

Chapter = 12

Electrostatics

**Electrostatics:**

The branch of physics which deals with behavior of electric charges at the state of rest is called electrostatics.

COULOMBS LAW:

According to this law the magnitude electrostatic force between two point charges is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them.

EXPLANATION:

Consider two point charges of magnitudes q_1 and q_2 at a distance r from each other. By coulomb's law the magnitude of electrostatic force F exerted by either of the charge on the other is given by.

$$F = \frac{Kq_1q_2}{r^2} \text{ ----- (1)}$$

Where K is a constant of proportionality and its value depends on the medium between the charges.

Vector form:

The force in the vector form is given by.

$$F_{12} = \frac{Kq_1q_2}{r^2} \gamma_{12}$$

Where F_{12} = force exerted by q_1 to q_2 .

r_{12} = unit vector along the line joining the two charges from q_1 to q_2 .

The value of K for free space is.

$$K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{col}^2$$

Putting the value of K in equation 1.

$$(1) \Rightarrow F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$$

Where ϵ_0 = permittivity of free space.

In the presence of insulating material (medium other than free space and air)

If instead of free space there is some insulating material or medium between the charges then.

$$K = \frac{1}{4\pi\epsilon}$$

Where ϵ = permittivity of the medium and $\epsilon = \epsilon_0\epsilon_r$. while ϵ_r = relative permittivity.

Thus equation (1) become,

$$(1) \Rightarrow F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{r^2}$$

The force will be attractive if the charges are dissimilar and repulsive if the charges are similar.

ELECTRIC FIELD:

The region around a charged body in which an other charge experiences an electric force is called electric field.

INTENSITY OF ELECTRIC FIELD:

The electrostatic force experienced per unit positive charge is called electric field intensity at a point. If a test charge q_0 experiences a force F then intensity of electric field E is given by.

$$E = \frac{F}{q}$$

It is a vector quantity. The direction of electric field intensity is away from the point charge if the point charge is positive and towards the point charge if the point charge is negative. The S.I unit of electric field intensity is N/C. The commercial unit of electric field intensity is volt/meter (V.m^{-1}).

ELECTRIC FIELD INTENSITY NEAR AN ISOLATED POINT CHARGE:

Consider a very small positive test charge q_0 placed at a distance r from a point charge q . the magnitude of the force on q due to the charge q is given by.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

The electric field intensity E at q_0 due to q is given by



$$E = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \frac{1}{q_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

COULOMB:

If two similar and equal charges are placed 1meter apart and a force of 9×10^9 N acts between them then each of the two charges will be equal to one coulomb. One coulomb is equal to the charge 6.25×10^{18} electrons.

ELECTRIC LINES OF FORCE:

1. An electric line of force is an imaginary curve which starts from charge. The direction of electric field can be at any point of the curve.
2. The electric lines of force do not intersect with each other.
3. No line of force originates or terminates in space surrounding the charge.

ELECTRIC FLUX:

The total number of electric lines of force passing through any area normally is called electric flux. It is denoted by $\Delta\Phi_e$ and measured in S.I. unit in $\text{N.m}^2/\text{C}$. Electric flux depends upon the following factors.

EXPLANATION:

Practical observations show that electric flux is directly proportional to the magnitude of electric field intensity 'E' and component of element area of closed surface parallel to electric field intensity $\Delta A \cos\theta$.

Mathematically,

$$\Delta\Phi_e \propto E$$

$$\Delta\Phi_e \propto \Delta A \cos\theta$$

By combining the above factors we get,

$$\Delta\Phi_e = E \Delta A \cos\theta$$

Where constant of proportionality is 1.

Now,

$$\Delta\Phi_e = \vec{E} \cdot \vec{A}$$

Thus electric flux is the dot product of electric field intensity and element area vector.

a) If $\theta = 0^\circ$ then equation 1 becomes $\Delta\Phi_e = EA(\cos 0) = EA(1) = EA$
i.e the flux is maximum.

b) If $\theta = 90^\circ$ the equation 1 becomes $\Delta\Phi_e$
i.e the flux is zero

c) flux is negative if $\theta > 90^\circ$.

FIELD INTENSITY:

Electric flux per unit area is called field intensity or flux density E .

i.e

$$E = \frac{\Phi}{A}$$

ELECTRIC FLUX THROUGH A SPHERE:

Consider an isolated point charge q^+ at the center of an imaginary sphere. The lines of force from q will spread uniformly in space around it radially cutting the surfaces of the sphere normally at all positions.

To find the electric flux through the sphere divide the sphere into small elements each of area ΔA . The flux through the sphere is given by

$$\Phi = \sum_{i=1}^n \vec{E} \cdot \vec{\Delta A} = \sum_{i=1}^n E \Delta A \cos \theta$$

But $\theta = 0^\circ$ and $\cos 0^\circ = 1$

Therefore
$$\Phi = \sum_{i=1}^n E \Delta A \text{ ----- (2)}$$

For constant electric field intensity at the surface of sphere,

$$\Phi = E \sum_{i=1}^n \Delta A$$

But

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \text{ And } \sum_{i=1}^n \Delta A = 4\pi r^2$$

$$\text{Now (2)} \Rightarrow \Phi = \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right) (4\pi r^2)$$

$$\Phi = \frac{q}{\epsilon_0}$$

This shows that the flux does not depend upon the radius or size or shape of the closed surface. If the charge is unity i.e $q=1\text{C}$ then.

$$\Phi = \frac{1}{\epsilon_0}$$

GAUSS LAW:

According to this law the electric flux through any closed surface is $\frac{1}{\epsilon_0}$ times the charge enclosed by the surface. i.e

$$\Phi = \frac{1}{\epsilon_0} (q_{\text{enclosed}})$$

Proof:

Consider a closed surface of any size and shape surrounding 'n' number of point charges of magnitudes $q_1, q_2, q_3, \dots, q_n$ as shown in the figure. The electric field vectors at different points of the closed surface will have different magnitudes and directions.

In order to determine the flux over the closed surface assume imaginary spheres around each point charge and as we know electric flux through these imaginary spheres will be the magnitude of point charge divided by permittivity of free space ϵ_0 .

Therefore total electric flux over the whole surface is equal to the sum of $\frac{q_1}{\epsilon_0}, \frac{q_2}{\epsilon_0}, \frac{q_3}{\epsilon_0} \dots \dots \dots \frac{q_n}{\epsilon_0}$.

Hence,

$$\text{Total electric flux} = \phi_e = \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \dots + \frac{q_n}{\epsilon_0}$$

$$\phi_e = \frac{1}{\epsilon_0} (q_1 + q_2 + q_3 + \dots + q_n)$$

$$\Phi_e = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i$$

Gaussian surface:

Any closed hypothetical surface which contains charges is called a Gaussian surface.

APPLICATIONS OF GAUSS LAW:

a) Electric field intensity due to a uniform spherical surface charge at a distance r from its center:

Let a charge q be uniformly distributed on the surface of a spherical shell or that of a metal sphere. Now consider a point P outside the shell at a distance r from the center 'O'.

To determine electric field intensity imagine a Gaussian surface in the form of a concentric sphere of radius r that contains the point P. due to symmetrical charge distribution with respect to every point of Gaussian surface the field has the same magnitude every where on this surface and it is perpendicular to the surface at each point. If "E" be the electric intensity at 'P' then. Flux over Gaussian surface is given by

$$\Phi = \sum_{i=1}^n \vec{E} \cdot \vec{\Delta A} = \sum_{i=1}^n E \Delta A \cos \theta$$

But $\theta = 0^\circ$ and $\cos 0^\circ = 1$

Therefore
$$\Phi = \sum_{i=1}^n E \Delta A \text{ ----- (1)}$$

For constant electric field intensity at point 'P',

$$\Phi = E \sum_{i=1}^n \Delta A$$

As, the surface area of Sphere,

$$\sum_{i=1}^n \Delta A = 4\pi r^2$$

$$\Phi = E(4\pi r^2)$$

Applying Gauss law,

$$\Phi = \frac{1}{\epsilon_0} (q_{\text{enclosed}})$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$



Conclusion:

This shows that the electric field due to a uniform spherical surface charge is the same as that produced by a point charge q .

Intensity at surface:

IF 'a' be the radius of the spherical shell then electric intensity at the surface of the shell is given by.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \text{-----} (3)$$

In the form of surface density:

If " σ " be the surface charge density (charge per unit area) of the shell. Then

$$\begin{aligned}\sigma &= \frac{q}{A} \\ q &= \sigma A \\ q &= \sigma(4\pi a^2) \\ q &= 4\pi a^2 \sigma\end{aligned}$$

Thus equation (3) becomes

$$E = \frac{1}{4\pi\epsilon_0} \frac{4\pi a^2 \sigma}{a^2}$$

$$E = \frac{\sigma}{\epsilon_0}$$



Intensity inside the surface:

If the point "p" is situated inside the charged sphere then imaginary sphere (gaussian surface) encloses no charge i.e ($q = 0$) therefore,

$$E = \frac{1}{4\pi\epsilon_0} \frac{(0)}{a^2}$$

$$E = 0$$

Electric intensity due to an infinite sheet of charge:

Consider a large non-conducting sheet of infinite extent on which charges are distributed uniformly. Let the surface charge density of charge is " σ ". As the sheet is of an infinite extent therefore electric field is perpendicular to the sheet at all points consider a point "P" close to the sheet. To find electric field intensity "E" at P imagine a Gaussian surface in the form of a cylinder in such a way that the point "p" lies in side the cylinder as shown in the figure.

The electric flux through the curved surface of the cylinder is zero because the angle between "E" and the vector area of the curved surface is 90° while electric flux through end faces of the cylinder is given by

$$\begin{aligned}\Phi &= \vec{E} \cdot \vec{\Delta A} = E\Delta A \cos \theta \\ \Phi &= E\Delta A \cos 0^\circ && \text{(E and } \Delta A \text{ are parallel to each other)} \\ \Phi &= E\Delta A(1) \\ \Phi &= E\Delta A\end{aligned}$$

Electric flux through both ends of the cylinder is the algebraic sum of flux through each face of cylinder. Therefore

$$\begin{aligned}\Phi &= E\Delta A + E\Delta A && \text{(Where } \Delta A = \text{cross section area of the end faces)} \\ \Phi &= 2E\Delta A && \text{-----} (1)\end{aligned}$$

Applying Gauss' Law

$$\Phi = \frac{1}{\epsilon_0} (q_{\text{enclosed}})$$

$$2E\Delta A = \frac{q}{\epsilon_0} \text{-----} (2)$$

$$\text{But, } q = \sigma A$$

Substituting this value in eqn (2) we get

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

ELECTRIC INTENSITY BETWEEN TWO OPPOSITELY CHARGED PLATES:

Consider two parallel oppositely charged metal plates separated by a small distance let the surface charge density of each plate is " σ " since the plates are very closed to each other therefore electric lines of force are parallel to each other except the edges.

Now consider a point "P" between plates. The magnitude of electric field intensity at "P" due to the positive plate is $E_1 = \frac{\sigma}{2\epsilon_0}$ and is directed towards the negative plate. Similarly the electric intensity at "p" due to the

negative plate is also $E_2 = \frac{\sigma}{2\epsilon_0}$ and is also directed towards the negative plate. As the electric field intensity at point "P" due to positive and negative charged plates are in the same direction therefore simply adding them to get net intensity " E " at point "P". Therefore

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

The electric intensity in vector form is given by.

$$E = \left(\frac{\sigma}{\epsilon_0}\right)\hat{r}$$

Where \hat{r} = unit vector directed from positive plate to the negative plate.

ELECTRIC POTENTIAL DIFFERENCE:

The work done against the electric field per unit test charge is called electric potential difference.

(OR)

The potential energy per unit test charge is called electric potential difference. It is denoted by ΔV and measured in S.I. unit in volt (V).

Explanation:

Consider test charge q_0^+ placed in the electric field of a point charge q^+ at distance r . Δr is the displacement against electric field and it is so small that the electric intensity for such a small displacement nearly remains constant. If q_0 is moved towards q through displacement Δr then the potential difference is given by,

$$\Delta V = \frac{\text{work}}{q_0}$$

$$\Delta V = \frac{\vec{F} \cdot \vec{\Delta r}}{q_0}$$

$$\Delta V = \frac{\vec{F}}{q_0} \cdot \vec{\Delta r}$$

Since, $\vec{E} = \frac{\vec{F}}{q_0}$

$$\Delta V = \vec{E} \cdot \vec{\Delta r}$$

$$= E \Delta r \cos 180^\circ$$

$$\Delta V = E \Delta r (-1)$$

$$E = -\frac{\Delta V}{\Delta r} = \text{negative of potential gradient}$$



This shows that negative of potential gradient is called electric field intensity. This is why the commercial unit of electric field intensity is volt per metre $V.m^{-1}$.

VOLT:

The potential difference between two points in an electric field will be 1 volt if the work done per unit charge in moving a test charge between these points is 1 joules.



ABSOLUTE ELECTRIC POTENTIAL:

The absolute electric potential at a point is the work done per unit charge when a test charge is moved from infinity to that point.

Derivation:

Consider two points A and B in a straight line at distances r_A and r_B respectively from a point charge q . let a test charge q_0 be placed at A in the electric field of q and we want to bring q from A to B. The work done in bringing the charge q from A to B can not be found directly in once step because electric intensity and hence force is not constant for direct displacement. To overcome this difficulty we divide the line joining A and B into "N" number of small equal displacement each of magnitude Δr as shown in the figure. Now move the charge q from A to 1. The potential difference will be,

$$\begin{aligned}\Delta V_{A-1} &= \vec{E} \cdot \vec{\Delta r} \\ &= E \Delta r \cos 180^\circ \\ \Delta V_{A-1} &= \frac{Kq}{\bar{r}^2} (r_A - r_1) (-1)\end{aligned}$$

$$\Delta V_{A-1} = Kq \left[\frac{r_1 - r_A}{\bar{r}^2} \right]$$

where \bar{r}^2 = Square of mean distance of points A and 1 from the point charge q and can be given by the formula of geometric mean $\bar{r}^2 = r_A r_1$

thus,

$$\Delta V_{A-1} = Kq \left[\frac{r_1 - r_A}{r_A r_1} \right]$$

$$\Delta V_{A-1} = Kq \left[\frac{r_1}{r_A r_1} - \frac{r_A}{r_A r_1} \right]$$

$$\Delta V_{A-1} = Kq \left[\frac{1}{r_A} - \frac{1}{r_1} \right]$$

Similarly, the potential difference from point 1 to 2,

$$\Delta V_{1-2} = Kq \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

Similarly, the potential difference from point 1 to 2,

$$\Delta V_{2-3} = Kq \left[\frac{1}{r_2} - \frac{1}{r_3} \right]$$

Now ,the potential difference from point N-1 to B,

$$\Delta V_{N-1 \rightarrow B} = Kq \left[\frac{1}{r_{N-1}} - \frac{1}{r_B} \right]$$

Finally for total potential difference from point A to B,

ΔV_{A-B} = sum of all potential differences

$$\Delta V_{A-B} = Kq \left[\frac{1}{r_A} - \frac{1}{r_1} \right] + Kq \left[\frac{1}{r_1} - \frac{1}{r_2} \right] + Kq \left[\frac{1}{r_2} - \frac{1}{r_3} \right] + \dots + Kq \left[\frac{1}{r_{N-1}} - \frac{1}{r_B} \right]$$



$$\Delta V_{A-B} = Kq \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

If A is situated at infinity then $r_A = \infty$ and $r_B = r$

$$V_{\text{abs}} = Kq \left[\frac{1}{\infty} - \frac{1}{r} \right]$$

$$V_{\text{abs}} = Kq \left[0 - \frac{1}{r} \right]$$

The absolute potential “V” at a distance “r” from the point charge “q” is given by

$$V_{\text{abs}} = - \frac{Kq}{r}$$

Potential is a scalar quantity and the potential at a point due to a number of charges is the algebraic sum of all potential at that point.

ELECTRON VOLT:

In atomic physics the energies of accelerated particles are frequently expressed in terms of electron volt. If a charge “q” is allowed to move through a potential difference of “V” then its potential energy will decrease while kinetic energy will increase according to the law of conservation of energy

Loss in P.E = gain in K.E

$$qV = \frac{1}{2}mv^2$$

This shows that the energy can be expressed as the product of charge and potential difference an electron volt is the energy required by an electron in falling through a potential difference of 1 volt.

1 electron – volt = (charge on one electron) (1 volt)

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

EQUIPOTENTIAL SURFACE:

- 1) An equipotential surface is that at all points of which the potential has the same value.
- 2) Since potential difference on equipotential surface is zero therefore no work is done in moving a charge from one place to the other on equipotential surface.
- 3) Electric lines of force are perpendicular to equipotential surface.
- 4) Two equipotential surface of different potentials never intersect with each other.
- 5) Equipotential surface may not be in the form of sphere.

CAPACITANCE:

The capacity of storing electric charge is called “capacitance”

The potential “V” of a conductor depends on its own charge “q” as well the charges on the neighboring bodies for an isolated conductor. Mathematically,

$$q \propto V$$

$$q = CV$$

Where C= capacity to store electric charge or capacitance of Capacitor

C = Measure the charge should be placed on a conductor to rise its potential to 1 volt

CAPACITOR:

Capacitor is a device used for storing electric charges. A capacitor consists of two non-conducting plates separated by air or any insulating material non-conductors have opposite charges.

CAPACITANCE OF CAPACITOR: It is the ratio of the charge of the conductor to the potential difference between them its unit is FARAD.

FARAD: If by giving 1 volt potential difference between two parallel plates of capacitor 1 coulomb charge is stored on the plate of capacitor then capacitance will be 1 Farad. It is large unit its sub-multiples are

Micro farad (μF) = $1 \mu\text{F} = 10^{-6} \text{ F}$

Pico Farad (pF) = micro-micro farad ($\mu\mu\text{F}$) = $1 \mu\mu\text{F} = 1\text{pF} = 10^{-12} \text{ F}$



PARALLEL PLATES CAPACITOR:

A parallel plate capacitor consists of two parallel metal plates separated by a small distance "d" the charges on each plate are uniformly distributed on the inner sides of the plates due to attraction between opposite charges the electric field "E" between the plates is uniform except at its outer boundary.

(i) Capacitance of a Parallel Plate Capacitor in the presence of air or absence of DIELECTRIC SLAB:

Consider a parallel plate capacitor consisting of two metallic plates each of area 'A' separated by a small distance 'd' if 'Q' be the charge on each plate then capacitance is given by,

$$Q = CV$$

$$C = \frac{Q}{V} \text{-----(i)}$$

For Electric Charge:

By the definition of charge density,

$$\sigma = \frac{Q}{A}$$

$$Q = \sigma A$$

For potential difference:

$$V = E \times d \text{-----(ii)}$$

If 'E' be the electric field intensity between the plates then

$$E = \frac{\sigma}{2\epsilon_0}$$

Now,

$$V = \frac{\sigma}{2\epsilon_0} \times d$$

Hence eqⁿ(i) becomes,

$$C = \frac{\sigma A}{\frac{\sigma}{2\epsilon_0} \times d}$$

$$C = \frac{A\epsilon_0}{d} \text{-----(iii)}$$

This expression is called expression for capacitance in the presence of air between two parallel plates of capacitor.

(ii) Capacitance of a Parallel Plate Capacitor in the presence of air or absence of DIELECTRIC SLAB:

Equation (iii) shows that the capacitance is

- (i) directly proportional to the area of the plates
 - (ii) inversely proportional to the distance between the plates and
 - (iii) it is enhanced by ϵ if an insulating material called DIELECTRIC is introduced between the plates
- Therefore, if we placed a dielectric slab of dielectric constant ϵ_r and of thickness $t=d$ between two parallel plates of identical capacitor in eqⁿ(iii) then capacitance will be,



$$C' = \frac{A\epsilon_0 \epsilon_r}{d} \text{-----(iv)}$$

Dielectric Constant: It is the ratio between the capacitance of a parallel plate capacitor in the presence of dielectric between the plates to its capacitance when there is free space between the plates. It has no unit.

$$\epsilon_r = \frac{C'}{C}$$

Effect of dielectric in capacitor:

(a) When capacitor is disconnected after being charged and a dielectric is introduced between the plates of the charged capacitor the electric intensity decreases as a result of which potential difference decreases and capacitance increases

(b) When capacitor is connected to a battery and a dielectric is introduced between the plates of the capacitor the atoms of the dielectric becomes polarized the plates accumulate more charges from the battery as a result of which potential difference remains constant and capacitance increases

Factors on which capacitance of a parallel plate capacitor depends:

Capacitance of a parallel plate capacitor depends upon the following factors

1-Area of the plates: Capacitance increases with the increases of the area of the plates.

2. Distance between the plates: Capacitance increases with the decrease of distance between the plates.

3. Dielectric between the plates: Capacitance increases with the introduction dielectric between the plates.

CAPACITORS IN PARALLEL:

Capacitors are said to be in parallel if potential differences across each capacitor is same or connections of Capacitors are ends to ends. Parallel combination is also known as current or charge divider network.

Explanation:

Consider three capacitor having capacitance C_1 , C_2 and C_3 are connected in parallel and there combination is connected with a battery of voltage V as shown in figure. Since the capacitors are connected in parallel therefore potential differences across each capacitor will remain same but charges will be,

Q_1 = the charge drawn by the capacitor of capacitance ' C_1 ' = $C_1 V$

Q_2 = the charge drawn by the capacitor of capacitance ' C_2 ' = $C_2 V$

Q_3 = the charge drawn by the capacitor of capacitance ' C_3 ' = $C_3 V$

If, C_e = the capacitance of equivalent capacitor of the network

Then, charge on plates of this capacitor = $Q = C_e V$

According to the property of parallel combination,

$$Q = Q_1 + Q_2 + Q_3 \text{-----(i)}$$

By substituting the values of all charges,

$$\Rightarrow C_e V = C_1 V + C_2 V + C_3 V$$

$$C_e V = V(C_1 + C_2 + C_3)$$

$$C_e = C_1 + C_2 + C_3$$

This shows that when two or more than two capacitors are connected in parallel the capacitance of combination is equal to the sum of the capacitance of individual capacitors.

CAPACITORS IN SERIES:

Capacitors are said to be in series if the charge on each capacitor is same or connections of Capacitors are end to end. Series combination is also known as voltage divider network.

Explanation:

Consider three capacitor having capacitance C_1 , C_2 and C_3 are connected in series and there combination is connected with a battery of voltage V as shown in figure. The plates 'a' of C_1 is connected to the positive terminal of the battery therefore it draws positive charge from the battery and the plate 'd' of C_3 is connected to the negative terminal of the battery therefore it draws negative charge. Plates b and c will not draw any charge from the battery. They get charged due to electrostatic induction. In this way each plate acquires same charge Q .

If, V_{ab} = the potential difference across C_1 , then $V_{ab} = \frac{Q}{C_1}$

V_{bc} = the potential difference across C_2 , then $V_{bc} = \frac{Q}{C_2}$

V_{cd} = the potential difference across C_3 , then $V_{cd} = \frac{Q}{C_3}$

V_{ad} = the potential difference across C_e (equivalent capacitance of the network), then $V_{ad} = \frac{Q}{C_e}$

According to the property of series combination,

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} \text{ -----(i)}$$

$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{Q}{C_e} = Q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

This shows that the reciprocal of the equivalent capacitance can be obtained by adding reciprocals of the capacitance of the individual capacitors. The resultant is less than the smallest individual capacitance i.e capacitance decreases.

(iii) CAPACITANCE OF PARALLEL PLATES CAPACITOR IN THE PARTIAL PRESENCE OF DIELECTRIC SLAB OF SAME AREA BUT THICKNESS LESS THAN SEPERATION BETWEEN PLATES ($t < d$) COMPOUND CAPACITOR:

A compound capacitor consists of more than one dielectric between the plates is called compound capacitor.

Consider two parallel metal plates each of area A separated by a small distance d . let a dielectric slab of area A and thickness $t < d$ is slipped between the plates.

Capacitance of air portion:

$$C_1 = \frac{A \epsilon_0}{(d-t)}$$

Capacitance of dielectric slab portion:

$$C_2 = \frac{A\epsilon_0\epsilon_r}{t}$$

Where ϵ_r = dielectric constant of material of slab

The equivalent capacitance C_d can be found by formula.

$$C_d = \frac{C_1 \times C_2}{C_1 + C_2}$$

Putting the values of C_1 and C_2 ,

$$C_d = \frac{\frac{A\epsilon_0}{(d-t)} \times \frac{A\epsilon_0\epsilon_r}{t}}{\frac{A\epsilon_0}{(d-t)} + \frac{A\epsilon_0\epsilon_r}{t}}$$

$$C_d = \frac{\frac{A^2\epsilon_0^2\epsilon_r}{t(d-t)}}{A\epsilon_0 \left[\frac{1}{(d-t)} + \frac{\epsilon_r}{t} \right]}$$

$$C_d = \frac{\frac{A\epsilon_0\epsilon_r}{t(d-t)}}{\left[\frac{t+(d-t)\epsilon_r}{t(d-t)} \right]}$$



$$C_d = \frac{A\epsilon_0\epsilon_r}{\left[(d-t) + \frac{t}{\epsilon_r} \right] \epsilon_r}$$

$$C_d = \frac{A\epsilon_0}{\left[(d-t) + \frac{t}{\epsilon_r} \right]}$$

DIFFERENT TYPES OF CAPACITOR:

A) Multiplates capacitor:

It consists of a large number of plates each of large area is designed to have large capacitance. When N plates are used there are N individual's capacitors in parallel. In high grade capacitors mica is used as dielectric. In expensive capacitors of capacitance up to $10\mu\text{F}$ is usually made of alternate layers of tins or aluminum foil and waxed paper. These are frequently wound into rolls under pressure and sealed into moisture resisting metal container.

B) Variable capacitor:

It is that capacitor whose capacitance can be changed. It is used for tuning radio sets. It consists of two sets of semi circular aluminum or brass plates separated by air. One set of plates is fixed and other is rotated by a knob to alter the effective area of the plates.

C) Electrolytic capacitor:

It consists of two sheets of aluminum equally separated by a dielectric soaked in a special solution of ammonium borate these are rolled up and sealed in an insulating container. Wires attached to the foil strips are then connected to a battery and a highly insulating thin film of aluminum oxide forms on the positive foil. A capacitor is thus formed in which film acts as the dielectric. Its capacitance is up to $1000\mu\text{F}$. Its terminals are marked positive and negative.