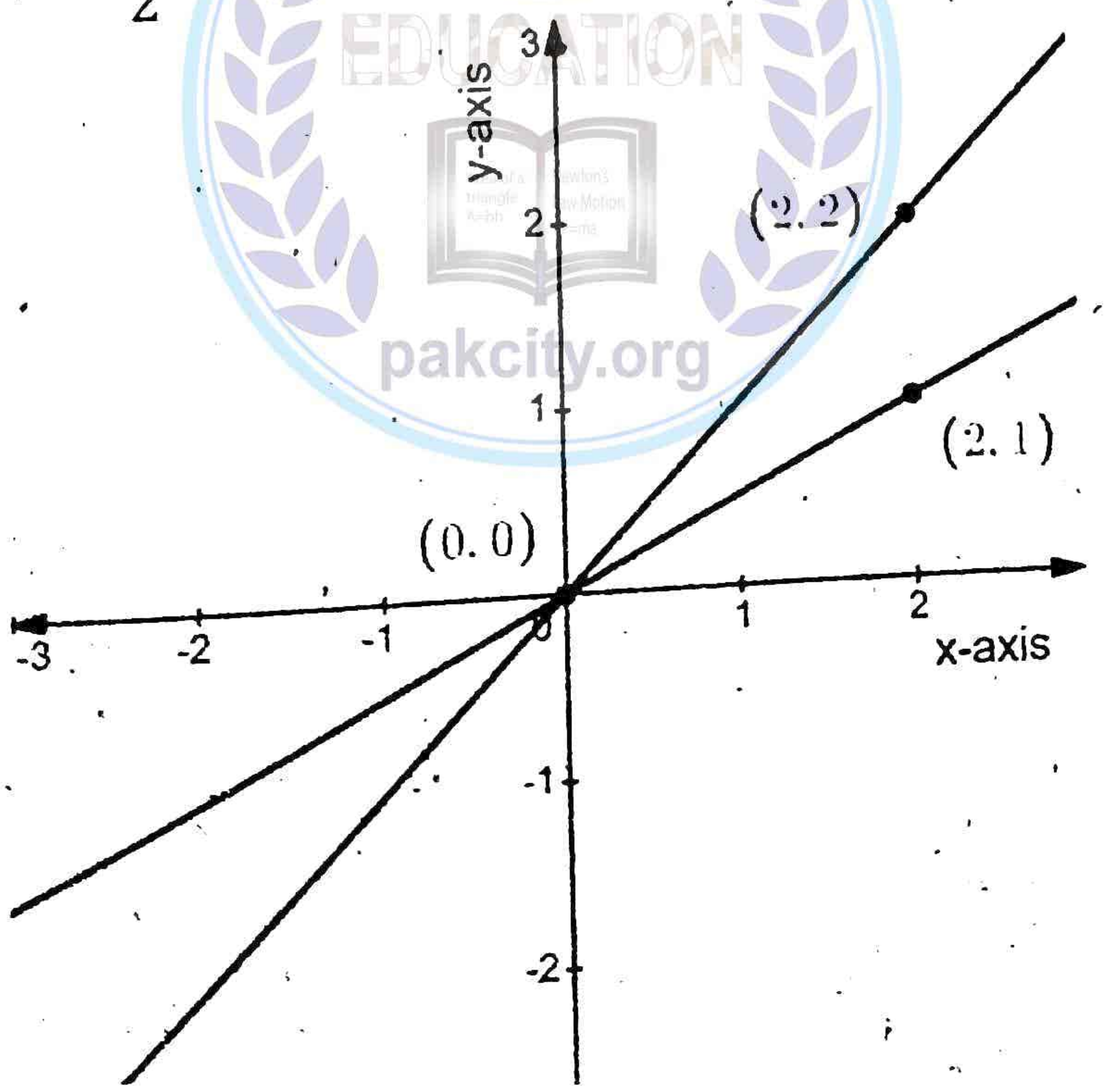
$x^2 - 2xy - xy + 2y^2 = 0$

$x(x - 2y) - y(x - 2y) = 0$

$(x - 2y)(x - y) = 0$

$x - y = 0 \Rightarrow y = x$

$x - 2y = 0 \Rightarrow y = \frac{x}{2}$



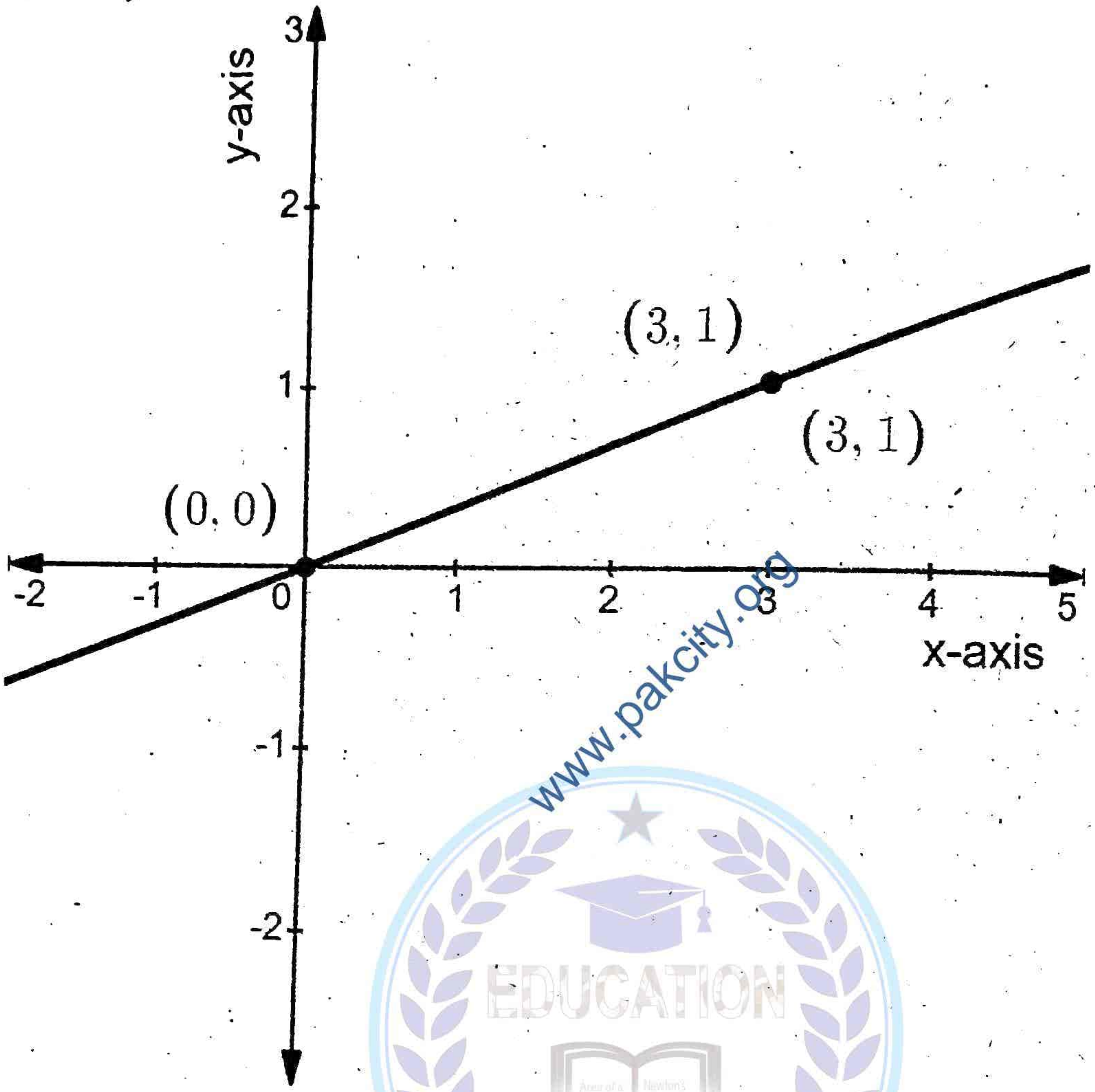
(ii) $x^2 - 6xy + 9y^2 = 0$

$(x)^2 - 2(x)(3y) + (3y)^2 = 0$

$(x - 3y)^2 = 0$

$(x - 3y)(x - 3y) = 0$

$x - 3y = 0$ and $x - 3y = 0$



(iii) $6x^2 - xy - y^2 = 0$

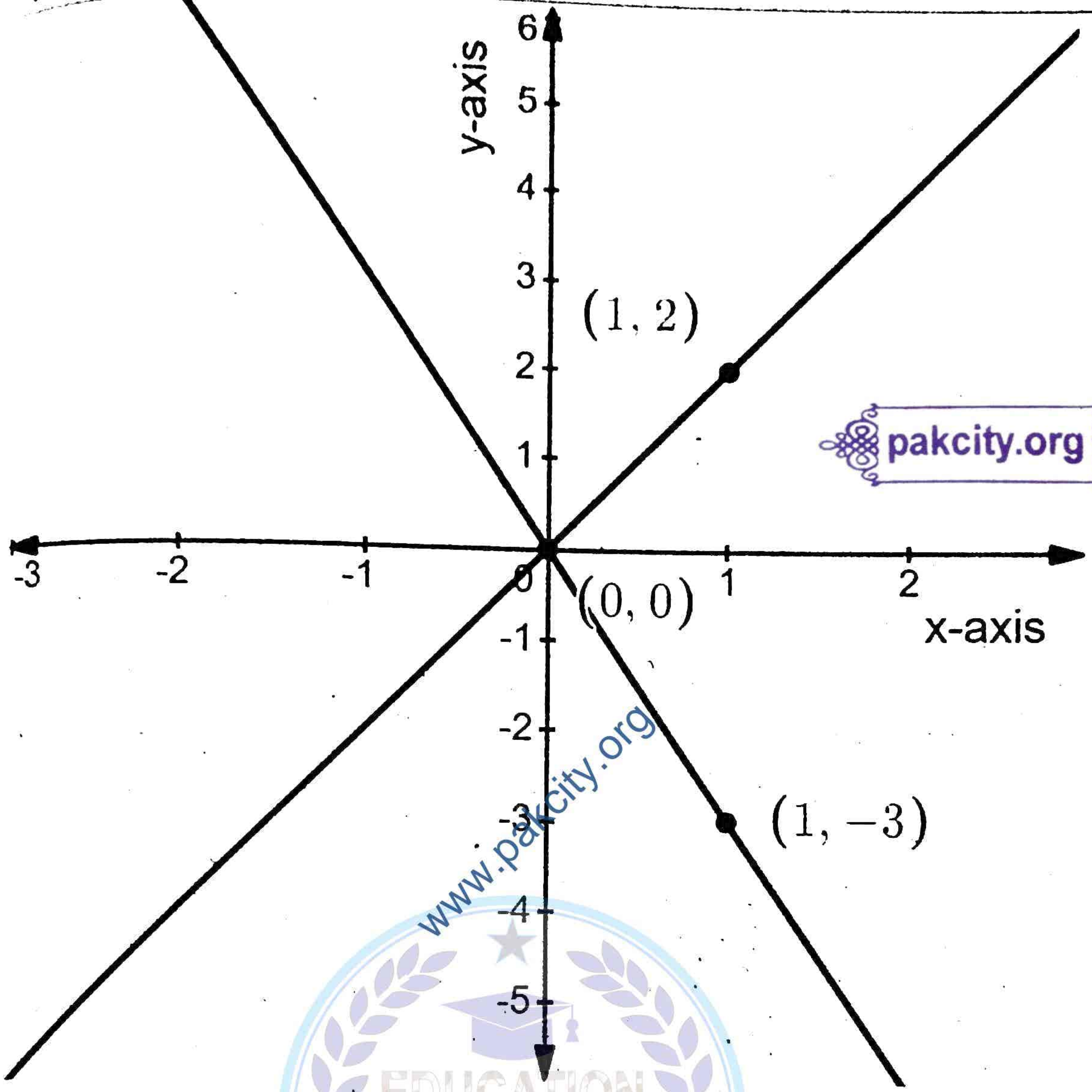
$6x^2 - 3xy + 2xy - y^2 = 0$

$3x(2x - y) + y(2x - y) = 0$

$(2x - y)(3x + y) = 0$

$2x - y = 0 \Rightarrow y = 2x$

$3x + y \Rightarrow y = -3x$



(iv) $8x^2 - 3xy - y^2 = 0$

$\div$ by x^2

$y^2 + 3xy - 8x^2 = 0$

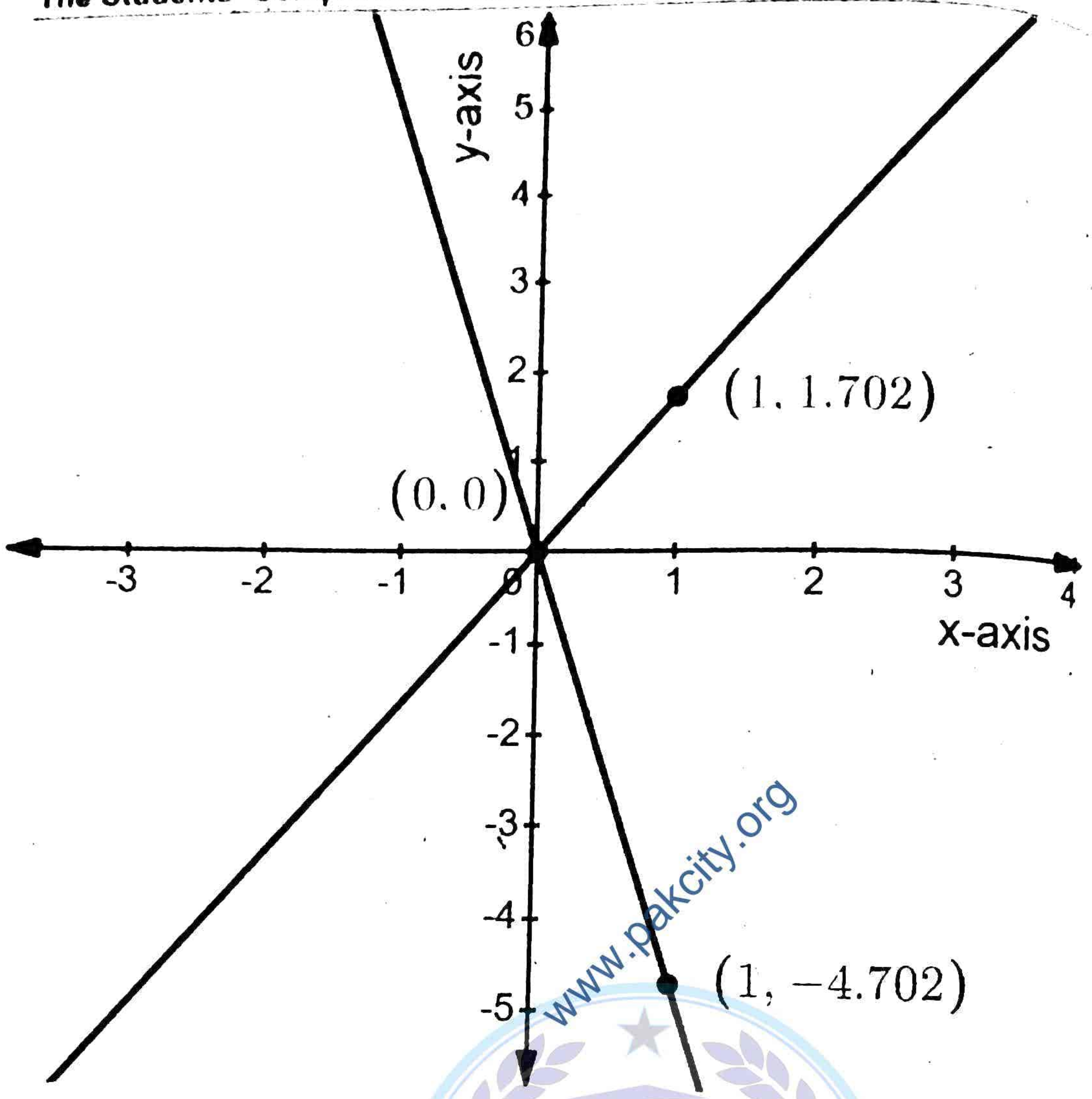
$\frac{y^2}{x^2} + 3\frac{y}{x} - 8 = 0$

$\frac{y}{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{y}{x} = \frac{-3 \pm \sqrt{3^2 - 4(1)(-8)}}{2(1)}$

$\frac{y}{x} = \frac{-3 \pm \sqrt{41}}{2}$

$y = \frac{-3 \pm \sqrt{41}}{2}x$



Q.4 Find the angle between the lines represented by:

(i) $x^2 - 5xy + 6y^2 = 0$ (ii) $3x^2 + 7xy + 2y^2 = 0$

(iii) $x^2 + 2xy - 3y^2 = 0$ (iv) $x^2 + xy - 2y^2 = 0$

Solution:

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

(i) $x^2 - 5xy + 6y^2 = 0$

$ax^2 + 2hxy + by^2 = 0$

$a = 1, b = 6, 2h = -5 \Rightarrow h = -\frac{5}{2}$

$$\tan\theta = \frac{2\sqrt{\left(-\frac{5}{2}\right)^2 - (1)(6)}}{1 + 6}$$

$\tan\theta = \frac{2}{7}\sqrt{\frac{1}{4}}$ …

$\theta = \tan^{-1}\frac{1}{7}$ …

(ii) $3x^2 + 7xy$ …

$ax^2 + 2hxy +$ …

$a = 3, b = 2,$ …

$\tan\theta = \frac{2\sqrt{(\frac{7}{2}\ldots}}{\ldots}$

$\tan\theta = \frac{2}{5}\sqrt{\frac{25}{4}\ldots}$

$\theta = \tan^{-1} 1 =$ …

(iii) $x^2 + 2xy$ …

$ax^2 + 2hxy +$ …

$a = 1, b = -$ …

$\tan\theta = \frac{2\sqrt{(1\ldots}}{\ldots}$

$\tan\theta = \frac{2\sqrt{4}}{-2}$ …

$\theta = \tan^{-1}(-$ …

(iv) $x^2 + xy -$ …

$ax^2 + 2hxy +$ …

$a = 1, b = -$ …

$\tan\theta = \frac{2\sqrt{(\frac{1}{2}\ldots}}{\ldots}$

$\tan\theta = \frac{2\sqrt{\frac{9}{4}}}{-1}$ …

$\theta = \tan^{-1}(-$ …

Q.5 The gradie…

that of the oth…

$\tan\theta = \frac{2}{7}\sqrt{\frac{1}{4}} = \frac{2}{7}\left(\frac{1}{2}\right) = \frac{1}{7}$

$\boxed{\theta = \tan^{-1}\frac{1}{7}}$

(ii) $3x^2 + 7xy + 2y^2 = 0$

$ax^2 + 2hxy + by^2 = 0$

$a = 3, b = 2, 2h = 7 \Rightarrow h = \frac{7}{2}$

$$\tan\theta = \frac{2\sqrt{\left(\frac{7}{2}\right)^2 - (3)(2)}}{3+2}$$

$\tan\theta = \frac{2}{5}\sqrt{\frac{25}{4}} = \frac{2}{5}\left(\frac{5}{2}\right) = 1$

$\theta = \tan^{-1} 1 \Rightarrow \boxed{\theta = 45°}$

(iii) $x^2 + 2xy - 3y^2 = 0$

$ax^2 + 2hxy + by^2 = 0$

$a = 1, b = -3, 2h = 2 \Rightarrow h = 1$

$$\tan\theta = \frac{2\sqrt{(1)^2 - (1)(-3)}}{1-3}$$

$\tan\theta = \frac{2\sqrt{4}}{-2} = -2$

$\boxed{\theta = \tan^{-1}(-2)}$

(iv) $x^2 + xy - 2y^2 = 0$

$ax^2 + 2hxy + by^2 = 0$

$a = 1, b = -2, 2h = 1 \Rightarrow h = \frac{1}{2}$

$$\tan\theta = \frac{2\sqrt{\left(\frac{1}{2}\right)^2 - (1)(-2)}}{1-2}$$

$\tan\theta = \frac{2\sqrt{\frac{9}{4}}}{-1} = -2\left(\frac{3}{2}\right) = -3$

$\boxed{\theta = \tan^{-1}(-3)}$

Q.5 The gradient of one of the lines of $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. Show that $8h^2 = 9ab$.

Solution:

$ax^2 + 2hxy + by^2 = 0$

$m_1 = \frac{-h + \sqrt{h^2 - ab}}{b}$ and $m_2 = \frac{-h - \sqrt{h^2 - ab}}{b}$

$m_1 = 2m_2$ or $m_2 = 2m_1$

$\frac{-h + \sqrt{h^2 - ab}}{b} = 2\left(\frac{-h - \sqrt{h^2 - ab}}{b}\right)$

$-h + \sqrt{h^2 - ab} = -2h - 2\sqrt{h^2 - ab}$

$2\sqrt{h^2 - ab} + \sqrt{h^2 - ab} = -2h + h$

$3\sqrt{h^2 - ab} = -h$

Squaring both sides

$9(h^2 - ab) = h^2$

$9h^2 - 9ab = h^2$

$9h^2 - h^2 = 9ab$

$8h^2 = 9ab$

REVIEW EXERCISE 7

1. Select the correct option.

(i) The slope of the line that passes through $(-3, -4)$ and $(7,6)$ is

(a) 0 (b) undefined

(c) -1 (d) ✓ 1

(ii) A line with a slope of $\frac{1}{2}$ and y-intercept 7 is

(a) $2y = x - 7$ (b) ✓ $y = \frac{1}{2}x + 7$

(c) $y = \frac{1}{2}x - 7$ (d) $x - 2y = 14$

(iii) Two lines are said to be parallel if and only if their slopes are

(a) ✓ equal (b) unequal

(c) does not exist (d) negative reciprocal of each other

(iv) Two lines l_1 and l_2 are said to be perpendicular if and only if

(a) ✓ $m_1m_2 = -1$ (b) $m_1m_2 = 1$

(c) $m_1 = -m_2$ (d) $m_1 = \frac{1}{m_2}$

(v) The slope of a line that is perpendicular to a vertical line is

(a) ✓ 0 (b) 1

(c) $90°$

(vi) The slope of a l

-5 is

(a) 2

(c) ✓ $\frac{1}{2}, -2$

(vii) The point on t

is

(a) $(-2,2)$

(c) $(1,-1)$

(viii) In a plane th

(a) they lie on a c

(c) ✓ they lie on

shape

(ix) If the line coi

(a) $y = b$

(c) ✓ $y = 0$

(x) The general e

(a) ✓ $ax + by +$

(c) $y - y_1 = m($

Q.2 If the distan

values of a.

Solution:

(i) $(x_1, y_1) = (5$

$d = \sqrt{(x_2 - x_1}$

$5^2 = (1 - 5)^2$

$25 = (-4)^2 +$

$25 = 16 + (a$

$25 - 16 = (a$

$(a + 2)^2 = 9$

$a + 2 = \pm 3$

$a + 2 = 3$

$a = 3 - 2$

$a = 1$

(c) 90° (d) undefined

(vi) The slope of a line which makes an angle 45° with the line $3x - y = -5$ is

(a) 2 (b) $\frac{1}{2}$

(c) ✓ $\frac{1}{2}, -2$ (d) -2

(vii) The point on the line $2x - 3y = 5$ is equidistant from (1,2) and (3,4) is

(a) $(-2,2)$ (b) ✓ $(4,1)$

(c) $(1,-1)$ (d) $(4,6)$

(viii) In a plane three or more points are said to be collinear if

(a) they lie on a circle (b) they form a closed loop together

(c) ✓ they lie on a straight line (d) they do not make any defined shape

(ix) If the line coincides with x-axis then its equation is

(a) $y = b$ (b) $y = -b$

(c) ✓ $y = 0$ (d) $y = \infty$

(x) The general equation of line also known as standard equation of lien is:

(a) ✓ $ax + by + c = 0$ (b) $y = ax + c$

(c) $y - y_1 = m(x - x_1)$ (d) $\frac{x}{a} + \frac{y}{b} = 1$

Q.2 If the distance between the points $(5,-2)$ and $(1,a)$ is 5, find the values of a.

Solution:

(i) $(x_1, y_1) = (5,-2)$ and $(x_2, y_2) = (1,a)$, $d = 5$ units

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5^2 = (1-5)^2 + (a+2)^2$

$25 = (-4)^2 + (a+2)^2$

$25 = 16 + (a+2)^2$

$25 - 16 = (a+2)^2$

$(a+2)^2 = 9$

$a + 2 = \pm 3$

$a + 2 = 3$	$a + 2 = -3$
$a = 3 - 2$	$a = -3 - 2$
$a = 1$	$a = -5$

Q.3 M(3,8) is the midpoint of the line AB. A has the coordinates (−2,3), find the coordinates of B.

Solution:

Midpoint $= M(3,8), (x_1, y_1) = (-2,3)$ and $B = (x_2, y_2) = ?$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M(3,8) = \left(\frac{-2 + x_2}{2}, \frac{3 + y_2}{2}\right)$$

$$\frac{-2 + x_2}{2} = 3 \Rightarrow -2 + x_2 = 6 \Rightarrow x_2 = 8$$

$$\frac{3 + y_2}{2} = 8 \Rightarrow 3 + y_2 = 16 \Rightarrow y_2 = 13$$

$B = (8,13)$

Q.4 The diameter of a circle has endpoints (2, −3) and (−6,5). Find the coordinates of the centre of this circle.

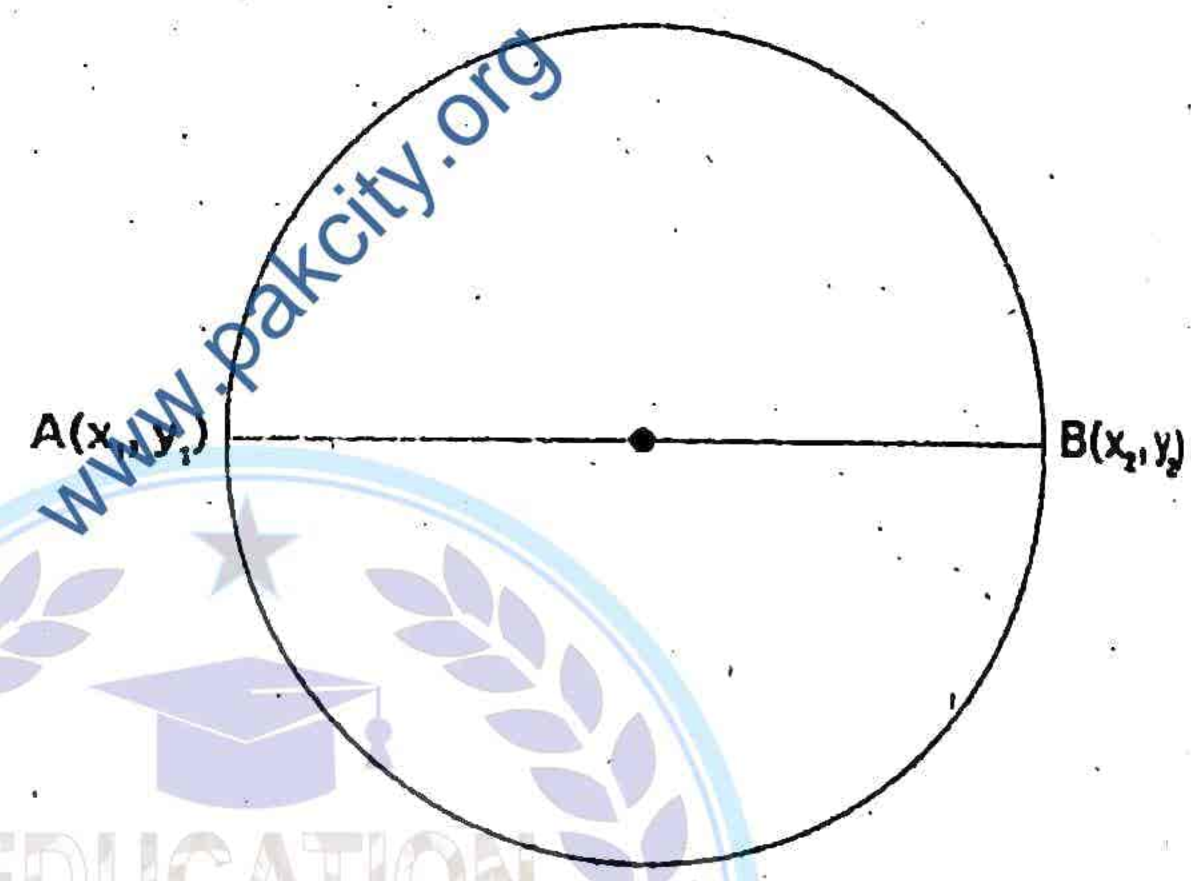
Solution:

Centre is midpoint of diameter

$(x_1, y_1) = (2, -3)$ and

$(x_2, y_2) = (-6,5)$

Centre = ?



$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Centre} = \left(\frac{2 - 6}{2}, \frac{-3 + 5}{2}\right)$$

Centre $= (-2,1)$

Q.5 Find the coordinates of the points which divides the join of P(−1,7) and Q(4, −3) in the ratio 2: 3.

Solution:

$(x_1, y_1) = P(-1,7)$ and $(x_2, y_2) = Q(4, -3)$ and $m{:}n = 2{:}3$

Point of division = ?

$$(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

$$(x, y) = \left(\frac{2(4) + 3(-1)}{2 + 3}, \frac{2(-3) + 3(7)}{2 + 3}\right)$$

$(x, y) = (1,3)$

The Studen...

Q.6 Find the ...
and B(10, −3...

Solution:

$(x_1, y_1) = A($...

$(x, y) = \Big(\frac{mx}{\dots}$...

For $m{:}n =$ ...

$(x, y) = \Big(\frac{1(}{\dots}$...

$P = \left(-\frac{2}{3}, \frac{20}{3}\right.$...

For $m{:}n = 2$...

$(x, y) = \Big(\frac{2(}{\dots}$...

$C = \left(\frac{14}{3}, \frac{5}{3}\right)$

Q.7 Two verti...
triangle is the...

Solution:

$(x_1, y_1) = (1,$...

$G(x, y) = \Big(\frac{x_1}{\dots}$...

$(0,0) = \Big(\frac{1 +}{\dots}$...

$\frac{4 + x_3}{2} = 0 \Rightarrow$ ...

$\frac{5 + y_3}{2} = 0 \Rightarrow$ ...

Third vertex = ...

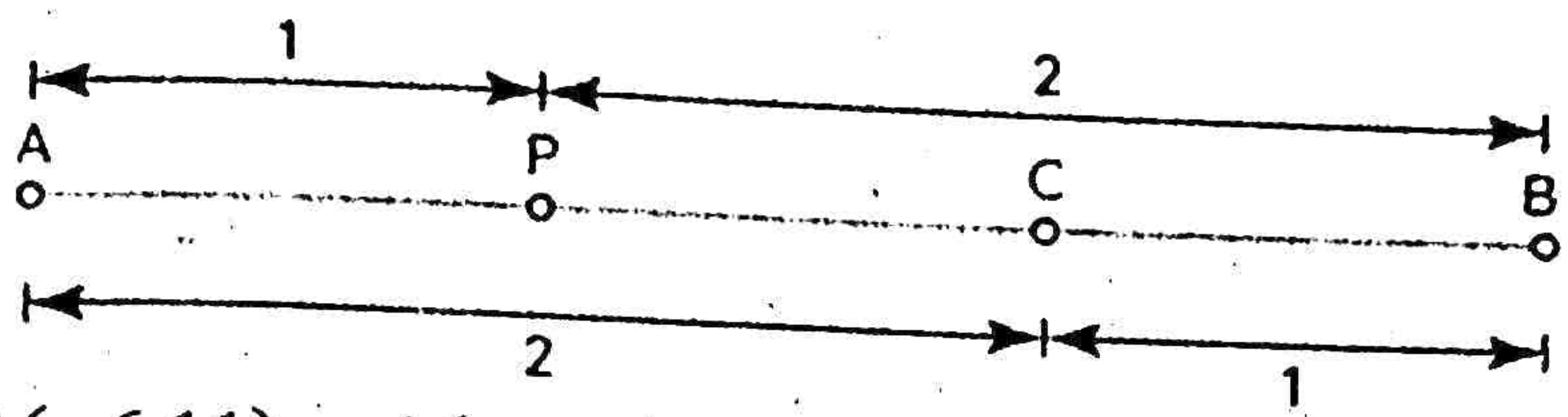
Q.8 Find the sl...
whose equatio...

Solution:

$-2y = -8x$ ...

Q.6 Find the points of trisection of the line segment AB, where $A(-6,11)$ and $B(10,-3)$.

Solution:



$(x_1,y_1) = A(-6,11)$ and $(x_2,y_2) = B(10,-3)$ and $m:n = 1:2$ or $2:1$

$$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

For $m:n = 1:2$

$$(x,y) = \left(\frac{1(10)+2(-6)}{1+2}, \frac{1(-3)+211}{1+2}\right)$$

$$P = \left(-\frac{2}{3}, \frac{208}{3}\right)$$

For $m:n = 2:1$

$$(x,y) = \left(\frac{2(10)+1(-6)}{2+1}, \frac{2(-3)+1(11)}{2+1}\right)$$

$$C = \left(\frac{14}{3}, \frac{5}{3}\right)$$

Q.7 Two vertices of a triangle are (1,4) and (3,1). If the centroid of the triangle is the origin, find the third vertex.

Solution:

$(x_1,y_1) = (1,4), (x_2,y_2) = (3,1)$ and $(x_3,y_3) = ?$ $G(x;y) = (0,0)$

$$G(x,y) = \left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$$

$$(0,0) = \left(\frac{1+3+x_3}{2}, \frac{4+1+y_3}{2}\right)$$

$$\frac{4+x_3}{2} = 0 \Rightarrow 4+x_3 = 0 \Rightarrow x_3 = -4$$

$$\frac{5+y_3}{2} = 0 \Rightarrow 5+y_3 = 0 \Rightarrow y_3 = -5$$

Third vertex $= (-4,-5)$

Q.8 Find the slope of the line which is perpendicular to the given line whose equation is $-2y = -8x + 9$.

Solution:

$$-2y = -8x + 9$$

$\div$ by -2

$y = 4x - \frac{9}{2}$

$y = mx + b$

$m_1 = 4$

$m_2 = -\frac{1}{4}$ $[m_1 m_2 = -1 \text{ for } \perp \text{ lines}]$

Q.9 If a straight line intercepts the x-axis at $(6,0)$ and intercepts the y-axis at $(0,5)$, write the equation of the straight line in two intercept form.

Solution:

$a = x$-intercept

$b = y$-intercept

Two intercept form is

$\frac{x}{a} + \frac{y}{b} = 1 \rightarrow (1)$

Here, $(a, 0) = (6,0)$ and $(0, b) = (0,5)$

$(1) \Rightarrow \frac{x}{6} + \frac{y}{5} = 1$

Q.10 Determine the slope (gradient) and y-intercept of each line:
(a) $y = 12x - 6$ (b) $y = 5 - 2x$ (c) $4x - y + 13 = 0$ (d) $y = 4x$

Solution:

Slope intercept form is: $y = mx + b$

$m =$ Slope of line and $b = y$-intercept

(a) $y = 12x - 6$

Comparing with $y = mx + b$

$m = 12$ and $b = -6$

(b) $y = 5 - 2x \Rightarrow y = -2x + 5$

Comparing with $y = mx + b$

$m = -2$ and $b = 5$

(c) $4x - y + 13 = 0$

$4x + 13 = y \Rightarrow y = 4x + 13$

Comparing with $y = mx + b$

$m = 4$ and $b = 13$

(d) $y = 4x$

$y = 4x + 0$

Comparing with $y = mx + b$

The Studer
$m = 4$ and
Q.11 Find tl
Solution:
$4x + 8y +$
$4x + 8y =$
$4x + 8y =$
$\div$ by -2
$-2x - 4y$
$\frac{x}{-\frac{1}{2}} + \frac{y}{-\frac{1}{4}}$
$\frac{x}{a} + \frac{y}{b} = 1$
$a = -\frac{1}{2}, b$
OR
$4x + 8y +$
For x-interc
$4x + 8(0)$
$4x = -2 \Rightarrow$
For x-interc
$4x + 8(0)$
$4x = -2 \Rightarrow$
For y-interc
$4(0) + 8y +$
$8y = -2 \Rightarrow$
Q.12 What i
parallel to th
Solution:
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$(x_1, y_1) = (3$
$m_1 = \frac{k - 7}{3 - 2}$
$(x_1, y_1) = (-$

epts the y-axis
ept form.
line:
(d) $y = 4x$

$m = 4$ and $b = 0$

Q.11 Find the x and y-intercept of the equation $4x + 8y + 2 = 0$.

Solution:

$4x + 8y + 2 = 0$

$4x + 8y = -2$

$4x + 8y = -2$

$\div$ by -2

$-2x - 4y = 1$

$$\frac{x}{-\frac{1}{2}} + \frac{y}{-\frac{1}{4}} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$a = -\frac{1}{2}, b = -\frac{1}{4}$$

OR

$4x + 8y + 2 = 0$

For x-intercept: $y = 0$

$4x + 8(0) + 2 = 0$

$$4x = -2 \Rightarrow x = -\frac{1}{2}$$

For x-intercept: $y = 0$

$4x + 8(0) + 2 = 0$

$$4x = -2 \Rightarrow x = -\frac{1}{2}$$

For y-intercept: $x = 0$

$4(0) + 8y + 2 = 0$

$$8y = -2 \Rightarrow x = -\frac{1}{4}$$

Q.12 What is the value of k so that the line through $(3, k)$ and $(2,7)$ is parallel to the line through $(-1,4)$ and $(0,6)$?

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$(x_1, y_1) = (3, k)$ and $(x_2, y_2) = (2,7)$

$$m_1 = \frac{k - 7}{3 - 2} = k - 7$$

$(x_1, y_1) = (-1,4)$ and $(x_2, y_2) = (0,6)$

$$m_2 = \frac{6-4}{0-(-1)} = \frac{2}{1} = 2$$

For parallel lines, $m_1 = m_2$

$k - 7 = 2$

$k = 2 + 7$

$k = 9$

Q.13 Tell whether the following pair of lines is parallel, perpendicular or neither.

(a) $y = 2x + 7$ and $y = 7x - 2$ (b) $x = 1$ and $x = 5$

(c) $x = -3$ and $y = -3$

Solution:

(a) $y = 2x + 7$ and $y = 7x - 2$

$y = mx + b$

$m_1 = 2$ and $m_2 = 7$

$m_1 \neq m_2$ and $m_1 m_2 \neq -1$

Neither parallel nor perpendicular

(b) $x = 1$ and $x = 5$

$x + 0y = 1$

$$m = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$$

$$m = -\frac{1}{0} = \text{undefined}$$

It is a line parallel to y-axis (vertical line)

$x + 0y = 5$

$$m = -\frac{1}{0} = \text{undefined}$$

It is a line parallel to y-axis (vertical line)

$m_1 = m_2$ i.e., Lines are parallel

(c) $x = -3$ and $y = -3$

$x + 0y = -3$

$$m = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$$

$$m = -\frac{1}{0}$$

It is a line parallel to y-axis (vertical line)

$0x + y = -3$

$m = -\frac{0}{1} = ($

It is a line pa

Both are per

-4

Q.14 For wh

$kx + 4$ be c

Solution:

$y = 3x - 1$

$2y = x + 3$

$3y = kx +$

The lines ar

$k(-5) + 3($

$-5k + 30 -$

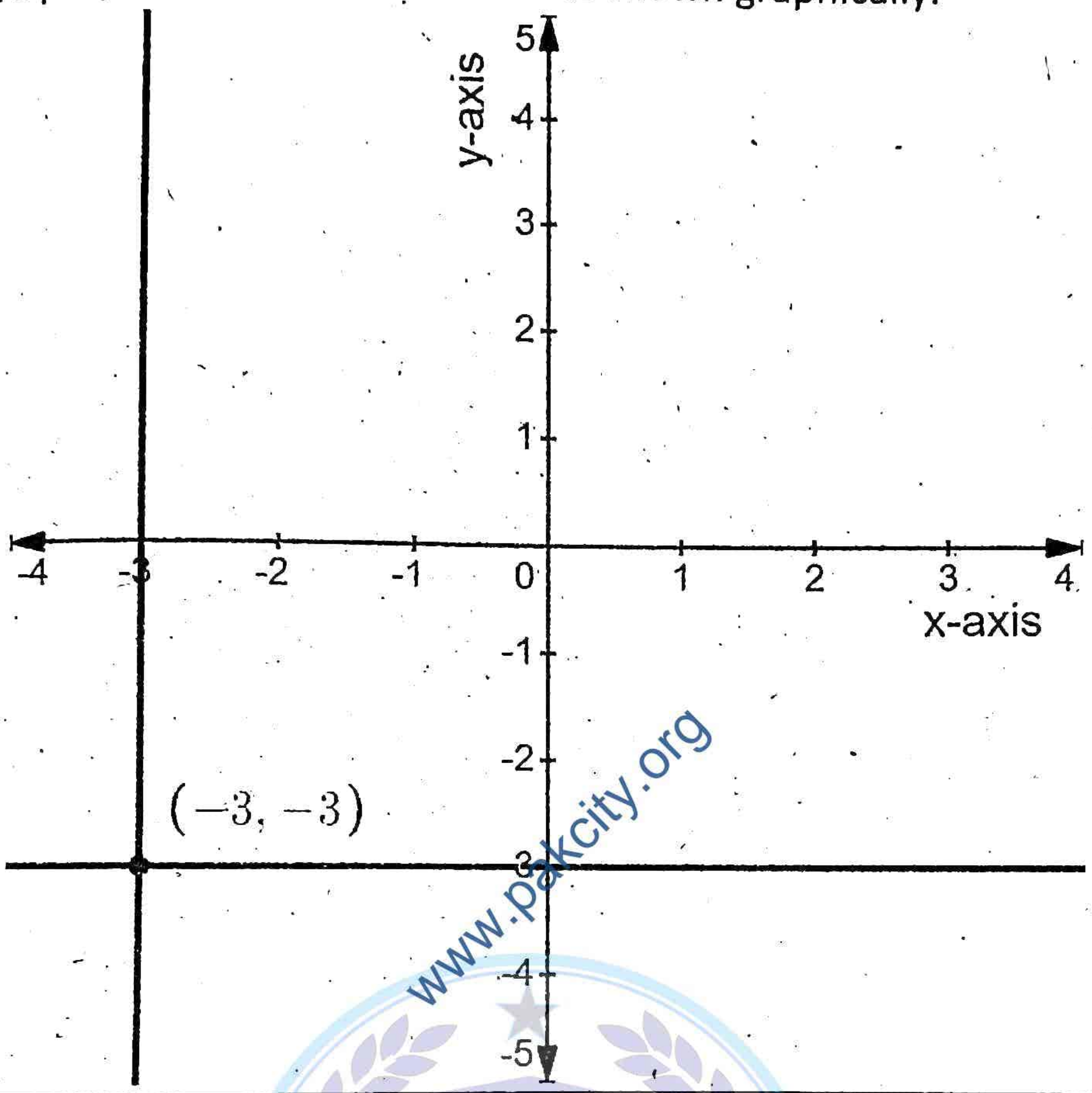
$10 = 5k$

$k = 2$

$m = -\frac{0}{1} = 0$

It is a line parallel to x-axis (horizontal line)

Both are perpendicular to each other as shown graphically.



Q.14 For what value of k the three lines $y = 3x - 1, 2y = x + 3$ and $3y = kx + 4$ be concurrent?

Solution:

$y = 3x - 1 \Rightarrow 3x - y - 1 = 0$

$2y = x + 3 \Rightarrow x - 2y + 3 = 0$

$3y = kx + 4 \Rightarrow kx - 3y + 4 = 0$

The lines are concurrent

$$\begin{vmatrix} 3 & -1 & -1 \\ 1 & -2 & 3 \\ k & -3 & 4 \end{vmatrix} = 0$$

$$k\begin{vmatrix} -1 & -1 \\ -2 & 3 \end{vmatrix} - (-3)\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} + 4\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} = 0$$

$k(-5) + 3(10) + 4(-5) = 0$

$-5k + 30 - 20 = 0$

$10 = 5k$

$\boxed{k = 2}$

Q.15 Find the value of a and b for which the lines given by $ax - 2y - 1 =$ 0 and $6x - 4y - b = 0$

(i) are parallel (ii) are perpendicular

(iii) coincide (iv) have no common point

Solution:

$ax - 2y - 1 = 0$ and $6x - 4y - b = 0$

(i) are parallel

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{a}{6} = \frac{-2}{-4} \neq \frac{-1}{-b}$$

$$\frac{a}{6} = \frac{1}{2} \neq \frac{1}{b}$$

$$\frac{a}{6} = \frac{1}{2} \Rightarrow a = \frac{6}{2} \Rightarrow \boxed{a = 3}$$

$$\frac{1}{2} \neq \frac{1}{b} \Rightarrow b \neq 2$$

(ii) are perpendicular

$$a_1a_2 + b_1b_2 = 0$$

$$a(6) + (-2)(-6) = 0$$

$$6a + 12 = 0$$

$$6a = -12 \Rightarrow \boxed{a = -2}$$

(iii) coincide

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{a}{6} = \frac{-2}{-4} = \frac{-1}{-b}$$

$$\frac{a}{6} = \frac{1}{2} = \frac{1}{b}$$

$$\frac{a}{6} = \frac{1}{2} \Rightarrow a = \frac{6}{2} \Rightarrow \boxed{a = 3}$$

$$\frac{1}{2} = \frac{1}{b} \Rightarrow b = 2$$

(iv) have no common point

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{a}{6} = \frac{-2}{-4} \neq \frac{-1}{-b}$$

The Studei

$\frac{a}{6} = \frac{1}{2} \neq \frac{1}{b}$

$\frac{a}{6} = \frac{1}{2} \Rightarrow a$

$\frac{1}{2} \neq \frac{1}{b} \Rightarrow b$

Q.16 Find t

liens $x + 2$

(i) parallel t

(ii) perpenc

Solution:

$l_1: x + 2y$

$l_2: 3x - 2y$

The line thı

$l_1 + kl_2 =$

$(x + 2y -$

$x + 2y - 5$

$x(1 + 3k)$

(i) parallel t

(1) is paral

$a_1b_2 - a_2b$

$3(1 + 3k)$

$3 + 9k - 8$

$17k - 5 =$

$\boxed{k = \frac{5}{17}}$

$(1) \Rightarrow (x +$

$17(x + 2y$

$17x + 34y$

$32x + 24y$

÷ by 8

$4x + 3y - 8$

(ii) perpendi

(1) is paralle

$a_1a_2 + b_1b_2$

$4(1 + 3k) +$

$\frac{a}{6} = \frac{1}{2} \neq \frac{1}{b}$

$\frac{a}{6} = \frac{1}{2} \Rightarrow a = \frac{6}{2} \Rightarrow \boxed{a = 3}$

$\frac{1}{2} \neq \frac{1}{b} \Rightarrow b \neq 2$

Q.16 Find the equation of the line passing through the intersection of the liens $x + 2y - 5 = 0$ and $3x - 2y + 1 = 0$ and
(i) parallel to the line $4x + 3y - 5 = 0$
(ii) perpendicular to the line $2x - 3y + 7 = 0$

Solution:

$l_1: x + 2y - 5 = 0$

$l_2: 3x - 2y + 1 = 0$

The line through the intersection of l_1 and l_2 is

$l_1 + kl_2 = 0$ or $l_2 + kl_1 = 0$

$(x + 2y - 5) + k(3x - 2y + 1) = 0 \rightarrow (1)$

$x + 2y - 5 + 3kx - 2ky + k = 0$

$x(1 + 3k) + y(2 - 2k) + k - 5 = 0$

(i) parallel to the line $4x + 3y - 5 = 0$

(1) is parallel to $4x + 3y - 5 = 0$

$a_1b_2 - a_2b_1 = 0$

$3(1 + 3k) - 4(2 - 2k) = 0$

$3 + 9k - 8 + 8k = 0$

$17k - 5 = 0$

$\boxed{k = \frac{5}{17}}$

$(1) \Rightarrow (x + 2y - 5) + \frac{5}{17}(3x - 2y + 1) = 0$

$17(x + 2y - 5) + 5(3x - 2y + 1) = 0$

$17x + 34y - 85 + 15x - 10y + 5 = 0$

$32x + 24y - 80 = 0$

÷ by 8

$4x + 3y - 8 = 0$

(ii) perpendicular to the line $2x - 3y + 7 = 0$

(1) is parallel to $4x + 3y - 5 = 0$

$a_1a_2 + b_1b_2 = 0$

$4(1 + 3k) + 3(2 - 2k) = 0$

$4 + 12k + 6 - 6k = 0$

$6k + 10 = 0 \Rightarrow 6k = -10$

$$\boxed{k = -\frac{5}{3}}$$

$(1) \Rightarrow (x + 2y - 5) - \frac{5}{3}(3x - 2y + 1) = 0$

$3(x + 2y - 5) - 5(3x - 2y + 1) = 0$

$3x + 6y - 15 - 15x + 10y - 5 = 0$

$-12x + 16y - 20 = 0$

$\div$ by -4

$3x - 4y + 5 = 0$

Q.17 Find the distance of the point from the line

(i) $15x - 8y - 5 = 0, (2,1)$ (ii) $2x - 7y + 1 = 0, (7,4)$

Solution:

(i) $15x - 8y - 5 = 0, (2,1)$

$(x_1, y_1) = (2,1)$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$d = \left|\frac{15(2) - 8(1) - 5}{\sqrt{15^2 + (-8)^2}}\right|$$

$$d = \left|\frac{17}{\sqrt{289}}\right|$$

$$d = \left|\frac{17}{17}\right|$$

$$\boxed{d = 1 \text{ units}}$$

(ii) $2x - 7y + 1 = 0, (7,4)$

$(x_1, y_1) = (7,4)$

$$d = \left|\frac{2(7) - 7(4) + 1}{\sqrt{2^2 + 7^2}}\right|$$

$$d = \left|\frac{-13}{\sqrt{53}}\right|$$

$$\boxed{d = \frac{13}{\sqrt{53}} \text{ units}}$$