CHAPTER 06

INTEGRATION

BASIC CONCEPTS AND FORMULAS

Partial Fraction

Case I: Linear factor

$$\frac{1}{(x+a)(x+b)(x+d)}=\frac{A}{x+a}+\frac{B}{x+b}+\frac{C}{x+d}$$

Case II: Repeated Linear factor

$$\frac{1}{(x+a)(x+b)^3}=\frac{A}{x+a}+\frac{B}{x+b}+\frac{C}{(x+b)^2}+\frac{D}{(x+b)^3}$$

Case III: Non repeated irreducible quadratic factors

$$\frac{1}{(x+a)(px^2+qx+r)}=\frac{A}{x+a}+\frac{Bx+C}{px^2+qx+r}$$

Case IV: Repeated irreducible quadratic factors

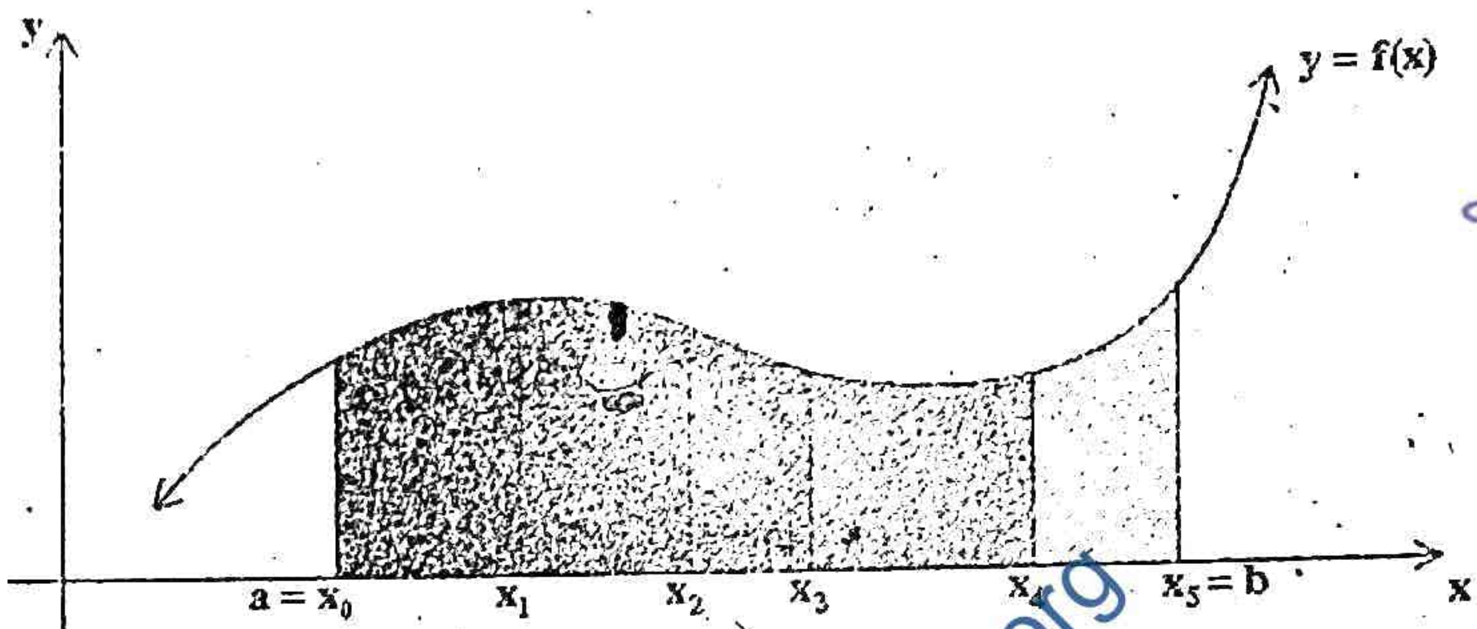
$$\frac{1}{(x+a)(px^2+qx+r)^2}$$
$$=\frac{A}{x+a}+\frac{Bx+C}{px^2+qx+r}+\frac{Dx+E}{(px^2+qx+r)^2}$$

The Riemann Integral

what is area? We are all familiar with determining the area of simple geometric figures such as rectangles and triangles. However, how do we determine the area of a region R whose boundary may consist of non rectilinear curves, such as a parabola? To see how this could be done let us consider the following process.

Suppose that a function f is continuous and non-negative on an interval $[a,b]$. We wish to know what it means to compute the area of the region R bounded above by the curve $y=f(x)$, below by the x-axis, and, on the sides, by the lines $x=a$ and $x=b$, in short, the area under the curve $y=f(x)$, as seen in the figure below.

We will obtain the area of the region R as the limit of a sum of areas of rectangles as follows: First, we divide the interval $[a, b]$ into n subintervals $[x_0, x_1]$, $[x_1, x_2]$, ..., $[x_{n-1}, x_n]$, where $a = x_0 < x_1 < x_2 < \cdots < x_n = b$. The intervals need not all be the same length. Let the lengths of these intervals be Δx_1, Δx_2, ..., Δx_n, respectively. This process divides the region R into n strips (see the figure below).

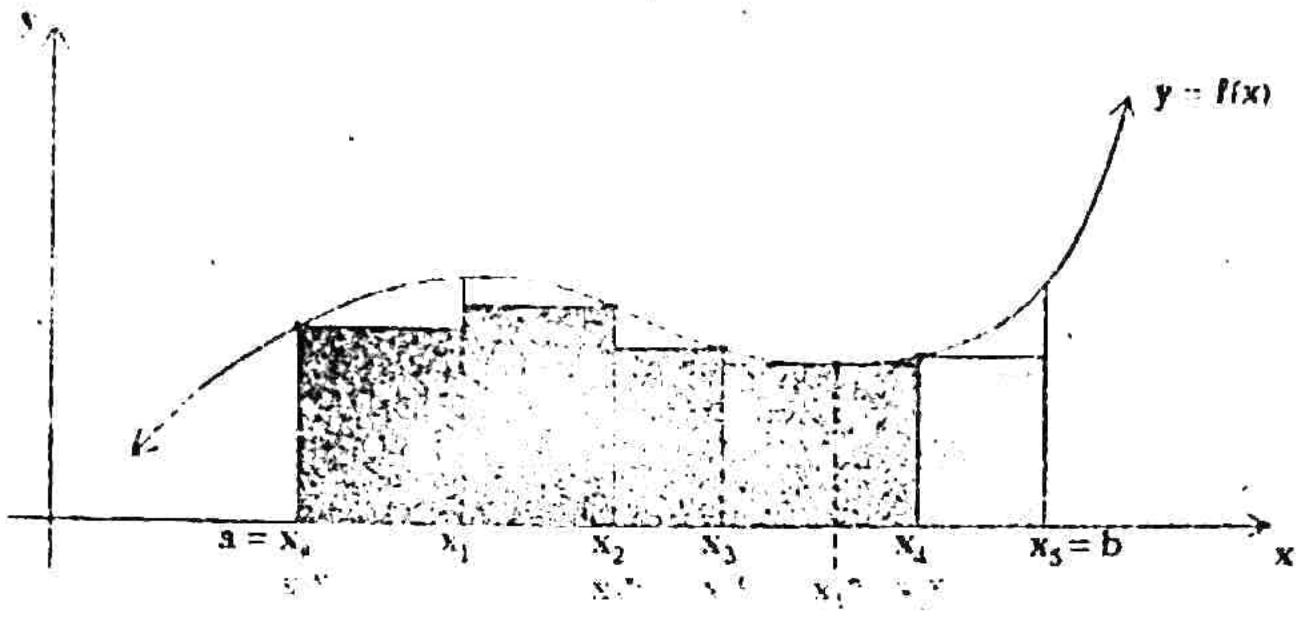


Next, let's approximate each strip by a rectangle with height equal to the height of the curve $y = f(x)$ at some arbitrary point in the subinterval. That is, for the first subinterval $[x_0, x_1]$ select some x_1^* contained in that subinterval and use $f(x_1^*)$ as the height of the first rectangle. The area of that rectangle is then $f(x_1^*)\Delta x_1$.

Similarly, for each remaining subinterval $[x_{k-1}, x_k]$, $2 \le k \le n$, we will choose some x_k^* and calculate the area of the corresponding rectangle to be $f(x_k^*)\Delta x_k$. The approximate area of the region R is then the sum of these rectangular areas, denoted by $S^*(P) = \sum_{k=1}^{n} f(x_k^*)\Delta x_k$.

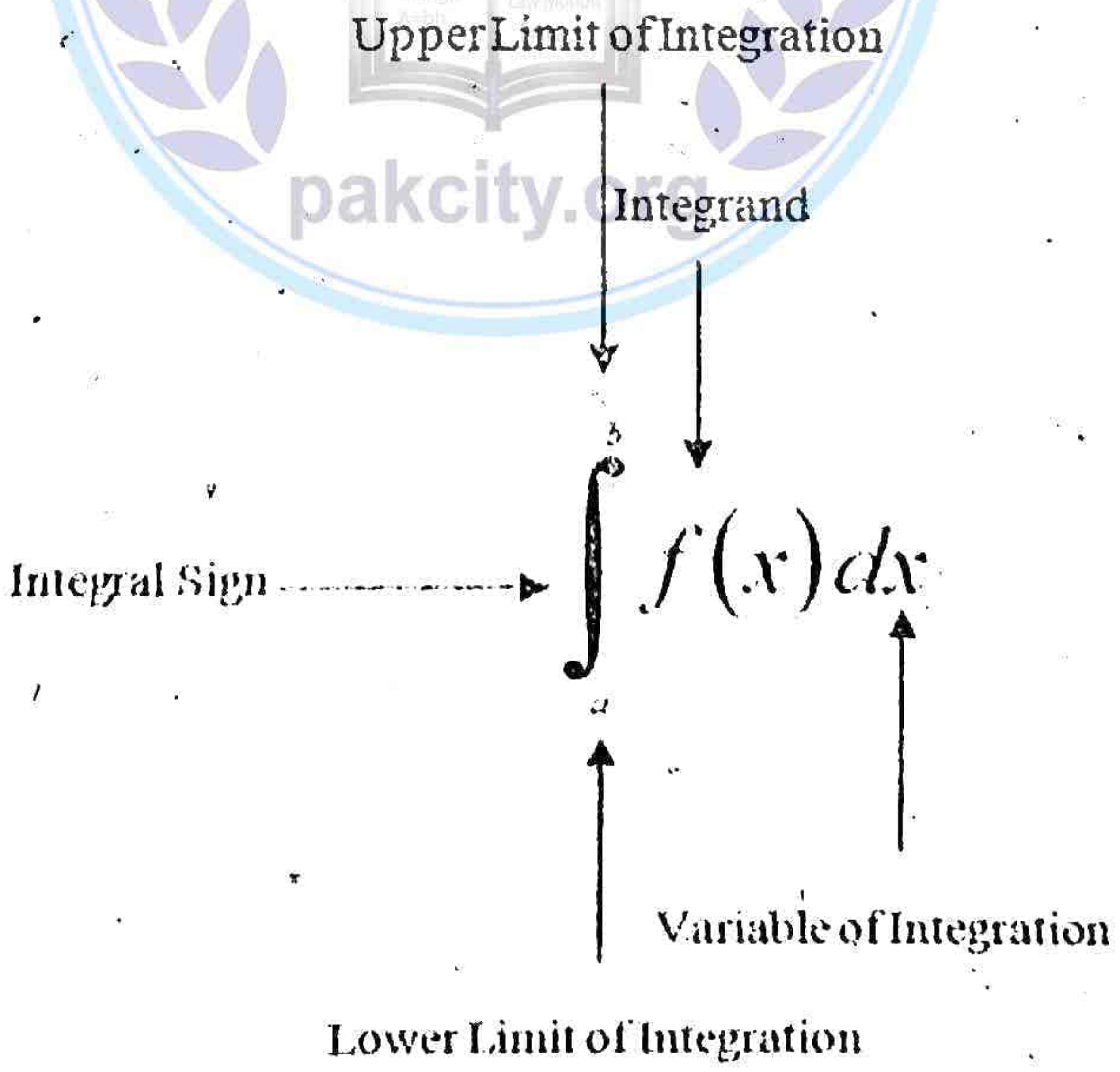
Depending on what points we select for the x_k^*'s, our estimate may be too large or too small. For example, if we choose each x_i^* to be the point in its subinterval giving the maximum height, we will overestimate the area of R, called the Upper Sum (see the figure below).

On the other hand, if we choose each x_k^* to be the point in its subinterval giving the minimum height, we will underestimate the area of R, called the Lower Sum (see the figure below).



Now, if the sum $\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x_k$ approaches a limit as the length of the subintervals $[x_{k-1}, x_k]$ approach zero, regardless of the starred points x_k^* chosen, we then define the area of the region R to be precisely this limit. Note the beauty of this definition. Since we really do not know what area really is, we let our intuition develop a process that we legitimize as the analytic meaning of area under a continuous curve. We will now formalize this process in the following development, called the Riemann Integral.

The components that make up the Definite/Riemann Integral are named as follows:



Fundamental Theorem of Integral Calculus (FTIC)

The relation $\int_a^b p(x)dx = \int p(x)dx \Big|_a^b$ in exercise 1 above is an example of the **Fundamental Theorem of Integral Calculus,** but for polynomials. We will now begin to show that this theorem also holds for any continuous f on $[a,b]$.

First Fundamental Theorem of Integral Calculus: Let f be continuous on the closed interval $[a,b]$, and let $F(x)=\int_a^x f$, $a \le x \le b$. Then

a) $F'_+(x)=f(x), x\in[a,b)$. In particular, $F'_+(a)=f(a)$.
b) $F'_-(x)=f(x), x\in(a,b]$. In particular, $F'_-(b)=f(b)$.
c) F is continuous on $[a,b]$.

$F'(x)=f(x), x\in(a,b)$.

Second Fundamental Theorem of Integral Calculus:
Let f be continuous on the closed interval $[a,b]$, and let $F(x)=\int_a^x f$, $a\le x\le b$. If G is an anti-derivative of f on $[a,b]$, then

$$\int_a^b f(x)dx = G(b)-G(a).$$

Name of Mathematician	Notation used for Integration
Leibniz	$\int y\,dx$
Newton	$\dot{x}$
Lagrange	$f^{(-1)}(x)$
Euler	$D^{-1}(x)$

General formulae of Integration

1) $\int x^n\,dx=\frac{x^{n+1}}{n+1}+c$, $(n\neq -1)$ $n\in Q$

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2) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \ (n \neq -1)$
3) $\int cf(x)\,dx = c\int f(x)\,dx$ (c is a constant)
4) $\int \frac{dx}{x} = \ln x + c$
5) $\int e^{mx}\,dx = \frac{e^{mx}}{m} + c$
6) $\int a^x\,dx = \frac{a^x}{\ln a} + c, \ (a > 0, a \neq 1)$
7) $\int (uv)\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$

(called integration by parts)

Integrals of Trigonometric Functions

1) $\int \sin x\,dx = -\cos x + c$
2) $\int \sin mx\,dx = -\frac{\cos mx}{m} + c$
3) $\int \cos x\,dx = \sin x + c$
4) $\int \cos mx\,dx = -\frac{\sin mx}{m} + c$
5) $\int \sec^2 x\,dx = \tan x + c$
6) $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
7) $\int \sec x \tan x\,dx = \sec x + c$
8) $\int \tan x\,dx = \ln \sec x + c$
9) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$
10) $\int \sec x\,dx = \ln|\sec x + \tan x| + c$
$= \ln \tan\left(x + \frac{\pi}{2}\right) + c$
11) $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x + \cot x| + c$
$= \ln \tan\left(\frac{x}{2}\right) + c$
12) $\int \tan^2 x\,dx = \tan x - x + c$
13) $\int \cot^2 x\,dx = -\cot x - x + c$

$\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$

Some integrals of Hyperbolic Functions

1) $\int \sinh x\,dx = \cosh x + c$
2) $\int \cosh x\,dx = \sinh x + c$
3) $\int \tanh x\,dx = \ln \cosh x + c$

Integrals including radicals

1) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$ **OR** $-\cos^{-1}\left(\frac{x}{a}\right) + c$
2) $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \sec^{-1}\left|\frac{x}{a}\right| + c$ **OR** $-\operatorname{cosec}^{-1}\left|\frac{x}{a}\right| + c; \ |x| > 1$
3) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

4) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$

5) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + c$

6) $\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

7) $\int\sqrt{x^2 + a^2}\,dx$
$= \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + c$

8) $\int\sqrt{x^2 - a^2}\,dx$
$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + c$

9) $\int\frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

10) $\int\frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

Integrals including exponential/logarithms

1) $\int\frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

2) $\int e^{ax}[af(x) + f'(x)]\,dx = e^{ax}f(x) + c$

3) $\int e^{ax}\cos bx\,dx = \frac{e^{x}(a\cos bx + b\sin bx)}{a^2+b^2} + c$

4) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + c$

Properties of definite integrals

1) $\int_a^b f(x)dx = |F(x)|_a^b = F(b) - F(a)$

2) $\int_a^a f(x)\,dx = 0$

3) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

4) $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$. (c is a constant)

5) $\int_a^b[f(x) \pm g(x)]\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)\,dx$

6) $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$, where $a < b < c$

7) $\int_{-a}^a f(x)dx = 0$, if f(x) is an odd function

8) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)$, if f(x) is an even function

9) $\int_a^b f(x)\,dx = \int_{a-c}^{b-c} f(x+c)\,dx$; [shift property]

10) $f(x) \geq g(x)$ in [a,b] then
$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$ [domain rule]

$a^2 - x^2$	$x = a\sin\theta$
$x^2 + a^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

APPLICATION OF INTEGRATION

1) Area under curves and the axes:

(i) If "A_1" is the area bounded by curve $y = f(x)$, x – axis and between the coordinates $x = a$, $x - b$ then $A_1 = \int_a^b y\,dx$

(ii) If "A_2" is the are bounded by the curve $y = f(x)$, y – axis and between the coordinates $y = a$, $y = b$, then $A_2 = \int_a^b y\,dx$

(iii) In choosing the limits of integration, the smaller values of x at ordinate is drawn will be taken as lower limit and the greater as upper limit. (i.e. you are to more from left to right on x-axis)

(iv) Only the numerical value of the area will be considered (i.e. if here comes negative sign with the area after calculating neglect it.)

(v) If the curve is symmetrical, then first are of one symmetrical portion and the multiply it by the number of symmetrical portions.)

To check symmetry:

(a) The given curve is symmetrical with respect to x-axis if equation $y = f(x)$ involves even powers of y and the curve is symmetrical about y-axis having even powers of x.

(b) If the equation involves the even powers of both x and y crave is symmetrical about axis.

2) Area Under two curves:

Let f(x) and g(x) be two curves such that f(x) > g(x) for every x ϵ (a, b) then the area of the region bounded by the graph of f(x) and g(x) and the ordinates x = a & x = b for a < x < b is given by A $\int_0^{\frac{\pi}{2}}\{f(x) - g(x)\}dx$

EXERCISE 6.1

Q.1 Evaluate the following indefinite integrals by using standard formulae.

(i) $\int 9x^5 dx$ (ii) $\int \frac{15}{x^3} dx$ (iii) $\int \frac{a}{\sqrt{bx}} dx$ (iv) $\int by^{\frac{2}{3}} dy$

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(v) $\int(3x^2-9x+5)dx$ (vi) $\int(2x^{-5}+3x^{-2})dx$

(vii) $\int\frac{x^5+3x^3-5x+6}{x^4}dx$ (viii) $\int(\cos x+2\sin x)dx$

(ix) $\int(3\sec x-\operatorname{cosec} x)dx$ (x) $\int(2\tan x-5\sec x)dx$

(xi) $\int(9e^x-3\cos x-5\sin x)dx$ (x) $\int(\sec^2 x+\operatorname{cosec}^2 x)dx$

Solution:

(i) $\int 9x^5dx$

$=9\left(\frac{x^6}{6}\right)+C$

$=\frac{3x^6}{2}+C$

(ii) $\int\frac{15}{x^3}dx$

$=15\int x^{-3}dx$

$=15\left(\frac{x^{-2}}{-2}\right)+C$

$=-\frac{15}{2x^2}+C$

(iii) $\int\frac{a}{\sqrt{bx}}dx$

$=\int\frac{a}{\sqrt{b}\sqrt{x}}dx$

$=\frac{a}{\sqrt{b}}\int\frac{1}{x^{\frac{1}{2}}}dx$

$=\frac{a}{\sqrt{b}}\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)+C$

$=\frac{2a}{\sqrt{b}}\sqrt{x}+C$

(iv) $\int by^{\frac{2}{3}}dy$

$=b\int y^{\frac{2}{3}}dy$

$=b\left(\frac{y^{\frac{5}{3}}}{\frac{5}{3}}\right)+C$

$=\frac{5b}{5}y^{\frac{5}{3}}+C$

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(v) $\int(3x^2-9x$ -

$=3\left(\frac{x^3}{3}\right)-9\Big($

$=x^3-\frac{9}{2}x^2+5$

(vi) $\int(2x^{-5}+3$

$=2\left(\frac{x^{-4}}{-4}\right)+3$

$=-\frac{x^{-4}}{2}-3x^{-}$

$=-\frac{1}{2x^4}-\frac{3}{x}+$

(vii) $\int\frac{x^5+3x^3-5x}{x^4}$

$=\int\left\{\frac{x^5}{x^4}+\frac{3x^3}{x^4}\right.$

$=\int\left\{x+\frac{3}{x}-5\right.$

$=\frac{x^2}{2}+3\ln x$ -

$=\frac{x^2}{2}+3\ln x$ +

(viii) $\int(\cos x+$

$=\int(\cos x+2$

$=\sin x-2\cos$

(ix) $\int(3\sec x-$

$=3\sec x\tan x$

(x) $\int(2\tan x-$

$=2\ln(\sec x)$ -

(xi) $\int(9e^x-3$

$=9e^x-3\sin$

$=9e^x-3\sin$

(x) $\int(\sec^2 x+$

$=\tan x-\cot x$

Q.2 Evaluate th

(i) $\int(3x^2+9x$

(v) $\int(3x^2-9x+5)dx$

$= 3\left(\frac{x^3}{3}\right)-9\left(\frac{x^2}{2}\right)+5x+C$

$= x^3-\frac{9}{2}x^2+5x+C$

(vi) $\int(2x^{-5}+3x^{-2})dx$

$= 2\left(\frac{x^{-4}}{-4}\right)+3\left(\frac{x^{-1}}{-1}\right)+C$

$= -\frac{x^{-4}}{2}-3x^{-1}+C$

$= -\frac{1}{2x^4}-\frac{3}{x}+C$

(vii) $\int\frac{x^5+3x^3-5x+6}{x^4}dx$

$= \int\left\{\frac{x^5}{x^4}+\frac{3x^3}{x^4}-\frac{5x}{x^4}+\frac{6}{x^4}\right\}dx$

$= \int\left\{x+\frac{3}{x}-5x^{-3}+6x^{-4}\right\}dx$

$= \frac{x^2}{2}+3\ln x-5\left(\frac{x^{-2}}{-2}\right)+6\left(\frac{x^{-3}}{-3}\right)+C$

$= \frac{x^2}{2}+3\ln x+\frac{5}{2x^2}=2x^{-3}+C$

(viii) $\int(\cos x+2\sin x)dx$

$= \int(\cos x+2\sin x)dx$

$= \sin x-2\cos x+C$

(ix) $\int(3\sec x-\operatorname{cosec} x)dx$

$= 3\sec x\tan x-\ln(\operatorname{cosec} x-\cot x)+C$

(x) $\int(2\tan x-5\sec x)dx$

$= 2\ln(\sec x)-5\ln(\sec x+\tan x)+C$

(xi) $\int(9e^x-3\cos x-5\sin x)dx$

$= 9e^x-3\sin x-5(-\cos x)+C$

$= 9e^x-3\sin x+5\cos x+C$

(x) $\int(\sec^2 x+\operatorname{cosec}^2 x)dx$

$= \tan x-\cot x+C$

Q.2 Evaluate the following indefinite integrals by using standard formulae.

(i) $\int(3x^2+9x+3)^{\frac{1}{2}}(6x+9)dx$ (ii) $\int\sqrt{ax^2+2bx+c}(ax+b)dx$

(iii) $\int \frac{6x+5}{\sqrt{3x^2+5x+2}}dx$ (iv) $\int (x^2+4x+3)^{-9}(2x+4)dx$

(v) $\int (x-2)(x-3)(x-4)dx$ (vi) $\int (2x^2-3)^2 dx$

(vii) $\int (x^3-3x^2+9)^{\frac{7}{2}}(x^2-2x)dx$ (viii) $\int (x^2-5)^3 dx$

(ix) $\int (\cos x+\sin x)^{\frac{3}{2}}(\cos x-\sin x)dx$

(x) $\int (\tan x+\sin x)(\sec^2 x+\cos x)dx$

Solution:

$$\int \{f(x)\}^n f'(x)dx = \frac{\{f(x)\}^{n+1}}{n+1}+C; n\neq -1$$

(i) $\int (3x^2+9x+3)^{\frac{1}{2}}(6x+9)dx$

$$=\frac{(3x^2+9x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=\frac{(3x^2+9x+3)^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{2}{3}(3x^2+9x+3)^{\frac{3}{2}}+C$$

(ii) $\int \sqrt{ax^2+2bx+c}(ax+b)dx$

$$=\frac{1}{2}\int (ax^2+2bx+c)^{\frac{1}{2}}(2ax+2b)dx$$

$$=\frac{\frac{1}{2}(ax^2+2bx+c)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=\frac{\frac{1}{2}(ax^2+2bx+c)^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{1}{3}(ax^2+2bx+c)^{\frac{3}{2}}+C$$

(iii) $\int \frac{6x+5}{\sqrt{3x^2+5x+2}}dx$

$$=\int (3x^2+5x+2)^{\frac{1}{2}}(6x+5)dx$$

$$=\frac{(3x^2+5x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$=\frac{(3x^2+5x ...}{\frac{3}{2}}$

$=\frac{2}{3}(3x^2+5x ...$

(iv) $\int (x^2+4x ...$

$=\frac{(x^2+4x+ ...}{-8}$

$=-\frac{1}{8(x^2+4 ...}$

(v) $\int (x-2)(...$

$=\int (x^2-5x ...$

$=\int (x^3-5x ...$

$=\int (x^3-9x ...$

$=\frac{x^4}{4}-\frac{9x^3}{3} ...$

$=\frac{x^4}{4}-3x^3 ...$

(vi) $\int (2x^2- ...$

$=\int (4x^4-1 ...$

$=4\left(\frac{x^5}{5}\right)- ...$

$=\frac{4x^5}{5}-4x^3 ...$

(vii) $\int (x^3-3 ...$

$=\frac{1}{3}\int (x^3- ...$

$=\frac{1}{3}\int (x^3- ...$

$=\frac{1}{3}\frac{(x^3-3x ...}{\frac{7}{2}}$

$+ 4)dx$

$= \dfrac{(3x^2 + 5x + 2)^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \dfrac{2}{3}(3x^2 + 5x + 2)^{\frac{3}{2}} + C$

(iv) $\int (x^2 + 4x + 3)^{-9}(2x + 4)dx$

$= \dfrac{(x^2 + 4x + 3)^{-8}}{-8} + C$

$= -\dfrac{1}{8(x^2 + 4x + 3)^8} + C$

(v) $\int (x - 2)(x - 3)(x - 4)dx$

$= \int (x^2 - 5x + 6)(x - 4)dx$

$= \int (x^3 - 5x^2 + 6x - 4x^2 + 20x - 24)dx$

$= \int (x^3 - 9x^2 + 26x - 24)dx$

$= \dfrac{x^4}{4} - \dfrac{9x^3}{3} + \dfrac{26x^2}{2} - 24x + C$

$= \dfrac{x^4}{4} - 3x^3 + 13x^2 - 24x + C$

(vi) $\int (2x^2 - 3)^2 dx$

$= \int (4x^4 - 12x^2 + 9)dx$

$= 4\left(\dfrac{x^5}{5}\right) - 12\left(\dfrac{x^3}{3}\right) + 9x + C$

$= \dfrac{4x^5}{5} - 4x^3 + 9x + C$

(vii) $\int (x^3 - 3x^2 + 9)^{\frac{7}{2}}(x^2 - 2x)dx$

$= \dfrac{1}{3}\int (x^3 - 3x^2 + 9)^{\frac{7}{2}} 3(x^2 - 2x)dx$

$= \dfrac{1}{3}\int (x^3 - 3x^2 + 9)^{\frac{7}{2}}(3x^2 - 6x)dx$

$= \dfrac{1}{3}\dfrac{(x^3 - 3x^2 + 9)^{\frac{7}{2}+1}}{\frac{7}{2} + 1} + C$

$$= \frac{1}{3}\frac{(x^3 - 3x^2 + 9)^{\frac{9}{2}}}{\frac{9}{2}} + C$$

$$= \frac{2}{27}(x^3 - 3x^2 + 9)^{\frac{9}{2}} + C$$

(viii) $\int (x^2 - 5)^3 dx$

$$= \int \{(x^2)^3 - 3(x^2)^2(5) + 3(x^2)(5)^2 - (5)^3\} dx$$

$$= \int (x^6 - 15x^4 + 75x^2 - 125) dx$$

$$= \frac{x^7}{7} - \frac{15x^5}{5} + \frac{75x^3}{3} - 125x + C$$

$$= \frac{x^7}{7} - 3x^5 + 25x^3 - 125x + C$$

(ix) $\int (\cos x + \sin x)^{\frac{3}{2}}(\cos x - \sin x) dx$

$$= \int (\sin x + \cos x)^{\frac{3}{2}}(\cos x - \sin x) dx$$

$$= \frac{(\sin x + \cos x)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{(\sin x + \cos x)^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{5}(\sin x + \cos x)^{\frac{5}{2}} + C$$

(x) $\int (\tan x + \sin x)(\sec^2 x + \cos x) dx$

$$= \frac{(\tan x + \sin x)^2}{2} + C$$

$$= \frac{1}{2}(\tan x + \sin x)^2 + C$$

Q.3 Evaluate by using standard formulae of integration.

(i) $\int \frac{x}{x^2+3} dx$ (ii) $\int \frac{\sec^2 x + \cos x}{\tan x + \sin x} dx$ (iii) $\int \frac{5x^4 + 4x^3 - 3x^2 + 2x}{x^5 + x^4 - x^3 + x^2} dx$

(iv) $\int \frac{e^x + \frac{1}{x}}{e^x + \ln x} dx$ (v) $\int (5x^3 - 3x^2 + 6x - 9)^{-1}(5x^2 - 2x + 2) dx$

(vi) $\int \frac{1}{x + \sqrt{x}} dx$

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Solution:

$\int \frac{f'(x)}{f(x)} dx$ ...

(i) $\int \frac{x}{x^2+3} dx$ ...

$= \frac{1}{2}\int \frac{2x}{x^2 + \ldots}$

$= \frac{1}{2}\ln|x^2 + \ldots$

(ii) $\int \frac{\sec^2 x + \ldots}{\tan x + \ldots}$

$= \ln|\tan x \ldots$

(iii) $\int \frac{5x^4 + 4\ldots}{x^5 + \ldots}$

$= \ln|x^5 + \ldots$

iv) $\int \frac{e^x + \frac{1}{x}}{e^x + \ln x}$

$= \ln|e^x + \ldots$

(v) $\int (5x^3 \ldots$

$= \frac{1}{3}\int \frac{3(\ldots}{5x^3 \ldots}$

$= \frac{1}{3}\int \frac{1\ldots}{5x^3 \ldots}$

$= \ln|5x^3 \ldots$

(vi) $\int \frac{1}{x + \sqrt{x}} \ldots$

$= \int \frac{\ldots}{\sqrt{x}\sqrt{x}\ldots}$

$= \int \frac{\ldots}{\sqrt{x}(\sqrt{\ldots}}$

$= 2\int \frac{\ldots}{(\sqrt{x}\ldots}$

$= 2\ln|\sqrt{x} \ldots$

Q.4 Evaluat...

(i) $\int e^x(\sin \ldots$

(iii) $\int e^x(\tan \ldots$

Solution:

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$$

(i) $\int \frac{x}{x^2+3}dx$

$= \frac{1}{2}\int \frac{2x}{x^2+3}dx$

$= \frac{1}{2}\ln|x^2+3| + C$

(ii) $\int \frac{\sec^2 x+\cos x}{\tan x+\sin x}dx$

$= \ln|\tan x + \sin x| + C$

(iii) $\int \frac{5x^4+4x^3-3x^2+2x}{x^5+x^4-x^3+x^2}dx$

$= \ln|x^5 + x^4 - x^3 + x^2| + C$

iv) $\int \frac{e^x+\frac{1}{x}}{e^x+\ln x}dx$

$= \ln|e^x + \ln x| + C$

(v) $\int (5x^3 - 3x^2 + 6x - 9)^{-1}(5x^2 - 2x + 2)dx$

$= \frac{1}{3}\int \frac{3(5x^2 - 2x + 2)}{5x^3 - 3x^2 + 6x - 9}dx$

$= \frac{1}{3}\int \frac{15x^2 - 6x + 6}{5x^3 - 3x^2 + 6x - 9}dx$

$= \ln|5x^3 - 3x^2 + 6x - 9| + C$

(vi) $\int \frac{1}{x+\sqrt{x}}dx$

$= \int \frac{1}{\sqrt{x}\sqrt{x} + \sqrt{x}}dx$

$= \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$

$= 2\int \frac{1}{(\sqrt{x}+1)}\left(\frac{1}{2\sqrt{x}}\right)dx$

$= 2\ln|\sqrt{x} + 1| + C$

Q.4 Evaluate the following integrals by using standard formulae:

(i) $\int e^x(\sin x + \cos x)dx$

(ii) $\int e^x\left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right)dx$

(iii) $\int e^x\left(\tan^{-1} x + \frac{1}{1+x^2}\right)dx$

(iv) $\int e^x(\sec^2 x + \tan x)dx$

dx

x

(v) $\int e^x\left(\frac{1}{x}+\ln x\right)dx$

Solution:

$$\frac{d}{dx}\{e^x f(x)\}=e^x\{f(x)+f'(x)\}$$

$$\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$$

(i) $\int e^x(\sin x+\cos x)dx$

$$\frac{d}{dx}\{e^x\sin x\}=e^x(\sin x+\cos x)$$

$$\int e^x(\sin x+\cos x)dx=e^x\sin x+C$$

(ii) $\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$

$$\frac{d}{dx}\{e^x\sin^{-1}x\}=e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)$$

$$\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx=e^x\sin^{-1}x+C$$

(iii) $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$

$$\frac{d}{dx}\{e^x\tan^{-1}x\}=e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)$$

$$\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx=e^x\tan^{-1}x+C$$

(iv) $\int e^x(\sec^2 x+\tan x)dx$

$$\frac{d}{dx}\{e^x\tan x\}=e^x(\sec^2 x+\tan x)$$

$$\int e^x(\sec^2 x+\tan x)dx=e^x\tan x+C$$

(v) $\int e^x\left(\frac{1}{x}+\ln x\right)dx$

$$\frac{d}{dx}\{e^x\ln x\}=e^x\left(\frac{1}{x}+\ln x\right)$$

$$\int e^x\left(\frac{1}{x}+\ln x\right)dx=e^x\ln x+C$$

Q.5 Evaluate the following integrals by using appropriate formulae:

(i) $\int\frac{dx}{x^2+9}$ (ii) $\int\frac{dt}{\sqrt{4-t^2}}$ (iii) $\int\frac{dy}{y\sqrt{y^2-9}}$

(iv) $\int\frac{dt}{4t^2-9}$ (v) $\int\frac{dx}{\sqrt{9x^2+16}}$ (vi) $\int\frac{dx}{\sqrt{16x^2-9}}$

(vii) $\int \frac{dx}{9-x^2}$ (viii) $\int \frac{dx}{x\sqrt{4x^2-16}}$ (ix) $\int \sqrt{9-4x^2}dx$

(x) $\int \sqrt{25+9x^2}dx$ (viii) $\int \frac{dy}{9y^2+81}$ (ix) $\int \frac{dx}{4x^2-16}$

Solution:

(i) $\int \frac{dx}{x^2+9}$

$$= \int \frac{dx}{x^2+3^2}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

$a = 3$

$$= \frac{1}{3}\tan^{-1}\frac{x}{3}+C$$

(ii) $\int \frac{dt}{\sqrt{4-t^2}}$

$$= \int \frac{dt}{\sqrt{2^2-t^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+C$$

$a = 3$

$$= \sin^{-1}\frac{t}{2}+C$$

(iii) $\int \frac{dy}{|y|\sqrt{y^2-9}}$

$$= \int \frac{dy}{|y|\sqrt{y^2-3^2}}$$

$$\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+C$$

$a = 3$

$$= \frac{1}{3}\sec^{-1}\frac{x}{3}+C$$

(iv) $\int \frac{dt}{4t^2-9}$

$$= \int \frac{dt}{4\left(t^2-\frac{9}{4}\right)}$$

$$= \frac{1}{4}\int \frac{dt}{t^2-\left(\frac{3}{2}\right)^2}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$a = \frac{3}{2}$$

$$= \left(\frac{1}{4}\right)\frac{1}{2\left(\frac{2}{3}\right)}\ln\left|\frac{t-\frac{3}{2}}{t+\frac{3}{2}}\right| + C$$

$$= 3\ln\left|\frac{2t-3}{2t+3}\right| + C$$

(v) $\int \frac{dx}{\sqrt{9x^2+16}}$

$$= \int \frac{dx}{\sqrt{(3x)^2 + 16}}$$

Let $y = 3x$

$$\frac{dy}{dx} = 3 \Rightarrow \frac{dy}{3} = dx$$

$$= \frac{1}{3}\int \frac{dy}{\sqrt{y^2 + 4^2}}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|\sqrt{x^2 + a^2}\right| + C$$

$$a = 4$$

$$= \frac{1}{3}\int \frac{dy}{\sqrt{y^2 + 4^2}}$$

$$= \frac{1}{3}\ln\left|\sqrt{y^2 + 16}\right| + C$$

$$= \frac{1}{3}\ln\left|\sqrt{9x^2 + 16}\right| + C$$

OR

$$= \int \frac{dx}{\sqrt{9\left(x^2 + \frac{16}{9}\right)}}$$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{x^2 + \left(\frac{4}{3}\right)^2}}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|\sqrt{x^2 + a^2}\right| + C$$

$$a = \frac{4}{3}$$

$= \frac{1}{3}\ln\left|\sqrt{\right.$

$= \frac{1}{3}\ln\left|\sqrt{\right.$

$= \frac{1}{3}\ln\left|\sqrt{\right.$

$= \frac{1}{3}\ln|\sqrt{}$

$= \frac{1}{3}\ln|\sqrt{}$

(vi) $\int \frac{}{\sqrt{16}}$

$= \int \frac{}{\sqrt{(4}}$

Let $y =$

$\frac{dy}{dx} = 4$

$I = \frac{1}{4}\int$

$\int \frac{dx}{\sqrt{x^2 -}}$

$a = 3$

$= \frac{1}{4}\ln|y$

(vii) $\int \frac{d}{9-}$

$= \int \frac{d}{3^2}$

$\int \frac{dx}{a^2 - x}$

$a = 3$

$= \frac{1}{2(3)}\ln$

$= \frac{1}{6}\ln\left|\frac{3}{3}\right.$

$$=\frac{1}{3}\ln\left|\sqrt{x^2+\left(\frac{4}{3}\right)^2}\right|+C$$

$$=\frac{1}{3}\ln\left|\sqrt{x^2+\frac{16}{9}}\right|+C$$

$$=\frac{1}{3}\ln\left|\sqrt{\frac{9x^2+16}{9}}\right|+C$$

$$=\frac{1}{3}\ln\left|\sqrt{9x^2+16}\right|-\frac{1}{3}\ln 3+C$$

$$=\frac{1}{3}\ln\left|\sqrt{9x^2+16}\right|+C$$

(vi) $\int \frac{dx}{\sqrt{16x^2-9}}$

$$=\int \frac{dx}{\sqrt{(4x)^2-9}}$$

Let $y=4x$

$$\frac{dy}{dx}=4 \Rightarrow \frac{dy}{4}=dx$$

$$I=\frac{1}{4}\int \frac{dy}{\sqrt{y^2-3^2}}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}}=\ln\left|x+\sqrt{x^2-a^2}\right|+C$$

$a=3$

$$=\frac{1}{4}\ln\left|y+\sqrt{y^2-9}\right|+C$$

(vii) $\int \frac{dx}{9-x^2}$

$$=\int \frac{dx}{3^2-x^2}$$

$$\int \frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$$

$a=3$

$$=\frac{1}{2(3)}\ln\left|\frac{3+x}{3-x}\right|+C$$

$$=\frac{1}{6}\ln\left|\frac{3+x}{3-x}\right|+C$$

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(viii) $\int \frac{dx}{x\sqrt{4x^2-16}}$

$$= \int \frac{dx}{x\sqrt{(2x)^2-16}}$$

Let $y = 2x \Rightarrow \frac{y}{2} = x$

$$\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{2} = dx$$

$$= \frac{1}{2}\int \frac{dy}{\left(\frac{y}{2}\right)\sqrt{y^2-16}}$$

$$= \int \frac{dy}{y\sqrt{y^2-4^2}}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + C$$

$a = 4$

$$= \frac{1}{4}\sec^{-1}\frac{y}{4} + C$$

(ix) $\int \sqrt{9-4x^2}dx$

$$= \int \sqrt{9-(2x)^2}dx$$

Let $y = 2x \Rightarrow \frac{y}{2} = x$

$$\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{2} = dx$$

$$= \int \sqrt{9-y^2}\left(\frac{dy}{2}\right)$$

$$= \frac{1}{2}\int \sqrt{3^2-y^2}dy$$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$a = 3$

$$= \frac{1}{2}\left\{\frac{y}{2}\sqrt{9-y^2} + \frac{9}{2}\sin^{-1}\frac{y}{3}\right\} + C$$

$$= \frac{y}{4}\sqrt{9-y^2} + \frac{9}{4}\sin^{-1}\frac{y}{3} + C$$

$$= \frac{2x}{4}\sqrt{9-(2x)^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$$

$$= \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$$

(x) $\int \sqrt{25+9x^2}dx$

$= \int \sqrt{25 + (3x)^2}dx$

Let $y = 3x \Rightarrow \frac{y}{3} = x$

$\frac{dy}{dx} = 3 \Rightarrow \frac{dy}{3} = dx$

$= \frac{1}{3}\int \sqrt{5^2 + y^2}dx$

$\int \sqrt{x^2 + a^2}dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + C$

$= \frac{1}{3}\left\{\frac{y}{2}\sqrt{y^2 + 25} + \frac{25}{2}\ln\left|y + \sqrt{y^2 + 25}\right|\right\} + C$

$= \frac{y}{6}\sqrt{y^2 + 25} + \frac{25}{6}\ln\left|y + \sqrt{y^2 + 25}\right| + C$

(viii) $\int \frac{dy}{9y^2+81}$

$= \int \frac{dy}{9(y^2 + 9)}$

$= \frac{1}{9}\int \frac{dy}{y^2 + 3^2}$

$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$a = 3$

$= \left(\frac{1}{9}\right)\frac{1}{3}\tan^{-1}\frac{x}{3} + C$

$= \frac{1}{327}\tan^{-1}\frac{x}{3} + C$

(ix) $\int \frac{dx}{4x^2-16}$

$= \int \frac{dx}{4x^2 - 16}$

$= \int \frac{dx}{4(x^2 - 4)}$

$= \frac{1}{4}\int \frac{dx}{x^2 - 2^2}$

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + C$

$a = 2$

$= \left(\frac{1}{4}\right)\frac{1}{2(2)}\ln\left|\frac{x - 2}{x + 2}\right| + C$

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$= \frac{1}{16}\ln\left|\frac{x-2}{x+2}\right| + C$

EXERCISE 6.2

Q.1 Evaluate the following integrals by substitution method:

(i) $\int \frac{3x dx}{\sqrt{x^2+7}}$ (ii) $\int x^{\frac{4}{3}}\left(a^{\frac{7}{3}} - x^{\frac{7}{3}}\right)^{\frac{5}{2}} dx$ (iii) $\int \frac{1+2x}{\sqrt{1-x}} dx$

(iv) $\int (2x^2+4x+5)^{\frac{3}{2}}(x+1)dx$ (v) $\int \frac{x dx}{\sqrt{1-x^2}}$

(vi) $\int (x^2+2x+5)^{-1}(x+1)dx$ (vii) $\int x^3(9+x^2)^{\frac{3}{2}}dx$

(viii) $\int (x^3-9)^{\frac{5}{2}}x^5 dx$ (ix) $\int x^9(x^5+3)^{\frac{2}{5}}dx$

(x) $\int (x^3+x^2+5x-1)^{-1}(3x^2+2x+5)dx$

Solution:

(i) $\int \frac{3x dx}{\sqrt{x^2+7}}$

$= \int \frac{3x dx}{(x^2+7)^{\frac{1}{2}}}$

$= 3\int (x^2+7)^{-\frac{1}{2}} x dx$

Let $y = x^2 + 7$.

$\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{2} = dx$

$= \frac{3}{2}\int y^{-\frac{1}{2}} dy$

$= \frac{3}{2}\left\{\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$

$= 3\left(x^2 + 7^{\frac{1}{2}}\right)^{\frac{1}{2}} + C$

OR

$\int \frac{3x dx}{\sqrt{x^2+7}} = \int \frac{3x dx}{(x^2+7)^{\frac{1}{2}}}$

$= \frac{3}{2}\int (x^2+7)^{-\frac{1}{2}} 2x dx$

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$= \frac{3}{2}\frac{(x^2+ \dots}{\frac{1}{2}}$

$= 3\sqrt{x^2+ \dots}$

(ii) $\int x^{\frac{4}{3}}\left(a^{\frac{7}{3}} \dots\right.$

Let $y = a^{\frac{7}{3}} \dots$

$\frac{dy}{dx} = -\frac{7}{3}x \dots$

$-\frac{3}{7}dy = x \dots$

$I = -\frac{3}{7}\int \dots$

$= -\frac{3}{7}\left\{\frac{y^{\frac{7}{2}}}{\frac{7}{2}} \dots\right.$

$= -\frac{6}{49}\left(a^{\frac{7}{3}} \dots\right.$

OR

$= \frac{3}{7}\int \frac{7}{3}x^{\frac{4}{3}} \dots$

$= \frac{3}{7}\frac{\left(a^{\frac{7}{3}} - \dots\right.}{\frac{5}{2} + \dots}$

$= \frac{3}{7}\frac{\left(a^{\frac{7}{3}} - \dots\right.}{\frac{7}{2}}$

$= \frac{6}{49}\left(a^{\frac{7}{3}} - \dots\right.$

(iii) $\int \frac{1+2x}{\sqrt{1-x}} d\dots$

$= \int \frac{1+2x}{(1-x) \dots}$

$= \int (1-x) \dots$

$$=\frac{3}{2}\frac{(x^2+7)^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=3\sqrt{x^2+7}+C$$

(ii) $\int x^{\frac{4}{3}}\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{5}{2}}dx$

Let $y=a^{\frac{7}{3}}-x^{\frac{7}{3}}$

$$\frac{dy}{dx}=-\frac{7}{3}x^{\frac{4}{3}}$$

$$-\frac{3}{7}dy=x^{\frac{4}{3}}dx$$

$$I=-\frac{3}{7}\int y^{\frac{5}{2}}dy$$

$$=-\frac{3}{7}\left\{\frac{y^{\frac{7}{2}}}{\frac{7}{2}}\right\}+C$$

$$=-\frac{6}{49}\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{7}{2}}+C$$

OR

$$=\frac{3}{7}\int\frac{7}{3}x^{\frac{4}{3}}\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{5}{2}}dx$$

$$=\frac{3}{7}\frac{\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{5}{2}+1}}{\frac{5}{2}+1}+C$$

$$=\frac{3}{7}\frac{\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{7}{2}}}{\frac{7}{2}}+C$$

$$=\frac{6}{49}\left(a^{\frac{7}{3}}-x^{\frac{7}{3}}\right)^{\frac{7}{2}}+C$$

(iii) $\int\frac{1+2x}{\sqrt{1-x}}dx$

$$=\int\frac{1+2x}{(1-x)^{\frac{1}{2}}}dx$$

$$=\int(1-x)^{-\frac{1}{2}}(1+2x)dx$$

Let $y = 1 - x \rightarrow (1)$

$\frac{dy}{dx} = -1 \Rightarrow \boxed{-dy = dx}$

$(1) \Rightarrow x = 1 - y$

$2x = 2 - 2y$

$1 + 2x = 1 + 2 - 2y$

$\boxed{1 + 2x = 3 - 2y}$

$$= \int y^{-\frac{1}{2}}(3 - 2y)dx$$

$$= \int \left(3y^{-\frac{1}{2}} - 2y^{\frac{1}{2}}\right)dx$$

$$= 3\left\{\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right\} - 2\left\{\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right\} + C$$

$$= 6(1 - x)^{\frac{1}{2}} - \frac{1}{3}(1 - x)^{\frac{3}{2}} + C$$

OR

$$\int \frac{1 + 2x}{\sqrt{1 - x}}dx$$

$$= -\int \frac{-2x - 1}{\sqrt{1 - x}}dx$$

$$= -2\int \frac{-x - \frac{1}{2}}{\sqrt{1 - x}}dx$$

$$= -2\int \frac{1 - x - 1 - \frac{1}{2}}{\sqrt{1 - x}}dx$$

$$= -2\int \left\{\frac{1 - x}{\sqrt{1 - x}} - \frac{\frac{3}{2}}{\sqrt{1 - x}}\right\}dx$$

$$= -2\int \left\{(1 - x)^{\frac{1}{2}} - \frac{3}{2}(1 - x)^{-\frac{1}{2}}\right\}dx$$

$$= -2\left\{-\frac{(1 - x)^{\frac{1}{2}+1}}{\frac{1}{2} + 1} + \frac{\frac{3}{2}(1 - x)^{-\frac{1}{2}+1}}{-\frac{1}{2} + 1}\right\} + C$$

$$= -2\left\{-\frac{(1-x)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{\frac{3}{2}(1-x)^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$$

$$= -2\left\{-\frac{2}{3}(1-x)^{\frac{3}{2}} + 3(1-x)^{\frac{1}{2}}\right\} + C$$

$$= \frac{4}{3}(1-x)^{\frac{3}{2}} - 6\sqrt{1-x} + C$$

(iv) $\int (2x^2 + 4x + 5)^{\frac{3}{2}}(x+1)dx$

$$\int (2x^2 + 4x + 5)^{\frac{3}{2}}(x+1)dx$$

Let $y = 2x^2 + 4x + 5$

$$\frac{dy}{dx} = 4x + 4$$

$$dy = 4(x+1)dx$$

$$\frac{dy}{4} = (x+1)dx$$

$$I = \frac{1}{4}\int y^{\frac{3}{2}}dy$$

$$= \frac{1}{4}\left\{\frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right\} + C$$

$$= \frac{1}{10}(2x^2 + 4x + 5)^{\frac{5}{2}} + C$$

OR

$$\int (2x^2 + 4x + 5)^{\frac{3}{2}}(x+1)dx$$

$$= \frac{1}{4}\int (2x^2 + 4x + 5)^{\frac{3}{2}}4(x+1)dx$$

$$= \frac{1}{4}\int (2x^2 + 4x + 5)^{\frac{3}{2}}(4x+4)dx$$

$$= \frac{1}{4}\frac{(2x^2 + 4x + 5)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{1}{4}\frac{(2x^2 + 4x + 5)^{\frac{5}{2}}}{\frac{5}{2}} + C$$

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$= \frac{1}{10}(2x^2 + 4x + 5)^{\frac{5}{2}} + C$

(v) $\int \frac{xdx}{\sqrt{1-x^2}}$

$= \int \frac{xdx}{(1-x^2)^{\frac{1}{2}}}$

$= \int (1-x^2)^{-\frac{1}{2}} xdx$

Let $y = 1 - x^2$

$\frac{dy}{dx} = -2x \Rightarrow -\frac{dy}{2} = dx$

$= -\frac{1}{2}\int y^{-\frac{1}{2}} dy$

$= -\frac{1}{2}\left\{\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$

$= -\sqrt{1-x^2} + C$

OR

$\int \frac{xdx}{\sqrt{1-x^2}}$

$= \int \frac{xdx}{(1-x^2)^{\frac{1}{2}}}$

$= \int (1-x^2)^{-\frac{1}{2}} xdx$

Let $y = 1 - x^2$

$\frac{dy}{dx} = -2x \Rightarrow -\frac{dy}{2} = xdx$

$= -\frac{1}{2}\int y^{-\frac{1}{2}} dy$

$= -\frac{1}{2}\left\{\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$

$= -\sqrt{1-x^2} + C$

OR

$\int \frac{xdx}{\sqrt{1-x^2}}$

The S

$= \int$

$= -\frac{1}{2}$

$= -\frac{1}{2}$

$= -\frac{1}{2}$

$= -\sqrt{}$

(vi) $\int ($

$= \int \frac{}{x}$

Let y

$\frac{dy}{dx} =$

$\frac{dy}{dx} =$

$I = \frac{1}{2}$

$= \frac{1}{2}\ln$

$= \frac{1}{2}\ln|$

$= \ln|x$

$= \ln\sqrt{}$

OR

$\int (x^2 +$

$= \int \frac{}{x^2}$

$= \frac{1}{2}\int \frac{}{x}$

$= \frac{1}{2}\ln|x$

$$=\int \frac{xdx}{(1-x^2)^{\frac{1}{2}}}$$

$$=-\frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x)dx$$

$$=-\frac{1}{2}\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$$

$$=-\frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=-\sqrt{1-x^2}+C$$

(vi) $\int (x^2+2x+5)^{-1}(x+1)dx$

$$=\int \frac{x+1}{x^2+2x+5}dx$$

Let $y=x^2+2x+5$

$$\frac{dy}{dx}=2x+2$$

$$\frac{dy}{dx}=2(x+1) \Rightarrow \frac{dy}{2}=(x+1)dx$$

$$I=\frac{1}{2}\int \frac{1}{y}dy$$

$$=\frac{1}{2}\ln|y|+C$$

$$=\frac{1}{2}\ln|x^2+2x+5|+C$$

$$=\ln|x^2+2x+5|^{\frac{1}{2}}+C$$

$$=\ln\sqrt{x^2+2x+5}+C$$

OR

$$\int (x^2+2x+5)^{-1}(x+1)dx$$

$$=\int \frac{x+1}{x^2+2x+5}dx$$

$$=\frac{1}{2}\int \frac{2x+2}{x^2+2x+5}dx$$

$$=\frac{1}{2}\ln|x^2+2x+5|+C$$

$$=\ln|x^2+2x+5|^{\frac{1}{2}}+C$$

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$= \ln\sqrt{x^2+2x+5}+C$

(vii) $\int x^3(9+x^2)^{\frac{3}{2}}dx$

Let $u = 9 + x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = 2xdx$

$\Rightarrow \boxed{u - 9 = x^2}$

$\int (9+x^2)^{\frac{3}{2}}x^2xdx$

$= \int u^{\frac{3}{2}}(u-9)\frac{du}{2}$

$= \frac{1}{2}\int\left(u^{\frac{5}{2}} - 9u^{\frac{3}{2}}\right)du$

$= \frac{1}{2}\left(\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{9u^{\frac{5}{2}}}{\frac{5}{2}}\right) + C$

$= \frac{1}{7}(9+x^2)^{\frac{7}{2}} - \frac{9}{5}(9+x^2)^{\frac{5}{2}} + C$

(viii) $\int (x^3-9)^{\frac{5}{2}}x^5dx$

$= \int (x^3-9)^{\frac{5}{2}}x^5dx$

Let $y = x^3 - 9 \rightarrow (1)$

$\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{3} = x^2dx$

$(1) \Rightarrow x^3 - 9 = y$

$x^3 = y + 9$

$\int (x^3-9)^{\frac{5}{2}}x^3x^2dx$

$= \frac{1}{3}\int y^{\frac{5}{2}}(y+9)dy$

$= \frac{1}{3}\int\left(y^{\frac{7}{2}} + 9y^{\frac{5}{2}}\right)dy$

$= \frac{1}{3}\left\{\frac{y^{\frac{9}{2}}}{\frac{9}{2}} + \frac{9y^{\frac{7}{2}}}{\frac{7}{2}}\right\} + C$

$= \frac{2}{27}(x^3-9)^{\frac{9}{2}} + \frac{6}{7}(x^3-9)^{\frac{7}{2}} + C$

The Students' C

(ix) $\int x^9(x^5+3)$

Let $y = x^5 + 3$ -

$\frac{dy}{dx} = 5x^4 \Rightarrow \frac{dy}{5}$

$(1) \Rightarrow x^5 + 3 =$

$x^5 = y - 3$

$\int (x^5+3)^{\frac{2}{5}}x^5x^4$

$= \frac{1}{5}\int y^{\frac{2}{5}}(y-3)$

$= \frac{1}{5}\int\left(y^{\frac{7}{5}} - 3y^{\frac{2}{5}}\right.$

$= \frac{1}{5}\left\{\frac{y^{\frac{12}{5}}}{\frac{12}{5}} - \frac{3y^{\frac{7}{5}}}{\frac{7}{5}}\right.$

$= \frac{1}{5}\frac{y^{\frac{12}{5}}}{\frac{12}{5}} - \frac{1}{5}\frac{3y^{\frac{7}{5}}}{\frac{7}{5}}$

$= \frac{1}{12}(x^5+3)^{\frac{12}{5}}$.

(x) $\int (x^3 + x^2 + 5$

$= \int \frac{3x^2+2x}{x^3+x^2+5x}$

Let $y = x^3 + x^2 +$

$\frac{dy}{dx} = 3x^2 + 2x +$

$dy = (3x^2 + 2x +$

$I = \int \frac{1}{y}dy$

$= \ln|y| + C$

$= \ln|x^3 + x^2 + 5x$

OR

$\int (x^3 + x^2 + 5x -$

$= \int \frac{3x^2+2x+}{x^3+x^2+5x}$

$= \ln|x^3 + x^2 + 5x$

(ix) $\int x^9(x^5+3)^{\frac{2}{5}}dx$

Let $y = x^5 + 3 \rightarrow (1)$

$\frac{dy}{dx} = 5x^4 \Rightarrow \frac{dy}{5} = x^4dx$

$(1) \Rightarrow x^5 + 3 = y$

$x^5 = y - 3$

$\int (x^5+3)^{\frac{2}{5}}x^5x^4dx$

$= \frac{1}{5}\int y^{\frac{2}{5}}(y-3)dy$

$= \frac{1}{5}\int \left(y^{\frac{7}{5}} - 3y^{\frac{2}{5}}\right)dy$

$= \frac{1}{5}\left\{\frac{y^{\frac{12}{5}}}{\frac{12}{5}} - \frac{3y^{\frac{7}{5}}}{\frac{7}{5}}\right\} + C$

$= \frac{1}{5}\frac{y^{\frac{12}{5}}}{\frac{12}{5}} - \frac{1}{5}\frac{3y^{\frac{7}{5}}}{\frac{7}{5}} + C$

$= \frac{1}{12}(x^5+3)^{\frac{12}{5}} - \frac{3}{7}(x^5+3)^{\frac{7}{5}} + C$

(x) $\int (x^3+x^2+5x-1)^{-1}(3x^2+2x+5)dx$

$= \int \frac{3x^2+2x+5}{x^3+x^2+5x-1}dx$

Let $y = x^3 + x^2 + 5x - 1$

$\frac{dy}{dx} = 3x^2 + 2x + 5$

$dy = (3x^2+2x+5)dx$

$I = \int \frac{1}{y}dy$

$= \ln|y| + C$

$= \ln|x^3+x^2+5x-1| + C$

OR

$\int (x^3+x^2+5x-1)^{-1}(3x^2+2x+5)dx$

$= \int \frac{3x^2+2x+5}{x^3+x^2+5x-1}dx$

$= \ln|x^3+x^2+5x-1| + C$

The Students ...

Q.2 Evaluate the following integrals by substitution method.

(i) $\int \frac{\ln x dx}{x}$ (ii) $\int \frac{dx}{x\ln x}$ (iii) $\int \frac{\tan x dx}{\ln(\cos x)}$

(iv) $\int \frac{\tan(\ln x)}{x}dx$ (v) $\int (1+e^{2x})^{-\frac{1}{2}}e^{2x}dx$

(vi) $\int \frac{5e^{5\tan x}}{\cos^2 x}dx$ (vii) $\int \frac{e^{2x}}{e^x+e^{-x}}dx$

(viii) $\int e^{\operatorname{cosec} 2x+1}(\operatorname{cosec} 2x\cot 2x)dx$

(ix) $\int 3^x dx$ (x) $\int e^{\sin x+\cos x+1}(\cos x-\sin x)dx$

Solution:

(i) $\int \frac{\ln x dx}{x}$

$= \int \ln x \frac{dx}{x}$

Let $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow dy = \frac{dx}{x}$

$= \int y dy$

$= \frac{y^2}{2} + C$

(ii) $\int \frac{dx}{x\ln x}$

$= \int \left(\frac{1}{\ln x}\right)\frac{dx}{x}$

Let $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow dy = \frac{dx}{x}$

$= \int \frac{1}{y}dy$

$= \ln|y| + C$

$= \ln|\ln x| + C$

(iii) $\int \frac{\tan x dx}{\ln(\cos x)}$

Let $y = \ln(\cos x)$

$\frac{dy}{dx} = \frac{1}{\cos x}\frac{d}{dx}(\cos x)$

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$

$-dy = \tan x\, dx$

The Students ...

$I = \int \frac{1}{\ln(\cos x)}t$

$= -\int \frac{1}{y}dy$

$= -\ln|y| + C$

$= -\ln|\ln(\cos x)$

(iv) $\int \frac{\tan(\ln x)}{x}dx$

$= \int \tan(\ln x)\frac{1}{x}$

Let $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow dy =$

$= \int \tan y\, dy$

$= \ln|\sec y| + C$

$= \ln|\sec(\ln x)|$ -

(v) $\int (1+e^{2x})^{-\frac{1}{2}}$

$= \int (1+e^{2x})^{-\frac{1}{2}}$

Let $y = 1+e^{2x}$

$\frac{dy}{dx} = 2e^{2x} \Rightarrow \frac{dy}{2}$

$= \frac{1}{2}\int y^{-\frac{1}{2}}dy$

$= \frac{1}{2}\left\{\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$

$= \sqrt{y} + C$

$= \sqrt{1+e^{2x}} + C$

(vi) $\int \frac{5e^{5\tan x}}{\cos^2 x}dx$

$= \int e^{5\tan x}(5\sec$

Let $y = 5\tan x$

$\frac{dy}{dx} = 5\sec^2 x \Rightarrow$

$= \int e^y dy$

$I = \int \frac{1}{\ln(\cos x)} \tan x\, dx$

$= -\int \frac{1}{y} dy$

$= -\ln|y| + C$

$= -\ln|\ln(\cos x)| + C$

(iv) $\int \frac{\tan(\ln x)}{x} dx$

$= \int \tan(\ln x) \frac{1}{x} dx$

Let $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow dy = \frac{dx}{x}$

$= \int \tan y\, dy$

$= \ln|\sec y| + C$

$= \ln|\sec(\ln x)| + C$

(v) $\int (1 + e^{2x})^{-\frac{1}{2}} e^{2x} dx$

$= \int (1 + e^{2x})^{-\frac{1}{2}} e^{2x} dx$

Let $y = 1 + e^{2x}$

$\frac{dy}{dx} = 2e^{2x} \Rightarrow \frac{dy}{2} = e^{2x} dx$

$= \frac{1}{2} \int y^{-\frac{1}{2}} dy$

$= \frac{1}{2} \left\{ \frac{y^{\frac{1}{2}}}{\frac{1}{2}} \right\} + C$

$= \sqrt{y} + C$

$= \sqrt{1 + e^{2x}} + C$

(vi) $\int \frac{5e^{5\tan x}}{\cos^2 x} dx$

$= \int e^{5\tan x} (5\sec^2 x) dx$

Let $y = 5\tan x$

$\frac{dy}{dx} = 5\sec^2 x \Rightarrow dy = 5\sec^2 x\, dx$

$= \int e^y dy$

The Stu...

$= e^{y} + C$

$= e^{5\tan x} + C$

(vii) $\int \frac{e^{2x}}{e^{x}+e^{-x}}dx$

$= \int \frac{e^{2x}}{e^{x}+\frac{1}{e^{x}}}dx$

Let $y = e^{x}$

$\frac{dy}{dx} = e^{x} \Rightarrow dy = e^{x}dx$

$\int \frac{e^{x}}{e^{x}+\frac{1}{e^{x}}}e^{x}dx$

$= \int \frac{y}{y+\frac{1}{y}}dy$

$= \int \frac{y}{\frac{y^2+1}{y}}dy$

$= \int \frac{y^2}{y^2+1}dy$

$= \int \frac{y^2+1-1}{y^2+1}dy$

$= \int \left\{\frac{y^2+1}{y^2+1} - \frac{1}{y^2+1}\right\}dy$

$= \int \left\{1 - \frac{1}{y^2+1}\right\}dy$

$= y - \tan^{-1}y + C$

$= e^{x} - \tan^{-1}e^{x} + C$

(viii) $\int e^{\operatorname{cosec}2x+1}(\operatorname{cosec}2x\cot 2x)dx$

Let $y = \operatorname{cosec}2x + 1$

$\frac{dy}{dx} = -2\operatorname{cosec}2x\cot 2x$

$-\frac{dy}{2} = \operatorname{cosec}2x\cot 2x\,dx$

$I = -\frac{1}{2}\int e^{y}dy$

$= -\frac{1}{2}e^{y} + C$

The Studen...

$= -\frac{1}{2}e^{\text{cosec}}$...

(ix) $\int 3^{x}dx$

Let $y = 3^{x}$

$\frac{dy}{dx} = 3^{x}(\ln$ ...

$\frac{dy}{\ln 3} = 3^{x}dx$

$= \frac{1}{\ln 3}\int dy$

$= \frac{1}{\ln 3}y + C$

$= \frac{3^{x}}{\ln 3} + C$

(x) $\int e^{\sin x + c}$...

$= \int e^{\sin x + c}$...

Let $y = \sin x$ ...

$\frac{dy}{dx} = \cos x -$ ...

$dy = (\cos x$ ...

$I = \int e^{y}dy$

$= e^{y} + C$

$= e^{\sin x+\cos x}$...

Q.3 Evaluate t...

(i) $\int \cos^{5}2x\,s$...

(iii) $\int(2+3\,\text{si}$...

(v) $\int \frac{\sin x+\cos x}{\sin x-\cos x}$...

(vii) $\int \frac{\operatorname{cosec}3x}{(a+b\,\text{cose}}$...

(ix) $\int \frac{\sec x}{3\sin x+4\,\text{c}}$...

Solution:

(i) $\int \cos^{5}2x\,\text{si}$...

$= \int(\cos 2x)^{5}$ ...

Let $y = \cos 2x$

$= -\frac{1}{2}e^{\operatorname{cosec} 2x+1} + C$

(ix) $\int 3^x dx$

Let $y = 3^x$

$\frac{dy}{dx} = 3^x(\ln 3)$

$\frac{dy}{\ln 3} = 3^x dx$

$= \frac{1}{\ln 3}\int dy$

$= \frac{1}{\ln 3}y + C$

$= \frac{3^x}{\ln 3} + C$

(x) $\int e^{\sin x+\cos x+1}(\cos x - \sin x)dx$

$= \int e^{\sin x+\cos x+1}(\cos x - \sin x)dx$

Let $y = \sin x + \cos x + 1$

$\frac{dy}{dx} = \cos x - \sin x$

$dy = (\cos x - \sin x)dx$

$I = \int e^y dy$

$= e^y + C$

$= e^{\sin x+\cos x+1} + C$

Q.3 Evaluate the following integrals by substitution method:

(i) $\int \cos^5 2x \sin 2x\, dx$ (ii) $\int \frac{\cot\sqrt{x}}{\sqrt{x}} dx$

(iii) $\int (2 + 3\sin 3x)^6 \cos 3x\, dx$ (iv) $\int \sin(ax + b)\, dx$

(v) $\int \frac{\sin x+\cos x}{\sin x-\cos x} dx$ (vi) $\int e^{2x} \sec^2 e^{2x}\, dx$

(vii) $\int \frac{\operatorname{cosec} 3x \cot 3x}{(a+b\operatorname{cosec} 3x)^2} dx$ (viii) $\int \frac{dx}{(2\cot x+3)\sin^2 x} dx$

(ix) $\int \frac{\sec x}{3\sin x+4\cos x} dx$ (x) $\int \cos(3x - 5)\, dx$

Solution:

(i) $\int \cos^5 2x \sin 2x\, dx$

$= \int (\cos 2x)^5 \sin 2x\, dx$

Let $y = \cos 2x$

$\frac{dy}{dx} = -2\sin 2x$

$-\frac{dy}{2} = \sin 2x\, dx$

$-\frac{1}{2}\int y^5 dx$

$= -\frac{1}{2}\left(\frac{y^6}{6}\right) + C$

$= -\frac{1}{12}(\cos 2x)^6 + C$

$= -\frac{1}{12}\cos^6 2x + C$

(ii) $\int \frac{\cot\sqrt{x}}{\sqrt{x}} dx$

$= \int \cot\sqrt{x}\frac{1}{\sqrt{x}} dx$

Let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow 2dy = \frac{1}{\sqrt{x}} dx$

$\text{I} = 2\int \cot y\, dy$

$= 2\ln|\sin y| + C$

$= 2\ln|\sin\sqrt{x}| + C$

(iii) $\int (2 + 3\sin 3x)^6 \cos 3x\, dx$

Let $y = 2 + 3\sin 3x$

$\frac{dy}{dx} = 0 + 3(3\cos 3x)$

$\frac{dy}{dx} = 9\cos 3x$

$\frac{dy}{9} = \cos 3x\, dx$

$\text{I} = \frac{1}{9}\int y^6 dy$

$= \frac{1}{9}\left\{\frac{y^7}{7}\right\} + C$

$= \frac{1}{63}(2 + 3\sin 3x)^7 + C$

(iv) $\int \sin(ax + b)\, dx$

Let $y = ax + b$

$\frac{dy}{dx} = a \Rightarrow \frac{dy}{a} =$

$\text{I} = \frac{1}{a}\int \sin y\, dy$

$= \frac{1}{a}(-\cos y) +$

$= -\frac{1}{a}\cos(ax +$

(v) $\int \frac{\sin x + \cos x}{\sin x - \cos x}$

Let $y = \sin x -$

$\frac{dy}{dx} = \sin x + c$

$dy = (\sin x +$

$\text{I} = \int \frac{1}{y} dy$

$= \ln|y| + C$

$= \ln|\sin x - c$

(vi) $\int e^{2x}\sec^2$

Let $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x} \Rightarrow$

$\text{I} = \frac{1}{2}\int \sec^2 y$

$= \frac{1}{2}\tan y + C$

$= \frac{1}{2}\tan e^{2x} +$

(vii) $\int \frac{\operatorname{cosec} 3x \cdots}{(a + b\operatorname{cosec}}$

$= \int (a + b\cos$

Let $y = a + b$

$\frac{dy}{dx} = 0 + b(-$

$\frac{dy}{dx} = -3b\operatorname{cose}$

$-\frac{dy}{3b} = \operatorname{cosec} 3$

$\frac{dy}{dx} = a \Rightarrow \frac{dy}{a} = dx$

$I = \frac{1}{a}\int \sin y\, dy$

$= \frac{1}{a}(-\cos y) + C$

$= -\frac{1}{a}\cos(ax+b) + C$

(v) $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$

Let $y = \sin x - \cos x$

$\frac{dy}{dx} = \sin x + \cos x$

$dy = (\sin x + \cos x)dx$

$I = \int \frac{1}{y} dy$

$= \ln|y| + C$

$= \ln|\sin x - \cos x| + C$

(vi) $\int e^{2x} \sec^2 e^{2x}\, dx$

Let $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x} \Rightarrow \frac{dy}{2} = e^{2x} dx$

$I = \frac{1}{2}\int \sec^2 y\, dx$

$= \frac{1}{2}\tan y + C$

$= \frac{1}{2}\tan e^{2x} + C$

(vii) $\int \frac{\operatorname{cosec} 3x \cot 3x}{(a + b\operatorname{cosec} 3x)^2} dx$

$= \int (a + b\operatorname{cosec} 3x)^{-2} \operatorname{cosec} 3x \cot 3x\, dx$

Let $y = a + b\operatorname{cosec} 3x$

$\frac{dy}{dx} = 0 + b(-3\operatorname{cosec} 3x \cot 3x)$

$\frac{dy}{dx} = -3b\operatorname{cosec} 3x \cot 3x$

$-\frac{dy}{3b} = \operatorname{cosec} 3x \cot 3x\, dx$

$$I = -\frac{1}{3b}\int y^{-2}dy$$

$$= -\frac{1}{3b}\left(\frac{y^{-1}}{-1}\right) + C$$

$$= \frac{1}{3by} + C$$

$$= \frac{1}{3b(a + b\,\text{cosec}\,3x)} + C$$

(viii) $\int \frac{1}{(2\cot x+3)\sin^2 x}dx$

$$= \int \frac{1}{(2\cot x + 3)}\left(\frac{1}{\sin^2 x}\right)dx$$

$$= \int \frac{1}{2\cot x + 3}\text{cosec}^2 x\,dx$$

Let $y = 2\cot x + 3$

$$\frac{dy}{dx} = -2\,\text{cosec}^2 x \Rightarrow -\frac{dy}{2} = \text{cosec}^2 x\,dx$$

$$= -\frac{1}{2}\int \frac{1}{y}dy$$

$$= -\frac{1}{2}\ln|y| + C$$

$$= -\frac{1}{2}\ln|2\cot x + 3| + C$$

(ix) $\int \frac{\sec x}{3\sin x+4\cos x}dx$

$$= \int \frac{\sec x}{\cos x\left(\frac{3\sin x}{\cos x} + 4\right)}dx$$

$$= \int \frac{\sec x \sec x}{3\tan x + 4}dx$$

$$= \int \frac{\sec^2 x}{3\tan x + 4}dx$$

Let $y = 3\tan x + 4$

$$\frac{dy}{dx} = 3\sec^2 x \Rightarrow \frac{dy}{3} = \sec^2 x\,dx$$

$$I = \frac{1}{3}\int \frac{1}{y}dy$$

$$= \frac{1}{3}\ln|y| + C$$

$= \frac{1}{3}\ln|3\tan$ ...

(x) $\int \cos(3x$ ...

$I = \int \cos(3x$ ...

Let $y = 3x$ ...

$\frac{dy}{dx} = 3 \Rightarrow$ ...

$I = \frac{1}{3}\int \cos$ ...

$= \frac{1}{3}\sin y +$ ...

$= \frac{1}{3}\sin(3x$ ...

Q.4 Evaluat...

(i) $\int \cos^2 2y$ ...

(iii) $\int \cos^3 x$ ...

(v) $\int \frac{\sin^3 x}{\sqrt{\cos x}}$ ...

(vii) $\int \sin^3 x$ ...

(ix) $\int \cos 3x$ ...

(xi) $\int \tan^2 x$ ...

(xiii) $\int \tan^7$ ...

(xv) $\int \tan^5$ ...

(xvii) $\int \sec^4$ ...

(xix) $\int \frac{\text{cosec}^4}{\sqrt{\cot}}$ ...

Solution:

(i) $\int \cos^2 2y$ ...

Let $t = 2y$

$\frac{dt}{dy} = 2 \Rightarrow$ ...

$I = \frac{1}{2}\int \cos$ ...

$= \frac{1}{2}\int\left(\frac{1+}{}\right.$ ...

$= \int\left(\frac{1}{}\right.$ ...

$= \frac{1}{3}\ln|3\tan x + 4| + C$

(x) $\int \cos(3x-5)\,dx$

$I = \int \cos(3x-5)\,dx$

Let $y = 3x - 5$

$\frac{dy}{dx} = 3 \Rightarrow \frac{dy}{3} = dx$

$I = \frac{1}{3}\int \cos y\,dx$

$= \frac{1}{3}\sin y + C$

$= \frac{1}{3}\sin(3x-5) + C$

Q.4 Evaluate the following integrals by substitution method:

(i) $\int \cos^2 2y\,dy$

(ii) $\int \sin^3(3x+5)\,dx$

(iii) $\int \cos^3 x\sqrt{\sin x}\,dx$

(iv) $\int \sin^4 x \cos^5 x\,dx$

(v) $\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$

(vi) $\int \cos^5 x \sin^7 x\,dx$

(vii) $\int \sin^3 x \cos^3 x\,dx$

(viii) $\int \sin 2x \cos 4x\,dx$

(ix) $\int \cos 3x \cos 5x\,dx$

(x) $\int \sin 3x \cos 7x\,dx$

(xi) $\int \tan^2 x\,dx$

(xii) $\int \cot^4 x\,dx$

(xiii) $\int \tan^7 x\,dx$

(xiv) $\int \sec^4 2x\,dx$

(xv) $\int \tan^5 3x \sec^3 3x\,dx$

(xvi) $\int \operatorname{cosec}^4 3x\,dx$

(xvii) $\int \sec^4 x\sqrt{\tan x}\,dx$

(xviii) $\int \cot 2x \operatorname{cosec}^4 2x\,dx$

(xix) $\int \frac{\operatorname{cosec}^4 x}{\sqrt{\cot x}}\,dx$

(xviii) $\int \sqrt{1+\cos x}\,dx$

Solution:

(i) $\int \cos^2 2y\,dy$

Let $t = 2y$

$\frac{dt}{dy} = 2 \Rightarrow \frac{dt}{2} = dy$

$I = \frac{1}{2}\int \cos^2 t\,dt$

$= \frac{1}{2}\int \left(\frac{1+\cos 2t}{2}\right)dt$

$= \int \left(\frac{1}{4} + \frac{1}{4}\cos 2t\right)dt$

The Students' ...

$$=\frac{1}{4}t+\frac{1}{4}\left(\frac{\sin 2t}{2}\right)+C$$

$$=\frac{1}{4}(2y)+\frac{1}{8}\sin 2(2y)+C$$

$$=\frac{y}{2}+\frac{1}{8}\sin 4y+C$$

(ii) $\int \sin^3(3x+5)\,dx$

Let $y=3x+5$

$$\frac{dy}{dx}=3\Rightarrow\frac{dy}{3}=dx$$

$$\mathrm{I}=\frac{1}{3}\int \sin^3 y\,dy$$

$$=\frac{1}{3}\int \sin y\sin^2 y\,dy$$

$$=\frac{1}{3}\int \sin y\,(1-\cos^2 y)dy$$

$$=\frac{1}{3}\int (\sin y-\cos^2 y\sin y)dy$$

$$=-\frac{1}{3}\cos y+\left(\frac{1}{3}\right)\frac{\cos^3 y}{3}+C$$

$$=-\frac{1}{3}\cos(3x+5)+\frac{1}{9}\cos^3(3x+5)+C$$

(iii) $\int \cos^3 x\sqrt{\sin x}\,dx$

$$=\int \cos^2 x\cos x\,(\sin x)^{\frac{1}{2}}dx$$

$$=\int \cos x\,(\sin x)^{\frac{1}{2}}(1-\sin^2 x)dx$$

$$=\int \left\{(\sin x)^{\frac{1}{2}}-(\sin x)^{\frac{5}{2}}\right\}\cos x\,dx$$

Let $y=\sin x$

$$\frac{dy}{dx}=\cos x\Rightarrow dy=\cos x\,dx$$

$$=\int \left\{y^{\frac{1}{2}}-y^{\frac{5}{2}}\right\}dy$$

$$=\frac{y^{\frac{3}{2}}}{\frac{3}{2}}-\frac{y^{\frac{7}{2}}}{\frac{7}{2}}+C$$

$$=\frac{2}{3}(\sin x)^{\frac{3}{2}}-\frac{2}{7}(\sin x)^{\frac{7}{2}}+C$$

The Students'

(iv) $\int \sin^4 x$ co...

$=\int \sin^4 x$ cos...

$=\int \sin^4 x$ (co...

$=\int \sin^4 x\,(1$...

$=\int \sin^4 x\,(1$...

$=\int (\sin^4 x-$...

Let $y=\sin x$

$\frac{dy}{dx}=\cos x\Rightarrow$...

$=\int (y^4-2y$...

$=\frac{y^5}{5}-\frac{2y^7}{7}$...

$=\frac{1}{5}(\sin x)^5$...

(v) $\int \frac{\sin^3 x}{\sqrt{\cos x}}dx$

$=\int \frac{\sin^2 x\sin...}{(\cos x)...}$

$=\int (\cos x)^{-...}$

$=\int (\cos x)^{-...}$

$=\int \{(\cos x)$...

Let $y=\cos x$

$\frac{dy}{dx}=-\sin x$

$=-\int \{y^{-\frac{1}{2}}$...

$=-\frac{y^{\frac{1}{2}}}{\frac{1}{2}}+\frac{y^{\frac{5}{2}}}{\frac{5}{2}}$...

(iv) $\int \sin^4 x \cos^5 x \, dx$

$= \int \sin^4 x \cos^4 x \cos x \, dx$

$= \int \sin^4 x (\cos^2 x)^2 \cos x \, dx$

$= \int \sin^4 x (1 - \sin^2 x)^2 \cos x \, dx$

$= \int \sin^4 x (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$

$= \int (\sin^4 x - 2\sin^6 x + \sin^8 x) \cos x \, dx$

Let $y = \sin x$

$\frac{dy}{dx} = \cos x \Rightarrow dy = \cos x \, dx$

$= \int (y^4 - 2y^6 + y^8) dy$

$= \frac{y^5}{5} - \frac{2y^7}{7} + \frac{y^9}{9} + C$

$= \frac{1}{5}(\sin x)^5 - \frac{2}{7}(\sin x)^7 + \frac{1}{9}(\sin x)^9 + C$

(v) $\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

$= \int \frac{\sin^2 x \sin x}{(\cos x)^{\frac{1}{2}}} dx$

$= \int (\cos x)^{-\frac{1}{2}} \sin^2 x \sin x \, dx$

$= \int (\cos x)^{-\frac{1}{2}} (1 - \cos^2 x) \sin x \, dx$

$= \int \left\{(\cos x)^{-\frac{1}{2}} - (\cos x)^{\frac{3}{2}}\right\} \sin x \, dx$

Let $y = \cos x$

$\frac{dy}{dx} = -\sin x \Rightarrow -dy = \sin x \, dx$

$= -\int \left\{y^{-\frac{1}{2}} - y^{\frac{3}{2}}\right\} dy$

$= -\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + \frac{y^{\frac{5}{2}}}{\frac{5}{2}} + C$

$= -2\sqrt{\cos x} + \frac{2}{5}(\cos x)^{\frac{5}{2}} + C$

(vi) $\int \cos^5 x \sin^7 x \, dx$

$= \int \cos^4 x \cos x \sin^7 x \, dx$

$= \int \sin^7 x (\cos^2 x)^2 \cos x \, dx$

$= \int \sin^7 x (1 - \sin^2 x)^2 \cos x \, dx$

$= \int \sin^7 x (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$

$= \int (\sin^7 x - 2\sin^9 x + \sin^{11} x) \cos x \, dx$

Let $y = \sin x$

$\frac{dy}{dx} = \cos x \Rightarrow dy = \cos x \, dx$

$= \int (y^7 - 2y^9 + y^{11}) dy$

$= \frac{y^8}{8} - \frac{2y^{10}}{10} + \frac{y^{12}}{12} + C$

$= \frac{\sin^8 x}{8} - \frac{\sin^{10} x}{5} + \frac{\sin^{12} x}{12} + C$

(vii) $\int \sin^3 x \cos^3 x \, dx$

$= \int \sin^3 x \cos^2 x \cos x \, dx$

$= \int \sin^3 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^3 x - \sin^5 x) \cos x \, dx$

Let $y = \sin x$

$\frac{dy}{dx} = \cos x \Rightarrow dy = \cos x \, dx$

$= \int (y^3 - y^5) dy$

$= \frac{y^4}{4} - \frac{y^6}{6} + C$

$= \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + C$

(viii) $\int \sin 2x \cos 4x \, dx$

$\sin 2x \cos 4x = \frac{1}{2}\{$

$= \frac{1}{2}\sin(6x) + \frac{1}{2}\text{si}$

$= \frac{1}{2}\sin 6x - \frac{1}{2}\sin$

$I = \int \left\{\frac{1}{2}\sin 6x -\right.$

$= \frac{1}{2}\left(-\frac{\cos 6x}{6}\right) -$

$= -\frac{1}{12}\cos 6x +$

(ix) $\int \cos 3x \cos 5$

$\cos 5x \cos 3x = \frac{1}{2}$

$= \frac{1}{2}\cos 8x + \frac{1}{2}\text{co}$

$I = \int \left\{\frac{1}{2}\cos 8x +\right.$

$= \frac{1}{2}\left(\frac{\sin 8x}{8}\right) + \frac{1}{2}$

$= \frac{1}{16}\sin 8x + \frac{1}{4}\text{s}$

(x) $\int \sin 3x \cos 7x$

$\sin 3x \cos 7x = \frac{1}{2}$

$= \frac{1}{2}\{\sin 10x + \text{si}$

$= \frac{1}{2}\sin 10x - \frac{1}{2}\text{s}$

$I = \int \left\{\frac{1}{2}\sin 10x\right.$

$= \frac{1}{2}\left(-\frac{\cos 10x}{10}\right)$

$= -\frac{1}{20}\cos 10x +$

(xi) $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) d$

$= \tan x - x + C$

$\sin 2x \cos 4x = \frac{1}{2}\{\sin(2x+4x)+\sin(2x-4x)\}$

$= \frac{1}{2}\sin(6x) + \frac{1}{2}\sin(-2x)$

$= \frac{1}{2}\sin 6x - \frac{1}{2}\sin 2x$

$I = \int \left\{\frac{1}{2}\sin 6x - \frac{1}{2}\sin 2x\right\} dx$

$= \frac{1}{2}\left(-\frac{\cos 6x}{6}\right) - \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + C$

$= -\frac{1}{12}\cos 6x + \frac{1}{4}\cos 2x + C$

(ix) $\int \cos 3x \cos 5x \, dx$

$\cos 5x \cos 3x = \frac{1}{2}\{\cos(5x+3x)+\cos(5x-3x)\}$

$= \frac{1}{2}\cos 8x + \frac{1}{2}\cos 2x$

$I = \int \left\{\frac{1}{2}\cos 8x + \frac{1}{2}\cos 2x\right\} dx$

$= \frac{1}{2}\left(\frac{\sin 8x}{8}\right) + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$

$= \frac{1}{16}\sin 8x + \frac{1}{4}\sin 2x + C$

(x) $\int \sin 3x \cos 7x \, dx$

$\sin 3x \cos 7x = \frac{1}{2}\{\sin(3x+7x)+\sin(3x-7x)\}$

$= \frac{1}{2}\{\sin 10x + \sin(-4x)\}$

$= \frac{1}{2}\sin 10x - \frac{1}{2}\sin 4x$

$I = \int \left\{\frac{1}{2}\sin 10x - \frac{1}{2}\sin 4x\right\} dx$

$= \frac{1}{2}\left(-\frac{\cos 10x}{10}\right) - \frac{1}{2}\left(-\frac{\cos 4x}{4}\right) + C$

$= -\frac{1}{20}\cos 10x + \frac{1}{8}\cos 4x + C$

(xi) $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) dx$

$= \tan x - x + C$

(xii) $\int \cot^4 x\, dx$

$$= \int \cot^2 x \cot^2 x\, dx$$

$$= \int \cot^2 x (\operatorname{cosec}^2 x - 1) dx$$

$$= \int (\cot^2 x \operatorname{cosec}^2 x - \cot^2 x) dx$$

$$= \int \left(\cot^2 x \operatorname{cosec}^2 x - (\operatorname{cosec}^2 x - 1)\right) dx$$

$$= \int (\cot^2 x \operatorname{cosec}^2 x - \operatorname{cosec}^2 x + 1) dx$$

$$= \int (\cot^2 x \operatorname{cosec}^2 x) dx + \int (-\operatorname{cosec}^2 x + 1) dx$$

$$= \int (\cot^2 x \operatorname{cosec}^2 x) dx - (-\cot x) + x + C$$

$$= \int (\cot^2 x \operatorname{cosec}^2 x) dx + \cot x + x$$

Let $y = \cot x$

$$\frac{dy}{dx} = -\operatorname{cosec}^2 x \Rightarrow -dy = \operatorname{cosec}^2 x\, dx$$

$$= -\int y^2 dx + \cot x + x$$

$$= -\frac{y^3}{3} + \cot x + x + C$$

$$= -\frac{1}{3}\cot^3 x + \cot x + x + C$$

(xiii) $\int \tan^7 x\, dx$

$$= \int \tan^7 x\, dx$$

$$= \int \tan^6 x \tan x\, dx$$

$$= \int (\tan^2 x)^3 \tan x\, dx$$

$$= \int (\sec^2 x - 1)^3 \tan x\, dx$$

$$= \int (\sec^6 x - 3\sec^4 x + 3\sec^2 x - 1) \tan x\, dx$$

$$= \int \{(\sec^6 x - 3\sec^4 x + 3\sec^2 x) \tan x - \tan x\} dx$$

$= \int (\sec^5 x - 3\, \mathrm{se}$

$= \int (\sec^5 x - 3\, \mathrm{se}$

Let $y = \sec x$

$\frac{dy}{dx} = \sec x \tan x$

$= \int (y^5 - 3y^3 +$

$= \left\{\frac{y^6}{6} - 3\left(\frac{y^4}{4}\right)\right.$

$= \frac{1}{6}(\sec x)^6 - \frac{3}{4}($

(xiv) $\int \sec^4 2x\, dx$

$= \int \sec^2 2x \sec^2$

$= \int \sec^2 2x (1 +$

$= \int (\sec^2 2x + \mathrm{ta}$

$= \int \sec^2 2x\, dx +$

$= \frac{\tan 2x}{2} + \int \tan$

Let $y = \tan 2x$

$\frac{dy}{dx} = 2\sec^2 2x \Rightarrow$

$(1) \Rightarrow \frac{\tan 2x}{2} + \frac{1}{2}$

$= \frac{\tan 2x}{2} + \frac{1}{2}\left(\frac{y^3}{3}\right.$

$= \frac{\tan 2x}{2} + \frac{1}{6}\tan^3$

(xv) $\int \tan^5 3x \sec^3$

$= \int \tan^4 3x \tan 3x$

$= \int (\tan^2 3x)^2 \tan$

$= \int (\sec^5 x - 3\sec^3 x + 3\sec x)\sec x \tan x\, dx - \int \tan x\, dx$

$= \int (\sec^5 x - 3\sec^3 x + 3\sec x)\sec x \tan x\, dx - \ln(\sec x)$

Let $y = \sec x$

$\frac{dy}{dx} = \sec x \tan x \Rightarrow dy = \sec x \tan x\, dx$

$= \int (y^5 - 3y^3 + 3y)dy - \ln(\sec x)$

$= \left\{\frac{y^6}{6} - 3\left(\frac{y^4}{4}\right) + 3\left(\frac{y^2}{2}\right)\right\} - \ln(\sec x) + C$

$= \frac{1}{6}(\sec x)^6 - \frac{3}{4}(\sec x)^4 + \frac{3}{2}(\sec x)^2 - \ln(\sec x) + C$

(xiv) $\int \sec^4 2x\, dx$

$= \int \sec^2 2x \sec^2 2x\, dx$

$= \int \sec^2 2x\,(1 + \tan^2 2x)dx$

$= \int (\sec^2 2x + \tan^2 2x \sec^2 2x)dx$

$= \int \sec^2 2x\, dx + \int \tan^2 2x \sec^2 2x\, dx$

$= \frac{\tan 2x}{2} + \int \tan^2 2x \sec^2 2x\, dx \rightarrow (1)$

Let $y = \tan 2x$

$\frac{dy}{dx} = 2\sec^2 2x \Rightarrow \frac{dy}{2} = \sec^2 2x\, dx$

$(1) \Rightarrow \frac{\tan 2x}{2} + \frac{1}{2}\int y^2 dy$

$= \frac{\tan 2x}{2} + \frac{1}{2}\left(\frac{y^3}{3}\right) + C$

$= \frac{\tan 2x}{2} + \frac{1}{6}\tan^3 2x + C$

(xv) $\int \tan^5 3x \sec^3 3x\, dx$

$= \int \tan^4 3x \tan 3x \sec^3 3x\, dx$

$= \int (\tan^2 3x)^2 \tan 3x \sec^3 3x\, dx$

$= \int \tan 3x \sec^3 3x\,(\sec^2 3x - 1)^2 dx$

$= \int \tan 3x \sec^2 3x \sec 3x\,(\sec^4 3x - 2\sec^2 3x + 1)dx$

$= \int \tan 3x \sec 3x\,(\sec^6 3x - 2\sec^4 3x + \sec^2 3x)dx \to (1)$

Let $y = \sec 3x$

$\frac{dy}{dx} = 3\sec 3x \tan 3x \Rightarrow \frac{dy}{3} = \sec 3x \tan 3x\, dx$

$(1) \Rightarrow \frac{1}{3}\int (y^6 - 2y^4 + y^2)dx$

$= \frac{1}{3}\left(\frac{y^7}{7} - \frac{2y^5}{5} + \frac{y^3}{3}\right) + C$

$= \frac{1}{21}(\sec 3x)^7 - \frac{2}{15}(\sec 3x)^5 + \frac{1}{9}(\sec 3x)^3 + C$

(xvi) $\int \operatorname{cosec}^4 3x\, dx$

$= \int \operatorname{cosec}^2 3x \operatorname{cosec}^2 3x\, dx$

$= \int \operatorname{cosec}^2 3x\,(1 + \cot^2 3x)dx$

$= \int (\operatorname{cosec}^2 3x + \cot^2 3x \operatorname{cosec}^2 3x)dx$

$= \int \operatorname{cosec}^2 3x\, dx + \int \cot^2 3x \operatorname{cosec}^2 3x\, dx$

$= -\frac{\cot 3x}{2} + \int \cot^2 3x \operatorname{cosec}^2 3x\, dx \to (1)$

Let $y = \cot 3x$

$\frac{dy}{dx} = -3\operatorname{cosec}^2 3x \Rightarrow -\frac{dy}{3} = \operatorname{cosec}^2 3x\, dx$

$(1) \Rightarrow -\frac{\cot 3x}{2} - \int y^2 dx$

$= -\frac{\cot 3x}{2} - \frac{y^3}{3} + C$

$= -\frac{\cot 3x}{2} - \frac{1}{3}(\cot 3x)^3 + C$

(xvii) $\int \sec^4 x \sqrt{\tan x}\, dx$

$= \int \sec^2 x \sec^2 x\,(\tan x)^{\frac{1}{2}} dx$

$= \int \sec^2$

$= \int \{(\tan$

Let $y = \tan$

$\frac{dy}{dx} = \sec$

$(1) \Rightarrow \int \{$

$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y}{}$

$= \frac{2}{3}(\tan x$

(xviii) $\int \cot$

$= \int \cot 2x$

$= \int \cot 2x$

$= \int (\cot 2x$

Let $y = \cot$

$\frac{dy}{dx} = -2\,\mathrm{c}$

$(1) \Rightarrow -\frac{1}{2}\int$

$= -\frac{1}{2}\left(\frac{y^2}{2}\right.$

$= -\frac{1}{4}y^2 -$

$= -\frac{1}{4}\cot^2 2$

(xix) $\int \frac{\operatorname{cosec}^4}{\sqrt{\cot x}}$

$= \int \frac{\operatorname{cosec}^2 x}{}$

$= \int (\cot x)^{\frac{1}{2}}($

$= \int \sec^2 x\,(\tan x)^{\frac{1}{2}}(1 + \tan^2 x)dx$

$= \int \left\{(\tan x)^{\frac{1}{2}} + (\tan x)^{\frac{5}{2}}\right\}\sec^2 x\,dx \to (1)$

Let $y = \tan x$

$\frac{dy}{dx} = \sec^2 x \Rightarrow dy = \sec^2 x\,dx$

$(1) \Rightarrow \int \left\{y^{\frac{1}{2}} + y^{\frac{5}{2}}\right\} dy \to (1)$

$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}} + C$

$= \frac{2}{3}(\tan x)^{\frac{3}{2}} + \frac{2}{7}(\tan x)^{\frac{7}{2}} + C$

(xviii) $\int \cot 2x\,\mathrm{cosec}^4\,2x\,dx$

$= \int \cot 2x\,\mathrm{cosec}^2\,2x\,\mathrm{cosec}^2\,2x\,dx$

$= \int \cot 2x\,\mathrm{cosec}^2\,2x\,(1 + \cot^2 2x)dx$

$= \int (\cot 2x + \cot^3 2x)\,\mathrm{cosec}^2\,2x\,dx \to (1)$

Let $y = \cot 2x$

$\frac{dy}{dx} = -2\,\mathrm{cosec}^2\,2x \Rightarrow -\frac{dy}{2} = \mathrm{cosec}^2\,2x\,dx$

$(1) \Rightarrow -\frac{1}{2}\int (y + y^3)dy$

$= -\frac{1}{2}\left(\frac{y^2}{2} + \frac{y^4}{4}\right) + C$

$= -\frac{1}{4}y^2 - \frac{1}{8}y^4 + C$

$= -\frac{1}{4}\cot^2 2x - \frac{1}{8}\cot^4 2x + C$

(xix) $\int \frac{\mathrm{cosec}^4\,x}{\sqrt{\cot x}}dx$

$= \int \frac{\mathrm{cosec}^2\,x\,\mathrm{cosec}^2\,x}{(\cot x)^{\frac{1}{2}}}dx$

$= \int (\cot x)^{\frac{1}{2}}(1 + \cot^2 x)\,\mathrm{cosec}^2\,x\,dx$

The Students' Compa...

$= \int \left\{(\cot x)^{\frac{1}{2}} + (\cot x)^{\frac{5}{2}}\right\} \operatorname{cosec}^2 x \, dx \rightarrow (1)$

Let $y = \cot x$

$\frac{dy}{dx} = -\operatorname{cosec}^2 x \Rightarrow -dy = \operatorname{cosec}^2 x \, dx$

$(1) \Rightarrow \int \left\{y^{\frac{1}{2}} + y^{\frac{5}{2}}\right\} dy$

$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}} + C$

$= \frac{2}{3}(\cot x)^{\frac{3}{2}} + \frac{2}{7}(\cot x)^{\frac{7}{2}} + C$

(xviii) $\int \sqrt{1 + \cos x} \, dx$

$\sqrt{1 + \cos x} = 2 \cos \frac{x}{2}$

$I = 2 \int \cos \frac{x}{2} \, dx \rightarrow (1)$

Let $y = \frac{x}{2}$

$\frac{dy}{dx} = \frac{1}{2} \Rightarrow 2dy = dx$

$(1) \Rightarrow 2 \int \cos y \, dy$

$= 2 \sin y + C$

$= 2 \sin \frac{x}{2} + C$

EXERCISE 6.3

Integrand	Substitution
$a^2 - x^2$	$x = a \sin \theta$
$x^2 + a^2$	$x = a \tan \theta$
$x^2 - a^2$	$x = a \sec \theta$

Evaluate by using trigonometric substitution:

Q.1 $\int \frac{x^3 dx}{\sqrt{9 - x^2}}$

Solution:

$I = \int \frac{x^3 dx}{\sqrt{9 - x^2}} = \int \frac{x^3 dx}{\sqrt{3^2 - x^2}}$

Let $x = 3 \sin \theta \rightarrow (1)$

The Students' C...

$dx = 3 \cos \theta \, d\theta$

$I = \int \frac{(3 \sin \theta)^3 (...}{\sqrt{3^2 - (...}}$

$= 3 \int \frac{27 \sin^3 \theta \, c...}{\sqrt{9 - 9 s...}}$

$= 81 \int \frac{\sin^3 \theta \cos...}{\sqrt{9(1 - s...}}$

$= 81 \int \frac{\sin^3 \theta \cos...}{3\sqrt{\cos...}}$

$= 27 \int \frac{\sin^3 \theta \cos...}{\cos \theta}$

$= 27 \int \sin^3 \theta \, d\theta$

$= 27 \int \sin \theta \sin^2 ...$

$= 27 \int \sin \theta (1 - ...$

$= 27 \int (\sin \theta - co...$

$= 27 \left\{-\cos \theta + \frac{c...}{...}\right.$

$(1) \Rightarrow \sin \theta = \frac{x}{3} = ...$

$\cos \theta = \frac{B}{H} = \frac{\sqrt{H^2 - ...}}{H}$

$(2) \Rightarrow 27 \left\{-\frac{\sqrt{9 - ...}}{3}\right.$

$= -9\sqrt{9 - x^2} + (9 ...$

Q.2 $\int \frac{6dx}{9 - x^2}$

Solution:

$I = \int \frac{6dx}{9 - x^2} = \int \frac{6...}{3^2 ...}$

Let $x = 3 \sin \theta \rightarrow (1)$

$dx = 3 \cos \theta \, d\theta$

$I = 6 \int \frac{3 \cos \theta \, d\theta}{9 - 9 \sin^2 \theta}$

$\boxed{dx = 3\cos\theta\, d\theta}$

$$I = \int \frac{(3\sin\theta)^3(3\cos\theta\, d\theta)}{\sqrt{3^2 - (3\sin\theta)^2}}$$

$$= 3\int \frac{27\sin^3\theta\cos\theta\, d\theta}{\sqrt{9 - 9\sin^2\theta}}$$

$$= 81\int \frac{\sin^3\theta\cos\theta\, d\theta}{\sqrt{9(1 - \sin^2\theta)}}$$

$$= 81\int \frac{\sin^3\theta\cos\theta\, d\theta}{3\sqrt{\cos^2\theta}}$$

$$= 27\int \frac{\sin^3\theta\cos\theta\, d\theta}{\cos\theta}$$

$$= 27\int \sin^3\theta\, d\theta$$

$$= 27\int \sin\theta\sin^2\theta\, d\theta$$

$$= 27\int \sin\theta\,(1 - \cos^2\theta)d\theta$$

$$= 27\int (\sin\theta - \cos^2\theta\sin\theta)d\theta$$

$$= 27\left\{-\cos\theta + \frac{\cos^3\theta}{3}\right\} + C \rightarrow (2)$$

$$(1) \Rightarrow \sin\theta = \frac{x}{3} = \frac{P}{H}$$

$$\cos\theta = \frac{B}{H} = \frac{\sqrt{H^2 - P^2}}{H} = \frac{\sqrt{9 - x^2}}{3}$$

$$(2) \Rightarrow 27\left\{-\frac{\sqrt{9 - x^2}}{3} + \frac{1}{3(3^3)}(9 - x^2)^{\frac{3}{2}}\right\} + C$$

$$= -9\sqrt{9 - x^2} + (9 - x^2)^{\frac{3}{2}} + C$$

Q.2 $\int \frac{6dx}{9 - x^2}$

Solution:

$$I = \int \frac{6dx}{9 - x^2} = \int \frac{6dx}{3^2 - x^2}$$

Let $x = 3\sin\theta \rightarrow (1)$

$\boxed{dx = 3\cos\theta\, d\theta}$

$$I = 6\int \frac{3\cos\theta\, d\theta}{9 - 9\sin^2\theta}$$

$$I = 18\int \frac{\cos\theta\, d\theta}{9(1-\sin^2\theta)}$$

$$I = 2\int \frac{\cos\theta\, d\theta}{\cos^2\theta}$$

$$I = 2\int \frac{d\theta}{\cos\theta}$$

$$I = 2\int \sec\theta\, d\theta$$

$$I = 2\ln(\sec\theta + \tan\theta) + C \rightarrow (2)$$

$$(1) \Rightarrow \sin\theta = \frac{x}{3} = \frac{\mathrm{P}}{\mathrm{H}}$$

$$\cos\theta = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\sqrt{\mathrm{H}^2 - P^2}}{\mathrm{H}} = \frac{\sqrt{9-x^2}}{3}$$

$$\tan\theta = \frac{\mathrm{P}}{\mathrm{B}} = \frac{x}{\sqrt{\mathrm{H}^2 - P^2}} = \frac{x}{\sqrt{9-x^2}}$$

$$(2) \Rightarrow 2\ln\left(\frac{3}{\sqrt{9-x^2}} + \frac{x}{\sqrt{9-x^2}}\right) + C$$

$$= 2\ln\left(\frac{3+x}{\sqrt{9-x^2}}\right) + C$$

$$= 2\ln\left\{\frac{3+x}{\sqrt{(3-x)(3+x)}}\right\} + C$$

$$= 2\ln\left\{\frac{\sqrt{3+x}}{\sqrt{3-x}}\right\} + C$$

$$= 2\ln\left\{\sqrt{\frac{3+x}{3-x}}\right\} + C$$

$$= 2\ln\left(\frac{3+x}{3-x}\right)^{\frac{1}{2}} + C$$

$$= \frac{2}{2}\ln\left(\frac{3+x}{3-x}\right) + C$$

$$= \ln\left(\frac{3+x}{3-x}\right) + C$$

Q.3 $\int x^2\sqrt{9-x^2}\,dx$

Solution:

$$I = \int x^2\sqrt{9-x^2}\,dx = \int x^2\sqrt{3^2-x^2}\,dx$$

Let $x = 3\sin\theta \rightarrow (1)$

$dx = 3\cos\theta$ ...

$I = \int 9\sin^2\theta$ ...

$= 27\int \sin^2\theta$ ...

$= 27(3)\int \sin$ ...

$= 81\int \sin^2\theta$ ...

$= 81\int \sin^2\theta$ ...

$= \frac{81}{4}\int 4\sin^2$ ...

$= \frac{81}{4}\int (2\cos$ ...

$= \frac{81}{4}\int (\sin 2\theta$ ...

$= \frac{81}{4}\int \sin^2 2\theta$ ...

$= \frac{81}{4}\int \left(\frac{1-c}{2}\right.$ ...

$= \frac{81}{8}\int (1-co$ ...

$= \frac{81}{8}\left(\theta - \frac{\sin}{4}\right.$ ...

$= \frac{81}{8}\left(\theta - \frac{2\sin}{}\right.$ ...

$= \frac{81}{8}\left\{\theta - \frac{(2\sin}{}\right.$ ...

$= \frac{81}{8}\{\theta - \sin\theta$ ...

$= \frac{81}{8}\theta - \frac{81}{8}\sin$ ...

$(1) \Rightarrow \sin\theta = \frac{x}{3}$ ...

$\sin\theta = \frac{x}{3} = \frac{\mathrm{P}}{\mathrm{H}}$

$\cos\theta = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\sqrt{\mathrm{H}^2}}{}$ ...

$\boxed{dx = 3\cos\theta\, d\theta}$

$$I = \int 9\sin^2\theta\sqrt{9 - 9\sin^2\theta}\,(3\cos\theta\, d\theta)$$

$$= 27\int \sin^2\theta\sqrt{9(1 - \sin^2\theta)}\cos\theta\, d\theta$$

$$= 27(3)\int \sin^2\theta\sqrt{\cos^2\theta}\cos\theta\, d\theta$$

$$= 81\int \sin^2\theta\cos\theta\cos\theta\, d\theta$$

$$= 81\int \sin^2\theta\cos^2\theta\, d\theta$$

$$= \frac{81}{4}\int 4\sin^2\theta\cos^2\theta\, d\theta$$

$$= \frac{81}{4}\int (2\cos\theta\sin\theta)^2 d\theta$$

$$= \frac{81}{4}\int (\sin 2\theta)^2 d\theta$$

$$= \frac{81}{4}\int \sin^2 2\theta\, d\theta$$

$$= \frac{81}{4}\int \left(\frac{1 - \cos 4\theta}{2}\right) d\theta$$

$$= \frac{81}{8}\int (1 - \cos 4\theta) d\theta$$

$$= \frac{81}{8}\left(\theta - \frac{\sin 4\theta}{4}\right) + C$$

$$= \frac{81}{8}\left(\theta - \frac{2\sin 2\theta\cos 2\theta}{4}\right) + C$$

$$= \frac{81}{8}\left\{\theta - \frac{(2\sin\theta\cos\theta)(1 - 2\sin^2\theta)}{2}\right\} + C$$

$$= \frac{81}{8}\{\theta - \sin\theta\cos\theta + 2\sin^3\theta\cos\theta\} + C$$

$$= \frac{81}{8}\theta - \frac{81}{8}\sin\theta\cos\theta + \frac{81}{4}\sin^3\theta\cos\theta + C \to (2)$$

$$(1) \Rightarrow \sin\theta = \frac{x}{3} \Rightarrow \boxed{\theta = \sin^{-1}\frac{x}{3}}$$

$$\sin\theta = \frac{x}{3} = \frac{P}{H}$$

$$\cos\theta = \frac{B}{H} = \frac{\sqrt{H^2 - P^2}}{H} = \frac{\sqrt{9 - x^2}}{3}$$

$$(2) \Rightarrow \frac{81}{8}\sin^{-1}\frac{x}{3} - \frac{81}{8}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + \frac{81}{4}\left(\frac{x}{3}\right)^3\left(\frac{\sqrt{9-x^2}}{3}\right) + c$$

$$= \frac{81}{8}\sin^{-1}\frac{x}{3} - \frac{9}{8}x\sqrt{9-x^2} + \frac{1}{4}x^3\sqrt{9-x^2} + C$$

Q.4 $\int \frac{5}{25x^2+9}dx$

Solution:

$$I = \int \frac{5}{25x^2+9}dx = \frac{5}{25}\int \frac{1}{x^2+\frac{9}{25}}dx$$

$$= \frac{1}{5}\int \frac{1}{x^2+\left(\frac{3}{5}\right)^2}dx$$

Let $x = \frac{3}{5}\tan\theta \rightarrow (1)$

$$\boxed{dx = \frac{3}{5}\sec^2\theta\, d\theta}$$

$$I = \frac{1}{5}\int \frac{\frac{3}{5}\sec^2\theta\, d\theta}{\frac{9}{25}\tan^2\theta + \frac{9}{25}}$$

$$= \frac{3}{25}\int \frac{\sec^2\theta\, d\theta}{\frac{9}{25}(\tan^2\theta + 1)}$$

$$= \frac{1}{3}\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \frac{1}{3}\int d\theta$$

$$= \frac{1}{3}\theta + C \rightarrow (2)$$

$$(1) \Rightarrow \tan\theta = \frac{5x}{3} \Rightarrow \boxed{\theta = \tan^{-1}\frac{5x}{3}}$$

$$(2) \Rightarrow \frac{1}{3}\tan^{-1}\frac{5x}{3} + C$$

Q.5 $\int \frac{dx}{(4+x^2)^{\frac{3}{2}}}$

Solution:

$$I = \int \frac{dx}{(4+x^2)^{\frac{3}{2}}} = \int \frac{dx}{(2^2+x^2)^{\frac{3}{2}}}$$

Let $x = 2\tan\theta \rightarrow (1)$

$dx = 2\sec^2\theta\, d$

$I = \int \frac{2\sec^2 ...}{(4+4\tan ...}$

$= \int \frac{2\sec^2\theta ...}{\{4(1+\tan ...}$

$= \int \frac{2\sec^2\theta ...}{(2^2)^{\frac{3}{2}}(\sec ...}$

$= \int \frac{2\sec^2\theta\, d ...}{8\sec^3\theta}$

$= \frac{1}{4}\int \frac{d\theta}{\sec\theta}$

$= \frac{1}{4}\int \cos\theta\, d ...$

$= \frac{1}{4}\sin\theta + C$

$(1) \Rightarrow \tan\theta = ...$

$\sin\theta = \frac{P}{H} = ...$

$(2) \Rightarrow \frac{1}{4}\frac{x}{\sqrt{x^2 + ...}}$

Q.6 $\int \frac{dx}{\sqrt{16+4x^2}}$

Solution:

$I = \int \frac{dx}{\sqrt{16+ ...}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{4+x^2 ...}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{2^2+x ...}}$

Let $x = 2\tan ...$

$dx = 2\sec^2\theta ...$

$I = \frac{1}{2}\int \frac{2\sec ...}{\sqrt{4+ ...}}$

$= \int \frac{\sec^2\theta ...}{\sqrt{4(1+\tan ...}}$

x^2 $) + c$

$dx = 2\sec^2\theta\, d\theta$

$$I = \int \frac{2\sec^2\theta\, d\theta}{(4+4\tan^2\theta)^{\frac{3}{2}}}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{\{4(1+\tan^2\theta)\}^{\frac{3}{2}}}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{(2^2)^{\frac{3}{2}}(\sec^2\theta)^{\frac{3}{2}}}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{8\sec^3\theta}$$

$$= \frac{1}{4}\int \frac{d\theta}{\sec\theta}$$

$$= \frac{1}{4}\int \cos\theta\, d\theta$$

$$= \frac{1}{4}\sin\theta + C \rightarrow (2)$$

$$(1) \Rightarrow \tan\theta = \frac{x}{2} = \frac{P}{B}$$

$$\sin\theta = \frac{P}{H} = \frac{P}{\sqrt{P^2+B^2}} = \frac{x}{\sqrt{x^2+4}}$$

$$(2) \Rightarrow \frac{1}{4}\frac{x}{\sqrt{x^2+4}} + C$$

Q.6 $\int \frac{dx}{\sqrt{16+4x^2}}$

Solution:

$$I = \int \frac{dx}{\sqrt{16+4x^2}} = \int \frac{dx}{\sqrt{4(4+x^2)}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{4+x^2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{2^2+x^2}}$$

Let $x = 2\tan\theta \rightarrow (1)$

$dx = 2\sec^2\theta\, d\theta$

$$I = \frac{1}{2}\int \frac{2\sec^2\theta\, d\theta}{\sqrt{4+4\tan^2\theta}}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sqrt{4(1+\tan^2\theta)}}$$

$$= \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\sqrt{\sec^2\theta}}$$

$$= \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \frac{1}{2}\int \sec\theta\, d\theta$$

$$= \frac{1}{2}\ln(\sec\theta + \tan\theta) + C \rightarrow (2)$$

$$(1) \Rightarrow \tan\theta = \frac{x}{2} = \frac{\text{P}}{\text{B}}$$

$$\cos\theta = \frac{\text{B}}{\text{H}} = \frac{\text{B}}{\sqrt{P^2+B^2}} = \frac{2}{\sqrt{x^2+4}}$$

$$(1) \Rightarrow \frac{1}{2}\ln\left(\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right) + C$$

$$= \frac{1}{2}\ln\left(\frac{\sqrt{x^2+4}+x}{2}\right) + C$$

$$= \frac{1}{2}\left\{\ln\left(\sqrt{x^2+4}+x\right) - \ln(2)\right\} + C$$

$$= \frac{1}{2}\ln\left(\sqrt{x^2+4}+x\right) - \frac{1}{2}\ln(2) + C$$

$$= \frac{1}{2}\ln\left(x+\sqrt{x^2+4}\right) + C$$

Q.7 $\int x^3\sqrt{9x^2-36}dx$

Solution:

$$\text{I} = \int x^3\sqrt{9x^2-36}dx$$

$$= \int x^3\sqrt{9(x^2-4)}dx$$

$$= 3\int x^3\sqrt{x^2-2^2}dx$$

Let $x = 2\sec\theta \rightarrow (1)$

$dx = 2\sec\theta\tan\theta\, d\theta$

$$\text{I} = 3\int 8\sec^3\theta\sqrt{4\sec^2\theta-4}(2\sec\theta\tan\theta)d\theta$$

$$= 48\int \sec^4\theta\sqrt{4(\sec^2\theta-1)}\tan\theta\, d\theta$$

$$= 48(2)\int \sec^4\theta\sqrt{\tan^2\theta}\tan\theta\, d\theta$$

$= 96\int \sec^4$

$= 96\int \sec^4$

$= 96\int \sec^2$

$= 96\int \sec^2$

$= 96\int (\sec$

$= 96\left(\frac{\tan^3}{3}\right.$

$= 32\tan^3\theta$

$(1) \Rightarrow \sec\theta$

$\cos\theta = \frac{2}{x} =$

$\tan\theta = \frac{P}{B} =$

$(2) \Rightarrow 32\{$

$= 32\left\{\frac{(x^2 -}{8}\right.$

$= 4(x^2-4$

Q.8 $\int \frac{dx}{\sqrt{a^2+x^2}}$

Solution:

$\text{I} = \int \frac{dx}{\sqrt{a^2+}}$

Let $x = a\tan$

$dx = a\sec^2$

$\text{I} = \int \frac{a\sec}{\sqrt{a^2+}}$

$= 96\int \sec^4\theta \tan\theta \tan\theta\, d\theta$

$= 96\int \sec^4\theta \tan^2\theta\, d\theta$

$= 96\int \sec^2\theta \sec^2\theta \tan^2\theta\, d\theta$

$= 96\int \sec^2\theta \tan^2\theta\,(1+\tan^2\theta)d\theta$

$= 96\int (\sec^2\theta \tan^2\theta + \sec^2\theta\tan^4\theta)d\theta$

$= 96\left(\frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5}\right) + C$

$= 32\tan^3\theta + \frac{96}{5}\tan^5\theta + C \rightarrow (2)$

$(1) \Rightarrow \sec\theta = \frac{x}{2}$

$\cos\theta = \frac{2}{x} = \frac{B}{H}$

$\tan\theta = \frac{P}{B} = \frac{\sqrt{H^2 - B^2}}{B} = \frac{\sqrt{x^2-4}}{2}$

$(2) \Rightarrow 32\left\{\frac{(x^2-4)^{\frac{1}{2}}}{2}\right\}^3 + \frac{96}{5}\left\{\frac{(x^2-4)^{\frac{1}{2}}}{2}\right\}^5 + C$

$= 32\left\{\frac{(x^2-4)^{\frac{3}{2}}}{8}\right\} + \frac{96}{5}\left\{\frac{(x^2-4)^{\frac{5}{2}}}{32}\right\} + C$

$= 4(x^2-4)^{\frac{3}{2}} + \frac{3}{5}(x^2-4)^{\frac{5}{2}} + C$

Q.8 $\int \frac{dx}{\sqrt{a^2+x^2}}$

Solution:

$I = \int \frac{dx}{\sqrt{a^2+x^2}}$

Let $x = a\tan\theta \rightarrow (1)$

$dx = a\sec^2\theta\, d\theta$

$I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2 + a^2\tan^2\theta}}$

The Students' Companion ...

$$= \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2(1+\tan^2\theta)}}$$

$$= \int \frac{a\sec^2\theta\, d\theta}{a\sqrt{\sec^2\theta}}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln(\sec\theta + \tan\theta) + C \rightarrow (2)$$

$$(1) \Rightarrow \tan\theta = \frac{x}{a} = \frac{\mathrm{P}}{\mathrm{B}}$$

$$\cos\theta = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{B}}{\sqrt{P^2+B^2}} = \frac{a}{\sqrt{x^2+a^2}}$$

$$(1) \Rightarrow \frac{1}{2}\ln\left(\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right) + C$$

$$= \frac{1}{2}\ln\left(\frac{\sqrt{x^2+a^2}+x}{a}\right) + C$$

$$= \frac{1}{2}\left\{\ln\left(\sqrt{x^2+a^2}+x\right) - \ln(a)\right\} + C$$

$$= \frac{1}{2}\ln\left(\sqrt{x^2+a^2}+x\right) - \frac{1}{2}\ln(a) + C$$

$$= \frac{1}{2}\ln\left(\sqrt{x^2+a^2}+x\right) + C$$

Q.9 $\int \frac{dx}{(16-x^2)^{\frac{5}{2}}}$

Solution:

$$I = \int \frac{dx}{(16-x^2)^{\frac{5}{2}}} = \int \frac{dx}{(4^2-x^2)^{\frac{5}{2}}}$$

Let $x = 4\sin\theta \rightarrow (1)$

$dx = 4\cos\theta\, d\theta$

$$I = \int \frac{4\cos\theta\, d\theta}{(16-16\sin^2\theta)^{\frac{5}{2}}}$$

$$= \int \frac{4\cos\theta\, d\theta}{\{16(1-\sin^2\theta)\}^{\frac{5}{2}}}$$

$$= \int \frac{4\cos\theta\, d\theta}{\{4^2(\cos^2\theta)\}^{\frac{5}{2}}}$$

The Studen...

$= \frac{4}{4^5}\int \frac{\cos ...}{\cos ...}$

$= \frac{1}{4^4}\int \frac{d\theta}{\cos^4 ...}$

$= \frac{1}{256}\int \sec ...$

$= \frac{1}{256}\int \sec ...$

$= \frac{1}{256}\int \sec ...$

$= \frac{1}{256}\int (\sec ...$

$= \frac{1}{256}\left(\tan ...\right.$

$(1) \Rightarrow \sin\theta ...$

$\tan\theta = \frac{\mathrm{P}}{\mathrm{B}} = ...$

$(2) \Rightarrow \frac{1}{256}[...$

Q.10 $\int \frac{x^5 dx}{\sqrt{x^2-9}}$

Solution:

$I = \int \frac{x^5 dx}{\sqrt{x^2 - ...}}$

Let $x = 3\sec ...$

$dx = 3\sec\theta ...$

$I = \int \frac{3^5\sec^5 ...}{...}$

$= 3^6\int \frac{\sec^6\theta ...}{\sqrt{9(\sec ...}}$

$= \frac{3^6}{3}\int \frac{\sec^6\theta ...}{\sqrt{\tan ...}}$

$= 3^5\int \frac{\sec^6\theta ...}{\tan ...}$

$= 243\int \sec^6 ...$

$$=\frac{4}{4^5}\int\frac{\cos\theta\,d\theta}{\cos^5\theta}$$

$$=\frac{1}{4^4}\int\frac{d\theta}{\cos^4\theta}$$

$$=\frac{1}{256}\int\sec^4\theta\,d\theta$$

$$=\frac{1}{256}\int\sec^2\theta\sec^2\theta\,d\theta$$

$$=\frac{1}{256}\int\sec^2\theta\,(1+\tan^2\theta)d\theta$$

$$=\frac{1}{256}\int(\sec^2\theta+\tan^2\theta\sec^2\theta)d\theta$$

$$=\frac{1}{256}\left(\tan\theta+\frac{\tan^3\theta}{3}\right)+C\rightarrow(2)$$

$$(1)\Rightarrow\sin\theta=\frac{x}{4}=\frac{\mathrm{P}}{\mathrm{H}}$$

$$\tan\theta=\frac{\mathrm{P}}{\mathrm{B}}=\frac{\mathrm{P}}{\sqrt{\mathrm{H}^2-P^2}}=\frac{x}{\sqrt{16-x^2}}$$

$$(2)\Rightarrow\frac{1}{256}\left[\frac{x}{\sqrt{16-x^2}}+\frac{x^3}{3(16-x^2)^{\frac{3}{2}}}\right]+C$$

Q.10 $\int\frac{x^5dx}{\sqrt{x^2-9}}$

Solution:

$$\mathrm{I}=\int\frac{x^5dx}{\sqrt{x^2-9}}=\int\frac{x^5dx}{\sqrt{x^2-3^2}}$$

Let $x=3\sec\theta\rightarrow(1)$

$dx=3\sec\theta\tan\theta\,d\theta$

$$\mathrm{I}=\int\frac{3^5\sec^5\theta\,(3\sec\theta\tan\theta)d\theta}{\sqrt{9\sec^2\theta-9}}$$

$$=3^6\int\frac{\sec^6\theta\tan\theta\,d\theta}{\sqrt{9(\sec^2\theta-1)}}$$

$$=\frac{3^6}{3}\int\frac{\sec^6\theta\tan\theta\,d\theta}{\sqrt{\tan^2\theta}}$$

$$=3^5\int\frac{\sec^6\theta\tan\theta\,d\theta}{\tan\theta}$$

$$=243\int\sec^6\theta\,d\theta$$

$$= 243\int \sec^2\theta \sec^4\theta\, d\theta$$

$$= 243\int \sec^2\theta (\sec^2\theta)^2 d\theta$$

$$= 243\int \sec^2\theta (1+\tan^2\theta)^2 d\theta$$

$$= 243\int \sec^2\theta (1+2\tan^2\theta+\tan^4\theta) d\theta$$

$$= 243\int (\sec^2\theta + 2\tan^2\theta\sec^2\theta + \tan^4\theta\sec^2\theta) d\theta$$

$$= 243\left(\tan\theta + \frac{2\tan^3\theta}{3} + \frac{\tan^5\theta}{5}\right) + C \rightarrow (2)$$

$(1) \Rightarrow \sec\theta = \dfrac{x}{3}$

$\cos\theta = \dfrac{3}{x} = \dfrac{B}{H}$

$\tan\theta = \dfrac{P}{B} = \dfrac{\sqrt{H^2 - B^2}}{B} = \dfrac{\sqrt{x^2-9}}{3}$

$$(2) \Rightarrow 243\left\{\frac{\sqrt{x^2-9}}{3} + \frac{2}{3}\left(\frac{\sqrt{x^2-9}}{3}\right)^3 + \frac{1}{5}\left(\frac{\sqrt{x^2-9}}{3}\right)^5\right\} + C$$

$$= 243\left\{\frac{\sqrt{x^2-9}}{3} + \left(\frac{2}{3}\right)\frac{(x^2-9)^{\frac{3}{2}}}{27} + \left(\frac{1}{5}\right)\frac{(x^2-9)^{\frac{5}{2}}}{243}\right\} + C$$

$$= 81\sqrt{x^2-9} + 6(x^2-9)^{\frac{3}{2}} + \frac{1}{5}(x^2-9)^{\frac{5}{2}} + C$$

Q.11 $\int \frac{dx}{x^2+4x+5}$

Solution:

$$I = \int \frac{dx}{x^2+4x+5}$$

$x^2 + 4x + 5 = x^2 + 4x + 4 + 1$

$= x^2 + 2(x)(2) + 2^2 + 1$

$= (x+2)^2 + 1$

$$I = \int \frac{dx}{(x+2)^2+1}$$

Let $x + 2 = \tan\theta \rightarrow (1)$

$dx = \sec^2\theta\, d\theta$

$I = \int \frac{\sec^2}{\tan^2}$

$= \int \frac{\sec^2\theta}{\sec^2}$

$= \int d\theta$

$= \theta + C \rightarrow$

$(1) \Rightarrow \tan$

$\theta = \tan^{-1}$

$(1) \Rightarrow \tan$

Q.12 $\int \frac{}{\sqrt{5+}}$

Solution:

$I = \int \frac{}{\sqrt{5 +}}$

$5 + 4x - x$

$= -\{x^2 -$

$= -\{x^2 -$

$= -\{x^2 -$

$= -\{(x -$

$= 9 - (x -$

$I = \int \frac{}{\sqrt{9 -}}$

Let $x - 2 =$

$dx = 3\cos$

$I = \int \frac{3\,c}{\sqrt{9 -}}$

$= 3\int \frac{c}{\sqrt{9(1}}$

$= \frac{3}{3}\int \frac{\cos\theta}{\sqrt{\cos}}$

$= \int \frac{\cos\theta\, d}{\cos\theta}$

$= \int d\theta$

$= \theta + C \rightarrow$

$$I = \int \frac{\sec^2\theta\, d\theta}{\tan^2\theta + 1}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \int d\theta$$

$= \theta + C \rightarrow (2)$

$(1) \Rightarrow \tan\theta = x + 2$

$\theta = \tan^{-1}(x + 2)$

$(1) \Rightarrow \tan^{-1}(x + 2) + C$

Q.12 $\int \frac{dx}{\sqrt{5+4x-x^2}}$

Solution:

$$I = \int \frac{dx}{\sqrt{5 + 4x - x^2}}$$

$5 + 4x - x^2$

$= -\{x^2 - 4x - 5\}$

$= -\{x^2 - 2(x)(2) + 4 - 4 - 5\}$

$= -\{x^2 - 2(x)(2) + 2^2 - 9\}$

$= -\{(x - 2)^2 - 9\}$

$= 9 - (x - 2)^2$

$$I = \int \frac{dx}{\sqrt{9 - (x - 2)^2}}$$

Let $x - 2 = 3\sin\theta \rightarrow (1)$

$dx = 3\cos\theta\, d\theta$

$$I = \int \frac{3\cos\theta\, d\theta}{\sqrt{9 - 9\sin^2\theta}}$$

$$= 3\int \frac{\cos\theta\, d\theta}{\sqrt{9(1 - \sin^2\theta)}}$$

$$= \frac{3}{3}\int \frac{\cos\theta\, d\theta}{\sqrt{\cos^2\theta}}$$

$$= \int \frac{\cos\theta\, d\theta}{\cos\theta}$$

$$= \int d\theta$$

$= \theta + C \rightarrow (2)$

The Students

$(1) \Rightarrow \sin\theta = \frac{x-2}{3}$

$\theta = \sin^{-1}\left(\frac{x-2}{3}\right)$

$(2) \Rightarrow \sin^{-1}\left(\frac{x-2}{3}\right) + C$

Q.13 $\int \frac{dx}{\sqrt{9x-x^2}}$

Solution:

$I = \int \frac{dx}{\sqrt{9x-x^2}}$

$9x - x^2 = -\{x^2 - 9x\}$

$= -\left\{x^2 - 2(x)\left(\frac{9}{2}\right) + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right\}$

$= -\left\{\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right\}$

$= \frac{81}{4} - \left(x - \frac{9}{2}\right)^2$

$I = \int \frac{dx}{\sqrt{\frac{81}{4} - \left(x-\frac{9}{2}\right)^2}}$

$= \int \frac{dx}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x-\frac{9}{2}\right)^2}}$

$x - \frac{9}{2} = \frac{9}{2}\sin\theta \rightarrow (1)$

$\boxed{dx = \frac{9}{2}\cos\theta\, d\theta}$

$I = \int \frac{\frac{9}{2}\cos\theta\, d\theta}{\sqrt{\frac{81}{4} - \frac{81}{4}\sin^2\theta}}$

$= \int \frac{\frac{9}{2}\cos\theta\, d\theta}{\sqrt{\frac{81}{4}(1-\sin^2\theta)}}$

$= \int \frac{\frac{9}{2}\cos\theta\, d\theta}{\frac{9}{2}\sqrt{\cos^2\theta}}$

The Student

$= \int \frac{\cos\theta\, d\ldots}{\cos\theta}$

$= \int d\theta$

$= \theta + C \rightarrow ($

$(1) \Rightarrow \frac{9}{2}\sin\ldots$

$\frac{9}{2}\sin\theta = \frac{2x\ldots}{\ldots}$

$\sin\theta = \frac{2x - \ldots}{9}$

$\theta = \sin^{-1}\left(\ldots\right.$

$(2) \Rightarrow \sin^{-1}$

Q.14 $\int \frac{\ldots}{(x+1)\sqrt{\ldots}}$

Solution:

$I = \int \frac{\ldots}{(x+1\ldots}$

$x^2 + 2x - 15\ldots$

$= (x+1)^2 - \ldots$

$I = \int \frac{\ldots}{(x+1)\ldots}$

$x + 1 = 4\sec\ldots$

$\boxed{dx = 4\sec\theta\ldots}$

$I = \int \frac{4\sec\ldots}{4\sec\theta\ldots}$

$= \int \frac{\tan\ldots}{\sqrt{16(\sec\ldots}}$

$= \frac{1}{4}\int \frac{\tan\theta\, d\ldots}{\sqrt{\tan^2\ldots}}$

$= \frac{1}{4}\int \frac{\tan\theta\, d\ldots}{\tan\theta}$

$= \frac{1}{4}\int d\theta$

$= \frac{1}{4}\theta + C \rightarrow (\ldots$

$= \int \frac{\cos\theta \, d\theta}{\cos\theta}$

$= \int d\theta$

$= \theta + C \rightarrow (2)$

$(1) \Rightarrow \frac{9}{2}\sin\theta = x - \frac{9}{2}$

$\frac{9}{2}\sin\theta = \frac{2x-9}{2}$

$\sin\theta = \frac{2x-9}{9}$

$\theta = \sin^{-1}\left(\frac{2x-9}{9}\right)$

$(2) \Rightarrow \sin^{-1}\left(\frac{2x-9}{9}\right) + C$

Q.14 $\int \frac{dx}{(x+1)\sqrt{x^2+2x-15}}$

Solution:

$I = \int \frac{dx}{(x+1)\sqrt{x^2+2x-15}}$

$x^2 + 2x - 15 = (x^2 + 2x + 1) - 16$

$= (x+1)^2 - 16$

$I = \int \frac{dx}{(x+1)\sqrt{(x+1)^2 - 4^2}}$

$x + 1 = 4\sec\theta \rightarrow (1)$

$\boxed{dx = 4\sec\theta\tan\theta \, d\theta}$

$I = \int \frac{4\sec\theta\tan\theta \, d\theta}{4\sec\theta\sqrt{16\sec^2\theta - 16}}$

$= \int \frac{\tan\theta \, d\theta}{\sqrt{16(\sec^2\theta - 1)}}$

$= \frac{1}{4}\int \frac{\tan\theta \, d\theta}{\sqrt{\tan^2\theta}}$

$= \frac{1}{4}\int \frac{\tan\theta \, d\theta}{\tan\theta}$

$= \frac{1}{4}\int d\theta$

$= \frac{1}{4}\theta + C \rightarrow (2)$

$(1) \Rightarrow \sec\theta = \dfrac{x+1}{4}$

$\theta = \sec^{-1}\left(\dfrac{x+1}{4}\right)$

Q.15 $\int \frac{dx}{(x-4)\sqrt{x^2-8x-9}}$

Solution:

$$I = \int \frac{dx}{(x-4)\sqrt{x^2-8x-9}}$$

$x^2 - 8x - 9 = x^2 - 2(x)(4) + (4)^2 - 16 - 9$

$= (x-4)^2 - 25$

$$I = \int \frac{dx}{(x-4)\sqrt{(x-4)^2 - 5^2}}$$

Let $x - 4 = 5\sec\theta \rightarrow (1)$

$\boxed{dx = 5\sec\theta\tan\theta\, d\theta}$

$$I = \int \frac{5\sec\theta\tan\theta\, d\theta}{5\sec\theta\sqrt{25\sec^2\theta - 25}}$$

$$= \int \frac{\tan\theta\, d\theta}{\sqrt{25(\sec^2\theta - 1)}}$$

$$= \frac{1}{5}\int \frac{\tan\theta\, d\theta}{\sqrt{\tan^2\theta}}$$

$$= \frac{1}{5}\int \frac{\tan\theta\, d\theta}{\tan\theta}$$

$$= \frac{1}{5}\int \frac{\tan\theta\, d\theta}{\tan\theta}$$

$$= \frac{1}{5}\int d\theta$$

$$= \frac{1}{5}\theta + C \rightarrow (2)$$

$(1) \Rightarrow \sec\theta = \dfrac{x-4}{5}$

$\theta = \sec^{-1}\left(\dfrac{x-4}{5}\right)$

$(2) \Rightarrow \sec^{-1}\left(\dfrac{x-4}{5}\right) + C$

Q.16 $\int \frac{(2x-5)dx}{\sqrt{8x-x^2}}$

Solution:

$I = \int \frac{(2x-5}{\sqrt{8x-}}$

$8x - x^2 = -\{$

$= -\{x^2 - 2(x$

$= -\{(x-4)^2$

$= 16 - (x-4$

$I = \int \frac{(2x-}{\sqrt{4^2-(}}$

Let $x - 4 = 4$

$\boxed{dx = 4\cos\theta}$

$(1) \Rightarrow x = 4 +$

$2x = 8 + 8\sin$

$2x - 5 = 8 -$

$\boxed{2x - 5 = 3 +}$

$I = \int \frac{(3 + 8\sin}{\sqrt{16}}$

$= 4\int \frac{(3+8\sin}{\sqrt{16(}}$

$= \frac{4}{4}\int \frac{(3+8\sin}{\sqrt{}}$

$= \int \frac{(3+8\sin}{\cos}$

$= \int (3 + 8\sin$

$= 3\theta - 8\cos\theta$

$(1) \Rightarrow \sin\theta =$

$\sin\theta = \frac{x-4}{4} =$

$\cos\theta = \frac{B}{H} = \frac{\sqrt{H}}{}$

$\cos\theta = \frac{\sqrt{16-(}}{}$

$\boxed{\cos\theta = \frac{\sqrt{x^2-}}{4}}$

$$I = \int \frac{(2x-5)dx}{\sqrt{8x - x^2}}$$

$8x - x^2 = -\{x^2 - 8x\}$

$= -\{x^2 - 2(x)(4) + (4)^2 - (4)^2\}$

$= -\{(x-4)^2 - 16\}$

$= 16 - (x-4)^2$

$$I = \int \frac{(2x-5)dx}{\sqrt{4^2 - (x-4)^2}}$$

Let $x - 4 = 4\sin\theta \rightarrow (1)$

$\boxed{dx = 4\cos\theta\, d\theta}$

$(1) \Rightarrow x = 4 + 4\sin\theta$

$2x = 8 + 8\sin\theta$

$2x - 5 = 8 - 5 + 8\sin\theta$

$\boxed{2x - 5 = 3 + 8\sin\theta}$

$$I = \int \frac{(3 + 8\sin\theta)(4\cos\theta)d\theta}{\sqrt{16 - 16\sin^2\theta}}$$

$$= 4\int \frac{(3 + 8\sin\theta)\cos\theta\, d\theta}{\sqrt{16(1 - \sin^2\theta)}}$$

$$= \frac{4}{4}\int \frac{(3 + 8\sin\theta)\cos\theta\, d\theta}{\sqrt{\cos^2\theta}}$$

$$= \int \frac{(3 + 8\sin\theta)\cos\theta\, d\theta}{\cos\theta}$$

$$= \int (3 + 8\sin\theta)d\theta$$

$= 3\theta - 8\cos\theta + C \rightarrow (2)$

$(1) \Rightarrow \sin\theta = \dfrac{x-4}{4} \Rightarrow \theta = \sin^{-1}\left(\dfrac{x-4}{4}\right)$

$\sin\theta = \dfrac{x-4}{4} = \dfrac{P}{H}$

$\cos\theta = \dfrac{B}{H} = \dfrac{\sqrt{H^2 - P^2}}{H} = \dfrac{\sqrt{4^2 - (x-4)^2}}{4}$

$\cos\theta = \dfrac{\sqrt{16 - (x^2 - 8x + 16)}}{4}$

$\boxed{\cos\theta = \dfrac{\sqrt{x^2 - 8x}}{4}}$

$$(2) \Rightarrow 3\sin^{-1}\left(\frac{x-4}{4}\right) - 8\left(\frac{\sqrt{x^2-8x}}{4}\right) + C$$

$$= 3\sin^{-1}\left(\frac{x-4}{4}\right) - 2\sqrt{x^2-8x} + C$$

Q.17 $\int \frac{(x+3)dx}{x^2+2x+5}$

Solution:

$$I = \int \frac{(x+3)dx}{x^2+2x+5}$$

$$x^2+2x+5 = (x^2+2x+1)+4$$

$$= (x+1)^2+4$$

$$I = \int \frac{(x+3)dx}{(x+1)^2+2^2}$$

Let $x+1 = 2\tan\theta \rightarrow (1)$

$$\boxed{dx = 2\sec^2\theta\, d\theta}$$

$$(1) \Rightarrow x+1 = 2\tan\theta$$

$$x+1+2 = 2+2\tan\theta$$

$$\boxed{x+3 = 2+2\tan\theta}$$

$$I = \int \frac{(2+2\tan\theta)(2\sec^2\theta)d\theta}{4\tan^2\theta+4}$$

$$= 2\int \frac{2(1+\tan\theta)\sec^2\theta\, d\theta}{4(\tan^2\theta+1)}$$

$$= \int \frac{(1+\tan\theta)\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \int (1+\tan\theta)d\theta$$

$$= \theta + \ln(\sec\theta) + C \rightarrow (2)$$

$$(1) \Rightarrow 2\tan\theta = x+1$$

$$\tan\theta = \frac{x+1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{x+1}{2}\right)$$

$$\tan\theta = \frac{x+1}{2} = \frac{P}{B}$$

$$\cos\theta = \frac{B}{H} = \frac{B}{\sqrt{P^2+B^2}} = \frac{2}{\sqrt{(x+1)^2+2^2}}$$

$$\cos\theta = \frac{2}{\sqrt{x^2+2x+5}}$$

$\sec\theta = \frac{\sqrt{x^2 ...}}{...}$

$(2) \Rightarrow \tan^{-1}(...$

$= \tan^{-1}\left(\frac{x+...}{2}\right. ...$

$= \tan^{-1}\left(\frac{x+...}{2}\right. ...$

Q.18 $\int \frac{(3x+9)...}{x^2+4x+...}$

Solution:

$I = \int \frac{(3x+ ...}{x^2+4 ...}$

$= \int \frac{3x ...}{x^2+2(...}$

$= \int \frac{3x+9}{(x+2)^2 ...}$

$= \int \frac{3x+6- ...}{(x+ ...}$

$= \int \frac{3(x+2) ...}{(x+2 ...}$

$= \int \left\{\frac{3(x+2 ...}{(x+2) ...}\right.$

$= \int \left\{\frac{3}{x+2} + ...\right.$

$= 3\ln(x+2) ...$

$= 3\ln(x+2) ...$

$= 3\ln(x+2) ...$

Q.19 $\int \frac{(4x+9) ...}{\sqrt{2x^2+8x ...}}$

Solution:

$I = \int \frac{(4x+ ...}{\sqrt{2x^2+ ...}}$

$2x^2+8x-10 ...$

$= 2\{(x)^2+2(...$

$$\sec\theta = \frac{\sqrt{x^2+2x+5}}{2}$$

$$(2) \Rightarrow \tan^{-1}\left(\frac{x+1}{2}\right) + \ln\left\{\frac{\sqrt{x^2+2x+5}}{2}\right\} + C$$

$$= \tan^{-1}\left(\frac{x+1}{2}\right) + \ln\left(\sqrt{x^2+2x+5}\right) - \ln(2) + C$$

$$= \tan^{-1}\left(\frac{x+1}{2}\right) + \ln\left(\sqrt{x^2+2x+5}\right) + C.$$

Q.18 $\int \frac{(3x+9)dx}{x^2+4x+4}$

Solution:

$$I = \int \frac{(3x+9)dx}{x^2+4x+4}$$

$$= \int \frac{3x+9}{x^2+2(x)(2)+2^2}dx$$

$$= \int \frac{3x+9}{(x+2)^2}dx$$

$$= \int \frac{3x+6-6+9}{(x+2)^2}dx$$

$$= \int \frac{3(x+2)+3}{(x+2)^2}dx$$

$$= \int \left\{\frac{3(x+2)}{(x+2)^2} + \frac{3}{(x+2)^2}\right\}dx$$

$$= \int \left\{\frac{3}{x+2} + 3(x+2)^{-2}\right\}dx$$

$$= 3\ln(x+2) + \frac{3(x+2)^{-2+1}}{-2+1} + C$$

$$= 3\ln(x+2) - 3(x+2)^{-1} + C$$

$$= 3\ln(x+2) - \frac{3}{(x+2)} + C$$

Q.19 $\int \frac{(4x+9)dx}{\sqrt{2x^2+8x-10}}$

Solution:

$$I = \int \frac{(4x+9)dx}{\sqrt{2x^2+8x-10}}$$

$2x^2 + 8x - 10 = 2\{x^2 + 4x - 5\}$

$= 2\{(x)^2 + 2(x)(2) + 2^2 - 4 - 5\}$

$= 2\{(x+2)^2 - 9\}$

$$I = \int \frac{(4x+9)dx}{\sqrt{2\{(x+2)^2 - 9\}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{(4x+9)dx}{\sqrt{(x+2)^2 - 3^2}}$$

Let $x + 2 = 3\sec\theta \rightarrow (1)$

$\boxed{dx = 3\sec\theta\tan\theta\, d\theta}$

$(1) \Rightarrow x + 2 = 3\sec\theta$

$4(x+2) = 12\sec\theta$

$4x + 8 = 12\sec\theta$

$4x + 8 + 1 = 1 + 12\sec\theta$

$\boxed{4x + 9 = 1 + 12\sec\theta}$

$$I = \frac{1}{\sqrt{2}} \int \frac{(1 + 12\sec\theta)(3\sec\theta\tan\theta\, d\theta)}{\sqrt{9\sec^2\theta - 9}}$$

$$= \frac{3\sqrt{2}}{2} \int \frac{(1 + 12\sec\theta)\sec\theta\tan\theta}{\sqrt{9(\sec^2\theta - 1)}} d\theta$$

$$= \frac{3\sqrt{2}}{2} \int \frac{(1 + 12\sec\theta)\sec\theta\tan\theta}{3\sqrt{\tan^2\theta}} d\theta$$

$$= \frac{\sqrt{2}}{2} \int \frac{(1 + 12\sec\theta)\sec\theta\tan\theta}{\tan\theta} d\theta$$

$$= \frac{\sqrt{2}}{2} \int (1 + 12\sec\theta)\sec\theta\, d\theta$$

$$= \frac{\sqrt{2}}{2} \int (\sec\theta + 12\sec^2\theta) d\theta$$

$$= \frac{\sqrt{2}}{2} \{\ln(\sec\theta + \tan\theta) + 12\tan\theta\} + C$$

$$= \frac{\sqrt{2}}{2} \ln(\sec\theta + \tan\theta) + 6\sqrt{2}\tan\theta + C \rightarrow (2)$$

$(1) \Rightarrow 3\sec\theta = x + 2$

$$\sec\theta = \frac{x+2}{3}$$

$$\cos\theta = \frac{3}{x+2} = \frac{B}{H}$$

$$\tan\theta = \frac{P}{B} = \frac{\sqrt{H^2 - B^2}}{B} = \frac{\sqrt{(x+2)^2 - 9}}{3}$$

$\tan\theta = \sqrt{x^2 \ldots}$

$(2) \Rightarrow \frac{\sqrt{2}}{2}\ln(\ldots$

$= \frac{\sqrt{2}}{2}\ln\left(\frac{x + \ldots}{3}\right.$

$= \frac{\sqrt{2}}{2}\ln\left(\frac{x + \ldots}{}\right.$

$= \frac{\sqrt{2}}{2}\{\ln(x \ldots$

$= \frac{\sqrt{2}}{2}\ln(x + \ldots$

$= \frac{\sqrt{2}}{2}\ln(x + \ldots$

Q.20 $\int \frac{(2x-5)\ldots}{\sqrt{5+4x\ldots}}$

Solution:

$I = \int \frac{(2x - \ldots}{\sqrt{5 + \ldots}}$

$5 + 4x - x^2$

$= -\{x^2 - 2(\ldots$

$= -\{(x-2)\ldots$

$= -\{(x-2)\ldots$

$= 9 - (x - \ldots$

$I = \int \frac{(2x \ldots}{\sqrt{3^2 - \ldots}}$

$x - 2 = 3\sin\ldots$

$\boxed{dx = 3\cos\ldots}$

$(1) \Rightarrow x - 2 \ldots$

$2(x-2) = 2\ldots$

$2x - 4 = 6\sin\ldots$

$2x - 4 - 1 = \ldots$

$\boxed{2x - 5 = -1\ldots}$

$I = \int \frac{(-1 + \ldots}{}$

$\tan\theta = \frac{\sqrt{x^2+4x-5}}{3}$

$(2) \Rightarrow \frac{\sqrt{2}}{2}\ln(\sec\theta + \tan\theta) + 6\sqrt{2}\tan\theta + C$

$= \frac{\sqrt{2}}{2}\ln\left(\frac{x+2}{3} + \frac{\sqrt{x^2+4x-5}}{3}\right) + \frac{6\sqrt{2}\sqrt{x^2+4x-5}}{3} + C$

$= \frac{\sqrt{2}}{2}\ln\left(\frac{x+2+\sqrt{x^2+4x-5}}{3}\right) + 2\sqrt{2}\sqrt{x^2+4x-5} + C$

$= \frac{\sqrt{2}}{2}\left\{\ln\left(x+2+\sqrt{x^2+4x-5}\right) - \ln 3\right\} + 2\sqrt{2}\sqrt{x^2+4x-5} + C$

$= \frac{\sqrt{2}}{2}\ln\left(x+2+\sqrt{x^2+4x-5}\right) - \frac{\sqrt{2}}{2}\ln 3 + 2\sqrt{2}\sqrt{x^2+4x-5} + C$

$= \frac{\sqrt{2}}{2}\ln\left(x+2+\sqrt{x^2+4x-5}\right) + 2\sqrt{2(x^2+4x-5)} + C$

Q.20 $\int \frac{(2x-5)dx}{\sqrt{5+4x-x^2}}$

Solution:

$I = \int \frac{(2x-5)dx}{\sqrt{5+4x-x^2}}$

$5+4x-x^2 = -\{x^2-4x-5\}$

$= -\{x^2 - 2(x)(2) + (2)^2 - 4 - 5\}$

$= -\{(x-2)^2 - 9\}$

$= -\{(x-2)^2 - 9\}$

$= 9 - (x-2)^2$

$I = \int \frac{(2x-5)dx}{\sqrt{3^2-(x-2)^2}}$

$x - 2 = 3\sin\theta \rightarrow (1)$

$\boxed{dx = 3\cos\theta\, d\theta}$

$(1) \Rightarrow\ x - 2 = 3\sin\theta$

$2(x-2) = 2(3\sin\theta)$

$2x - 4 = 6\sin\theta$

$2x - 4 - 1 = -1 + 6\sin\theta$

$\boxed{2x - 5 = -1 + 6\sin\theta}$

$I = \int \frac{(-1+6\sin\theta)(3\cos\theta\, d\theta)}{\sqrt{9-9\sin^2\theta}}$

The Students' Compan...

$$= 3\int \frac{(-1+6\sin\theta)\cos\theta\, d\theta}{\sqrt{9(1-\sin^2\theta)}}$$

$$= 3\int \frac{(-1+6\sin\theta)\cos\theta\, d\theta}{3\sqrt{\cos^2\theta}}$$

$$= \int \frac{(-1+6\sin\theta)\cos\theta\, d\theta}{\cos\theta}$$

$$= \int (-1+6\sin\theta)d\theta$$

$$= -\theta - 6\cos\theta + C \rightarrow (2)$$

$$(1) \Rightarrow \sin\theta = \frac{x-2}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{x-2}{3}\right)$$

$$\sin\theta = \frac{x-2}{3} = \frac{\text{P}}{\text{H}}$$

$$\cos\theta = \frac{\text{B}}{\text{H}} = \frac{\sqrt{\text{H}^2 - P^2}}{\text{H}} = \frac{\sqrt{9-(x-2)^2}}{3}$$

$$\cos\theta = \frac{\sqrt{5+4x-x^2}}{3}$$

$$(2) \Rightarrow -\sin^{-1}\left(\frac{x-2}{3}\right) - 6\left(\frac{\sqrt{5+4x-x^2}}{3}\right) + C$$

$$= -\sin^{-1}\left(\frac{x-2}{3}\right) - 2\sqrt{5+4x-x^2} + C$$

EXERCISE 6.4

Q.1 Integrate by parts the following:

(i) $\int x^2 e^x dx$ (ii) $\int x^3 e^x dx$
(iii) $\int x\cos x\, dx$ (iv) $\int \ln x\, dx$
(v) $\int x^2 \sin x\, dx$ (vi) $\int x^2 \operatorname{cosec}^2 x\, dx$
(vii) $\int x\sec^2 x\, dx$ (viii) $\int (\ln x)^2 dx$

Solution:

$$\text{I} = \begin{pmatrix}\text{First}\\ \text{function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix} - \int \begin{pmatrix}\text{Derivative of}\\ \text{first function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix}$$

(i) $\int x^2 e^x dx$

First function $= x^2$

Second function $= e^x$

$$\text{I} = (x^2)(e^x) - \int (2x)(e^x)dx$$

The Students' ...

$= (x^2)(e^x) - \int$ ...
First function = ...
Second functio...
$= x^2e^x - \{(2x$ ...
$= x^2e^x - 2xe$ ...
(ii) $\int x^3 e^x dx$
First function = ...
Second functio...
$\text{I} = (x^3)(e^x) -$ ...
$= x^3e^x - \int (3$ ...
First function = ...
Second functio...
$= x^3e^x - \{(3x$ ...
$= x^3e^x - 3x^2e$ ...
First function = ...
Second functio...
$= x^3e^x - 3x^2e$ ...
$= x^3e^x - 3x^2e$ ...
$= x^3e^x - 3x^2e$ ...
$= e^x(x^3 - 3x^2$ ...
(iii) $\int x\cos x\, dx$
First function = ...
Second functio...
$\text{I} = (x)(\sin x) -$ ...
$= x\sin x - \int \sin$ ...
$= x\sin x - (-c$ ...
$= x\sin x + \cos$ ...
(iv) $\int \ln x\, dx$

$= (x^2)(e^x) - \int (2x)(e^x)dx$

First function $= 2x$

Second function $= e^x$

$= x^2e^x - \left\{(2x)(e^x) - \int (2)(e^x)dx\right\}$

$= x^2e^x - 2xe^x + 2e^x + C$

(ii) $\int x^3e^x dx$

First function $= x^3$

Second function $= e^x$

$I = (x^3)(e^x) - \int (3x^2)(e^x)dx$

$= x^3e^x - \int (3x^2)(e^x)dx$

First function $= 3x^2$

Second function $= e^x$

$= x^3e^x - \left\{(3x^2)(e^x) - \int (6x)(e^x)dx\right\}$

$= x^3e^x - 3x^2e^x + \int (6x)(e^x)dx$

First function $= 6x$

Second function $= e^x$

$= x^3e^x - 3x^2e^x + \left\{(6x)(e^x) - \int (6)(e^x)dx\right\}$

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

$= e^x(x^3 - 3x^2 + 6x - 6) + C$

(iii) $\int x\cos x\, dx$

First function $= x$

Second function $= \cos x$

$I = (x)(\sin x) - \int (1)(\sin x)dx$

$= x\sin x - \int \sin x\, dx$

$= x\sin x - (-\cos x) + C$

$= x\sin x + \cos x + C$

(iv) $\int \ln x\, dx$

The Students

$= \int (1) \ln x \, dx$

First function $= \ln x$

Second function $= 1$

$$I = (\ln x)(x) - \int \left(\frac{1}{x}\right)(x) dx$$

$$= x \ln x - \int dx$$

$= x \ln x - x + C$

$= x(\ln x - 1) + C$

(v) $\int x^2 \sin x \, dx$

First function $= x^2$

Second function $= \sin x$

$$I = (x^2)(-\cos x) - \int (2x)(-\cos x) dx$$

$$= -x^2 \cos x + \int (2x)(\cos x) dx$$

First function $= 2x$

Second function $= \cos x$

$$= -x^2 \cos x + (2x)(\sin x) - \int (2)(\sin x) dx$$

$= -x^2 \cos x + 2x \sin x - (2)(-\cos x) + C$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

(vi) $\int x \operatorname{cosec}^2 x \, dx$

First function $= x$

Second function $= \operatorname{cosec}^2 x$

$$I = (x)(-\cot x) - \int (1)(-\cot x) dx$$

$$= -x \cot x + \int \cot x \, dx$$

$= -x \cot x + \ln|\sin x| + C$

(vii) $\int x \sec^2 x \, dx$

First function $= x$

Second function $= \sec^2 x$

$$I = (x)(\tan x) - \int (1)(\tan x) dx$$

$$= x \tan x - \int \tan x \, dx$$

The Stud

$= x \tan x$

$= x \tan x$

(viii) $\int (\ln$

$= \int (\ln x$

First func

Second f

$\frac{d}{dx}(\ln x)$

$I = (\ln x$

$= (\ln x)^2$

First func

Second f

$= (\ln x)^2$

$= (\ln x)^2$

$= (\ln x)^2$

Q.2 Integr

(i) $\int (x +$

(iii) $\int x^{-3}$

(v) $\int \sin x$

(vii) $\int \cot$

(ix) $\int \operatorname{cose}$

(xi) $\int \sec^3$

Solution:

(i) $\int (x + 1$

First functi

Second fun

$I = \ln(x +$

$= \frac{1}{2} \ln(x +$

$= x\tan x - \{-\ln|\cos x|\} + C$

$= x\tan x + \ln|\cos x| + C$

(viii) $\int (\ln x)^2 dx$

$= \int (\ln x)^2 (1) dx$

First function $= (\ln x)^2$

Second function $= 1$

$$\frac{d}{dx}(\ln x)^2 = 2(\ln x)\frac{d}{dx}(\ln x) = 2(\ln x)\left(\frac{1}{x}\right) = \frac{2\ln x}{x}$$

$$I = (\ln x)^2(x) - \int \left(\frac{2\ln x}{x}\right)(x)dx$$

$$= (\ln x)^2(x) - \int 2\ln x\, dx$$

First function $= \ln x$

Second function $= 2$

$$= (\ln x)^2(x) - \left\{\ln x\,(2x) - \int \left(\frac{1}{x}\right)(2x)dx\right\}$$

$$= (\ln x)^2(x) - 2x\ln x + 2\int dx$$

$$= (\ln x)^2(x) - 2x\ln x + 2x + C$$

Q.2 Integrate by parts the following:

(i) $\int (x+1)\ln(x+1)\,dx$ (ii) $\int x\ln x\,dx$

(iii) $\int x^{-3}\ln x\,dx$ (iv) $\int x^{-4}\ln x^2\,dx$

(v) $\int \sin x\cos x\ln(\sin x)\,dx$ (vi) $\int \frac{\tan x}{\cos^2 x}\ln(\tan x)\,dx$

(vii) $\int \cot x\,\text{cosec}^2 x\ln(\cot x)\,dx$ (viii) $\int \sec^3 x\tan x\ln(\sec x)\,dx$

(ix) $\int \text{cosec}\, x\cot x\ln(\text{cosec}\, x)\,dx$ (x) $\int \frac{\ln x^2}{x^2}dx$

(xi) $\int \sec^3 x\,dx$ (xii) $\int \text{cosec}^3 x\,dx$

Solution:

(i) $\int (x+1)\ln(x+1)\,dx$

First function $= \ln(x+1)$

Second function $= (x+1)$

$$I = \ln(x+1)\left\{\frac{(x+1)^2}{2}\right\} - \int \left(\frac{1}{x+1}\right)\left\{\frac{(x+1)^2}{2}\right\}dx$$

$$= \frac{1}{2}\ln(x+1)\,(x+1)^2 - \frac{1}{2}\int (x+1)dx$$

$$=\frac{1}{2}\ln(x+1)\,(x+1)^2-\frac{1}{2}\left\{\frac{(x+1)^2}{2}\right\}+C$$

$$=\frac{1}{2}\ln(x+1)\,(x+1)^2-\frac{1}{4}(x+1)^2+C$$

(ii) $\int x\ln x\,dx$

First function $=\ln x$

Second function $=x$

$$\text{I}=(\ln x)\left(\frac{x^2}{2}\right)-\int\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right)dx$$

$$=\frac{x^2}{2}\ln x-\frac{1}{2}\int x dx$$

$$=\frac{x^2}{2}\ln x-\frac{1}{2}\left(\frac{x^2}{2}\right)+C$$

$$=\frac{x^2}{2}\ln x-\frac{1}{4}x^2+C$$

$$=\frac{2x^2\ln x-x^2}{4}+C$$

$$=\frac{x^2}{4}(2\ln x-1)+C$$

$$=\frac{x^2}{4}(\ln x^2-1)+C$$

(iii) $\int x^{-3}\ln x\,dx$

First function $=\ln x$

Second function $=x^{-3}$

$$\text{I}=(\ln x)\left(\frac{x^{-2}}{-2}\right)-\int\left(\frac{1}{x}\right)\left(\frac{x^{-2}}{-2}\right)dx$$

$$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\int\frac{1}{x}dx$$

$$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\ln x+C$$

(iv) $\int x^{-4}\ln x^2\,dx$

$$=\int x^{-4}(2\ln x)dx$$

$$=\int(2x^{-4})\ln x\,dx$$

First function $=\ln x$

Second function $=2x^{-4}$

$\text{I}=(\ln x)\big($

$=-\frac{2}{3x^3}\ln$

$=-\frac{2}{3x^3}\ln$

$=-\frac{2}{3x^3}\ln$

$=-\frac{2}{3x^3}\ln$

$=-\frac{2}{3x^3}\ln$

(v) $\int\sin x$ (

First functi

Second fun

$\text{I}=\ln(\sin$

$=\frac{1}{2}\sin^2 x$ l

$=\frac{1}{2}\sin^2 x$ l

$=\frac{1}{2}\sin^2 x$ l

(vi) $\int\frac{\tan x}{\cos^2 x}$

$=\int\ln(\tan$

First functic

Second fun

$\text{I}=\ln(\tan$

$=\frac{1}{2}\tan^2 x$ l

$=\frac{1}{2}\tan^2 x$ l

$=\frac{1}{2}\tan^2 x$ l

(vii) $\int\cot x$ (

$$I = (\ln x)\left(\frac{2x^{-3}}{-3}\right) - \int \left(\frac{1}{x}\right)\left(\frac{2x^{-3}}{-3}\right) dx$$

$$= -\frac{2}{3x^3}\ln x + \frac{2}{3}\int x^{-1}(x^{-3})dx$$

$$= -\frac{2}{3x^3}\ln x + \frac{2}{3}\int x^{-4}dx$$

$$= -\frac{2}{3x^3}\ln x + \frac{2}{3}\left(\frac{x^{-3}}{-3}\right) + C$$

$$= -\frac{2}{3x^3}\ln x - \frac{2}{9}\left(\frac{1}{x^3}\right) + C$$

$$= -\frac{2}{3x^3}\ln x - \frac{2}{9x^3} + C$$

(v) $\int \sin x \cos x \ln(\sin x)\, dx$

First function $= \ln(\sin x)$

Second function $= \sin x \cos x$

$$I = \ln(\sin x)\left(\frac{\sin^2 x}{2}\right) - \int \left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin^2 x}{2}\right) dx$$

$$= \frac{1}{2}\sin^2 x \ln(\sin x) - \frac{1}{2}\int \sin x \cos x\, dx$$

$$= \frac{1}{2}\sin^2 x \ln(\sin x) - \frac{1}{2}\left(\frac{\sin^2 x}{2}\right) + C$$

$$= \frac{1}{2}\sin^2 x \ln(\sin x) - \frac{1}{4}\sin^2 x + C$$

(vi) $\int \frac{\tan x}{\cos^2 x}\ln(\tan x)\, dx$

$$= \int \ln(\tan x) \tan x \sec^2 x\, dx$$

First function $= \ln(\tan x)$

Second function $= \tan x \sec^2 x$

$$I = \ln(\tan x)\left(\frac{\tan^2 x}{2}\right) - \int \left(\frac{\sec^2 x}{\tan x}\right)\left(\frac{\tan^2 x}{2}\right) dx$$

$$= \frac{1}{2}\tan^2 x \ln(\tan x) - \frac{1}{2}\int \tan x \sec^2 x\, dx$$

$$= \frac{1}{2}\tan^2 x \ln(\tan x) - \frac{1}{2}\left(\frac{\tan^2 x}{2}\right) + C$$

$$= \frac{1}{2}\tan^2 x \ln(\tan x) - \frac{1}{4}\tan^2 x + C$$

(vii) $\int \cot x \operatorname{cosec}^2 x \ln(\cot x)\, dx$

First function $= \ln(\cot x)$

Second function $= \cot x \operatorname{cosec}^2 x$

$$I = \ln(\cot x)\left(-\frac{\cot^2 x}{2}\right) - \int\left(-\frac{\operatorname{cosec}^2 x}{\cot x}\right)\left(-\frac{\cot^2 x}{2}\right)dx$$

$$= -\frac{1}{2}\cot^2 x\ln(\cot x) + \frac{1}{2}\int \cot x\,(-\operatorname{cosec}^2 x)dx$$

$$= -\frac{1}{2}\cot^2 x\ln(\cot x) + \frac{1}{2}\left(\frac{\cot^2 x}{2}\right) + C$$

$$= -\frac{1}{2}\cot^2 x\ln(\cot x) + \frac{1}{4}\cot^2 x + C$$

(viii) $\int \sec^3 x\tan x\ln(\sec x)\,dx$

$$= \int \sec^2 x\,(\sec x\tan x)\ln(\sec x)\,dx$$

First function $= \ln(\sec x)$

Second function $= \sec^2 x\,(\sec x\tan x)$

$$I = \ln(\sec x)\left(\frac{\sec^3 x}{3}\right) - \int\left(\frac{\sec x\tan x}{\sec x}\right)\left(\frac{\sec^3 x}{3}\right)dx$$

$$= \frac{1}{3}\sec^3 x\ln(\sec x) - \frac{1}{3}\int(\sec x\tan x)(\sec^2 x)dx$$

$$= \frac{1}{3}\sec^3 x\ln(\sec x) - \frac{1}{3}\left(\frac{\sec^3 x}{3}\right) + C$$

$$= \frac{1}{3}\sec^3 x\ln(\sec x) - \frac{1}{9}\sec^3 x + C$$

(ix) $\int \operatorname{cosec} x\cot x\ln(\operatorname{cosec} x)\,dx$

First function $= \ln(\operatorname{cosec} x)$

Second function $= \operatorname{cosec} x\cot x$

$$I = \ln(\operatorname{cosec} x)\,(-\operatorname{cosec} x) - \int\left(-\frac{\operatorname{cosec} x\cot x}{\operatorname{cosec} x}\right)(-\operatorname{cosec} x)dx$$

$$= -\operatorname{cosec} x\ln(\operatorname{cosec} x) - \int \cot x\operatorname{cosec} x\,dx$$

$$= -\operatorname{cosec} x\ln(\operatorname{cosec} x) + \operatorname{cosec} x + C$$

(x) $\int \frac{\ln x^2}{x^2}dx$

$$= \int x^{-2}(2\ln x)dx$$

First function $= 2\ln x$

Second function $= x^{-2}$

$$I = 2\ln x\left(\frac{x^{-1}}{-1}\right) - \int\left(\frac{2}{x}\right)\left(\frac{x^{-1}}{-1}\right)dx$$

$$= -\frac{2\ln x}{x} + 2\int x^{-2}dx$$

$$= -\frac{2\ln x}{x} + 2\left(\frac{x^{-1}}{-1}\right) + C$$

$$= -\frac{2\ln x}{x} - \frac{2}{x} + C$$

(xi) $\int \sec^3 x\, dx$

$I = \int \sec^3 x\, dx$

$I = \int \sec^2 x \sec x\, dx$

First function $= \sec x$

Second function $= \sec^2 x$

$$I = \sec x(\tan x) - \int (\sec x \tan x)(\tan x)dx$$

$$I = \sec x(\tan x) - \int \sec x \tan^2 x\, dx$$

$$I = \sec x(\tan x) - \int \sec x(\sec^2 x - 1)dx$$

$$I = \sec x(\tan x) - \int (\sec^3 x - \sec x)dx$$

$$I = \sec x(\tan x) - \int \sec^3 x\, dx + \int \sec x\, dx$$

$$I = \sec x(\tan x) - I + \ln(\sec x + \tan x) + C$$

$$2I = \sec x(\tan x) + \ln(\sec x + \tan x) + C$$

$$I = \frac{1}{2}\sec x(\tan x) + \frac{1}{2}\ln(\sec x + \tan x) + C$$

$$I = \frac{1}{2}\sec x(\tan x) + \ln(\sec x + \tan x)^{\frac{1}{2}} + C$$

$$I = \frac{1}{2}\sec x(\tan x) + \ln\sqrt{\sec x + \tan x} + C$$

(xii) $\int \text{cosec}^3 x\, dx$

$$I = \int \text{cosec}^3 x\, dx$$

$$I = \int \text{cosec}^2 x\, \text{cosec}\, x\, dx$$

First function $= \text{cosec}\, x$

Second function $= \text{cosec}^2 x$

$$I = \operatorname{cosec} x\,(-\cot x) - \int (-\operatorname{cosec} x \cot x)(-\cot x)dx$$

$$= -\cot x \operatorname{cosec} x - \int \operatorname{cosec} x \cot^2 x\, dx$$

$$= -\cot x \operatorname{cosec} x - \int \operatorname{cosec} x\,(\operatorname{cosec}^2 x - 1)dx$$

$$= -\cot x \operatorname{cosec} x - \int \operatorname{cosec}^3 x\, dx + \int \operatorname{cosec} x\, dx$$

$$I = -\cot x \operatorname{cosec} x - I + \ln(\operatorname{cosec} x - \cot x) + C$$

$$2I = -\cot x \operatorname{cosec} x + \ln(\operatorname{cosec} x - \cot x) + C$$

$$I = -\frac{1}{2}\cot x \operatorname{cosec} x + \frac{1}{2}\ln|\operatorname{cosec} x - \cot x| + C$$

$$I = -\frac{1}{2}\cot x \operatorname{cosec} x + \ln|\operatorname{cosec} x - \cot x|^{\frac{1}{2}} + C$$

$$I = -\frac{1}{2}\cot x \operatorname{cosec} x + \ln\sqrt{\operatorname{cosec} x - \cot x} + C$$

Q.3 Integrate by parts the following:

(i) $\int 3x\cos(3x)\,dx$ (ii) $\int x^2 \sin x\,dx$

(iii) $\int \frac{x}{\cot^2 x}dx$ (iv) $\int x\sec^2 x\,dx$

(v) $\int \frac{5x}{\sin^2 2x}dx$ (vi) $\int \cos\sqrt{x}\,dx$

(vii) $\int e^{2x}\sin 2x\,dx$ (viii) $\int e^{-x}\cos 2x\,dx$

(ix) $\int \cos(\ln x)\,dx$ (x) $\int e^{ax}\sin bx\,dx$

Solution:

(i) $\int 3x\cos(3x)\,dx$

First function $= 3x$

Second function $= \cos(3x)$

$$I = 3x\left(\frac{\sin 3x}{3}\right) - \int (3)\left(\frac{\sin 3x}{3}\right)dx$$

$$= x\sin 3x - \int \sin 3x\,dx$$

$$= x\sin 3x - \left(-\frac{\cos(3x)}{3}\right) + C$$

$$= x\sin 3x + \frac{1}{3}\cos(3x) + C$$

(ii) $\int x^2 \sin x\,dx$

First function $= x^2$

Second function $= \sin x$

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$I = x^2(-\cos x)$ -

$= -x^2\cos x + \int$

First function = 2

Second function :

$= -x^2\cos x + (2$

$= -x^2\cos x + 2$

$= -x^2\cos x + 2$

$= (2 - x^2)\cos x$

(iii) $\int \frac{x}{\cot^2 x}dx$

$= \int x\tan^2 x\,dx$

$= \int x(\sec^2 x - 1$

$= \int (x\sec^2 x - x$

$= \int x\sec^2 x\,dx$ -

First function $= x$

Second function =

$I = x(\tan x) - \int$

$= x\tan x - \ln(se$

(iv) $\int x\sec^2 x\,dx$

First function $= x$

Second function =

$I = x(\tan x) - \int($

$= x\tan x - \{-\ln($

$= x\tan x + \ln(\cos$

(v) $\int \frac{5x}{\sin^2 2x}dx$

$= \int 5x\operatorname{cosec}^2 2x$

First function $= 5x$

Second function =

$I = x^2(-\cos x) - \int (2x)(-\cos x)dx$

$= -x^2 \cos x + \int (2x)(\cos x)dx$

First function $= 2x$

Second function $= \cos x$

$= -x^2 \cos x + (2x)(\sin x) - \int (2)(\sin x)dx$

$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

$= (2 - x^2)\cos x + 2x \sin x + C$

(iii) $\int \frac{x}{\cot^2 x} dx$

$= \int x \tan^2 x\, dx$

$= \int x(\sec^2 x - 1)dx$

$= \int (x \sec^2 x - x)dx$

$= \int x \sec^2 x\, dx - \frac{x^2}{2}$

First function $= x$

Second function $= \sec^2 x$

$I = x(\tan x) - \int (1)(\tan x)dx$

$= x\tan x - \ln(\sec x) - \frac{x^2}{2} + C$

(iv) $\int x \sec^2 x\, dx$

First function $= x$

Second function $= \sec^2 x$

$I = x(\tan x) - \int (1)(\tan x)dx$

$= x\tan x - \{-\ln(\cos x)\} + C$

$= x\tan x + \ln(\cos x) + C$

(v) $\int \frac{5x}{\sin^2 2x} dx$

$= \int 5x \operatorname{cosec}^2 2x\, dx$

First function $= 5x$

Second function $= \operatorname{cosec}^2 2x$

$$I = 5x\left(-\frac{\cot 2x}{2}\right) - \int (5)\left(-\frac{\cot 2x}{2}\right) dx$$

$$= -\frac{5x}{2}\cot 2x + \frac{5}{2}\int \cot 2x\, dx$$

$$= -\frac{5x}{2}\cot 2x + \frac{5}{2}\left\{\frac{\ln(\sin 2x)}{2}\right\} + C$$

$$= -\frac{5x}{2}\cot 2x + \frac{5}{4}\ln(\sin 2x) + C$$

$$= \frac{5}{4}\{-2x\cot 2x + \ln(\sin 2x)\} + C$$

(vi) $\int \cos\sqrt{x}\, dx$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow 2\sqrt{x}dy = dx \Rightarrow 2ydy = dx$$

$$I = \int \cos y\,(2y)dy$$

$$= \int \cos y\,(2y)dy$$

First function $= 2y$

Second function $= \cos y$

$$I = 2y(\sin y) - \int (2)(\sin y)dx$$

$$= 2y\sin y - 2(-\cos y) + C$$

$$= 2y\sin y + 2\cos y + C$$

$$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$$

(vii) $\int e^{2x}\sin 2x\, dx$

$$I = \int e^{2x}\sin 2x\, dx$$

First function $= \sin 2x$

Second function $= e^{2x}$

$$I = \sin 2x\left(\frac{e^{2x}}{2}\right) - \int (2\cos 2x)\left(\frac{e^{2x}}{2}\right) dx$$

$$I = \frac{1}{2}e^{2x}\sin 2x - \int e^{2x}\cos 2x\, dx$$

First function $= \cos 2x$

Second function $= e^{2x}$

$I = \frac{1}{2}e^{2x}\sin 2x -$

$I = \frac{1}{2}e^{2x}\sin 2x -$

$I = \frac{1}{2}e^{2x}\sin 2x -$

$2I = \frac{e^{2x}}{2}\{\sin 2x -$

$I = \frac{e^{2x}}{4}\{\sin 2x - c$

(viii) $\int e^{-x}\cos 2x\, d$

$I = \int e^{-x}\cos 2x\, d$

First function $= \cos$

Second function $=$

$I = \cos 2x\,(-e^{-x})$

$I = -e^{-x}\cos 2x -$

First function $= 2\,\mathrm{si}$

Second function $=$

$I = -e^{-x}\cos 2x - \{$

$I = -e^{-x}\cos 2x + 2$

$I = e^{-x}(-\cos 2x +$

$5I = e^{-x}(-\cos 2x +$

$I = \frac{e^{-x}}{5}(-\cos 2x +$

(ix) $\int \cos(\ln x)\, dx$

Let $y = \ln x \to (1)$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow xdy = dx$

$(1) \Rightarrow \ln x = y(1)$

$\ln x = y(\ln e)$

$\ln x = \ln e^{y} \Rightarrow x = e^{y}$

$e^{y}dy = dx$

$$I=\frac{1}{2}e^{2x}\sin 2x-\left\{\cos 2x\left(\frac{e^{2x}}{2}\right)-\int(-2\sin 2x)\left(\frac{e^{2x}}{2}\right)dx\right\}$$

$$I=\frac{1}{2}e^{2x}\sin 2x-\frac{1}{2}e^{2x}\cos 2x-\int e^{2x}\sin 2x\,dx$$

$$I=\frac{1}{2}e^{2x}\sin 2x-\frac{1}{2}e^{2x}\cos 2x-I$$

$$2I=\frac{e^{2x}}{2}\{\sin 2x-\cos 2x\}$$

$$I=\frac{e^{2x}}{4}\{\sin 2x-\cos 2x\}$$

(viii) $\int e^{-x}\cos 2x\,dx$

$$I=\int e^{-x}\cos 2x\,dx$$

First function $=\cos 2x$

Second function $=e^{-x}$

$$I=\cos 2x\,(-e^{-x})-\int(-2\sin 2x)(-e^{-x})dx$$

$$I=-e^{-x}\cos 2x-\int e^{-x}(2\sin 2x)dx$$

First function $=2\sin 2x$

Second function $=e^{-x}$

$$I=-e^{-x}\cos 2x-\left\{(2\sin 2x)(-e^{-x})-\int(4\cos 2x)(-e^{-x})dx\right\}$$

$$I=-e^{-x}\cos 2x+2e^{-x}\sin 2x-4\int e^{-x}\cos 2x\,dx$$

$$I=e^{-x}(-\cos 2x+2\sin 2x)-4I$$

$$5I=e^{-x}(-\cos 2x+2\sin 2x)$$

$$I=\frac{e^{-x}}{5}(-\cos 2x+2\sin 2x)$$

(ix) $\int\cos(\ln x)\,dx$

Let $y=\ln x\rightarrow(1)$

$$\frac{dy}{dx}=\frac{1}{x}\Rightarrow xdy=dx$$

$(1)\Rightarrow\ \ln x=y(1)$

$\ln x=y(\ln e)$

$\ln x=\ln e^{y}\Rightarrow x=e^{y}$

$\boxed{e^{y}dy=dx}$

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$$I = \int e^y \cos y \, dy$$

First function $= \cos y$

Second function $= e^y$

$$I = \sin y\,(e^y) - \int (\sin y)(e^y)dy$$

$$I = e^y \cos y - \int e^y \sin y\, dy$$

First function $= \sin y$

Second function $= e^y$

$$I = e^y \cos y - \left\{\sin y\,(e^y) - \int (-\cos y)e^y dy\right\}$$

$$I = e^y \cos y + e^y \sin y - \int e^y \cos y\, dy$$

$$I = e^y(\cos y + \sin y) - I$$

$$2I = e^y(\cos y + \sin y)$$

$$I = \frac{e^y}{2}(\cos y + \sin y)$$

$$I = \frac{e^{\ln x}}{2}\{\cos(\ln x) + \sin(\ln x)\}$$

$$I = \frac{x}{2}\{\cos(\ln x) + \sin(\ln x)\}$$

(x) $\int e^{ax} \sin bx\, dx$

First function $= \sin bx$

Second function $= e^{ax}$

$$I = \sin bx\left(\frac{e^{ax}}{a}\right) - \int (b\cos bx)\left(\frac{e^{ax}}{a}\right)dx$$

$$I = \frac{e^{ax}}{a}\sin bx - \int\left(\frac{b}{a}\cos bx\right)e^{ax}dx$$

First function $= \frac{b}{a}\cos bx$

Second function $= e^{ax}$

$$I = \frac{e^{ax}}{a}\sin bx - \left\{\left(\frac{b}{a}\cos bx\right)\left(\frac{e^{ax}}{a}\right) - \int\left(-\frac{b^2}{a}\sin bx\right)\left(\frac{e^{ax}}{a}\right)dx\right\}$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\sin bx\, dx$$

$$I = \frac{e^{ax}}{a}\sin bx - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}I$$

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$I + \frac{b^2}{a^2}I = \frac{ae^{\ldots}}{\ldots}$

$I\left(\frac{a^2+b^2}{a^2}\right) = \ldots$

$I = \frac{e^{ax}}{a^2+b^2}(\ldots$

Q.4 Integrate

(i) $\int \sin^{-1} 3x \ldots$

(iii) $\int \tan^{-1} 2\ldots$

(v) $\int 3x^2 \sin^{-}\ldots$

(vii) $\int 6x \operatorname{cose}\ldots$

Solution:

(i) $\int \sin^{-1} 3x$

$= \int (1) \sin^{-1}\ldots$

$\frac{d}{dx}(\sin^{-1} 3x)$

First function

Second functi

$I = (\sin^{-1} 3x \ldots$

$= x \sin^{-1} 3x \ldots$

$= x \sin^{-1} 3x \ldots$

$= x \sin^{-1} 3x \ldots$

$= x \sin^{-1} 3x \ldots$

(ii) $\int x^4 \tan^{-1}\ldots$

First function :

Second functic

$I = \tan^{-1} x\left(\frac{x\ldots}{\ldots}\right.$

$= \frac{1}{5}x^5 \tan^{-1} x \ldots$

$$I + \frac{b^2}{a^2}I = \frac{ae^{ax}\sin bx - be^{ax}\cos bx}{a^2}$$

$$I\left(\frac{a^2+b^2}{a^2}\right) = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2}$$

$$I = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

Q.4 Integrate by parts the following:

(i) $\int \sin^{-1} 3x\, dx$ (ii) $\int x^4 \tan^{-1} x\, dx$

(iii) $\int \tan^{-1} 2x\, dx$ (iv) $\int x\cos^{-1} x\, dx$

(v) $\int 3x^2 \sin^{-1} 3x\, dx$ (vi) $\int 2x \sec^{-1} x\, dx$

(vii) $\int 6x\,\text{cosec}^{-1} 2x\, dx$ (viii) $\int x^2 \cot^{-1} x\, dx$

Solution:

(i) $\int \sin^{-1} 3x\, dx$

$$= \int (1)\sin^{-1} 3x\, dx$$

$$\frac{d}{dx}(\sin^{-1} 3x) = \frac{3}{\sqrt{1-9x^2}}$$

First function $= \sin^{-1} 3x$

Second function $= 1$

$$I = (\sin^{-1} 3x)(x) - \int \left(\frac{3}{\sqrt{1-9x^2}}\right)(x)dx$$

$$= x\sin^{-1} 3x - \int (1-9x^2)^{-\frac{1}{2}} 3x\,dx$$

$$= x\sin^{-1} 3x + \frac{1}{6}\int (1-9x^2)^{-\frac{1}{2}}(-18x)dx$$

$$= x\sin^{-1} 3x + \frac{1}{6}\left\{\frac{(1-9x^2)^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$$

$$= x\sin^{-1} 3x + \frac{1}{3}\sqrt{1-9x^2} + C$$

(ii) $\int x^4 \tan^{-1} x\, dx$

First function $= \tan^{-1} x$

Second function $= x^4$

$$I = \tan^{-1} x\left(\frac{x^5}{5}\right) - \int \left(\frac{1}{1+x^2}\right)\left(\frac{x^5}{5}\right)dx$$

$$= \frac{1}{5}x^5 \tan^{-1} x - \frac{1}{5}\int \frac{x^5}{1+x^2}dx$$

$$\begin{array}{r|l} & x^3 - x \\ \hline x^2+1 & x^5 \\ & \pm x^5 \pm x^3 \\ \hline & -x^3 \\ & \mp x^3 \mp x \\ \hline & x \end{array}$$

$$= \frac{1}{5}x^5 \tan^{-1} x - \frac{1}{5}\int\left(x^3 - x + \frac{x}{1+x^2}\right)dx$$

$$= \frac{1}{5}x^5 \tan^{-1} x - \frac{1}{5}\int x^3 dx + \frac{1}{5}\int x dx - \frac{1}{5}\left(\frac{1}{2}\right)\int\left(\frac{2x}{1+x^2}\right)dx$$

$$= \frac{1}{5}x^5 \tan^{-1} x - \frac{1}{5}\left(\frac{x^4}{4}\right) + \frac{1}{5}\left(\frac{x^2}{2}\right) - \frac{1}{10}\ln|1+x^2| + C$$

$$= \frac{1}{5}x^5 \tan^{-1} x - \frac{1}{20}x^4 + \frac{1}{10}x^2 - \frac{1}{10}\ln|1+x^2| + C$$

(iii) $\int \tan^{-1} 2x \, dx$

$$= \int (1)\tan^{-1} 2x \, dx$$

$$\frac{d}{dx}(\tan^{-1} 2x) = \frac{2}{1+4x^2}$$

First function = $\tan^{-1} 2x$

Second function = 1

$$\text{I} = \tan^{-1} 2x\,(x) - \int\left(\frac{2}{1+4x^2}\right)(x)dx$$

$$= x\tan^{-1} 2x - \frac{1}{4}\int \frac{8x}{1+4x^2}dx$$

$$= x\tan^{-1} 2x - \frac{1}{4}\ln|1+4x^2| + C$$

(iv) $\int x\cos^{-1} x \, dx$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

First function = $\cos^{-1} x$

Second function = x

$$\text{I} = (\cos^{-1} x)\left(\frac{x^2}{2}\right) - \int\left(-\frac{1}{\sqrt{1-x^2}}\right)\left(\frac{x^2}{2}\right)dx$$

$$= \frac{1}{2}x^2\cos^{-1} x + \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

$= \frac{1}{2}x^2\cos^{-1} x$

(v) $\int 3x^2 \sin^{-1}$

$\frac{d}{dx}(\sin^{-1} 3x)$

First function =

Second functio

$\text{I} = (\sin^{-1} 3x)$

$= x^3\sin^{-1} 3x$

$= x^3\sin^{-1} 3x$

Let $y = 1 - 9x$

$\frac{dy}{dx} = -18x \Rightarrow$

$(2) \Rightarrow 9x^2 = 1$

$x^2 = \frac{1-y}{9}$

$(1) \Rightarrow x^3\sin^{-1}$

$= x^3\sin^{-1} 3x +$

$= x^3\sin^{-1} 3x +$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}\right\}dx$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\int\left\{\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right\}dx$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right\}+C$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}x\right\}+C$

$=\frac{1}{2}x^2\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}x+C$

(v) $\int 3x^2\sin^{-1}3x\,dx$

$\frac{d}{dx}(\sin^{-1}3x)=\frac{3}{\sqrt{1-9x^2}}$

First function $=\sin^{-1}3x$

Second function $=3x^2$

$I=(\sin^{-1}3x)\left(\frac{3x^3}{3}\right)-\int\left(\frac{3}{\sqrt{1-9x^2}}\right)\left(\frac{3x^3}{3}\right)dx$

$=x^3\sin^{-1}3x-3\int\frac{x^3}{\sqrt{1-9x^2}}dx$

$=x^3\sin^{-1}3x-3\int(1-9x^2)^{-\frac{1}{2}}x^2\,xdx\rightarrow(1)$

Let $y=1-9x^2\rightarrow(2)$

$\frac{dy}{dx}=-18x\Rightarrow-\frac{dy}{18}=xdx$

$(2)\Rightarrow 9x^2=1-y$

$x^2=\frac{1-y}{9}$

$(1)\Rightarrow x^3\sin^{-1}3x-3\int y^{-\frac{1}{2}}\left(\frac{1-y}{9}\right)\left(-\frac{dy}{18}\right)$

$=x^3\sin^{-1}3x+\frac{3}{18(9)}\int\left(y^{-\frac{1}{2}}-y^{\frac{1}{2}}\right)dy$

$=x^3\sin^{-1}3x+\frac{1}{54}\left(\frac{y^{\frac{1}{2}}}{\frac{1}{2}}-\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$

$$= x^3 \sin^{-1} 3x + \frac{1}{54}\left(2\sqrt{y} - \frac{2}{3}y^{\frac{3}{2}}\right) + C$$

$$= x^3 \sin^{-1} 3x + \frac{1}{27}\sqrt{1-9x^2} - \frac{1}{81}(1-9x^2)^{\frac{3}{2}} + C$$

(vi) $\int 2x \sec^{-1} x \, dx$

First function $= \sec^{-1} x$

Second function $= 2x$

$$I = (\sec^{-1} x)\left(\frac{2x^2}{2}\right) - \int \left(\frac{1}{x\sqrt{x^2-1}}\right)\left(\frac{2x^2}{2}\right) dx$$

$$= x^2 \sec^{-1} x - \int \frac{x}{\sqrt{x^2-1}} dx$$

$$= x^2 \sec^{-1} x - \int (x^2-1)^{-\frac{1}{2}} x dx$$

$$= x^2 \sec^{-1} x - \frac{1}{2}\int (x^2-1)^{-\frac{1}{2}}(2x) dx$$

$$= x^2 \sec^{-1} x - \frac{1}{2}\left\{\frac{(x^2-1)^{\frac{1}{2}}}{\frac{1}{2}}\right\} + C$$

$$= x^2 \sec^{-1} x - \sqrt{x^2-1} + C$$

(vii) $\int 6x \operatorname{cosec}^{-1} 2x \, dx$

First function $= \operatorname{cosec}^{-1} 2x$

Second function $= 6x$

$$I = (\operatorname{cosec}^{-1} 2x)\left(\frac{6x^2}{2}\right) - \int \left(-\frac{2}{2x\sqrt{4x^2-1}}\right)\left(\frac{6x^2}{2}\right) dx$$

$$= 3x^2 \operatorname{cosec}^{-1} 2x + 3\int \frac{x}{\sqrt{4x^2-1}} dx$$

$$= 3x^2 \operatorname{cosec}^{-1} 2x + 3\int (4x^2-1)^{-\frac{1}{2}} x dx$$

$$= 3x^2 \operatorname{cosec}^{-1} 2x + \frac{3}{8}\int (4x^2-1)^{-\frac{1}{2}}(8x) dx$$

$$= 3x^2 \operatorname{cosec}^{-1} 2x + \frac{3}{8}\frac{(4x^2-1)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 3x^2 \operatorname{cosec}^{-1} 2x + \frac{3}{4}\sqrt{4x^2-1} + C$$

(viii) $\int x^2 \cot^{-1} x \, dx$

First function $= \cot^{-1} x$

Second function

$I = (\cot^{-1} x)\left(\frac{x}{3}\right.$

$= \frac{1}{3}x^3 \cot^{-1} x +$

x

$x^2+1 \;\big|\; x^3$

$\pm x$

$-x$

$= \frac{1}{3}x^3 \cot^{-1} x$

$= \frac{1}{3}x^3 \cot^{-1} x$

$= \frac{1}{3}x^3 \cot^{-1} x$

$= \frac{1}{3}x^3 \cot^{-1} x$

Q.5 Integrate b

(i) $\int \sqrt{9-x^2} dx$

(iii) $\int \sqrt{x^2-25}$

(i) $\int \sqrt{9-x^2} dx$

$= \int (1)\sqrt{9-x}$

$\frac{d}{dx}(9-x^2)^{\frac{1}{2}} =$

First function =

Second functio

$I = \left(\sqrt{9-x^2}\right)$

$= x\sqrt{9-x^2} +$

$= x\sqrt{9-x^2} -$

$= x\sqrt{9-x^2} -$

Second function $= x^2$

$$I = (\cot^{-1}x)\left(\frac{x^3}{3}\right) - \int\left(-\frac{1}{1+x^2}\right)\left(\frac{x^3}{3}\right)dx$$

$$= \frac{1}{3}x^3\cot^{-1}x + \frac{1}{3}\int\frac{x^3}{1+x^2}dx$$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & \pm x^3 \pm x \\ \hline & -x \end{array}$$

$$= \frac{1}{3}x^3\cot^{-1}x + \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$$

$$= \frac{1}{3}x^3\cot^{-1}x + \frac{1}{3}\int xdx - \frac{1}{3}\int\frac{x}{1+x^2}dx$$

$$= \frac{1}{3}x^3\cot^{-1}x + \frac{1}{3}\left(\frac{x^2}{2}\right) - \frac{1}{3(2)}\int\frac{2x}{1+x^2}dx$$

$$= \frac{1}{3}x^3\cot^{-1}x + \frac{1}{6}x^2 - \frac{1}{6}\ln(1+x^2) + C$$

Q.5 Integrate by parts the following:

(i) $\int\sqrt{9-x^2}dx$ (ii) $\int\sqrt{16+4x^2}dx$

(iii) $\int\sqrt{x^2-25}dx$

(i) $\int\sqrt{9-x^2}dx$

$$= \int(1)\sqrt{9-x^2}dx$$

$$\frac{d}{dx}(9-x^2)^{\frac{1}{2}} = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{9-x^2}}$$

First function $= \sqrt{9-x^2}$

Second function $= 1$

$$I = \left(\sqrt{9-x^2}\right)(x) - \int\left(-\frac{x}{\sqrt{9-x^2}}\right)(x)dx$$

$$= x\sqrt{9-x^2} + \int\frac{x^2}{\sqrt{9-x^2}}dx$$

$$= x\sqrt{9-x^2} - \int\frac{-x^2}{\sqrt{9-x^2}}dx$$

$$= x\sqrt{9-x^2} - \int\frac{9-x^2-9}{\sqrt{9-x^2}}dx$$

$$= x\sqrt{9-x^2} - \int\left\{\frac{9-x^2}{\sqrt{9-x^2}} - \frac{9}{\sqrt{9-x^2}}\right\}dx$$

$$= x\sqrt{9-x^2} - \int\left\{\sqrt{3^2-x^2} - \frac{9}{\sqrt{3^2-x^2}}\right\}dx$$

$$= x\sqrt{9-x^2} - \left\{\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right\} + 9\sin^{-1}\frac{x}{3} + C$$

$$= x\sqrt{9-x^2} - \frac{x}{2}\sqrt{9-x^2} - \frac{9}{2}\sin^{-1}\frac{x}{3} + 9\sin^{-1}\frac{x}{3} + C$$

$$= \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + C$$

(ii) $\int \sqrt{16+4x^2}dx$

$$= \int \sqrt{4(4+x^2)}dx$$

$$= \int 2\sqrt{4+x^2}dx$$

$$\frac{d}{dy}(4+x^2)^{\frac{1}{2}} = \frac{1}{2}(4+x^2)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{4+x^2}}$$

First function $= \sqrt{4+x^2}$

Second function $= 2$

$$\text{I} = \left(\sqrt{4+x^2}\right)(2x) - \int\left(\frac{x}{\sqrt{4+x^2}}\right)(2x)dx$$

$$= 2x\sqrt{4+x^2} - 2\int\frac{x^2}{\sqrt{4+x^2}}dx$$

$$= 2x\sqrt{4+x^2} - 2\int\frac{x^2+4-4}{\sqrt{4+x^2}}dx$$

$$= 2x\sqrt{4+x^2} - 2\int\left\{\frac{x^2+4}{\sqrt{4+x^2}} - \frac{4}{\sqrt{4+x^2}}\right\}dx$$

$$= 2x\sqrt{4+x^2} - 2\int\left\{\sqrt{4+x^2} - \frac{4}{\sqrt{4+x^2}}\right\}dx$$

$$= 2x\sqrt{4+x^2} - 2\int\sqrt{4+x^2}dx + 8\int\frac{1}{\sqrt{4+x^2}}dx$$

$$= 2x\sqrt{4+x^2} - 2\left\{\frac{x}{2}\sqrt{4+x^2} + \frac{4}{2}\ln\left|x+\sqrt{4+x^2}\right|\right\} + 8\ln\left|x+\sqrt{4+x^2}\right| + C$$

$$= 2x\sqrt{4+x^2} - x\sqrt{4+x^2} - 4\ln\left|x+\sqrt{4+x^2}\right| + 8\ln\left|x+\sqrt{4+x^2}\right| + C$$

$= x\sqrt{4+x^2} + 4$ ...

(iii) $\int\sqrt{x^2-25}d$...

$\frac{d}{dx}(x^2-25)^{\frac{1}{2}} =$ ...

First function = ...

Second function ...

$\text{I} = \left(\sqrt{x^2-25}\right)$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= x\sqrt{x^2-25} -$ ...

$= \frac{x}{2}\sqrt{x^2-25}$ ...

Evaluate the foll...

Q.1 $\int\frac{(5x-2)dx}{(x-3)(x+7)}$

Q.4 $\int\frac{5dx}{x^2-2x-15}$

Q.7 $\int\frac{(2x+7)d}{(x-1)(x-5)(}$ ...

Q.10 $\int\frac{(2x+1)dx}{(x-3)(x^2+}$ ...

Q.13 $\int\frac{(3x+7)dx}{(2x-1)(x-}$ ...

$= x\sqrt{4+x^2} + 4\ln\left|x+\sqrt{4+x^2}\right| + C$

(iii) $\int \sqrt{x^2-25}\,dx$

$\frac{d}{dx}(x^2-25)^{\frac{1}{2}} = \frac{1}{2}(x^2-25)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2-25}}$

First function $= \sqrt{x^2-25}$

Second function $= 1$

$$I = \left(\sqrt{x^2-25}\right)(x) - \int\left(\frac{x}{\sqrt{x^2-25}}\right)(x)dx$$

$$= x\sqrt{x^2-25} - \int \frac{x^2}{\sqrt{x^2-25}}dx$$

$$= x\sqrt{x^2-25} - \int \frac{x^2-25+25}{\sqrt{x^2-25}}dx$$

$$= x\sqrt{x^2-25} - \int\left\{\frac{x^2-25}{\sqrt{x^2-25}} + \frac{25}{\sqrt{x^2-25}}\right\}dx$$

$$= x\sqrt{x^2-25} - \int\left\{\sqrt{x^2-25} + \frac{25}{\sqrt{x^2-25}}\right\}dx$$

$$= x\sqrt{x^2-25} - \left\{\frac{x}{2}\sqrt{x^2-25} - \frac{25}{2}\ln\left|x+\sqrt{x^2-25}\right| + 25\ln\left|x+\sqrt{x^2-25}\right|\right\}$$

$$= x\sqrt{x^2-25} - \left\{\frac{x}{2}\sqrt{x^2-25} + \frac{25}{2}\ln\left|x+\sqrt{x^2-25}\right|\right\}$$

$$= x\sqrt{x^2-25} - \frac{x}{2}\sqrt{x^2-25} - \frac{25}{2}\ln\left|x+\sqrt{x^2-25}\right|$$

$$= \frac{x}{2}\sqrt{x^2-25} - \frac{25}{2}\ln\left|x+\sqrt{x^2-25}\right|$$

EXERCISE 6.5

Evaluate the following integrals by using partial fraction:

Q.1 $\int \frac{(5x-2)dx}{(x-3)(x+7)}$ Q.2 $\int \frac{(7x-25)dx}{(x-3)(x-4)}$ Q.3 $\int \frac{dx}{a^2-x^2}$

Q.4 $\int \frac{5dx}{x^2-2x-15}$ Q.5 $\int \frac{(x^2+2x+3)dx}{x^3-x}$ Q.6 $\int \frac{5dx}{x^2-2x-15}$

Q.7 $\int \frac{(2x+7)dx}{(x-1)(x-5)(x+3)}$ Q.8 $\int \frac{(5x+6)dx}{(x+3)(x-2)^2}$ Q.9 $\int \frac{(7x^2-2x+5)dx}{(x-6)(x-3)^3}$

Q.10 $\int \frac{(2x+1)dx}{(x-3)(x^2+1)}$ Q.11 $\int \frac{\sec^2 x\,dx}{(1+\tan x)(2+\tan x)}$ Q.12 $\int \frac{\text{cosec}^2 x\,dx}{\cot x(2+\cot x)}$

Q.13 $\int \frac{(3x+7)dx}{(2x-1)(x-4)^2}$ Q.14 $\int \frac{(7x-4)dx}{(x-3)(x^2+2)}$ Q.15 $\int \frac{(2x^2+5x+1)dx}{x^2+5x+6}$

Q.16 $\int \frac{(x^3+3x+1)dx}{x^2+5x+6}$

Q.1 $\int \frac{(5x-2)dx}{(x-3)(x+7)}$

Solution:

$$\int \frac{(5x-2)}{(x-3)(x+7)}dx$$

Let $\frac{(5x-2)}{(x-3)(x+7)} = \frac{A}{x-3} + \frac{B}{x+7}$

$$\frac{(5x-2)}{(x-3)(x+7)} = \frac{A(x+7)+B(x-3)}{(x-3)(x+7)}$$

$$(5x-2) = A(x+7) + B(x-3)$$

Let $x = 3$

$$5(3) - 2 = A(3+7) + 0 \Rightarrow 10A = 13 \Rightarrow A = \frac{13}{10}$$

Let $x = -7$

$$5(-7) - 2 = 0 + B(-7-3) \Rightarrow 10A = 37 \Rightarrow A = \frac{37}{10}$$

$$\mathrm{I} = \frac{13}{10}\int \frac{1}{x-3}dx + \frac{37}{10}\int \frac{1}{x+7}dx$$

$$= \frac{13}{10}\ln|x-3| + \frac{37}{10}\ln|x+7| + C$$

$$= \frac{1}{10}\ln|x-3|^{13} + \frac{1}{10}\ln|x+7|^{37} + C$$

$$= \frac{1}{10}\{\ln|x-3|^{13} + \ln|x+7|^{37}\} + C$$

$$= \frac{1}{10}\{\ln|x-3|^{13}\ln|x+7|^{37}\} + C$$

Q.2 $\int \frac{(7x-25)dx}{(x-3)(x-4)}$

Solution:

$$\int \frac{(7x-25)}{(x-3)(x-4)}dx$$

$$\frac{(7x-25)}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$\frac{(7x-25)}{(x-3)(x-4)} = \frac{A(x-4)+B(x-3)}{(x-3)(x-4)}$$

$$(7x-25) = A(x-4) + B(x-3)$$

Let $x = 3$

$7(3) - 25 = A($…

Let $x = 4$

$7(4) - 25 = 0 +$…

$\mathrm{I} = 4\int \frac{1}{x-3}dx$…

$= 4\ln|x-3| +$…

$= \ln|x-3|^4 +$…

$= \ln|(x-3)^4(x$…

Q.3 $\int \frac{dx}{a^2-x^2}$

Solution:

$$\int \frac{1}{a^2-x^2}dx$$

$= \int \frac{1}{(a-x)(a}$…

$\frac{1}{(a-x)(a+x)}$…

$\frac{1}{(a-x)(a+x)}$…

$1 = A(a-x) +$…

Let $x = a$

$1 = 0 + B(a +$…

Let $x = -a$

$1 = A(a+a) +$…

$\mathrm{I} = \frac{1}{2a}\int \frac{1}{a-x}$…

$= -\frac{1}{2a}\ln|a-$…

$= \frac{1}{2a}\{\ln|a+x|$…

$= \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$…

Q.4 $\int \frac{dx}{x^2-a^2}$

Solution:

Let $x = 3$

$7(3) - 25 = A(3 - 4) + 0 \Rightarrow A = 4$

Let $x = 4$

$7(4) - 25 = 0 + B(4 - 3) \Rightarrow B = 3$

$I = 4\int \frac{1}{x-3}dx + 3\int \frac{1}{x-4}dx$

$= 4\ln|x-3| + 3\ln|x-4| + C$

$= \ln|x-3|^4 + \ln|x-4|^3 + C$

$= \ln|(x-3)^4(x-4)^3| + C$

Q.3 $\int \frac{dx}{a^2-x^2}$

Solution:

$\int \frac{1}{a^2-x^2}dx$

$= \int \frac{1}{(a-x)(a+x)}dx$

$\frac{1}{(a-x)(a+x)} = \frac{A}{a-x} + \frac{B}{a+x}$

$\frac{1}{(a-x)(a+x)} = \frac{A(a-x)+B(a+x)}{(a-x)(a+x)}$

$1 = A(a-x) + B(a+x)$

Let $x = a$

$1 = 0 + B(a+a) \Rightarrow B = \frac{1}{2a}$

Let $x = -a$

$1 = A(a+a) + 0 \Rightarrow A = \frac{1}{2a}$

$I = \frac{1}{2a}\int \frac{1}{a-x}dx + \frac{1}{2a}\int \frac{1}{a+x}dx$

$= -\frac{1}{2a}\ln|a-x| + \frac{1}{2a}\ln|a+x| + C$

$= \frac{1}{2a}\{\ln|a+x| - \ln|a-x|\} + C$

$= \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

Q.4 $\int \frac{dx}{x^2-a^2}$

Solution:

$$\int \frac{1}{x^2 - a^2} dx$$

$$\int \frac{1}{(x-a)(x+a)} dx$$

$$\frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$$

$$\frac{1}{(x-a)(x+a)} = \frac{A(x+a) + B(x-a)}{(x-a)(x+a)}$$

$$1 = A(x+a) + B(x-a)$$

Let $x = a$

$$1 = A(a+a) + 0 \Rightarrow A = \frac{1}{2a}$$

Let $x = -a$

$$1 = 0 + B(-a-a) \Rightarrow A = -\frac{1}{2a}$$

$$\mathrm{I} = \frac{1}{2a}\int \frac{1}{x-a}dx - \frac{1}{2a}\int \frac{1}{x+a}dx$$

$$= \frac{1}{2a}\ln|x-a| - \frac{1}{2a}\ln|x+a| + C$$

$$= \frac{1}{2a}\{\ln|x-a| - \ln|x+a|\} + C$$

$$= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

Q.5 $\int \frac{(x^2+2x+3)dx}{x^3-x}$

Solution:

$$\int \frac{x^2+2x+3}{x^3-x} dx$$

$$= \int \frac{(x^2+2x+3)}{x(x^2-1)} dx$$

$$= \int \frac{x^2+2x+3}{x(x-1)(x+1)} dx$$

$$\frac{x^2+2x+3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\frac{x^2+2x+3}{x(x-1)(x+1)} = \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)}$$

$$x^2 + 2x + 3 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

Let $x = 0$

The Students' C

$0 + 0 + 3 = A($

$3 = A(-1)(1) =$

Let $x = 1$

$1 + 2 + 3 = 0 -$

Let $x = -1$

$1 - 2 + 3 = 0 \cdot$

$\mathrm{I} = -\int \frac{3}{x}dx +$

$= -3\ln|x| + 3$

$= -\ln|x|^3 + \ln$

$= \ln\left|\frac{(x-1)^3}{x^3}\right.$

Q.6 $\int \frac{5dx}{x^2-2x-15}$

Solution:

$\int \frac{5}{x^2-2x-1}$

$x^2 - 2x - 15$

$= x(x-5) +$

$= (x-5)(x+$

$\mathrm{I} = \int \frac{5}{(x-5)}$

$\mathrm{I} = 5\int \frac{}{(x-5}$

$\frac{1}{(x-5)(x+3}$

$\frac{1}{(x-5)(x+3}$

$1 = A(x+3)$

Let $x = -3$

$1 = 0 + B(-3$

Let $x = 5$

$1 = A(5+3)$

$= 5\left\{\frac{1}{8}\int \frac{1}{x-}\right.$

$0 + 0 + 3 = A(0-1)(0+1) + 0 + 0$

$3 = A(-1)(1) \Rightarrow A = -3$

Let $x = 1$

$1 + 2 + 3 = 0 + B(1+1) + 0 \Rightarrow 6 = 2B \Rightarrow B = 3$

Let $x = -1$

$1 - 2 + 3 = 0 + 0 + C(-1)(-1-1) \Rightarrow 2 = 2C \Rightarrow C = 1$

$$I = -\int \frac{3}{x}dx + \int \frac{3}{x-1}dx + \int \frac{1}{x+1}dx$$

$= -3\ln|x| + 3\ln|x-1| + \ln|x+1| + C$

$= -\ln|x|^3 + \ln|x-1|^3 + \ln|x+1| + C$

$$= \ln\left|\frac{(x-1)^3(x+1)}{x^3}\right| + C$$

Q.6 $\int \frac{5dx}{x^2-2x-15}$

Solution:

$$\int \frac{5}{x^2-2x-15}dx$$

$x^2 - 2x - 15 = x^2 - 5x + 3x - 15$

$= x(x-5) + 3(x-5)$

$= (x-5)(x+3)$

$$I = \int \frac{5}{(x-5)(x+3)}dx$$

$$I = 5\int \frac{1}{(x-5)(x+3)}dx$$

$$\frac{1}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}$$

$$\frac{1}{(x-5)(x+3)} = \frac{A(x+3) + B(x-5)}{(x-5)(x+3)}$$

$1 = A(x+3) + B(x-5)$

Let $x = -3$

$1 = 0 + B(-3-5) \Rightarrow B = -\frac{1}{8}$

Let $x = 5$

$1 = A(5+3) + 0 \Rightarrow A = \frac{1}{8}$

$$= 5\left\{\frac{1}{8}\int \frac{1}{x-5}dx - \frac{1}{8}\int \frac{1}{x+3}dx\right\}$$

$$=\frac{5}{8}\ln|x-5|-\frac{5}{8}\ln|x+3|+C$$
$$=\frac{5}{8}\{\ln|x-5|-\ln|x+3|\}+C$$
$$=\frac{5}{8}\ln\left|\frac{x-5}{x+3}\right|+C$$

Q.7 $\int\frac{(2x+7)dx}{(x-1)(x-5)(x+3)}$

Solution:

$$\int\frac{(2x+7)}{(x-1)(x-5)(x+3)}dx$$
$$\frac{(2x+7)}{(x-1)(x-5)(x+3)}=\frac{A}{x-1}+\frac{B}{x-5}+\frac{C}{x+3}$$
$$\frac{(2x+7)}{(x-1)(x-5)(x+3)}=\frac{A(x-5)(x+3)+B(x-1)(x+3)+C(x-1)(x-5)}{(x-1)(x-5)(x+3)}$$
$$2x+7=A(x-5)(x+3)+B(x-1)(x+3)+C(x-1)(x-5)$$

Let $x=1$

$$2(1)+7=A(1-5)(1+3)+0+0\Rightarrow A=-\frac{9}{16}$$

Let $x=-3$

$$2(-3)+7=0+0+C(-3-1)(-3-5)\Rightarrow C=\frac{1}{32}$$

Let $x=5$

$$2(5)+7=0+B(5-1)(5+3)+0\Rightarrow B=\frac{17}{32}$$

$$\text{I}=-\frac{9}{16}\int\frac{1}{x-1}dx+\frac{17}{32}\int\frac{1}{x-5}dx+\frac{1}{32}\int\frac{1}{x+3}dx$$
$$=-\frac{9}{16}\ln|x-1|+\frac{17}{32}\ln|x-5|+\frac{1}{32}\ln|x+3|+C$$
$$=-\frac{9}{16}\ln|x-1|+\frac{1}{32}\{17\ln|x-5|+\ln|x+3|\}+C$$

Q.8 $\int\frac{(5x+6)dx}{(x+3)(x-2)^2}$

Solution:

$$\int\frac{5x+6}{(x+3)(x-2)^2}dx$$

$\frac{5x+6}{(x+3)(x-2)^2}=$ …

$\frac{5x+6}{(x+3)(x-2)^2}=$ …

$5x+6=A(x-2)^2$ …

Let $x=2$

$5(2)+6=0+0+$ …

Let $x=-3$

$5(-3)+6=A(-3$ …

$5x+6=A(x-2)$ …

$x=0$

$6=A(-2)^2+B(3$ …

$6=4A-6B+3C$ …

$6B=4\left(-\frac{9}{25}\right)-$ …

$B=\frac{54}{25(6)}\Rightarrow B=$ …

$\text{I}=-\frac{9}{25}\int\frac{1}{(x+3}$ …

$=-\frac{9}{25}\ln|x+3|$ …

$=\frac{9}{25}\{\ln|x-2|-$ …

$=\frac{9}{25}\ln\left|\frac{x-2}{x+3}\right|-$ …

Q.9 $\int\frac{(7x^2-2x+5)d}{(x-6)(x-3)^3}$ …

Solution:

$\int\frac{(7x^2-2x+5}{(x-6)(x-3)}$ …

$\frac{7x^2-2x+5}{(x-6)(x-3)^3}$ …

$\frac{7x^2-2x+5}{(x-6)(x-3)^3}$ …

$$\frac{5x+6}{(x+3)(x-2)^2} = \frac{A}{(x+3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{5x+6}{(x+3)(x-2)^2} = \frac{A(x-2)^2 + B(x+3)(x-2) + C(x+3)}{(x+3)(x-2)^2}$$

$5x+6 = A(x-2)^2 + B(x+3)(x-2) + C(x+3)$

Let $x = 2$

$5(2)+6 = 0+0+C(2+3) \Rightarrow C = \frac{16}{5}$

Let $x = -3$

$5(-3)+6 = A(-3-2)^2 + 0 + 0 \Rightarrow A = -\frac{9}{25}$

$5x+6 = A(x-2)^2 + B(x+3)(x-2) + C(x+3)$

$x = 0$

$6 = A(-2)^2 + B(3)(-2) + C(3)$

$6 = 4A - 6B + 3C$

5) $6B = 4\left(-\frac{9}{25}\right) - 6 + 3\left(\frac{16}{5}\right)$

$B = \frac{54}{25(6)} \Rightarrow B = \frac{9}{25}$

$I = -\frac{9}{25}\int \frac{1}{(x+3)}dx + \frac{9}{25}\int \frac{1}{(x-2)}dx + \frac{16}{5}\int (x-2)^{-2}dx$

$= -\frac{9}{25}\ln|x+3| + \frac{9}{25}\ln|x-2| + \frac{16}{5}\left\{\frac{(x-2)^{-1}}{-1}\right\} + C$

$= \frac{9}{25}\{\ln|x-2| - \ln|x+3|\} - \frac{16}{5(x-2)} + C$

$= \frac{9}{25}\ln\left|\frac{x-2}{x+3}\right| - \frac{16}{5(x-2)} + C$

Q.9 $\int \frac{(7x^2-2x+5)dx}{(x-6)(x-3)^3}$

Solution:

$\int \frac{(7x^2-2x+5)}{(x-6)(x-3)^3}dx$

$$\frac{7x^2-2x+5}{(x-6)(x-3)^3} = \frac{A}{(x-6)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)^3}$$

$$\frac{7x^2-2x+5}{(x-6)(x-3)^3} = \frac{A(x-3)^3 + B(x-6)(x-3)^2 + C(x-6)(x-3) + D(x-6)}{(x-6)(x-3)^3}$$

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$$7x^2-2x+5=A(x-3)^3+B(x-6)(x-3)^2+C(x-6)(x-3)+D(x-6)$$

Let $x=3$

$$7(3)^2-2(3)+5=0+0+0+D(3-6)\Rightarrow D=\frac{62}{3}$$

Let $x=6$

$$7(6)^2-2(6)+5=A(6-3)^3+0+0+0\Rightarrow A=\frac{245}{27}$$

$$7x^2-2x+5=A(x^3-9x^2+27x-27)+B(x^3-12x^2+45x-54)+C(x^2-9x+18)+D(x-6)$$

$$7x^2-2x+5=A(x^3-9x^2+27x-27)+B(x^3-12x^2+45x-54)+C(x^2-9x+18)+D(x-6)$$

$$7x^2-2x+5=x^3(A+B)+x^2(-9A-12B+C)+x(27A+45B-9C+D)-27A-54B+18C-6D$$

$$A+B=0\Rightarrow B=-\frac{245}{27}$$

$$-9A-12B+C=7$$

$$C=7+9A+12B\Rightarrow C=7+9\left(\frac{245}{27}\right)+12\left(-\frac{245}{27}\right)=-\frac{182}{9}$$

$$\mathrm{I}=\frac{245}{27}\int\frac{1}{(x-6)}dx-\frac{245}{27}\int\frac{1}{(x-3)}dx-\frac{182}{9}\int\frac{1}{(x-3)^2}dx-\frac{62}{3}\int\frac{1}{(x-3)^3}dx$$

$$=\frac{245}{27}\ln|x-6|-\frac{245}{27}\ln|x-3|-\frac{182}{9}\int(x-3)^{-2}dx-\frac{62}{3}\int(x-3)^{-3}dx.$$

$$=\frac{245}{27}\{\ln|x-6|-\ln|x-3|\}-\frac{182}{9}\left\{\frac{(x-3)^{-1}}{-1}\right\}-\frac{62}{3}\left\{\frac{(x-3)^{-2}}{-2}\right\}+C$$

$$=\frac{245}{27}\{\ln|x-6|-\ln|x-3|\}+\frac{182}{9(x-3)}+\frac{31}{3(x-3)^2}+C$$

Q.10 $\int\frac{(2x+1)dx}{(x-3)(x^2+1)}$

The Stu...

Solution:

$\int\frac{(2x+1}{(x-3)(x^2}$

$\frac{(2x+1)}{(x-3)(x^2+}$

$\frac{(2x+1)}{(x-3)(x^2+}$

$2x+1=A($

Let $x=3$

$2(3)+1=A$

$(1)\Rightarrow 2x+1$

$2x+1=x^2($

$A+B=0\Rightarrow$

$-3B+C=$

$\mathrm{I}=\frac{7}{10}\int\frac{1}{x-}$

$=\frac{7}{10}\ln|x-$

$=\frac{7}{10}\ln|x-$

$=\frac{7}{10}\ln|x-$

$=\frac{7}{10}\ln|x-$

$=\frac{1}{20}\{14\ln|x$

Q.11 $\int\frac{\sec}{(1+\tan x}$

Solution:

$\int\frac{\sec}{(1+\tan x)}$

Let $y=\tan x$

Solution:

$$\int \frac{(2x+1)}{(x-3)(x^2+1)}dx$$

$$\frac{(2x+1)}{(x-3)(x^2+1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+1}$$

$$\frac{(2x+1)}{(x-3)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x-3)}{(x-3)(x^2+1)}$$

$$2x+1 = A(x^2+1)+(Bx+C)(x-3) \rightarrow (1)$$

Let $x = 3$

$$2(3)+1 = A((3)^2+1)+0 \Rightarrow A = \frac{7}{10}$$

$$(1) \Rightarrow 2x+1 = Ax^2 + A + Bx^2 - 3Bx + Cx - 3C$$

$$2x+1 = x^2(A+B) + x(-3B+C) + A - 3C$$

$$A+B = 0 \Rightarrow B = -\frac{7}{10}$$

$$-3B+C = 2 \Rightarrow C = 2+3B \Rightarrow C = -\frac{1}{10}$$

$$I = \frac{7}{10}\int\frac{1}{x-3}dx + \int\frac{-\frac{7}{10}x - \frac{1}{10}}{x^2+1}dx$$

$$= \frac{7}{10}\ln|x-3| - \frac{7}{10}\int\frac{x+\frac{1}{7}}{x^2+1}dx$$

$$= \frac{7}{10}\ln|x-3| - \frac{7}{10}\left(\frac{1}{2}\right)\int\frac{2x+\frac{2}{7}}{x^2+1}dx$$

$$= \frac{7}{10}\ln|x-3| - \frac{7}{20}\int\left\{\frac{2x}{x^2+1} + \frac{\frac{2}{7}}{x^2+1}\right\}dx$$

$$= \frac{7}{10}\ln|x-3| - \frac{7}{20}\left\{\ln|x^2+1| + \frac{2}{7}\left(\frac{1}{1}\right)\tan^{-1}\frac{x}{1}\right\} + C$$

$$= \frac{1}{20}\left\{14\ln|x-3| - \ln|x^2+1| - \frac{2}{7}\tan^{-1}x\right\} + C$$

Q.11 $\int \frac{\sec^2 x dx}{(1+\tan x)(2+\tan x)}$

Solution:

$$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$$

Let $y = \tan x \Rightarrow dy = \sec^2 x\, dx$

$$I = \int \frac{1}{(1+y)(2+y)} dy$$

$$\frac{1}{(1+y)(2+y)} = \frac{A}{1+y} + \frac{B}{2+y}$$

$$\frac{1}{(1+y)(2+y)} = \frac{A(2+y)+B(1+y)}{(1+y)(2+y)}$$

$$1 = A(2+y) + B(1+y)$$

Let $y = -1$

$$1 = A(2-1) + 0 \Rightarrow A = 1$$

Let $y = -2$

$$1 = 0 + B(1-2) \Rightarrow B = -1$$

$$I = \int \frac{1}{1+y} dy - \int \frac{1}{2+y} dy$$

$$= \ln|1+y| - \ln|2+y| + C$$

$$= \ln\left|\frac{1+y}{2+y}\right| + C$$

$$= \ln\left|\frac{1+\tan x}{2+\tan x}\right| + C$$

Q.12 $\int \frac{\operatorname{cosec}^2 x dx}{\cot x(2+\cot x)}$

Solution:

$$\int \frac{\operatorname{cosec}^2 x}{\cot x\,(2+\cot x)} dx$$

Let $y = \cot x \Rightarrow -dy = \operatorname{cosec}^2 x\, dx$

$$I = -\int \frac{1}{y(2+y)} dy$$

$$\frac{1}{y(2+y)} = \frac{A}{y} + \frac{B}{2+y}$$

$$\frac{1}{y(2+y)} = \frac{A(2+y)+By}{y(2+y)}$$

$$1 = A(2+y) + By$$

Let $y = 0$

$$1 = A(2+0) + 0 \Rightarrow A = \frac{1}{2}$$

Let $y = -2$

$$1 = 0 - 2B \Rightarrow B = -\frac{1}{2}$$

$I = -\left\{\frac{1}{2}\int\right.$

$= -\frac{1}{2}\int \frac{1}{y}$

$= -\frac{1}{2}\ln|y|$

$= \frac{1}{2}\{\ln|2+$

$= \frac{1}{2}\ln\left|\frac{2+}{y}\right.$

$= \frac{1}{2}\ln\left|\frac{2+}{c}\right.$

$= \frac{1}{2}\ln\left|\frac{2}{\cot}\right.$

$= \frac{1}{2}\ln|2\ \text{ta}$

$= \ln|2\tan$

$= \ln\sqrt{2}\ \text{ta}$

Q.13 $\int \frac{(3}{(2x-}$

Solution:

$\int \frac{(3x}{(2x-1)}$

$\frac{(3x+}{(2x-1)(x}$

$\frac{(3x+}{(2x-1)(x}$

$3x + 7 = A$

Let $x = 4$

$3(4) + 7 =$

Let $2x - 1$

$3\left(\frac{1}{2}\right) + 7 =$

Let $x = 0$

$(1) \Rightarrow 0 + 7$

$7 = 16A +$

$I = -\left\{\frac{1}{2}\int\frac{1}{y}dy - \frac{1}{2}\int\frac{1}{2+y}dy\right\}$

$= -\frac{1}{2}\int\frac{1}{y}dy + \frac{1}{2}\int\frac{1}{2+y}dy$

$= -\frac{1}{2}\ln|y| + \frac{1}{2}\ln|2+y| + C$

$= \frac{1}{2}\{\ln|2+y| - \ln|y|\} + C$

$= \frac{1}{2}\ln\left|\frac{2+y}{y}\right| + C$

$= \frac{1}{2}\ln\left|\frac{2+\cot x}{\cot x}\right| + C$

$= \frac{1}{2}\ln\left|\frac{2}{\cot x} + \frac{\cot x}{\cot x}\right| + C$

$= \frac{1}{2}\ln|2\tan x + 1| + C$

$= \ln|2\tan x + 1|^{\frac{1}{2}} + C$

$= \ln\sqrt{2\tan x + 1} + C$

Q.13 $\int\frac{(3x+7)dx}{(2x-1)(x-4)^2}$

Solution:

$$\int\frac{(3x+7)}{(2x-1)(x-4)^2}dx$$

$$\frac{(3x+7)}{(2x-1)(x-4)^2} = \frac{A}{2x-1} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$$

$$\frac{(3x+7)}{(2x-1)(x-4)^2} = \frac{A(x-4)^2 + B(2x-1)(x-4) + C(2x-1)}{(2x-1)(x-4)^2}$$

$3x+7 = A(x-4)^2 + B(2x-1)(x-4) + C(2x-1) \rightarrow (1)$

Let $x = 4$

$3(4)+7 = 0+0+C(2(4)-1) \Rightarrow C = \frac{19}{7}$

Let $2x-1 = 0 \Rightarrow x = \frac{1}{2}$

$3\left(\frac{1}{2}\right) + 7 = A\left(\frac{1}{2}-4\right)^2 + 0 + 0 \Rightarrow A = \frac{34}{49}$

Let $x = 0$

$(1) \Rightarrow 0+7 = A(0-4)^2 + B(0-1)(0-4) + C(0-1)$

$7 = 16A + 4B - C$

$4B = 7 + C - 16A \Rightarrow 4B = 7 + \frac{\;}{7} - 16\left(\frac{\;}{49}\right) \Rightarrow B = -\frac{\;}{49}$

$$I = \frac{34}{49}\int \frac{1}{2x-1}dx - \frac{17}{49}\int \frac{1}{x-4}dx + \frac{19}{7}\int (x-4)^{-2}dx$$

$$= \frac{34}{49(2)}\ln|2x-1| - \frac{17}{49}\ln|x-4| + \frac{19}{7}\left\{\frac{(x-4)^{-1}}{-1}\right\} + C$$

$$= \frac{17}{49}\ln|2x-1| - \frac{17}{49}\ln|x-4| - \frac{19}{7(x-4)} + C$$

$$= \frac{17}{49}\{\ln|2x-1| - \ln|x-4|\} - \frac{19}{7(x-4)} + C$$

$$= \frac{17}{49}\ln\left|\frac{2x-1}{x-4}\right| - \frac{19}{7(x-4)} + C$$

Q.14 $\int \frac{(7x-4)dx}{(x-3)(x^2+2)}$

Solution:

$$\int \frac{(7x-4)}{(x-3)(x^2+2)}dx$$

$$\frac{(7x-4)}{(x-3)(x^2+2)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+2}$$

$$\frac{(7x-4)}{(x-3)(x^2+2)} = \frac{A(x^2+2) + (Bx+C)(x-3)}{(x-3)(x^2+2)}$$

$7x - 4 = A(x^2+2) + (Bx+C)(x-3) \rightarrow (1)$

Let $x = 3$

$$7(3) - 4 = A((3)^2+2) + 0 \Rightarrow A = \frac{17}{11}$$

$(1) \Rightarrow 7x - 4 = A(x^2+2) + (Bx+C)(x-3)$

$7x - 4 = Ax^2 + 2A + Bx^2 - 3Bx + Cx - 3C$

$7x - 4 = x^2(A+B) + x(-3B+C) + 2A - 3C$

$$A + B = 0 \Rightarrow B = -\frac{17}{11}$$

$$-3B + C = 7 \Rightarrow C = 7 + 3\left(-\frac{17}{11}\right) \Rightarrow C = \frac{26}{11}$$

$$I = \frac{17}{11}\int \frac{1}{x-3}dx + \int \frac{-\frac{17}{11}x + \frac{26}{11}}{x^2+2}dx$$

$$= \frac{17}{11}\ln|x-3| - \frac{17}{11}\int \frac{x}{x^2+2}dx + \frac{26}{11}\int \frac{1}{x^2+2}dx$$

$$= \frac{17}{11}\ln|x-3| - \frac{17}{11(2)}\ln|x^2+2| + \frac{26}{11}\left(\frac{1}{\sqrt{2}}\right)\tan^{-1}\frac{x}{\sqrt{2}} + C$$

$= \frac{17}{11}\ln|x-3| - \frac{1}{1}$ …

$= \frac{17}{11}\ln|x-3| - \frac{1}{1}$ …

$= \frac{17}{11}\{\ln|x-3| - \ln$ …

$= \frac{17}{11}\ln\left|\frac{x-3}{\sqrt{x^2+2}}\right|$ …

Q.15 $\int \frac{(2x^2+5x+1)dx}{x^2+5x+6}$

Solution:

$\int \frac{(2x^2+5x+1)}{x^2+5x+6}$ …

$\frac{2x^2+5x+1}{x^2+5x+6} = \frac{2}{}$ …

$= \frac{2(x^2+5x+6)}{x^2+5x}$ …

$= \frac{2(x^2+5x+6)}{x^2+5x+6}$ …

$= 2 - \frac{5x+11}{x^2+5x+}$ …

$x^2 + 5x + 6 = x^2$ …

$= x(x+2) + 3(x$ …

$= (x+2)(x+3)$

$\int \frac{(2x^2+5x+1)}{x^2+5x+6}$ …

$= 2x - \int \frac{5x}{(x+2)}$ …

$\frac{5x+11}{(x+2)(x+3)} =$ …

$\frac{5x+11}{(x+2)(x+3)} =$ …

$5x + 11 = A(x+$ …

Let $x = -2$

$5(-2) + 11 = A($ …

Let $x = -3$

$$=\frac{17}{11}\ln|x-3|-\frac{17}{11}\ln|x^2+2|^{\frac{1}{2}}+\frac{26}{11}\left(\frac{\sqrt{2}}{2}\right)\tan^{-1}\frac{x}{\sqrt{2}}+C$$

$$=\frac{17}{11}\ln|x-3|-\frac{17}{11}\ln\left|\sqrt{x^2+2}\right|+\frac{13\sqrt{2}}{11}\tan^{-1}\frac{x}{\sqrt{2}}+C$$

$$=\frac{17}{11}\left\{\ln|x-3|-\ln\left|\sqrt{x^2+2}\right|\right\}+\frac{13\sqrt{2}}{11}\tan^{-1}\frac{x}{\sqrt{2}}+C$$

$$=\frac{17}{11}\ln\left|\frac{x-3}{\sqrt{x^2+2}}\right|+\frac{13\sqrt{2}}{11}\tan^{-1}\frac{x}{\sqrt{2}}+C$$

Q.15 $\int\frac{(2x^2+5x+1)dx}{x^2+5x+6}$

Solution:

$$\int\frac{(2x^2+5x+1)}{x^2+5x+6}dx$$

$$\frac{2x^2+5x+1}{x^2+5x+6}=\frac{2(x^2+5x+6)-2x^2-10x-12+(2x^2+5x+1)}{x^2+5x+6}$$

$$=\frac{2(x^2+5x+6)-5x-11}{x^2+5x+6}$$

$$=\frac{2(x^2+5x+6)}{x^2+5x+6}-\frac{5x+11}{x^2+5x+6}$$

$$=2-\frac{5x+11}{x^2+5x+6}$$

$$x^2+5x+6=x^2+2x+3x+6$$

$$=x(x+2)+3(x+2)$$

$$=(x+2)(x+3)$$

$$\int\frac{(2x^2+5x+1)}{x^2+5x+6}dx=\int\left\{2-\frac{5x+11}{(x+2)(x+3)}\right\}dx$$

$$=2x-\int\frac{5x+11}{(x+2)(x+3)}dx\rightarrow(1)$$

$$\frac{5x+11}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$$

$$\frac{5x+11}{(x+2)(x+3)}=\frac{A(x+3)+B(x+2)}{(x+2)(x+3)}$$

$$5x+11=A(x+3)+B(x+2)$$

Let $x=-2$

$$5(-2)+11=A(-2+3)+0\Rightarrow A=1$$

Let $x=-3$

$5(-3) + 11 = 0 + B(-3+2) \Rightarrow B = 4$

$(1) \Rightarrow 2x - \int \frac{1}{x+2}dx - 4\int \frac{1}{x+3}dx$

$= 2x - \ln|x+2| - 4c + C$

$= 2x - \{\ln|x+2| + \ln|x+3|^4\} + C$

$= 2x - \{\ln|(x+2)(x+3)^4|\} + C$

Q.16 $\int \frac{(x^3+3x+1)dx}{x^2+5x-14}$

Solution:

$$\int \frac{x^3+3x+1}{x^2+5x-14}dx$$

$$\begin{array}{r|l} & x-5 \\ \hline x^2+5x-14 & x^3+0x^2+3x+1 \\ & \pm x^3 \pm 5x^2 \mp 14x \\ \hline & -5x^2+17x+1 \\ & \mp 5x^2 \mp 25x \pm 70 \\ \hline & 42x-69 \end{array}$$

$$\frac{x^3+3x+1}{x^2+5x-14} = (x-5) + \frac{42x-69}{x^2+5x-14} \rightarrow (1)$$

$x^2+5x-14 = x^2+7x-2x-14$

$= x(x+7) - 2(x+7)$

$= (x+7)(x-2)$

$$(1) \Rightarrow \frac{x^3+3x+1}{x^2+5x-14} = (x-5) + \frac{42x-69}{(x+7)(x-2)}$$

$$I = \int (x-5)dx + \int \frac{42x-69}{(x+7)(x-2)}dx$$

$$= \frac{x^2}{2} - 5x + \int \frac{42x-69}{(x+7)(x-2)}dx \rightarrow (2)$$

$$\frac{42x-69}{(x+7)(x-2)} = \frac{A}{x+7} + \frac{B}{x-2}$$

$$\frac{42x-69}{(x+7)(x-2)} = \frac{A(x-2)+B(x+7)}{(x+7)(x-2)}$$

Let $x = 2$

$42(2) - 69 = 0 + B(2+7) \Rightarrow A = \frac{5}{3}$

Let $x = -7$

$42(-7) - 69 = A(-7-2) + 0 \Rightarrow B = \frac{121}{3}$

$(2) \Rightarrow I = \frac{x^2}{2} -$

$= \frac{x^2}{2} - 5x + \frac{5}{3}l$

$= \frac{x^2}{2} - 5x + \frac{1}{3}\{$

Q.1 Evaluate th

(i) $\int_0^2 (4x^3 + 3x$

(iii) $\int_1^2 (\theta + \sqrt{\theta})$

(v) $\int_0^4 \frac{dx}{\sqrt{2+x}+\sqrt{x}}$

Solution:

(i) $\int_0^2 (4x^3 + 3x$

$= \left[4\left(\frac{x^4}{4}\right) + 3\right.$

$= [x^4 + x^3 +$

$= (2^4 - 0) + ($

$= 16 + 8 + 10$

$= 34$

(ii) $\int_{-1}^2 (x^2 + 1)$

$= \int_{-1}^2 (x^4 + 2x$

$= \left[\frac{x^5}{5} + 2\left(\frac{x^3}{3}\right.\right.$

$= \frac{1}{5}\{2^5 - (-1$

$= \frac{1}{5}\{33\} + \frac{2}{3}\{$

$= \frac{33}{5} + 6 + 3$

$= \frac{78}{5}$

(iii) $\int_1^2 (\theta + \sqrt{\theta}$

(2) $\Rightarrow I = \dfrac{x^2}{2} - 5x + \dfrac{5}{3}\int \dfrac{1}{x+7}dx + \dfrac{121}{3}\int \dfrac{1}{x-2}dx$

$= \dfrac{x^2}{2} - 5x + \dfrac{5}{3}\ln|x+7| + \dfrac{121}{3}\ln|x-2| + C$

$= \dfrac{x^2}{2} - 5x + \dfrac{1}{3}\{5\ln|x+7| + 121\ln|x-2|\} + C$

EXERCISE 6.6

Q.1 Evaluate the following definite integrals.

(i) $\int_0^2 (4x^3 + 3x^2 + 5)dx$

(ii) $\int_{-1}^2 (x^2 + 1)^2 dx$

(iii) $\int_1^2 \left(\theta + \sqrt{\theta}\right)^3 d\theta$

(iv) $\int_1^3 \left(y + \frac{1}{\sqrt{y}}\right)^2 dy$

(v) $\int_0^4 \frac{dx}{\sqrt{2+x}+\sqrt{x}}$

Solution:

(i) $\int_0^2 (4x^3 + 3x^2 + 5)dx$

$= \left[4\left(\dfrac{x^4}{4}\right) + 3\left(\dfrac{x^3}{3}\right) + 5x\right]_0^2$

$= [x^4 + x^3 + 5x]_0^2$

$= (2^4 - 0) + (2^3 - 0) + 5(2 - 0)$

$= 16 + 8 + 10$

$= 34$

(ii) $\int_{-1}^2 (x^2 + 1)^2 dx$

$= \displaystyle\int_{-1}^{2} (x^4 + 2x^2 + 1)dx$

$= \left[\dfrac{x^5}{5} + 2\left(\dfrac{x^3}{3}\right) + x\right]_{-1}^2$

$= \dfrac{1}{5}\{2^5 - (-1)^5\} + \dfrac{2}{3}\{2^3 - (-1)^3\} + 2 - (-1)$

$= \dfrac{1}{5}\{33\} + \dfrac{2}{3}\{9\} + 3$

$= \dfrac{33}{5} + 6 + 3$

$= \dfrac{78}{5}$

(iii) $\int_1^2 \left(\theta + \sqrt{\theta}\right)^3 d\theta$

$$=\int_1^2\left(\theta+\theta^{\frac{1}{2}}\right)^3 d\theta$$

$$=\int_1^2\left\{(\theta)^3+3(\theta)^2\left(\theta^{\frac{1}{2}}\right)+3(\theta)\left(\theta^{\frac{1}{2}}\right)^2+\left(\theta^{\frac{1}{2}}\right)^3\right\}d\theta$$

$$=\int_1^2\left\{\theta^3+3\theta^2\left(\theta^{\frac{1}{2}}\right)+3\theta(\theta)+\theta^{\frac{3}{2}}\right\}d\theta$$

$$=\int_1^2\left\{\theta^3+3\theta^{\frac{5}{2}}+3\theta^2+\theta^{\frac{3}{2}}\right\}d\theta$$

$$=\left[\frac{\theta^4}{4}+\frac{3\theta^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\theta^3}{3}+\frac{\theta^{\frac{5}{2}}}{\frac{5}{2}}\right]_1^2$$

$$=\left[\frac{\theta^4}{4}+\frac{6}{7}\theta^{\frac{7}{2}}+\theta^3+\frac{2}{5}\theta^{\frac{5}{2}}\right]_1^2$$

$$=\frac{1}{4}(2^4-1^4)+\frac{6}{7}\left(2^{\frac{7}{2}}-1^{\frac{7}{2}}\right)+(2^3-1^3)+\frac{2}{5}\left(2^{\frac{5}{2}}-1^{\frac{5}{2}}\right)$$

$$=\frac{15}{4}+\frac{6}{7}\left(2^3\left(2^{\frac{1}{2}}\right)-1\right)+(7)+\frac{2}{5}\left(2^2\left(2^{\frac{1}{2}}\right)-1\right)$$

$$=\frac{43}{4}+\frac{6(8\sqrt{2}-1)}{7}+\frac{2(4\sqrt{2}-1)}{5}$$

$$=\frac{43(35)+20(48\sqrt{2}-6)+28(8\sqrt{2}-2)}{140}$$

$$=\frac{1505+(960\sqrt{2}-120)+(224\sqrt{2}-56)}{140}$$

$$=\frac{1329+1184\sqrt{2}}{140}$$

$$=21.45$$

(iv) $\int_1^3\left(y+\frac{1}{\sqrt{y}}\right)^2 dy$

$$=\int_1^3\left(y+\frac{1}{\sqrt{y}}\right)^2 dy$$

$$=\int_1^3\left\{(y)^2+2(y)\left(\frac{1}{\sqrt{y}}\right)+\left(\frac{1}{\sqrt{y}}\right)^2\right\}dy$$

$$=\int_1^3\left\{y^2+2\sqrt{y}+\frac{1}{y}\right\}dy$$

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$$=4\sqrt{3}+\ln 3$$

(v) $\int_0^4 \frac{dx}{\sqrt{2+x}+\sqrt{x}}$

$$=\int_1^3\left\{y^2+2y^{\frac{1}{2}}+\frac{1}{y}\right\}dy$$

$$=\left[\frac{y^3}{3}+\frac{2y^{\frac{3}{2}}}{\frac{3}{2}}+\ln y\right]_1^3$$

$$=\frac{1}{3}(3^3-1^3)+\frac{4}{3}\left(3^{\frac{3}{2}}-1^{\frac{3}{2}}\right)+\ln 3-\ln 1$$

$$=\frac{26}{3}+\frac{4}{3}(3\sqrt{3}-1)+\ln 3-0$$

$$=\frac{26}{3}+4\sqrt{3}-\frac{4}{3}+\ln 3$$

$$=4\sqrt{3}+\ln 3+\frac{22}{3}$$

(v) $\int_0^4\frac{dx}{\sqrt{2+x}+\sqrt{x}}$

$$=\int_0^4\frac{1}{\sqrt{2+x}+\sqrt{x}}\left(\frac{\sqrt{2+x}-\sqrt{x}}{\sqrt{2+x}-\sqrt{x}}\right)dx$$

$$=\int_0^4\frac{\sqrt{2+x}-\sqrt{x}}{\left(\sqrt{2+x}\right)^2-\left(\sqrt{x}\right)^2}dx$$

$$=\int_0^4\frac{\sqrt{2+x}-\sqrt{x}}{2+x-x}dx$$

$$=\int_0^4\frac{\sqrt{2+x}-\sqrt{x}}{2}dx$$

$$=\frac{1}{2}\int_0^4\left\{(2+x)^{\frac{1}{2}}-x^{\frac{1}{2}}\right\}dx$$

$$=\frac{1}{2}\left[\frac{(2+x)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$=\frac{1}{2}\left(\frac{2}{3}\right)\left[(2+4)^{\frac{3}{2}}-(2+0)^{\frac{3}{2}}-\left\{(2^2)^{\frac{3}{2}}-0^{\frac{3}{2}}\right\}\right]$$

$$=\frac{1}{3}\left[(6)^{\frac{3}{2}}-(2)^{\frac{3}{2}}-\{8-0\}\right]$$

$$=\frac{1}{3}\left[6\sqrt{6}-2\sqrt{2}-8\right]$$

$$=\frac{2}{3}\left[3\sqrt{6}-\sqrt{2}-4\right]$$

Q.2 Evaluate the following definite integrals by formula.

(i) $\int_1^4 x(x^2+9)^{\frac{3}{2}}dx$

(ii) $\int_2^5 \frac{xdx}{7x^2+2}$

(iii) $\int_0^3 \frac{(2x+3)dx}{\sqrt{2x^2+6x+5}}$

(iv) $\int_1^2 (x^3+2x)^{-\frac{1}{2}}(3x^2+2)dx$

(v) $\int_0^{\frac{\pi}{2}} \cos^3 x\,dx$

(vi) $\int_{\frac{\pi^2}{4}}^{\frac{\pi^2}{36}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

(vii) $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\sec^2 x\,dx$

(viii) $\int_0^2 e^{5x-2}dx$

(ix) $\int_0^{\frac{\pi}{3}} \cos^3 3x \sin^2 3x\,dx$

(viii) $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^4 x\,dx$

Solution:

(i) $\int_1^4 x(x^2+9)^{\frac{3}{2}}dx$

$$=\frac{1}{2}\int_1^4 (x^2+9)^{\frac{3}{2}}(2x)dx$$

$$=\frac{1}{2}\left[\frac{(x^2+9)^{\frac{5}{2}}}{\frac{5}{2}}\right]_1^4$$

$$=\frac{1}{5}\left[(x^2+9)^{\frac{5}{2}}\right]_1^4$$

$$=\frac{1}{5}\left[(4^2+9)^{\frac{5}{2}}-(1+9)^{\frac{5}{2}}\right]$$

$$=\frac{1}{5}\left[(5^2)^{\frac{5}{2}}-(10)^{\frac{5}{2}}\right]$$

$$=\frac{1}{5}\left[(5)^5-(10)^2(10)^{\frac{1}{2}}\right]$$

$$=\frac{1}{5}\left[(5)^2(5)^3-(2)^2(5)^2(10)^{\frac{1}{2}}\right]$$

$$=\frac{(5)^2}{5}\left[(5)^3-(2)^2\sqrt{10}\right]$$

$$=5\left[125-4\sqrt{10}\right]$$

(ii) $\int_2^5 \frac{xdx}{7x^2+2}$

$$=\frac{1}{14}\int_2^5 \frac{14xdx}{7x^2+2}$$

$$=\frac{1}{14}\left[\ln|7x^2+2|\right]_2^5$$

$$=\frac{1}{14}\left[\ln|7(5^2)+2|-\ln|7(2^2)+2|\right]$$

$$=\frac{1}{14}[\ln 177 - \ldots$$

$$=\frac{1}{14}\ln\left(\frac{177}{30}\right)$$

$$=\frac{1}{14}\ln\left(\frac{59}{10}\right)$$

(iii) $\int_0^3 \frac{(2x+3)dx}{\sqrt{2x^2+6x+\ldots}}$

$$=\int_0^3 \frac{2x+\ldots}{(2x^2+6\ldots}$$

$$=\frac{1}{2}\int_0^3 (2x^2+\ldots$$

$$=\frac{1}{2}\int_0^3 (2x^2+\ldots$$

$$=\frac{1}{2}\left[\frac{(2x^2+6x\ldots}{\frac{1}{2}}\right.$$

$$=\left[\sqrt{2x^2+6x\ldots}\right.$$

$$=\sqrt{2(3)^2+6(\ldots}$$

$$=\sqrt{41}-\sqrt{5}$$

(iv) $\int_1^2 (x^3+2x)\ldots$

$$=\left[\frac{(x^3+2x)^{\frac{1}{2}}}{\frac{1}{2}}\right]$$

$$=2\left[\sqrt{2^3+2(2\ldots}\right.$$

$$=2\left[\sqrt{12}-\sqrt{3}\right]$$

$$=2\left[2\sqrt{3}-\sqrt{3}\right]$$

$$=2\left[\sqrt{3}\right]$$

$$=2\sqrt{3}$$

(v) $\int_0^{\frac{\pi}{2}} \cos^3 x\,dx$

$$=\int_0^{\frac{\pi}{2}} \cos x \cos^2 \ldots$$

2) dx

$= \frac{1}{14}[\ln 177 - \ln 30]$

$= \frac{1}{14}\ln\left(\frac{177}{30}\right)$

$= \frac{1}{14}\ln\left(\frac{59}{10}\right)$

(iii) $\int_0^3 \frac{(2x+3)dx}{\sqrt{2x^2+6x+5}}$

$$= \int_0^3 \frac{2x+3}{(2x^2+6x+5)^{\frac{1}{2}}}dx$$

$$= \frac{1}{2}\int_0^3 (2x^2+6x+5)^{-\frac{1}{2}}(2)(2x+3)dx$$

$$= \frac{1}{2}\int_0^3 (2x^2+6x+5)^{-\frac{1}{2}}(4x+6)dx$$

$$= \frac{1}{2}\left[\frac{(2x^2+6x+5)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^3$$

$$= \left[\sqrt{2x^2+6x+5}\right]_0^3$$

$$= \sqrt{2(3)^2+6(3)+5} - \sqrt{0+0+5}$$

$$= \sqrt{41} - \sqrt{5}$$

(iv) $\int_1^2 (x^3+2x)^{-\frac{1}{2}}(3x^2+2)dx$

$$= \left[\frac{(x^3+2x)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^2$$

$$= 2\left[\sqrt{2^3+2(2)} - \sqrt{(1)^3+2(1)}\right]$$

$$= 2[\sqrt{12} - \sqrt{3}]$$

$$= 2[2\sqrt{3} - \sqrt{3}]$$

$$= 2[\sqrt{3}]$$

$$= 2\sqrt{3}$$

(v) $\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$

$$= \int_0^{\frac{\pi}{2}} \cos x \cos^2 x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x\,(1 - \sin^2 x)dx$$

$$= \int_0^{\frac{\pi}{2}} (\cos x - \sin^2 x \cos x)dx$$

$$= \left[\sin x - \frac{\sin^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin 0 - \frac{1}{3}\left(\sin^3\frac{\pi}{2} - \sin^3 0\right)$$

$$= 1 - 0 - \frac{1}{3}(1 - 0)$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

(vi) $\int_{\frac{\pi^2}{4}}^{\frac{\pi^2}{36}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

$$= 2\int_{\frac{\pi^2}{4}}^{\frac{\pi^2}{36}} \sin\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)dx$$

$$= -2\left[\cos\sqrt{x}\right]_{\frac{\pi^2}{4}}^{\frac{\pi^2}{36}}$$

$$= -2\left[\cos\sqrt{\frac{\pi^2}{36}} - \cos\sqrt{\frac{\pi^2}{4}}\right]$$

$$= -2\left[\cos\frac{\pi}{6} - \cos\frac{\pi}{2}\right]$$

$$= -2\left[\frac{\sqrt{3}}{2} - 0\right]$$

$$= -\sqrt{3}$$

(vii) $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\sec^2 x\,dx$

$$= \int_0^{\frac{\pi}{4}} (\tan x)^{\frac{1}{2}}\sec^2 x\,dx$$

$$= \left[\frac{(\tan x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{\pi}{4}}$$

$= \frac{2}{3}\left[\left(\tan\frac{\pi}{4}\right)^{\frac{3}{2}}\right.$

$= \frac{2}{3}[1 - 0]$

$= \frac{2}{3}$

(viii) $\int_0^2 e^{5x-2}$

$= \frac{1}{5}\int_0^2 e^{5x-2}$

$= \frac{1}{5}[e^{5x-2}]_0^2$

$= \frac{1}{5}[e^{5(2)-2}$

$= \frac{1}{5}[e^8 - e^-$

$= \frac{1}{5}\left[e^8 - \frac{1}{e^2}\right.$

$= \frac{1}{5}\left[\frac{e^{10} - 1}{e^2}\right.$

$= \frac{e^{10} - 1}{5e^2}$

(ix) $\int_0^{\frac{\pi}{3}} \cos^3 3$

$= \int_0^{\frac{\pi}{3}} \cos^2 3$

$= \int_0^{\frac{\pi}{3}} \sin^2 3x$

$= \int_0^{\frac{\pi}{3}} (\sin^2 3$

$= \frac{1}{3}\int_0^{\frac{\pi}{3}} \{\sin^2$

$= \frac{1}{3}\left[\frac{\sin^3 3x}{3}\right.$

$= \frac{1}{3}\left[\frac{1}{3}(\sin^3 3\right.$

$$=\frac{2}{3}\left[\left(\tan\frac{\pi}{4}\right)^{\frac{3}{2}}-(\tan 0)^{\frac{3}{2}}\right]$$

$$=\frac{2}{3}[1-0]$$

$$=\frac{2}{3}$$

(viii) $\int_0^2 e^{5x-2}dx$

$$=\frac{1}{5}\int_0^2 e^{5x-2}(5)dx$$

$$=\frac{1}{5}[e^{5x-2}]_0^2$$

$$=\frac{1}{5}[e^{5(2)-2}-e^{5(0)-2}]$$

$$=\frac{1}{5}[e^8-e^{-2}]$$

$$=\frac{1}{5}\left[e^8-\frac{1}{e^2}\right]$$

$$=\frac{1}{5}\left[\frac{e^{10}-1}{e^2}\right]$$

$$=\frac{e^{10}-1}{5e^2}$$

(ix) $\int_0^{\frac{\pi}{3}}\cos^3 3x\sin^2 3x\,dx$

$$=\int_0^{\frac{\pi}{3}}\cos^2 3x\cos 3x\sin^2 3x\,dx$$

$$=\int_0^{\frac{\pi}{3}}\sin^2 3x\cos 3x\,(1-\sin^2 3x)dx$$

$$=\int_0^{\frac{\pi}{3}}(\sin^2 3x\cos 3x-\sin^4 3x\cos 3x)dx$$

$$=\frac{1}{3}\int_0^{\frac{\pi}{3}}\{\sin^2 3x\,(3\cos 3x)-\sin^4 3x\,(3\cos 3x)\}\,dx$$

$$=\frac{1}{3}\left[\frac{\sin^3 3x}{3}-\frac{\sin^5 3x}{5}\right]_0^{\frac{\pi}{3}}$$

$$=\frac{1}{3}\left[\frac{1}{3}\left(\sin^3 3\frac{\pi}{3}-\sin^3 0\right)-\frac{1}{5}\left(\sin^5 3\frac{\pi}{3}-\sin^5 0\right)\right]$$

$= \frac{1}{3}\left[\frac{1}{3}(0-0) - \frac{1}{5}(0-0)\right]$

$= \frac{1}{3}[0]$

$= 0$

(viii) $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^4 x\, dx$

$= \int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \sec^2 x\, dx$

$= \int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x\,(1+\tan^2 x)dx$

$= \int_0^{\frac{\pi}{3}} (\tan^2 x \sec^2 x + \tan^4 x \sec^2 x)dx$

$= \left[\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5}\right]_0^{\frac{\pi}{3}}$

$= \frac{1}{3}\left(\tan^3\frac{\pi}{3} - \tan^3 0\right) + \frac{1}{5}\left(\tan^5\frac{\pi}{3} - \tan^5 0\right)$

$= \frac{1}{3}\left((\sqrt{3})^3 - 0\right) + \frac{1}{5}\left((\sqrt{3})^5 - 0\right)$

$= \frac{1}{3}(3\sqrt{3}) + \frac{1}{5}(9\sqrt{3})$

$= \sqrt{3} + \frac{9\sqrt{3}}{5}$

$= \frac{5\sqrt{3}+9\sqrt{3}}{5}$

$= \frac{14\sqrt{3}}{5}$

Q.3 Evaluate the following definite integrals by using basic properties.

(i) $\int_2^2 (x^4+x+3)^{\frac{5}{2}}(2x^3+1)dx$

(ii) $\int_{-50}^{50}(10x^9 - 8x^7 + 6x^5 - 4x^3 + 2x)dx$

(iii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \cos^6 x\, dx$

(iv) $\int_{-\pi}^{\pi} \sec^8 x \tan x\, dx$

(v) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x\, dx$

(vi) $\int_{-\pi}^{\pi} \tan^2 x\, dx$

(vii) $\int_{-2}^{2}(x^4+2x^2)dx$

Solution:

(i) $\int_2^2 (x^4 + x +$

$\int_a^a f(x)dx = 0$

$\int_2^2 (x^4 + x + 3$

(ii) $\int_{-50}^{50}(10x^9 -$

$f(x) = 10x^9 -$

$f(-x) = 10(-$

$f(-x) = -(10$

$f(-x) = -f(x$

$f(x)$ is odd fun

If $f(x)$ is odd f

$\int_{-50}^{50}(10x^9 - 8$

(iii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x\, c$

$f(x) = \sin^9 x\, c$

$f(-x) = \sin^9($

$f(-x) = -\sin$

$f(-x) = -f($

$f(x)$ is odd fun

If $f(x)$ is odd f

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \cos^6$

(iv) $\int_{-\pi}^{\pi} \sec^8 x\, t$

$f(x) = \sec^8 x\, t$

$f(-x) = \sec^8($

$f(-x) = -\sec^8$

$f(-x) = -f(x$

$f(x)$ is odd func

If $f(x)$ is odd fu

Solution:

(i) $\int_{-2}^{2}(x^4+x+3)^{\frac{5}{2}}(2x^3+1)dx$

$\int_{-a}^{a}f(x)dx=0$

$\int_{-2}^{2}(x^4+x+3)^{\frac{5}{2}}(2x^3+1)dx=0$

(ii) $\int_{-50}^{50}(10x^9-8x^7+6x^5-4x^3+2x)dx$

$f(x)=10x^9-8x^7+6x^5-4x^3+2x$

$f(-x)=10(-x)^9-8(-x)^7+6(-x)^5-4(-x)^3+2(-x)$

$f(-x)=-(10x^9-8x^7+6x^5-4x^3+2x)$

$f(-x)=-f(x)$

$f(x)$ is odd function

If $f(x)$ is odd function then $\int_{-a}^{a}f(x)dx=0$

$$\int_{-50}^{50}(10x^9-8x^7+6x^5-4x^3+2x)dx=0$$

(iii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^9 x\cos^6 x\,dx$

$f(x)=\sin^9 x\cos^6 x$

$f(-x)=\sin^9(-x)\cos^6(-x)$

$f(-x)=-\sin^9 x\cos^6 x$

$f(-x)=-f(x)$

$f(x)$ is odd function

If $f(x)$ is odd function then $\int_{-a}^{a}f(x)dx=0$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^9 x\cos^6 x\,dx=0$$

(iv) $\int_{-\pi}^{\pi}\sec^8 x\tan x\,dx$

$f(x)=\sec^8 x\tan x$

$f(-x)=\sec^8(-x)\tan(-x)$

$f(-x)=-\sec^8 x\tan x$

$f(-x)=-f(x)$

$f(x)$ is odd function

If $f(x)$ is odd function then $\int_{-a}^{a}f(x)dx=0$

$$\int_{-\pi}^{\pi} \sec^8 x \tan x \, dx = 0$$

(v) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x \, dx$

$f(x) = \sin^2 x \cos^2 x$

$f(-x) = \sin^2(-x) \cos^2(-x)$

$f(-x) = \sin^2 x \cos^2 x$

$f(-x) = f(x)$

$f(x)$ is even function

If $f(x)$ is even function then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x \, dx = 2\int_{0}^{\frac{\pi}{2}} \sin^2 x \cos^2 x \, dx$$

$$= \frac{2}{4}\int_{0}^{\frac{\pi}{2}} 4\sin^2 x \cos^2 x \, dx$$

$$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} (2\sin x \cos x)^2 dx$$

$$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} (\sin 2x)^2 dx$$

$$= \frac{1}{2}\int_{0}^{\frac{\pi}{2}} \left(\frac{1-\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\int_{0}^{\frac{\pi}{2}} (1-\cos 4x) dx$$

$$= \frac{1}{4}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{2} - 0\right) - \frac{1}{4}\left(\sin 4\frac{\pi}{2} - \sin 0\right)\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2} - \frac{1}{4}(0-0)\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2}\right]$$

$$= \frac{\pi}{8}$$

(vi) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x \, dx$

The Students' (

$f(x) = \tan^2 x$

$f(-x) = \tan^2($

$f(-x) = \tan^2 x$

$f(-x) = f(x)$

$f(x)$ is even fu

If $f(x)$ is even

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x \, dx =$

$= 2\int_{0}^{\frac{\pi}{4}} (\sec^2 x$

$= 2[\tan x - x]$

$= 2\left[\tan\frac{\pi}{4} - t\right.$

$= 2\left[1 - 0 - \frac{\pi}{4}\right.$

$= 2 - \frac{\pi}{2}$

(vii) $\int_{-2}^{2} (x^4 + 2$

$f(x) = x^4 + 2$

$f(-x) = (-x)$

$f(-x) = x^4 +$

$f(-x) = f(x)$

$f(x)$ is even fu

If $f(x)$ is even

$\int_{-2}^{2} (x^4 + 2x^2)$

$= 2\left[\frac{x^5}{5} + \frac{2x^3}{3}\right]$

$= 2\left[\frac{1}{5}(2^5 - 0^5\right.$

$= 2\left[\frac{32}{5} + \frac{16}{3}\right]$

$= 2\left[\frac{176}{15}\right]$

$f(x) = \tan^2 x$
$f(-x) = \tan^2(-x)$
$f(-x) = \tan^2 x$
$f(-x) = f(x)$
$f(x)$ is even function

If $f(x)$ is even function then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\, dx = 2\int_{0}^{\frac{\pi}{4}} \tan^2 x\, dx$$

$$= 2\int_{0}^{\frac{\pi}{4}} (\sec^2 x - 1)dx$$

$$= 2[\tan x - x]_0^{\frac{\pi}{4}}$$

$$= 2\left[\tan\frac{\pi}{4} - \tan 0 - \left(\frac{\pi}{4} - 0\right)\right]$$

$$= 2\left[1 - 0 - \frac{\pi}{4}\right]$$

$$= 2 - \frac{\pi}{2}$$

(vii) $\int_{-2}^{2}(x^4 + 2x^2)dx$

$f(x) = x^4 + 2x^2$
$f(-x) = (-x)^4 + 2(-x)^2$
$f(-x) = x^4 + 2x^2$
$f(-x) = f(x)$
$f(x)$ is even function

If $f(x)$ is even function then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

$$\int_{-2}^{2}(x^4 + 2x^2)dx = 2\int_{0}^{2}(x^4 + 2x^2)dx$$

$$= 2\left[\frac{x^5}{5} + \frac{2x^3}{3}\right]_0^2$$

$$= 2\left[\frac{1}{5}(2^5 - 0^5) + \frac{2}{3}(2^3 - 0^5)\right]$$

$$= 2\left[\frac{32}{5} + \frac{16}{3}\right]$$

$$= 2\left[\frac{176}{15}\right]$$

$$= \frac{352}{15}$$

Q.4 Given that $\frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{(1+x^2)^{\frac{3}{2}}}$ then evaluate $\int_0^3 \frac{1}{(1+x^2)^{\frac{3}{2}}}dx$.

Solution:

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{(1+x^2)^{\frac{3}{2}}}$$

$$d\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{(1+x^2)^{\frac{3}{2}}}dx$$

Integrating we get

$$\int d\left(\frac{x}{\sqrt{1+x^2}}\right) = \int \frac{1}{(1+x^2)^{\frac{3}{2}}}dx$$

$$\frac{x}{\sqrt{1+x^2}} = \int \frac{1}{(1+x^2)^{\frac{3}{2}}}dx$$

$$\int \frac{1}{(1+x^2)^{\frac{3}{2}}}dx = \frac{x}{\sqrt{1+x^2}} \to (1)$$

$$\int_0^3 \frac{1}{(1+x^2)^{\frac{3}{2}}}dx = ?$$

From (1)

$$\int_0^3 \frac{1}{(1+x^2)^{\frac{3}{2}}}dx = \left[\frac{x}{\sqrt{1+x^2}}\right]_0^3$$

$$= \frac{3}{\sqrt{1+3^2}} - 0$$

$$= \frac{3}{\sqrt{10}}$$

Q.5 Given that $\frac{d}{dx}(F(x)) = \frac{2+x^2}{1+x^2}$, evaluate $F(\sqrt{3}) - F(1)$. If $F(1) = \pi$, find $F(x)$.

Solution:

$$\frac{d}{dx}(F(x)) = \frac{2+x^2}{1+x^2}$$

$$d(F(x)) = \frac{2+x^2}{1+x^2}dx$$

Integrating we get

$\frac{1}{(1+x^2)^{\frac{3}{2}}}dx.$

$$\int d(F(x)) = \int \frac{2+x^2}{1+x^2}dx$$

$$F(x)+C = \int \frac{2+x^2}{1+x^2}dx$$

$$\int \frac{2+x^2}{1+x^2}dx = F(x) \to (1)$$

$$\int \frac{1+1+x^2}{1+x^2}dx = F(x)$$

$$\int \left\{\frac{1}{1+x^2}+\frac{1+x^2}{1+x^2}\right\}dx = F(x)$$

$$\int \left\{\frac{1}{1+x^2}+1\right\}dx + C = F(x)$$

$$\tan^{-1}x + x + C = F(x)$$

$$F(x) = \tan^{-1}x + x + C \to (1)$$

$$F(\sqrt{3}) - F(1) = \{\tan^{-1}\sqrt{3}+\sqrt{3}+C\} - \{\tan^{-1}1+1+C\}$$

$$= \frac{\pi}{3}+\sqrt{3}+C-\frac{\pi}{4}-1-C$$

$$= \frac{\pi}{3}-\frac{\pi}{4}+\sqrt{3}-1$$

$$F(\sqrt{3}) - F(1) = \frac{\pi}{12}+\sqrt{3}-1$$

$$F(1) = \pi$$

$$(1) \Rightarrow \tan^{-1}1+1+C = F(1)$$

$$\frac{\pi}{4}+1+C = \pi$$

$$C = -\frac{\pi}{4}-1+\pi$$

$$C = \frac{-\pi-4+4\pi}{4}$$

$$\boxed{C = \frac{3\pi-4}{4}}$$

$F(1) = \pi,$

$$(1) \Rightarrow F(x) = \tan^{-1}x + x + \frac{3\pi-4}{4}$$

Q.6 Given that $\int_{-2}^{3} f(x)dx = 4$ and $\int_{5}^{3} f(x)dx = 7$ then evaluate by using suitable properties.

(i) $\int_{3}^{-2} f(x)dx$ (ii) $\int_{-2}^{3} f(y)dy$ (iii) $\int_{3}^{5} f(y)dy$ (iv) $\int_{2}^{2} f(x)dx$

(v) $\int_{-2}^{5} f(x)dx$ (vi) $\int_{5}^{-2} f(y)dy$

Solution:

$\int_{-2}^{3} f(x)dx = 4$ and $\int_{5}^{3} f(x)dx = 7$

(i) $\int_{3}^{-2} f(x)dx$

$$\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$$

$$\int_{3}^{-2} f(x)dx = -\int_{3}^{-2} f(x)dx = -4$$

(ii) $\int_{-2}^{3} f(y)dy$

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(y)dy$$

$$\int_{-2}^{3} f(y)dy = \int_{-2}^{3} f(x)dx = 4$$

(iii) $\int_{3}^{5} f(y)dy$

$$\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$$

$$\int_{3}^{5} f(y)dy = -\int_{5}^{3} f(y)dy = -7$$

(iv) $\int_{2}^{2} f(x)dx$

$$\int_{a}^{a} f(x)dx = 0$$

$$\int_{2}^{2} f(x)dx = 0$$

(v) $\int_{-2}^{5} f(x)dx$

$$\int_{a}^{c} f(x)dx = \int_{a}^{b} f(x)dx + \int_{b}^{c} f(x)dx$$

$$\int_{-2}^{5} f(x)dx = \int_{-2}^{3} f(x)dx + \int_{3}^{5} f(y)dy = 4 - 7 = -3$$

(vi) $\int_{5}^{-2} f(y)dy$

$$\int_{5}^{-2} f(y)dy = -\int_{5}^{-2} f(y)dy = -(-3) = 3$$

Q.7 Evaluate the following integrals by using trigonometric substitutions.

(i) $\int_{0}^{2} \frac{dx}{\sqrt{16-x^2}}$ (ii) $\int_{-1}^{1} \frac{dx}{4-x^2}$

(iii) $\int_{3}^{2\sqrt{3}} \frac{x\,dx}{\sqrt{x^2+4}}$ (iv) $\int_{0}^{\sqrt{3}} x^2\sqrt{3-x^2}\,dx$

Solution:

(i) $\int_{0}^{2} \frac{dx}{\sqrt{16-x^2}}$

$$= \int_{0}^{2} \frac{dx}{\sqrt{4^2-x^2}}$$

Let $x = 4\cos\theta \rightarrow (1)$

$\boxed{dx = 4\cos\theta\, d\theta}$

For limits:

$(1) \Rightarrow 4\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

$4\sin\theta = 2 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$I = \int_{0}^{\frac{\pi}{6}} \frac{4\cos\theta\, d\theta}{\sqrt{16-16\sin^2\theta}}$$

$$= \int_{0}^{\frac{\pi}{6}} \frac{4\cos\theta\, d\theta}{\sqrt{16(1-\sin^2\theta)}}$$

$$= \int_{0}^{\frac{\pi}{6}} \frac{4\cos\theta\, d\theta}{4\sqrt{\cos^2\theta}}$$

$$= \int_{0}^{\frac{\pi}{6}} \frac{\cos\theta\, d\theta}{\cos\theta}$$

$$= \int_{0}^{\frac{\pi}{6}} d\theta$$

$$= [\theta]_{0}^{\frac{\pi}{6}}$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

(ii) $\int_{-1}^{1} \frac{dx}{4-x^2}$

$$f(x) = \frac{1}{4-x^2}$$

$$f(-x) = f(x) = \frac{1}{4-x^2}$$

$f(x)$ is even function

itutions.

$$I = 2\int_0^1 \frac{dx}{2^2 - x^2}$$

Let $x = 2\sin\theta \rightarrow (1)$

$\boxed{dx = 2\cos\theta\, d\theta}$

For limits:

$(1) \Rightarrow 2\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

$2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$I = 2\int_0^{\frac{\pi}{6}} \frac{2\cos\theta\, d\theta}{4 - 4\sin^2\theta}$$

$$= 4\int_0^{\frac{\pi}{6}} \frac{\cos\theta\, d\theta}{4(1-\sin^2\theta)}$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta\, d\theta}{\cos^2\theta}$$

$$= \int_0^{\frac{\pi}{6}} \frac{d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{6}} \sec\theta\, d\theta$$

$$= [\ln|\sec\theta + \tan\theta|]_0^{\frac{\pi}{6}}$$

$$= \ln\left|\sec\frac{\pi}{6} + \tan\frac{\pi}{6}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln\left|\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right| - \ln|1 + 0|$$

$$= \ln\left|\frac{3}{\sqrt{3}}\right| - \ln|1|$$

$$= \ln|\sqrt{3}| - 0$$

$$= \ln\left|3^{\frac{1}{2}}\right|$$

$$= \frac{1}{2}\ln 3$$

(iii) $\int_2^{2\sqrt{3}} \frac{x^3 dx}{\sqrt{x^2+4}}$

$$= \int_2^{2\sqrt{3}} \frac{x^3 dx}{\sqrt{x^2 + 2^2}}$$

Let $x = 2\tan\theta \rightarrow (1)$

$\boxed{dx = 2\sec^2\theta\, d}$

For limits:

$(1) \Rightarrow 2\tan\theta =$

$2\tan\theta = 2\sqrt{3} =$

$\int_2^{2\sqrt{3}} \frac{x^3 dx}{\sqrt{x^2 + 2^2}}$

$= 16\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta}{\sqrt{4(ta}}$

$= \frac{16}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta}{\sqrt{s}}$

$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta\, s}{se}$

$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3\theta\, s$

$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2\theta\, t$

$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec\theta\, ta$

$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec^2\theta$

$= 8\left[\frac{\sec^3\theta}{3} - s\right.$

$= 8\left[\frac{\sec^3\theta - 3}{3}\right.$

$= \frac{8}{3}\left[\left(\sec^3\frac{\pi}{3} - \right.\right.$

$= \frac{8}{3}\left[\left((2)^3 - (\sqrt{}\right.\right.$

$= \frac{8}{3}[8 - 2\sqrt{2} -$

$dx = 2\sec^2\theta\, d\theta$

For limits:

$(1) \Rightarrow 2\tan\theta = 2 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$2\tan\theta = 2\sqrt{3} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$$\int_2^{2\sqrt{3}} \frac{x^3 dx}{\sqrt{x^2+2^2}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{8\tan^3\theta\,(2\sec^2\theta\, d\theta)}{\sqrt{4\tan^2\theta+4}}$$

$$= 16\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta\sec^2\theta\, d\theta}{\sqrt{4(\tan^2\theta+1)}}$$

$$= \frac{16}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta\sec^2\theta\, d\theta}{\sqrt{\sec^2\theta}}$$

$$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3\theta\sec^2\theta\, d\theta}{\sec\theta}$$

$$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3\theta\sec\theta\, d\theta$$

$$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2\theta\tan\theta\sec\theta\, d\theta$$

$$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec\theta\tan\theta\,(\sec^2\theta-1)d\theta$$

$$= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec^2\theta\sec\theta\tan\theta - \sec\theta\tan\theta)d\theta$$

$$= 8\left[\frac{\sec^3\theta}{3} - \sec\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= 8\left[\frac{\sec^3\theta - 3\sec\theta}{3}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{8}{3}\left[\left(\sec^3\frac{\pi}{3} - \sec^3\frac{\pi}{4}\right) - 3\left(\sec\frac{\pi}{3} - \sec\frac{\pi}{4}\right)\right]$$

$$= \frac{8}{3}\left[\left((2)^3 - \left(\sqrt{2}\right)^3\right) - 3\left(2-\sqrt{2}\right)\right]$$

$$= \frac{8}{3}\left[8 - 2\sqrt{2} - 6 + 3\sqrt{2}\right]$$

$= \frac{8}{3}(2+\sqrt{2})$

(iv) $\int_0^{\sqrt{3}} x^2\sqrt{3-x^2}\,dx$

$= \int_0^{\sqrt{3}} x^2\sqrt{(\sqrt{3})^2 - x^2 dx}$

Let $x = \sqrt{3}\sin\theta \rightarrow (1)$

$\boxed{dx = \sqrt{3}\cos\theta\, d\theta}$

For limits:

$(1) \Rightarrow \sqrt{3}\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

$\sqrt{3}\sin\theta = \sqrt{3} \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\mathrm{I} = \int_0^{\frac{\pi}{2}} 3\sin^2\theta\sqrt{3-3\sin^2\theta}\,(\sqrt{3}\cos\theta)d\theta$

$= 3\sqrt{3}\int_0^{\frac{\pi}{2}} \sin^2\theta\sqrt{3(1-\sin^2\theta)}\cos\theta\, d\theta$

$= 3\sqrt{3}(\sqrt{3})\int_0^{\frac{\pi}{2}} \sin^2\theta\sqrt{\cos^2\theta}\cos\theta\, d\theta$

$= 9\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta\cos\theta\, d\theta$

$= 9\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\, d\theta$

$= \frac{9}{4}\int_0^{\frac{\pi}{2}} 4\sin^2\theta\cos^2\theta\, d\theta$

$= \frac{9}{4}\int_0^{\frac{\pi}{2}} (2\sin\theta\cos\theta)^2 d\theta$

$= \frac{9}{4}\int_0^{\frac{\pi}{2}} \sin^2 2\theta\, d\theta$

$= \frac{9}{4}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$

$= \frac{9}{8}\int_0^{\frac{\pi}{2}} (1-\cos 4\theta)d\theta$

The Studen

$= \frac{9}{8}\Big[\theta - \frac{\text{si}}{}$

$= \frac{9}{8}\Big[\Big(\frac{\pi}{2} - ($

$= \frac{9}{8}\Big[\frac{\pi}{2} - \frac{1}{4}$

$= \frac{9}{8}\Big[\frac{\pi}{2} - 0$

$= \frac{9}{8}\Big[\frac{\pi}{2}\Big]$

$= \frac{9\pi}{16}$

Q.8 Compu

(i) $\int_5^9 xe^{4x}$

(iii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\,\text{si}$

Solution:

(i) $\int_5^9 xe^{4x}$

First integr

$\int xe^{4x}dx$

First functi

Second fur

$\mathrm{I} = \Big($ Fir funct

$= (x)\Big(\frac{e^{4}}{4}$

$= \frac{1}{4}xe^{4x}$ -

$= \frac{1}{4}xe^{4x}$ -

$= \frac{1}{4}xe^{4x}$ -

$= \frac{e^{4x}}{16}(4x$

Applying lir

$=\frac{9}{8}\left[\theta-\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$

$=\frac{9}{8}\left[\left(\frac{\pi}{2}-0\right)-\frac{1}{4}\left(\sin 4\frac{\pi}{2}-\sin 0\right)\right]$

$=\frac{9}{8}\left[\frac{\pi}{2}-\frac{1}{4}(0-0)\right]$

$=\frac{9}{8}\left[\frac{\pi}{2}-0\right]$

$=\frac{9}{8}\left[\frac{\pi}{2}\right]$

$=\frac{9\pi}{16}$

Q.8 Compute the definite integrals by using integration by parts.

(i) $\int_5^9 xe^{4x}dx$

(ii) $\int_1^4 x^2\ln x\,dx$

(iii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x\sin 2x\,dx$

(iv) $\int_0^1 \tan^{-1}x\,dx$

Solution:

(i) $\int_5^9 xe^{4x}dx$

First integrate without limits

$\int xe^{4x}dx$

First function $= x$

Second function $= e^{4x}$

$I=\begin{pmatrix}\text{First}\\\text{function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\\text{IInd function}\end{pmatrix}-\int\begin{pmatrix}\text{Derivative of}\\\text{first function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\\text{IInd function}\end{pmatrix}$

$=(x)\left(\frac{e^{4x}}{4}\right)-\int(1)\left(\frac{e^{4x}}{4}\right)dx$

$=\frac{1}{4}xe^{4x}-\frac{1}{4}\int e^{4x}dx$

$=\frac{1}{4}xe^{4x}-\frac{1}{4}\left(\frac{e^{4x}}{4}\right)+C$

$=\frac{1}{4}xe^{4x}-\frac{1}{16}e^{4x}+C$

$=\frac{e^{4x}}{16}(4x-1)+C$

Applying limits

$$\int_5^9 xe^{4x}dx = \left[\frac{e^{4x}}{16}(4x-1)\right]_5^9$$

$$= \frac{1}{16}\{e^{4(9)}(4(9)-1) - e^{4(5)}(4(5)-1)\}$$

$$= \frac{1}{16}\{35e^{36} - 19e^{20}\}$$

(ii) $\int_1^4 x^2 \ln x \, dx$

First integrate without limits

$$\int x^2 \ln x \, dx$$

First function $= \ln x$

Second function $= x^2$

$$\mathrm{I} = (\ln x)\left(\frac{x^3}{3}\right) - \int \left(\frac{1}{x}\right)\left(\frac{x^3}{3}\right)dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

$$= \frac{3x^3 \ln x - x^3}{9} + C$$

$$= \frac{1}{9}x^3(3\ln x - 1) + C$$

Applying limits

$$= \left[\frac{1}{9}x^3(3\ln x - 1)\right]_1^4$$

$$= \frac{1}{9}\{4^3(3\ln 4 - 1) - 1^3(3\ln 1 - 1)\}$$

$$= \frac{1}{9}\{64(3\ln 2^2 - 1) - (0 - 1)\}$$

$$= \frac{1}{9}\{64(6\ln 2 - 1) + 1\}$$

$$= \frac{1}{9}\{64(6\ln 2) - 64 + 1\}$$

$$= \frac{384\ln 2}{9} - \frac{63}{9}$$

The Students

$= \frac{128\ln 2}{3} -$

(iii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin 2$

First integrat

$\int x \sin 2x \, d$

First function

Second funct

$\mathrm{I} = (x)\left(-\frac{c}{}\right.$

$= -\frac{1}{2}x\cos 2$

$= -\frac{1}{2}x\cos 2$

$= -\frac{1}{2}x\cos 2$

$= \frac{1}{4}\{-2x\cos$

Applying limi

$= \left[\frac{1}{4}\{-2x\cos\right.$

$= \frac{1}{4}\left[-\left\{2\frac{\pi}{2}\right.\right.$

$= \frac{1}{4}\left[-\{\pi(-\right.$

$= \frac{1}{4}\left[-\{-\pi\right.$

$= \frac{1}{4}\left[\frac{7\pi}{6} - \frac{\sqrt{3}}{2}\right.$

$= \frac{1}{4}\left[\frac{7\pi - 3\sqrt{}}{6}\right.$

$= \frac{7\pi - 3\sqrt{3}}{24}$

(iv) $\int_0^1 \tan^{-1} x$

$= \frac{128 \ln 2}{3} - 7$

(iii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin 2x \, dx$

First integrate without limits

$\int x \sin 2x \, dx$

First function $= x$

Second function $= \sin 2x$

$I = (x)\left(-\frac{\cos 2x}{2}\right) - \int (1)\left(-\frac{\cos 2x}{2}\right) dx$

$= -\frac{1}{2} x \cos 2x + \frac{1}{2}\int \cos 2x \, dx$

$= -\frac{1}{2} x \cos 2x + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$

$= -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C$

$= \frac{1}{4}\{-2x \cos 2x + \sin 2x\} + C$

Applying limits

$= \left[\frac{1}{4}\{-2x \cos 2x + \sin 2x\}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \frac{1}{4}\left[-\left\{2\frac{\pi}{2}\cos 2\frac{\pi}{2} - 2\frac{\pi}{6}\cos 2\frac{\pi}{6}\right\} + \left\{\sin 2\frac{\pi}{2} - \sin 2\frac{\pi}{6}\right\}\right]$

$= \frac{1}{4}\left[-\left\{\pi(-1) - \frac{\pi}{3}\left(\frac{1}{2}\right)\right\} + \left\{0 - \frac{\sqrt{3}}{2}\right\}\right]$

$= \frac{1}{4}\left[-\left\{-\pi - \frac{\pi}{6}\right\} - \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{4}\left[\frac{7\pi}{6} - \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{4}\left[\frac{7\pi - 3\sqrt{3}}{6}\right]$

$= \frac{7\pi - 3\sqrt{3}}{24}$

(iv) $\int_0^1 \tan^{-1} x \, dx$

First integrate without limits

$$\int (1)\tan^{-1}x\,dx$$

First function $= \tan^{-1}x$

Second function $= 1$

$$I = (\tan^{-1}x)(x) - \int \left(\frac{1}{1+x^2}\right)(x)dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C$$

Applying limits

$$= \left[x\tan^{-1}x - \frac{1}{2}\ln|1+x^2|\right]_0^1$$

$$= 1\tan^{-1}1 - 0 - \frac{1}{2}\{\ln|1+1^2| - \ln|1+0|\}$$

$$= \frac{\pi}{4} - \frac{1}{2}\{\ln 2 - \ln 1\}$$

$$= \frac{\pi}{4} - \frac{1}{2}\{\ln 2 - 0\}$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2} = \frac{\pi - 2\ln 2}{4}$$

$$= \frac{\pi - \ln 2^2}{4} = \frac{\pi - \ln 4}{4}$$

EXERCISE 6.7

Find the area, above the x-axis under the following curves, between the given ordinates.

Q.1 $y = 3x^2 + 2$ $\quad x = 1, x = 2$

Q.2 $y = \frac{1}{\sqrt{4-x^2}}$ $\quad x = \frac{1}{2}, x = \frac{\sqrt{3}}{2}$

Q.3 $y = \ln x$ $\quad x = 1, x = 3$

Q.4 $y = x\sin x$ $\quad x = \frac{\pi}{3}, x = \frac{\pi}{2}$

Q.5 $y = \frac{1}{9+x^2}$ $\quad x = -\sqrt{3}, x = \sqrt{3}$

Q.6 $y = 4x^3 + 3x^2 + 2x + 1$ $\quad x = 0, x = 2$

Q.7 $y = 3\sec^2 x$ $\quad x = \frac{\pi}{6}, x = \frac{\pi}{3}$

Q.8 $y = 6\sin^2 x$ $\quad x = 0, x = \frac{\pi}{...}$

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Q.9 $y = 5e^{5x}$

Q.10 $y = \cos^4 x$

Q.11 $y = \frac{4}{\sin^2 x}$

Q.12 $x^2 + y^2 = 3$

Q.13 $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Q.14 $y^2 = 2x + 5$

Q.15 $y^2 = \tan^4 x$

Q.1 $y = 3x^2 + 2$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= \int_1^2 (3x^2 + 2)dx$$

$$= \left[3\left(\frac{x^3}{3}\right) + 2x\right]_1^2$$

$$= (2^3 - 1^3) + 2(2$$

$$= 7 + 2$$

$= 9$ Square units

Q.2 $y = \frac{1}{\sqrt{4-x^2}}$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{4-x^2}}dx$$

$a = 2$

$$= \left[\sin^{-1}\frac{x}{2}\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \sin^{-1}\left(\frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$$

Q.9 $y = 5e^{5x}$	$x = -2, x = 3$
Q.10 $y = \cos^4 x$	$x = 0, x = \frac{\pi}{2}$
Q.11 $y = \frac{4}{\sin^2 x}$	$x = \frac{\pi}{6}, x = \frac{\pi}{4}$
Q.12 $x^2 + y^2 = 36$	$x = -1, x = 1$
Q.13 $\frac{x^2}{4} + \frac{y^2}{9} = 1$	$x = -1, x = 1$
Q.14 $y^2 = 2x + 5$	$x = 1, x = 2$
Q.15 $y^2 = \tan^4 x$	$x = \frac{\pi}{6}, x = \frac{\pi}{4}$

Q.1 $y = 3x^2 + 2$ $\quad x = 1, x = 2$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= \int_1^2 (3x^2 + 2)dx$$

$$= \left[3\left(\frac{x^3}{3}\right) + 2x\right]_1^2$$

$$= (2^3 - 1^3) + 2(2 - 1)$$

$$= 7 + 2$$

$= 9$ Square units

Q.2 $y = \frac{1}{\sqrt{4-x^2}}$ $\quad x = \frac{1}{2}, x = \frac{\sqrt{3}}{2}$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{4 - x^2}}dx$$

$a = 2$

$$= \left[\sin^{-1}\frac{x}{2}\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \sin^{-1}\left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2} \times \frac{1}{2}\right)$$

$= \sin^{-1}\left(\frac{\sqrt{3}}{4}\right) - \sin^{-1}\left(\frac{1}{4}\right)$

$= 0.195$ Square units

Q.3 $y = \ln x$ $\quad x = 1, x = 3$

Solution:

Area $= \int_a^b y dx$

$= \int_1^3 \ln x \, dx$

Integrating by parts without limits

$\int \ln x \, dx$

First function $= \ln x$

Second function $= 1$

$\text{I} = \binom{\text{First}}{\text{function}}\binom{\text{Integral of}}{\text{IInd function}} - \int \binom{\text{Derivative of}}{\text{first function}}\binom{\text{Integral of}}{\text{IInd function}}$

$= (\ln x)(x) - \int \left(\frac{1}{x}\right)(x)dx$

$= x\ln x - \int dx$

$= x\ln x - x + C$

$= x(\ln x - 1) + C$

Applying limits from $x = 1$ to $x = 3$

Area $= [x(\ln x - 1)]_1^3$

$= 1(\ln 1 - 1) - 3(\ln 3 - 1)$

$= (0 - 1) - 3(\ln 3 - 1)$

$= 1.29$ Square units

Q.4 $y = x\sin x$ $\quad x = \frac{\pi}{3}, x = \frac{\pi}{2}$

Solution:

Area $= \int_a^b y dx$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x\sin x \, dx$

Integrating by parts without limits

$\int x\sin x \, dx$

First function $= x$

Second function =

$\text{I} = \binom{\text{First}}{\text{function}}($

$= (x)(-\cos x) -$

$= -x\cos x + \int$

$= -x\cos x + \sin$

Applying limits fro

$= [-x\cos x + \sin$

$= -\left\{\frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{3}\right.$

$= -\left\{0 - \frac{\pi}{3}\left(\frac{1}{2}\right)\right\}$

$= \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}$

$= 0.657$ Square u

Q.5 $y = \frac{1}{9+x^2}$

Solution:

Area $= \int_a^b y dx$

$= \int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{9+x^2} dx$

$f(x)$ is even

$= 2\int_0^{\sqrt{3}} \frac{1}{9+x^2} dx$

$a = 3$

$= 2\left(\frac{1}{3}\right)\left[\tan^{-1}\frac{x}{3}\right]_0^{\sqrt{}}$

$= \frac{2}{3}\left[\tan^{-1}\frac{\sqrt{3}}{3} - \text{ta}\right.$

$\int x \sin x \, dx$

First function $= x$

Second function $= \sin x$

$I = \begin{pmatrix}\text{First}\\ \text{function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix} - \int \begin{pmatrix}\text{Derivative of}\\ \text{first function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix}$

$= (x)(-\cos x) - \int (1)(-\cos x)dx$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$

Applying limits from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{2}$

$= [-x\cos x + \sin x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= -\left\{\frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{3}\cos\frac{\pi}{3}\right\} + \sin\frac{\pi}{2} - \sin\frac{\pi}{3}$

$= -\left\{0 - \frac{\pi}{3}\left(\frac{1}{2}\right)\right\} + 1 - \frac{\sqrt{3}}{2}$

$= \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}$

$= 0.657$ Square units

Q.5 $y = \frac{1}{9+x^2}$ $\quad x = -\sqrt{3}, x = \sqrt{3}$

Solution:

$\text{Area} = \int_a^b y dx$

$= \int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{9+x^2} dx$

$f(x)$ is even

$= 2\int_0^{\sqrt{3}} \frac{1}{9+x^2} dx$

$a = 3$

$= 2\left(\frac{1}{3}\right)\left[\tan^{-1}\frac{x}{3}\right]_0^{\sqrt{3}}$

$= \frac{2}{3}\left[\tan^{-1}\frac{\sqrt{3}}{3} - \tan^{-1} 0\right]$

$= \frac{2}{3}\left[\frac{\pi}{6} - 0\right]$

$= \frac{\pi}{9}$ Square units

Q.6 $y = 4x^3 + 3x^2 + 2x + 1 \qquad x = 0, x = 2$

Solution:

$$\text{Area} = \int_a^b y dx$$

$$= \int_0^2 (4x^3 + 3x^2 + 2x + 1)dx$$

$$= \left[\frac{4x^4}{4} + \frac{3x^3}{3} + \frac{2x^2}{2} + x\right]_0^2$$

$$= [x^4 + x^3 + x^2 + x]_0^2$$

$$= (2^4 - 0^4) + (2^3 - 0^3) + (2^2 - 0^2) + (2 - 0)$$

$$= 16 + 8 + 4 + 2$$

$= 30$ Square units

Q.7 $y = 3\sec^2 x \qquad x = \frac{\pi}{6}, x = \frac{\pi}{3}$

Solution:

$$\text{Area} = \int_a^b y dx$$

$$= 3\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\, dx$$

$$= 3[\tan x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= 3\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$$

$$= 3\left[\sqrt{3} - \frac{\sqrt{3}}{3}\right]$$

$$= 3\left[\frac{2\sqrt{3}}{3}\right]$$

$= 2\sqrt{3}$ Square units

Q.8 $y = 6\sin^2 x \qquad x = 0, x = \frac{\pi}{3}$

Solution:

$$\text{Area} = \int_a^b y dx$$

$$= 6\int_a^b \sin^2 x\, dx$$

$= 6\int_0^{\frac{\pi}{3}} \Big(\frac{1 - \cos}{2}$ …

$= 3\int_0^{\frac{\pi}{3}} (1 - \cos$ …

$= 3\left[x - \frac{\sin 2x}{2}\right.$ …

$= 3\left[\left(\frac{\pi}{3} - 0\right) - \right.$ …

$= 3\left[\frac{\pi}{3} - \frac{1}{2}\Big(\frac{\sqrt{3}}{2}\right.$ …

$$= 3\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$$

$$= 3\left[\frac{4\pi - 3\sqrt{3}}{12}\right]$$

$= \frac{4\pi - 3\sqrt{3}}{4}$ Sq…

Q.9 $y = 5e^{5x}$

Solution:

$$\text{Area} = \int_a^b y dx$$

$$= 5\int_{-2}^{3} e^{5x} dx$$

$$= 5\left[\frac{e^{5x}}{5}\right]_{-2}^{3}$$

$$= e^{5(3)} - e^{5(-2)}$$

$$= e^{15} - e^{-10}$$

$= 3269017.37$ S…

Q.10 $y = \cos^4 x$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= 6\int_a^b \sin^2 x\,dx$$

$$= 6\int_0^{\frac{\pi}{3}} \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= 3\int_0^{\frac{\pi}{3}} (1-\cos 2x)dx$$

$$= 3\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{3}}$$

$$= 3\left[\left(\frac{\pi}{3} - 0\right) - \frac{1}{2}\left(\sin\frac{2\pi}{3} - \sin 0\right)\right]$$

$$= 3\left[\frac{\pi}{3} - \frac{1}{2}\left(\frac{\sqrt{3}}{2} - 0\right)\right]$$

$$= 3\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$$

$$= 3\left[\frac{4\pi - 3\sqrt{3}}{12}\right]$$

$$= \frac{4\pi - 3\sqrt{3}}{4} \text{ Square units}$$

Q.9 $y = 5e^{5x}$ $\quad x = -2, x = 3$

Solution:

$$\text{Area} = \int_a^b y\,dx$$

$$= 5\int_{-2}^{3} e^{5x} dx$$

$$= 5\left[\frac{e^{5x}}{5}\right]_{-2}^{3}$$

$$= e^{5(3)} - e^{5(-2)}$$

$$= e^{15} - e^{-10}$$

$$= 3269017.37 \text{ Square units}$$

Q.10 $y = \cos^4 x$ $\quad x = 0, x = \frac{\pi}{2}$

Solution:

$y = \cos^4 x$

$y = (\cos^2 x)^2$

$y = \left(\frac{1+\cos 2x}{2}\right)^2$

$y = \frac{1 + 3\cos 2x + \cos^2 2x}{4}$

$y = \frac{1}{4} + \frac{3}{4}\cos 2x + \frac{\cos^2 2x}{4}$

$y = \frac{1}{4} + \frac{3}{4}\cos 2x + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$

$y = \frac{1}{4} + \frac{3}{4}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$

$y = \frac{3}{8} + \frac{3}{4}\cos 2x + \frac{1}{8}\cos 4x$

$\text{Area} = \int_0^{\frac{\pi}{2}} \left(\frac{3}{8} + \frac{3}{4}\cos 2x + \frac{1}{8}\cos 4x\right) dx$

$= \left[\frac{3}{8}x + \frac{3}{4}\left(\frac{\sin 2x}{2}\right) + \frac{1}{8}\left(\frac{\sin 4x}{4}\right)\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{3}{8}x + \frac{3}{8}\sin 2x + \frac{1}{32}\sin 4x\right]_0^{\frac{\pi}{2}}$

$= \left[\frac{3}{8}\left(\frac{\pi}{2} - 0\right) + \frac{3}{8}\left(\sin\frac{2\pi}{2} - \sin 0\right) + \frac{1}{32}\left(\sin 4\frac{\pi}{2} - \sin 0\right)\right]$

$= \left[\frac{3\pi}{16} + \frac{3}{8}(0-0) + \frac{1}{32}(0-0)\right]$

$= \frac{3\pi}{16} + 0 + 0$

$= \frac{3\pi}{16}$ Square units

Q.11 $y = \frac{4}{\sin^2 x}$ $\quad x = \frac{\pi}{6}, x = \frac{\pi}{4}$

Solution:

$\text{Area} = \int_a^b y\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{4}{\sin^2 x} dx$

$= 4\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}^2 x\, dx$

The Students'

$= -4[\cot x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= -4\left[\cot\frac{\pi}{4} - \right.$

$= -4[1 - \sqrt{3}]$

$= 4(\sqrt{3} - 1)$ S

Q.12 $x^2 + y^2$

Solution:

$x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \sqrt{36 - x^2}$

$\text{Area} = \int_a^b y d$

$= \int_a^b \sqrt{6^2 - x}$

$\int \sqrt{a^2 - x^2} d$

$a = 6$

$= \left[\frac{x}{2}\sqrt{36 - x\right.$

$= \left[\frac{1}{2}\sqrt{36 - (1\right.$

$= \left[\frac{1}{2}\sqrt{35} + \frac{1}{2}\right.$

$= 11.94$ Squar

Q.13 $\frac{x^2}{4} + \frac{y^2}{9} =$

Solution:

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\frac{y^2}{9} = 1 - \frac{x^2}{4}$

$\frac{y^2}{9} = \frac{4 - x^2}{4}$

$y^2 = \frac{9}{4}(4 - x^2)$

$= -4[\cot x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= -4\left[\cot\frac{\pi}{4} - \cot\frac{\pi}{6}\right]$

$= -4[1 - \sqrt{3}]$

$= 4(\sqrt{3} - 1)$ Square units

Q.12 $x^2 + y^2 = 36$ $x = -1, x = 1$

Solution:

$x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \sqrt{36 - x^2}$

$\text{Area} = \int_a^b y\,dx$

$= \int_a^b \sqrt{6^2 - x^2}\,dx$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$a = 6$

$= \left[\frac{x}{2}\sqrt{36 - x^2} + \frac{36}{2}\sin^{-1}\frac{x}{6}\right]_{-1}^{1}$

$= \left[\frac{1}{2}\sqrt{36 - (1)^2} - \left(-\frac{1}{2}\right)\sqrt{36 - (-1)^2} + 18\left(\sin^{-1}\frac{1}{6} - \sin^{-1}\frac{-1}{6}\right)\right]$

$= \left[\frac{1}{2}\sqrt{35} + \frac{1}{2}\sqrt{35} + 18\left(\sin^{-1}\frac{1}{6} - \sin^{-1}\frac{-1}{6}\right)\right]$

$= 11.94$ Square units

Q.13 $\frac{x^2}{4} + \frac{y^2}{9} = 1$ $x = -1, x = 1$

Solution:

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\frac{y^2}{9} = 1 - \frac{x^2}{4}$

$\frac{y^2}{9} = \frac{4 - x^2}{4}$

$y^2 = \frac{9}{4}(4 - x^2)$

$y = \frac{3}{2}\sqrt{4-x^2}$

$\text{Area} = \int_a^b y dx$

$\text{Area} = \frac{3}{2}\int_{-1}^{1}\sqrt{4-x^2}dx$

$$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$a = 2$

$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{-1}^{1}$

$= \frac{3}{2}\left[\frac{1}{2}\sqrt{4-(1)^2} - \left(-\frac{1}{2}\right)\sqrt{4-(-1)^2} + 2\left(\sin^{-1}\frac{1}{2} - \sin^{-1}\frac{-1}{2}\right)\right]$

$= \frac{3}{2}\left[\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} + 2\left(\frac{\pi}{6} + \frac{\pi}{6}\right)\right]$

$= \frac{3}{2}\left[\sqrt{3} + 2\left(\frac{\pi}{3}\right)\right]$

$= 5.98$ Square units

Q.14 $y^2 = 2x + 5$ $\quad x = 1, x = 2$

Solution:

$y^2 = 2x + 5$

$y = \sqrt{2x+5}$

$\text{Area} = \int_a^b y dx$

$= \int_1^2 \sqrt{2x+5}dx$

$= \frac{1}{2}\int_1^2 (2x+5)^{\frac{1}{2}}(2)dx$

$= \frac{1}{2}\left[\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2$

$= \frac{1}{3}\left[(2\times 2+5)^{\frac{3}{2}} - (2\times 1+5)^{\frac{3}{2}}\right]$

$= \frac{1}{3}\left[(3^2)^{\frac{3}{2}} - (7)^{\frac{3}{2}}\right]$

$= \frac{1}{3}[27 - 7\sqrt{7}]$

$= 2.82$ Square un

Q.15 $y^2 = \tan^4 x$

Solution:

$y^2 = \tan^4 x$

$y = \tan^2 x$

$y = \sec^2 x - 1$

$\text{Area} = \int_a^b y dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\sec^2 x - 1$

$= \tan\frac{\pi}{4} - \tan\frac{\pi}{6} -$

$= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$

$= \frac{12 - 4\sqrt{3} - \pi}{12}$

$= 0.160$ Square u

Q.16 Write MAPL

(i) $f(x) = e^{2x}$

(iii) $f(x) = \cos 2$

Solution:

(i) $f(x) = e^{2x}$

$> \int e^{2x} dx$

(ii) $f(x) = \sin x$

$> \int \sin x\, dx$

(iii) $f(x) = \cos 2x$

$> \int \cos 2x\, dx$

(iv) $f(x) = \ln(1 +$

$= \frac{1}{3}[27 - \ldots]$

$= 2.82$ Square units

Q.15 $y^2 = \tan^4 x$ $x = \frac{\pi}{6}, x = \frac{\pi}{4}$

Solution:

$y^2 = \tan^4 x$

$y = \tan^2 x$

$y = \sec^2 x - 1$

$\text{Area} = \int_a^b y\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\sec^2 x - 1)\,dx$

$= \tan\frac{\pi}{4} - \tan\frac{\pi}{6} - \left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$

$= \frac{12 - 4\sqrt{3} - \pi}{12}$

$= 0.160$ Square units

Q.16 Write MAPLE command to find integration of the functions.

(i) $f(x) = e^{2x}$ (ii) $f(x) = \sin x$

(iii) $f(x) = \cos 2x$ (iv) $f(x) = \ln(1 + x)$

Solution:

(i) $f(x) = e^{2x}$

> $\int e^{2x}\,dx$

$\frac{1}{2}\frac{e^{2x}}{\ln(e)}$

(ii) $f(x) = \sin x$

> $\int \sin x\,dx$

$-\cos(x)$

(iii) $f(x) = \cos 2x$

> $\int \cos 2\,x\,dx$

$\frac{1}{2}\cos(2)\,x^2$

(iv) $f(x) = \ln(1 + x)$

REVIEW EXERCISE 6

1. Choose the correct option.

(i) $\int\{f(x)\}^n f'(x)dx$, where $n \neq -1$, is

(a) $\frac{\{f(x)\}^{n+1}}{n+1}$
(b) $\frac{\{f(x)\}^{n-1}}{n-1}+C$
(c) $n\{f(x)\}^{n-1}+C$
(d) ✓ $\frac{\{f(x)\}^{n+1}}{n+1}+C$

(ii) $\int\{f(x)\}^n f'(x)dx$, where $n=-1$, is

(a) $\frac{\{f(x)\}^{n+1}}{n+1}+C$
(b) $\ln|\{f(x)\}^n|+C$
(c) ✓ $\ln|f(x)|+C$
(d) $n\{f(x)\}^{n-1}+C$

(iii) $\int x^n dx$, where $n=-1$, is

(a) $\frac{x^{n+1}}{n+1}+C$
(b) $nx^{n-1}+C$
(c) $\frac{x^{n-1}}{n-1}+C$
(d) ✓ $\ln x+C$

(iv) $\int \sin x\cos x\,dx=$:

(a) $\sin x+C$
(b) $\cos x+C$
(c) $\frac{1}{4}\cos 2x+C$
(d) ✓ $-\frac{1}{4}\cos 2x+C$

(v) $\int x^2\ln(e^{x^2})\,dx=$:

(a) $\frac{x^2}{4}+C$
(b) ✓ $\frac{x^5}{5}+C$
(c) $\ln(e^{x^2})+C$
(d) $\ln(x^x)+C$

(vi) $\int e^{\ln x^3}dx=$:

(a) $e^{x^3}+C$
(b) $\frac{x^3}{3}+C$
(c) ✓ $\frac{x^4}{4}+C$
(d) $\ln(x^3)+C$

(vii) $\int(1+\tan^2 x)dx=$:

(a) ✓ $\tan x+C$
(b) $\sin^2 x+C$
(c) $\frac{\tan^2 x}{2}+C$
(d) $\ln\sec x+C$

(viii) $\int\frac{2e^x}{1+e^x}dx=$:

(a) $\ln(1+e^x)+C$
(b) ✓ $\ln(1+e^x)^2+C$
(c) $(1+e^x)^{-2}+C$
(d) $\frac{e^x}{2}+C$

(ix) $\int \frac{e^{x+3\ln x}}{x^3}dx =$:

(a) $\frac{1}{3}e^{x+3\ln x}+C$
(b) ✓ $e^x + C$
(c) $e^{x+3\ln x}+C$
(d) $3\ln x + C$

(x) $\int \ln(e^x . e^{\sin x})\, dx =$:

(a) $\frac{1}{e^{x+\sin x}}+C$
(b) $\ln(\sin x)+C$
(c) ✓ $\frac{x^2}{2}-\cos x + C$
(d) $x\ln(\sin x)+C$

(xi) $\int \frac{\frac{1}{x\sqrt{x^2-1}}}{\text{cosec}^{-1}x}dx =$:

(a) ✓ $\ln(\text{cosec}^{-1}x)^{-1}+C$
(b) $(\text{cosec}^{-1}x)^{-2}+C$
(c) $\text{cosec}^{-1}x + C$
(d) $\ln(\text{cosec}^{-1}x)+C$

(xii) If F is an antiderivative of $f(x)$ then $\int_a^b f(x)dx =$:

(a) $F(a)-F(b)$
(b) ✓ $F(b)-F(a)$
(c) $f(b)-f(a)$
(d) $\frac{f(b)}{f(a)}$

(xiii) $\int_{-50}^{50}(x^3+x)dx =$:

(a) ✓ 0
(b) 1000
(c) 2000
(d) 3000

(xiv) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \cos^{11} x\, dx =$:

(a) 1
(b) 3
(c) ✓ 0
(d) $2\int_0^{\frac{\pi}{2}} \sin^9 x \cos^{11} x\, dx$:

(xv) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^{10} x \cos^{11} x\, dx =$:

(a) 0
(b) 1
(c) 3
(d) ✓ $2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^{10} x \cos^{11} x\, dx$

(xvi) Area bounded by the curve $y=\ln e^{x^2}$ from $x=-1$ to $x=1$ is

(a) ✓ $\frac{2}{3}$
(b) 1
(c) $\ln 2$
(d) $\ln 3$

(xvii) $\int_a^b f(x)dx =$:

(a) $-\int_a^b f(x)dx$
(b) $\int_b^a f(x)dx$
(c) ✓ $-\int_b^a f(x)dx$
(d) 0

(xviii) $\int_2^2 \left(x^3+3x^2-5x^{-\frac{1}{2}}\right) dx =$:

(a) ✓ 0

(b) $12-10\sqrt{2}$

(c) $24-20\sqrt{2}$

(d) $20\sqrt{2}$

(xix) $\int_{-2}^{2}(x^5-x^3+x)^5(5x^4-3x^2+1)dx =$:

(a) $\frac{(26)^6}{3}$

(b) ✓ 0

(c) $2(26)^6$

(d) $\frac{(26)^6}{6}$

(xx) $\int \frac{dx}{a^2+x^2}$

(a) $\tan^{-1}\frac{x}{a}$

(b) $\frac{1}{a}\sec^{-1}\frac{x}{a}+C$

(c) ✓ $\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

(d) $\sin^{-1}\frac{x}{a}+C$

Q.2 Evaluate the following integrals:

(i) $\int 3x^5 dx$

(ii) $\int x \ln x^n dx$

(iii) $\int \sec^5 x\, dx$

(iv) $\int \frac{y^2 dy}{\sqrt{1-y^2}}$

(v) $\int \sqrt{1-\sin 2x}\, dx$

(vi) $\int \tan^5 x \sec^{\frac{5}{2}} x\, dx$

(vii) $\int \frac{\cos x}{(2+\sin x)(3+\sin x)} dx$

(viii) $\int x^2 \sin x\, dx$

(ix) $\int \frac{d\theta}{\sqrt{1+\cos\frac{5\theta}{2}}}$

(x) $\int \frac{dx}{x^2-81}$

(xi) $\int \cot^5 x \operatorname{cosec}^{\frac{3}{2}} x\, dx$

(xii) $\int \cos^5 x \sin^5 x\, dx$

(xiii) $\int_0^2 6(x^2+3x+2)^5(2x+3)dx$

(xiv) $\int_0^{\frac{\pi}{2}} \cos^3 x\sqrt{\sin x}\, dx$

(xv) $\int_{-a}^{a} \frac{dx}{\sqrt{x^2-a^2}}$

(xvi) $\int_0^{a} \frac{dx}{x^2+a^2}$

Solution:

(i) $\int 3x^5 dx$

$$= 3\int x^5 dx = 3\left(\frac{x^6}{6}\right)+C = \frac{x^6}{2}+C$$

(ii) $\int x \ln x^n\, dx$

$$= \int x(n\ln x)dx$$

$$= \int nx\ln x\, dx$$

First function $= \ln x$

Second function $= nx$

$I = \left(\begin{matrix}\text{First}\\ \text{function}\end{matrix}\right)\left(\begin{matrix}\text{Int}\\ \text{IInd}\end{matrix}\right.$

$I = \ln x\left(\frac{nx^2}{2}\right) - \int ($

$= \ln x\left(\frac{nx^2}{2}\right) - \int ($

$= \frac{nx^2\ln x}{2} - \frac{n}{2}\int xd$

$= \frac{nx^2\ln x}{2} - \frac{n}{2}\left(\frac{x^2}{2}\right.$

$= \frac{nx^2\ln x}{2} - \frac{nx^2}{4} +$

(iii) $\int \sec^5 x\, dx$

$I = \int \sec^n x\, dx$

$= \int \sec^{n-2} x\, (\sec^2$

First function $= \sec$

Second function $=$ s

$\frac{d}{dx}(\sec x)^{n-2} = (n$

$\frac{d}{dx}(\sec x)^{n-2} = (n$

$I = \left(\begin{matrix}\text{First}\\ \text{function}\end{matrix}\right)\left(\begin{matrix}\text{In}\\ \text{IIn}\end{matrix}\right.$

$I = \sec^{n-2} x\,(\tan x)$

$= \sec^{n-2} x \tan x - ($

$= \sec^{n-2} x \tan x - ($

$= \sec^{n-2} x \tan x - ($

$I = \sec^{n-2} x \tan x -$

$\int \sec^5 x\, dx$

$n = 5$

$I = \begin{pmatrix}\text{First}\\ \text{function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix} - \int \begin{pmatrix}\text{Derivative of}\\ \text{first function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix}$

$I = \ln x\left(\frac{nx^2}{2}\right) - \int\left(\frac{1}{x}\right)\left(\frac{nx^2}{2}\right)dx$

$= \ln x\left(\frac{nx^2}{2}\right) - \int\left(\frac{1}{x}\right)\left(\frac{nx^2}{2}\right)dx$

$= \frac{nx^2\ln x}{2} - \frac{n}{2}\int x dx$

$= \frac{nx^2\ln x}{2} - \frac{n}{2}\left(\frac{x^2}{2}\right) + C$

$= \frac{nx^2\ln x}{2} - \frac{nx^2}{4} + C$

(iii) $\int \sec^5 x\, dx$

$I = \int \sec^n x\, dx$

$= \int \sec^{n-2} x\,(\sec^2 x)dx$

First function $= \sec^{n-2} x$

Second function $= \sec^2 x$

$\frac{d}{dx}(\sec x)^{n-2} = (n-2)(\sec x)^{n-3}(\sec x\tan x)$

$\frac{d}{dx}(\sec x)^{n-2} = (n-2)(\sec x)^{n-2}\tan x$

$I = \begin{pmatrix}\text{First}\\ \text{function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix} - \int \begin{pmatrix}\text{Derivative of}\\ \text{first function}\end{pmatrix}\begin{pmatrix}\text{Integral of}\\ \text{IInd function}\end{pmatrix}$

$I = \sec^{n-2} x\,(\tan x) - \int (n-2)(\sec x)^{n-2}\tan x\tan x\, dx$

$= \sec^{n-2} x\tan x - (n-2)\int (\sec x)^{n-2}\tan^2 x\, dx$

$= \sec^{n-2} x\tan x - (n-2)\int (\sec x)^{n-2}(\sec^2 x - 1)dx$

$= \sec^{n-2} x\tan x - (n-2)\int \{(\sec x)^{n-2}\sec^2 x - (\sec x)^{n-2}\}dx$

$I = \sec^{n-2} x\tan x - (n-2)\frac{(\sec x)^{n-1}}{n-1} + (n-2)\int (\sec x)^{n-2}dx$

$\int \sec^5 x\, dx$

$n = 5$

$$I = \sec^{5-2} x \tan x - \frac{(5-2)}{(5-1)} (\sec x)^{5-1} + (5-2) \int (\sec x)^{5-2} dx$$

$$= \sec^3 x \tan x - \frac{3}{4} (\sec x)^4 + 3 \int (\sec x)^3 dx$$

$n = 3$

$$= \sec^3 x \tan x - \frac{3}{4} (\sec x)^4 +$$

$$3 \left\{ \sec^{3-2} x \tan x - \frac{3-2}{3-1} (\sec x)^{3-1} + (3-2) \int (\sec x)^{3-2} dx \right\}$$

$$= \sec^3 x \tan x - \frac{3}{4} (\sec x)^4 +$$

$$3 \left\{ \sec x \tan x - \frac{1}{2} (\sec x)^2 + \int \sec x \, dx \right\}$$

$$= \sec^3 x \tan x - \frac{3}{4} (\sec x)^4 +$$

$$3 \sec x \tan x - \frac{3}{2} (\sec x)^2 + 3 \ln(\sec x + \tan x)$$

(iv) $\int \frac{y^2 dy}{\sqrt{1-y^2}}$

$$= -\int \frac{-y^2}{\sqrt{1-y^2}} dy$$

$$= -\int \frac{1-y^2-1}{\sqrt{1-y^2}} dy$$

$$= -\int \left\{ \frac{1-y^2}{\sqrt{1-y^2}} - \frac{1}{\sqrt{1-y^2}} \right\} dy$$

$$= -\int \left\{ \sqrt{1-y^2} - \frac{1}{\sqrt{1-y^2}} \right\} dy$$

$$= \int \left\{ \frac{1}{\sqrt{1-y^2}} - \sqrt{1-y^2} \right\} dy$$

$$= \sin^{-1} y - \left\{ \frac{y}{2} \sqrt{1-y^2} + \frac{1}{2} \sin^{-1} y \right\} + C$$

$$= \sin^{-1} y - \frac{y}{2} \sqrt{1-y^2} - \frac{1}{2} \sin^{-1} y + C$$

$$= \sin^{-1} y - \frac{y}{2} \sqrt{1-y^2} - \frac{1}{2} \sin^{-1} y + C$$

$$= \frac{1}{2} \sin^{-1} y - \frac{y}{2} \sqrt{1-y^2} + C$$

$= \frac{1}{2}\{\sin^{-1} y - \ldots$

(v) $\int \sqrt{1 - \sin 2 \ldots}$

$= \int \sqrt{1 - 2\sin \ldots}$

$= \int \sqrt{\sin^2 x + \ldots}$

$= \int \sqrt{(\sin x)^2 \ldots}$

$= \int \sqrt{(\sin x - \ldots}$

$= \int |\sin x - \cos \ldots$

$= |-\cos x - \sin \ldots$

$= |-(\cos x + \sin \ldots$

$= |\cos x + \sin \ldots$

(vi) $\int \tan^5 x \sec \ldots$

$\int \tan^5 x \sec^{\frac{5}{2}} x \ldots$

$= \int \tan x \tan^4 \ldots$

$= \int \tan x (\sec^2 \ldots$

$= \int \tan x (\sec^4 \ldots$

$= \int \tan x \left(\sec \ldots\right.$

$= \int \left(\sec^{\frac{13}{2}} x \tan \ldots\right.$

$= \int \left(\sec^{\frac{11}{2}} x \sec \ldots\right.$

$= \frac{\sec^{\frac{13}{2}} x}{\frac{13}{2}} - \frac{2 \ldots}{\ldots}$

$= \frac{2}{13} \sec^{\frac{13}{2}} x - \ldots$

(vii) $\int \frac{\cos x}{(2+\sin x)(3+\ldots)}$

$\int \frac{\cos x}{(2+\sin x)(3 \ldots)}$

$= \frac{1}{2}\{\sin^{-1} y - y\sqrt{1-y^2}\} + C$

(v) $\int \sqrt{1 - \sin 2x}\, dx$

$= \int \sqrt{1 - 2\sin x \cos x}\, dx$

$= \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x}\, dx$

$= \int \sqrt{(\sin x)^2 - 2\sin x \cos x + (\cos x)^2}\, dx$

$= \int \sqrt{(\sin x - \cos x)^2}\, dx$

$= \int |\sin x - \cos x|\, dx$

$= |-\cos x - \sin x| + C$

$= |-(\cos x + \sin x)| + C$

$= |\cos x + \sin x| + C$

(vi) $\int \tan^5 x \sec^{\frac{5}{2}} x\, dx$

$\int \tan^5 x \sec^{\frac{5}{2}} x\, dx$

$= \int \tan x \tan^4 x \sec^{\frac{5}{2}} x\, dx$

$= \int \tan x (\sec^2 x - 1)^2 \sec^{\frac{5}{2}} x\, dx$

$= \int \tan x (\sec^4 x - 2\sec^2 x + 1) \sec^{\frac{5}{2}} x\, dx$

$= \int \tan x \left(\sec^{\frac{13}{2}} x - 2\sec^{\frac{9}{2}} x + \sec^{\frac{5}{2}} x\right) dx$

$= \int \left(\sec^{\frac{13}{2}} x \tan x - 2\sec^{\frac{9}{2}} x \tan x + \sec^{\frac{5}{2}} x \tan x\right) dx$

$= \int \left(\sec^{\frac{11}{2}} x \sec x \tan x - 2\sec^{\frac{7}{2}} x \sec x \tan x + \sec^{\frac{3}{2}} x \sec x \tan x\right) dx$

$= \frac{\sec^{\frac{13}{2}} x}{\frac{13}{2}} - \frac{2\sec^{\frac{9}{2}} x}{\frac{9}{2}} + \frac{\sec^{\frac{5}{2}} x}{\frac{5}{2}} + C$

$= \frac{2}{13}\sec^{\frac{13}{2}} x - \frac{4}{9}\sec^{\frac{9}{2}} x + \frac{2}{5}\sec^{\frac{5}{2}} x + C$

(vii) $\int \frac{\cos x}{(2+\sin x)(3+\sin x)}\, dx$

$\int \frac{\cos x}{(2+\sin x)(3+\sin x)}\, dx$

Let $u = \sin x$

$du = \cos x\, dx$

$$I = \int \frac{du}{(2+u)(3+u)}$$

$$= \int \frac{du}{6+5u+u^2}$$

$$= \int \frac{du}{u^2+5u+6}$$

$$u^2+5u+6 = (u)^2 + 2(u)\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6$$

$$= \left(u+\frac{5}{2}\right)^2 - \frac{1}{4}$$

$$I = \int \frac{du}{\left(u+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\boxed{\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C}$$

$$a = \frac{1}{2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)}\ln\left(\frac{u+\frac{5}{2}-\frac{1}{2}}{u+\frac{5}{2}+\frac{1}{2}}\right) + C$$

$$= \ln\left(\frac{u+2}{u+3}\right) + C$$

$$= \ln\left(\frac{\sin x+2}{\sin x+3}\right) + C$$

(viii) $\int x^2 \sin x\, dx$

First function $= x^2$

Second function $= \sin x$

$$I = \binom{\text{First}}{\text{function}}\binom{\text{Integral of}}{\text{IInd function}} - \int \binom{\text{Derivative of}}{\text{first function}}\binom{\text{Integral of}}{\text{IInd function}}$$

$$\int x^2 \sin x\, dx$$

$$= (x^2)(-\cos x) - \int (2x)(-\cos x)dx$$

$$= -x^2\cos x + \int 2x\cos x\, dx$$

First function $= 2x$

Second function ...

$= -x^2 \cos x + ($...

$= -x^2 \cos x + 2$...

$= -x^2 \cos x + 2$...

(ix) $\int \frac{d\theta}{\sqrt{1+\cos\frac{5\theta}{2}}}$

$\sqrt{1+\cos x} = \sqrt{2}$...

$\sqrt{1+\cos\frac{5\theta}{2}} = $...

$\sqrt{1+\cos\frac{5\theta}{2}} = $...

$\int \frac{d\theta}{\sqrt{1+\cos\frac{5\theta}{2}}} =$...

$= \frac{1}{\sqrt{2}}\int \sec\left(\frac{5\theta}{4}\right.$...

$= \frac{\sqrt{2}}{2}\left(\frac{4}{5}\right)\ln\{\sec$...

$= \frac{2\sqrt{2}}{5}\ln\{\sec($...

(x) $\int \frac{dx}{x^2-81}$

$= \int \frac{dx}{x^2-9^2}$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}$...

$a = 9$

$= \frac{1}{2(9)}\ln\left(\frac{x-9}{x+9}\right.$...

$= \frac{1}{18}\ln\left(\frac{x-9}{x+9}\right)$...

(xi) $\int \cot^5 x \operatorname{cosec}$...

$= \int \cot^4 x \cot x$...

Second function = $\cos x$,

$= -x^2 \cos x + (2x)(\sin x) - \int (2)(\sin x) dx$

$= -x^2 \cos x + 2x \sin x - 2(-\cos x)$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

(ix) $\int \frac{d\theta}{\sqrt{1+\cos\frac{5\theta}{2}}}$

$$\sqrt{1+\cos x} = \sqrt{2}\cos\frac{x}{2}$$

$$\sqrt{1+\cos\frac{5\theta}{2}} = \sqrt{2}\cos\left\{\frac{1}{2}\left(\frac{5\theta}{2}\right)\right\}$$

$$\sqrt{1+\cos\frac{5\theta}{2}} = \sqrt{2}\cos\left(\frac{5\theta}{4}\right)$$

$$\int \frac{d\theta}{\sqrt{1+\cos\frac{5\theta}{2}}} = \int \frac{d\theta}{\sqrt{2}\cos\left(\frac{5\theta}{4}\right)}$$

$$= \frac{1}{\sqrt{2}}\int \sec\left(\frac{5\theta}{4}\right) d\theta$$

$$= \frac{\sqrt{2}}{2}\left(\frac{4}{5}\right)\ln\left\{\sec\left(\frac{5\theta}{4}\right) + \tan\left(\frac{5\theta}{4}\right)\right\} + C$$

$$= \frac{2\sqrt{2}}{5}\ln\left\{\sec\left(\frac{5\theta}{4}\right) + \tan\left(\frac{5\theta}{4}\right)\right\} + C$$

(x) $\int \frac{dx}{x^2-81}$

$$= \int \frac{dx}{x^2-9^2}$$

$$\boxed{\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + C}$$

$a = 9$

$$= \frac{1}{2(9)}\ln\left(\frac{x-9}{x+9}\right) + C$$

$$= \frac{1}{18}\ln\left(\frac{x-9}{x+9}\right) + C$$

(xi) $\int \cot^5 x \operatorname{cosec}^{\frac{3}{2}} x \, dx$

$$= \int \cot^4 x \cot x \operatorname{cosec}^{\frac{3}{2}} x \, dx$$

$$= \int \cot x \operatorname{cosec}^{\frac{3}{2}} x\,(\cot^2 x)^2 dx$$

$$= \int \cot x \operatorname{cosec}^{\frac{3}{2}} x\,(\operatorname{cosec}^2 x - 1)^2 dx$$

$$= \int \cot x \operatorname{cosec}^{\frac{3}{2}} x\,(\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x + 1) dx$$

$$= \int \left(\cot x \operatorname{cosec}^{\frac{11}{2}} x - 2\cot x \operatorname{cosec}^{\frac{7}{2}} x + \cot x \operatorname{cosec}^{\frac{3}{2}} x\right) dx$$

$$= \operatorname{cosec}^{\frac{9}{2}} x\,(\operatorname{cosec} x \cot x) - 2\operatorname{cosec}^{\frac{5}{2}} x\,(\operatorname{cosec} x \cot x) + \operatorname{cosec}^{\frac{1}{2}} x\,(\operatorname{cosec} x \cot x) + C$$

$$= \frac{\operatorname{cosec}^{\frac{11}{2}} x}{\frac{11}{2}} - \frac{2\operatorname{cosec}^{\frac{7}{2}} x}{\frac{7}{2}} + \frac{\operatorname{cosec}^{\frac{3}{2}} x}{\frac{3}{2}} + C$$

$$= \frac{2}{11}\operatorname{cosec}^{\frac{11}{2}} x - \frac{4}{7}\operatorname{cosec}^{\frac{7}{2}} x + \frac{2}{3}\operatorname{cosec}^{\frac{3}{2}} x + C$$

(xii) $\int \cos^5 x \sin^5 x\, dx$

$$= \int \cos^4 x \cos x \sin^5 x\, dx$$

$$= \int \sin^5 x \cos x\,(\cos^2 x)^2 dx$$

$$= \int \sin^5 x \cos x\,(1 - \sin^2 x)^2 dx$$

$$= \int \sin^5 x \cos x\,(1 - 2\sin^2 x + \sin^4 x) dx$$

$$= \int (\sin^5 x \cos x - 2\sin^7 x \cos x + \sin^9 x \cos x) dx$$

$$= \frac{\sin^6 x}{6} - \frac{2\sin^8 x}{8} + \frac{\sin^{10} x}{10} + C$$

$$= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{4} + \frac{\sin^{10} x}{10} + C$$

(xiii) $\int_0^2 6(x^2+3x+2)^5(2x+3)dx$

$$\frac{d}{dx}(x^2+3x+2) = 2x+3$$

$$= 6\left[\frac{(x^2+3x+2)^6}{6}\right]_0^2$$

$$= (2^2 + 3\times 2 + 2)^6 - (0+0+2)^6$$

$$= (12)^6 - (2)^6$$

$= 2985920$

(xiv) $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt{\sin x}$

$$= \int_0^{\frac{\pi}{2}} \cos^2 x \cos x\,(s$$

$$= \int_0^{\frac{\pi}{2}} \cos x\,(\sin x)^{\frac{1}{2}}($$

$$= \int_0^{\frac{\pi}{2}} \{\cos x\,(\sin x)^{\frac{1}{2}}$$

$$= \left[\frac{(\sin x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(\sin}{\frac{7}{2}}\right.$$

$$= \frac{2}{3}\left\{\left(\sin\frac{\pi}{2}\right)^{\frac{3}{2}} - (\sin\right.$$

$$= \frac{2}{3}\left\{(1)^{\frac{3}{2}} - 0\right\} - \frac{2}{7}\{$$

$$= \frac{2}{3} - \frac{2}{7}$$

$$= \frac{8}{21}$$

(xv) $\int_{-a}^{a} \frac{dx}{x\sqrt{x^2-a^2}}$

The function is not c

$$f(x) = \frac{1}{x\sqrt{x^2-a^2}}$$

$$f(a) = \frac{1}{a\sqrt{a^2-a^2}} =$$

$$f(-a) = \frac{1}{-a\sqrt{(-a)^2}}$$

$$f(0) = \frac{1}{0\sqrt{0-a^2}} =$$

$$\int_{-a}^{a} \frac{dx}{x\sqrt{x^2-a^2}} = \text{div}$$

(xvi) $\int_0^a \frac{dx}{x^2+a^2}$

$$= \frac{1}{a}\left[\tan^{-1}\frac{x}{a}\right]_0^a$$

$= 2985920$

(xiv) $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt{\sin x}\, dx$

$= \int_0^{\frac{\pi}{2}} \cos^2 x \cos x\, (\sin x)^{\frac{1}{2}} dx$

$= \int_0^{\frac{\pi}{2}} \cos x\, (\sin x)^{\frac{1}{2}} (1 - \sin^2 x) dx$

$= \int_0^{\frac{\pi}{2}} \left\{ \cos x\, (\sin x)^{\frac{1}{2}} - \cos x\, (\sin x)^{\frac{5}{2}} \right\} dx$

$= \left[\frac{(\sin x)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(\sin x)^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^{\frac{\pi}{2}}$

$= \frac{2}{3}\left\{ \left(\sin\frac{\pi}{2}\right)^{\frac{3}{2}} - (\sin 0)^{\frac{3}{2}} \right\} - \frac{2}{7}\left\{ \left(\sin\frac{\pi}{2}\right)^{\frac{7}{2}} - (\sin 0)^{\frac{7}{2}} \right\}$

$= \frac{2}{3}\left\{ (1)^{\frac{3}{2}} - 0 \right\} - \frac{2}{7}\left\{ (1)^{\frac{7}{2}} - 0 \right\}$

$= \frac{2}{3} - \frac{2}{7}$

$= \frac{8}{21}$

(xv) $\int_{-a}^{a} \frac{dx}{x\sqrt{x^2 - a^2}}$

The function is not continuous at $[-a, a]$

$f(x) = \frac{1}{x\sqrt{x^2 - a^2}}$

$f(a) = \frac{1}{a\sqrt{a^2 - a^2}} = \frac{1}{a\sqrt{0}} = \frac{1}{0}$

$f(-a) = \frac{1}{-a\sqrt{(-a)^2 - a^2}} = \frac{1}{-a\sqrt{a^2 - a^2}} = \frac{1}{0}$

$f(0) = \frac{1}{0\sqrt{0 - a^2}} = \frac{1}{0}$

$\int_{-a}^{a} \frac{dx}{x\sqrt{x^2 - a^2}} = \text{diverges}$

(xvi) $\int_0^{a} \frac{dx}{x^2 + a^2}$

$= \frac{1}{a}\left[\tan^{-1}\frac{x}{a} \right]_0^{a}$

$$= \frac{1}{a}\left[\tan^{-1}\frac{a}{a} - \tan^{-1} 0\right]$$

$$= \frac{1}{a}[\tan^{-1} 1 - 0]$$

$$= \frac{1}{a}\left[\frac{\pi}{4}\right] = \frac{\pi}{4a}$$