

Unit: 12
"Introduction To Numerical Methods"

Pg (1)

Exercise: 12.1

1. Use bisection method to find a real root of the following equations.

i) $f(x) = 2x^3 - 2x - 5$, $[1, 2]$ up to three iterations.

Solve:-

$f(x) = 2x^3 - 2x - 5 = 0$

Your Success is our win

Taking $a = 1$ & $b = 2$

Here $f(1) = 2(1)^3 - 2(1) - 5 = -5 < 0$

$f(2) = 2(2)^3 - 2(2) - 5 = 7 > 0$

Since $f(1) \cdot f(2) < 0$

$\therefore$ root lies between 1 & 2

1st iteration:

Now,

$$c = \frac{1+2}{2} = 1.5$$

$\Rightarrow f(c) = f(1.5) = 2(1.5)^3 - 2(1.5) - 5 = -1.25 < 0$

$\because f(1.5) \cdot f(2) < 0$

$\therefore$ root lies between 1.5 & 2

2nd Iteration:

Now,

$$c = \frac{1.5+2}{2} = 1.75$$

$\Rightarrow f(c) = f(1.75) = 2(1.75)^3 - 2(1.75) - 5 = 2.2188 > 0$

$\because f(1.75) \cdot f(1.5) < 0$

$\therefore$ root lies between 1.75 & 1.5

3rd Iteration:

Now,

$$c = \frac{1.75+1.5}{2} = 1.625$$

$\Rightarrow f(c) = f(1.625) = 2(1.625)^3 - 2(1.625) - 5 = 0.332 > 0$ Pg (2)

~~$\because f(1.625)\cdot f(1.5) < 0$~~

~~$\therefore$ root lies between 1.625 & 1.5~~

Hence 1.625 is the approximate root of $2x^3-2x-5=0$ after three iterations.

(ii) $f(x) = x^3-2x-5$, $[1.5, 2.5]$ up to four iterations.

Solution:

$f(x) = x^3 - 2x - 5 = 0$

Taking $a = 1.5$ & $b = 2.5$

Here, $f(1.5) = (1.5)^3 - 2(1.5) - 5 = -4.625 < 0$

$f(2.5) = (2.5)^3 - 2(2.5) - 5 = 5.625 > 0$

$\because f(1.5)\cdot f(2.5) < 0$

$\therefore$ Root lies between 1.5 & 2.5.

1st Iteration:-

Now,

$$c = \frac{1.5+2.5}{2} = 2$$

$\Rightarrow f(c) = f(2) = 2^3 - 2(2) - 5 = -1 < 0$

$\because f(2)\cdot f(2.5) < 0$

$\therefore$ Root lies between 2 & 2.5

2nd Iteration:

Now,

$$c = \frac{2+2.5}{2} = 2.25$$

$\Rightarrow f(c) = f(2.25) = (2.25)^3 - 2(2.25) - 5 = 1.8906 > 0$

$\because f(2.25)\cdot f(2) > 0$

$\therefore$ root lies b/w 2.25 & 2

3rd Iteration: Pg 3

Now,

$$c = \frac{2.25+2}{2} = 2.125$$

$\Rightarrow f(c) = f(2.125) = (2.125)^3 - 2(2.125) - 5 = 0.3457 > 0$

$\because f(2.125) \cdot f(2) < 0$

$\therefore$ Root lies between 2.125 & 2.

4th Iteration:-

Your Success is our Win

Now,

$$c = \frac{2.125+2}{2} = 2.0625$$

$\Rightarrow f(c) = f(2.0625) = (2.0625)^3 - 2(2.0625) - 5 = -0.3513 < 0$

$\therefore \cancel{f(2.0625)} \cdot f(2.125) < 0$

~~$\therefore$ root lies b/w 2.0625~~

Hence 2.0625 is the approximate root of $x^3 - 2x - 5 = 0$ after four iterations.

(iii) $f(x) = x^3 - x + 1$, $[-2, -1]$ up to five iterations.

Solution:-

$f(x) = x^3 - x + 1 = 0$

Taking $a = -2$ & $b = -1$

Here, $f(-2) = (-2)^3 - (-2) + 1 = -5 < 0$

$f(-1) = (-1)^3 - (-1) + 1 = 1 > 0$

$\because f(-2) \cdot f(-1) < 0$

$\therefore$ root lies between -2 & -1

1st Iteration:-

$$c = \frac{-2-1}{2} = -1.5$$

$f(c) = f(-1.5) = (-1.5)^3 - (-1.5) + 1 = -0.875 < 0$ Pg 04 (iv)

$\therefore f(-1.5) \cdot f(-1) < 0$

$\because$ Root lies b/w -1.5 & -1

2nd iteration:-

now,

$$c = \frac{-1.5 - 1}{2} = -1.25$$

$\Rightarrow f(c) = f(-1.25) = (-1.25)^3 - (-1.25) + 1 = 0.2969 > 0$

$\therefore f(-1.25) \cdot f(-1.5) < 0$

$\therefore$ root lies b/w -1.25 & -1.5

3rd iteration:-

$$c = \frac{-1.25 - 1.5}{2} = -1.375$$

$\Rightarrow f(c) = f(-1.375) = (-1.375)^3 - (-1.375) + 1 = -0.2246 < 0$

$\therefore f(-1.375) \cdot f(-1.25) < 0$

$\therefore$ Root lies b/w -1.375 & -1.25

4th iteration:-

$$c = \frac{-1.375 - 1.25}{2} = -1.3125$$

$\Rightarrow f(c) = f(-1.3125) = (-1.3125)^3 - (-1.3125) + 1 = 0.0515 > 0$

$\because f(-1.3125) \cdot f(-1.375) < 0$

$\because$ root lies b/w -1.3125 & -1.375

5th iteration:-

$$c = \frac{-1.3125 - 1.375}{2} = -1.34375$$

$\Rightarrow f(c) = f(-1.34375) = (-1.34375)^3 - (-1.34375) + 1 = -0.0826 < 0$

$\because$ Hence -1.34375 is the approximate root of $x^3 - x + 1 = 0$ after five iterations.

Pg 05

(iv) $f(x) = \cos x$, $[1, 2]$ up to one decimal place (five iterations)

Solution:-

$f(x) = \cos x = 0$

Taking $a = 1$ & $b = 2$

Here, $f(1) = \cos(1) = 0.5403 > 0$

$f(2) = \cos(2) = -0.4161 < 0$

$\therefore f(1) \cdot f(2) < 0$

$\therefore$ root lies b/w 1 & 2

1st Iteration:-

$$c = \frac{1+2}{2} = 1.5$$

$f(c) = f(1.5) = \cos(1.5) = 0.0707 > 0$

$\therefore f(1.5) \cdot f(2) < 0$

$\therefore$ Root lies b/w 1.5 & 2

2nd Iteration:-

$$c = \frac{1.5+2}{2} = 1.75$$

$f(c) = f(1.75) = \cos(1.75) = -0.1782 < 0$

$\therefore f(1.75) \cdot f(1.5) < 0$

$\therefore$ root lies b/w 1.75 & 1.5

3rd Iteration:-

$$c = \frac{1.75+1.5}{2} = 1.625$$

$f(c) = f(1.625) = \cos(1.625) = -0.05417 < 0$

$\therefore f(1.625) \cdot f(1.5) < 0$

$\therefore$ Root lies b/w 1.625 & 1.5

Pg 06

4th Iteration:

$$c = \frac{1.625 + 1.5}{2} = 1.5625$$

$f(c) = f(1.5625) = \cos(1.5625) = 0.00829 > 0$

$\because f(1.5625) \cdot f(1.625) < 0$

$\therefore$ root lies b/w 1.5625 & 1.625.

5th Iteration:

$$c = \frac{1.5625 + 1.625}{2} = 1.59375$$

$f(c) = f(1.59375) = \cos(1.59375) = -0.0229 < 0$

~~$\because f(1.59375)$~~ Hence 1.59375 is the approximate root of $\cos x$ after five iterations.

(v) $f(x) = 3x - e^x$, $[0,1]$ up to three decimal places (eleven iterations).

Solve:-

$f(x) = 3x - e^x = 0$

Taking $a = 0$ & $b = 1$

Here, $f(0) = 3(0) - e^0 = -1 < 0$

$f(1) = 3(1) - e^1 = 0.2817 > 0$

$\because f(0) \cdot f(1) < 0$

$\therefore$ root lies b/w (0 & 1)

1st Iteration:-

$$c = \frac{0 + 1}{2} = 0.5$$

$\Rightarrow f(c) = f(0.5) = 3(0.5) - e^{0.5} = -0.1487 < 0$

$\because f(0.5) \cdot f(1) < 0$

$\therefore$ Root lies b/w 0.5 & 1.

2nd Iteration: Pg 07

$c = \frac{1+0.5}{2} = 0.75$

$\Rightarrow f(c) = f(0.75) = 3(0.75) - e^{0.75} = 0.1329 > 0$

$\because f(0.75) \cdot f(0.5) < 0$

$\therefore$ Root lies between 0.75 & 0.5

3rd Iteration:

$c = \frac{0.75+0.5}{2} = 0.625$

$\Rightarrow f(c) = f(0.625) = 3(0.625) - e^{0.625} = 0.0067 > 0$

$\because f(0.625) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w 0.625 & 0.5.

4th Iteration.

$c = \frac{0.625+0.5}{2} = 0.5625$

$\Rightarrow f(c) = f(0.5625) = 3(0.5625) - e^{0.5625} = -0.0675 < 0$

$\because f(0.5625) \cdot f(0.625) < 0$

$\therefore$ root lies b/w 0.5625 & 0.625

5th Iteration:-

$c = \frac{0.5625+0.625}{2} = 0.59375$

$\Rightarrow f(c) = f(0.59375) = 3(0.59375) - e^{0.59375} = -0.0295 < 0$

$\because f(0.59375) \cdot f(0.625) < 0$

$\therefore$ Root lies b/w 0.59375 & 0.625

6th Iteration:

$c = \frac{0.59375+0.625}{2} = 0.6094$

$f(c) = f(0.6094) = 3(0.6094) - e^{0.6094} = -0.0111 < 0$

$\because f(0.6094) \cdot f(0.625) < 0$

Pg 08

∴ Root lies b/w 0.6094 & 0.625.

7th Iteration:

$$c = \frac{0.6094 + 0.625}{2} = 0.6172$$

$f(c) = f(0.6172) = 3(0.6172) - e^{0.6172} = -0.0021 < 0$

∵ $f(0.6172) \cdot f(0.625) < 0$

∴ root lies b/w 0.6172 & 0.625.

8th Iteration:

$$c = \frac{0.6172 + 0.625}{2} = 0.6211$$

$f(c) = f(0.6211) = 3(0.6211) - e^{0.6211} = 0.0023 > 0$

∵ $f(0.6211) \cdot f(0.6172) < 0$

∴ root lies b/w 0.6211 & 0.6172.

9th Iteration:

$$c = \frac{0.6211 + 0.6172}{2} = 0.6192$$

$f(c) = f(0.6192) = 3(0.6192) - e^{0.6192} = 0.00015 > 0$

∵ $f(0.6192) \cdot f(0.6172) < 0$

∴ Root lies b/w 0.6192 & 0.6172.

10th Iteration:

$$c = \frac{0.6192 + 0.6172}{2} = 0.6182$$

$\Rightarrow f(c) = f(0.6182) = 3(0.6182) - e^{0.6182} = -0.00098 < 0$

∵ $f(0.6182) \cdot f(0.6192) < 0$

∴ root lies b/w 0.6182 & 0.6192.

11th iteration:

$$c = \frac{0.6182 + 0.6192}{2} = 0.6187$$

Pg 09

$f(c) = f(0.6187) = 3(0.6187) - e^{0.6187} = -0.00041 < 0$

$\because f(0.6187) \cdot f(0.6192) < 0$

Hence 0.6187 is the approximate root of $3x - e^x$ after eleven iterations.

xi) $f(x) = 3x - \sqrt{1+\sin x}$, $[0,1]$ up to three decimal places (thirteen iterations).

Solution:-

$f(x) = 3x - \sqrt{1+\sin x} = 0$

Taking $a = 0$ & $b = 1$

$f(0) = 3(0) - \sqrt{1+\sin(0)} = -1 < 0$

$f(1) = 3(1) - \sqrt{1+\sin(1)} = 1.6429 > 0$

$\because f(0) \cdot f(1) < 0$

$\therefore$ Root lies b/w 0 & 1

1st Iteration:-

$c = \dfrac{0+1}{2} = 0.5$

$\Rightarrow f(c) = 3(0.5) - \sqrt{1+\sin(0.5)} = 0.2836 > 0$

$\because f(0.5) \cdot f(0) < 0$

$\therefore$ Root lies b/w 0.5 & 0

2nd Iteration:

$c = \dfrac{0.5+0}{2} = 0.25$

$\Rightarrow f(0.25) = 3(0.25) - \sqrt{1+\sin(0.25)} = -0.3668 < 0$

$\because f(0.25) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w 0.25 & 0.5

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3rd Iteration:

$c = \frac{0.25+0.5}{2} = 0.375$

$\Rightarrow f(c) = f(0.375) = 3(0.375) - \sqrt{1+\sin(0.375)} = -0.0438 < 0$

$\because f(0.375) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w 0.375 & 0.5.

4th Iteration:-

$c = \frac{0.375+0.5}{2} = 0.4375$

$\Rightarrow f(c) = f(0.4375) = 3(0.4375) - \sqrt{1+\sin(0.4375)} = 0.1193 > 0$

$\because f(0.4375) \cdot f(0.375) < 0$

$\therefore$ Root lies b/w 0.4375 & 0.375

5th Iteration:-

$c = \frac{0.4375+0.375}{2} = 0.40625$

$\Rightarrow f(c) = f(0.40625) = 3(0.40625) - \sqrt{1+\sin(0.40625)}$

$= 0.0375 > 0$

$\because f(0.40625) \cdot f(0.375) < 0$

$\therefore$ Root lies b/w 0.40625 & 0.375.

6th Iteration:-

$c = \frac{0.4062+0.375}{2} = 0.3906$

$\Rightarrow f(c) = f(0.3906) = 3(0.3906) - \sqrt{1+\sin(0.3906)} = -0.00325 < 0$

$\because f(0.40625) \cdot f(0.3906) < 0$

$\therefore$ Root lies between 0.40625 & 0.3906.

7th Iteration:- Pg 11

$c = \dfrac{0.4062 + 0.3906}{2} = 0.3984$

$\Rightarrow f(c) = 3(0.3984) - \sqrt{1 + \sin(0.3984)} = 0.01708 > 0$

$\because f(0.3984) \cdot f(0.3906) < 0$

$\therefore$ Root lies b/w 0.3984 & 0.3906.

8th Iteration:-

$c = 0.3945$

$\Rightarrow f(c) = 3(0.3945) - \sqrt{1 + \sin(0.3945)} = 0.0069 > 0$

$\because f(0.3945) \cdot f(0.3906) < 0$

$\therefore$ Root lies b/w 0.3945 & 0.3906.

9th Iteration:-

$c = \dfrac{0.3945 + 0.3906}{2} = 0.3925$

$\Rightarrow f(c) = f(0.3925) = 3(0.3925) - \sqrt{1 + \sin(0.3925)} = 0.0017 > 0$

$\because f(0.3925) \cdot f(0.3906) < 0$

$\therefore$ Root lies b/w 0.3925 & 0.3906.

10th Iteration:-

$c = \dfrac{0.3925 + 0.3906}{2} = 0.3915$

$\Rightarrow f(c) = f(0.3915) = 3(0.3915) - \sqrt{1 + \sin(0.3915)} = -0.0009$

$\because f(0.3915) \cdot f(0.3925) < 0$

$\therefore$ Root lies b/w 0.3915 & 0.3925.

11th iteration:-

$c = \dfrac{0.3915 + 0.3925}{2} = 0.392$

$\Rightarrow f(c) = f(0.392) = 3(0.392) - \sqrt{1 + \sin(0.392)} = 0.000399 > 0$

$\because f(0.392) \cdot f(0.3915) < 0$

$\therefore$ Root lies b/w 0.392 & 0.3915.

Pg 12

12th iteration:

$$c = \frac{0.392 + 0.3915}{2} = 0.3917$$

$f(c) = 3(0.3917) - \sqrt{1+\sin(0.3917)} = -0.00038 < 0$

$\Rightarrow f(0.3917)\cdot f(0.392) < 0$

$\therefore$ Root lies b/w 0.3917 & 0.392.

13th iteration:

$$c = \frac{0.3917 + 0.392}{2} = 0.39185$$

$\Rightarrow f(c) = f(0.39185) = 3(0.39185) - \sqrt{1+\sin(0.39185)} = 0.000008 > 0$

Hence 0.39185 is the approximate root of $3x - \sqrt{1+\sin x}$ after thirteen iterations.

Exercise: 12.2

Use Regula Falsi method to find a root of following equations:

1. $x^3 - 2x - 5 = 0$, $[2, 3]$ up to three iterations.

Solution:

$f(x) = x^3 - 2x - 5 = 0$

So, $f(2) = (2)^3 - 2(2) - 5 = -1 < 0$

$f(3) = (3)^3 - 2(3) - 5 = 16 > 0$

Therefore, root lies b/w $a = 2$, & $b = 3$

since $f(1)\cdot f(2) < 0$

1st iteration:

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2(16) - 3(-1)}{16+1} = \frac{35}{17} = 2.0588$$

$f(x_1) = (2.0588)^3 - 2(2.0588) - 5 = -0.3910 < 0$

2nd Iteration: Pg 13

Here $f(2.0588) = -0.3910 < 0$ & $f(3) = 16 > 0$

Root lies b/w $a = 2.0588$ & $b = 3$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.0588(16) - 3(-0.3910)}{16 + 0.3910}$$

$x_2 = 2.0812$

$f(x_2) = (2.0812)^3 - 2(2.0812) - 5 = -0.1479 < 0$

3rd Iteration:-

Here $f(2.0812) = -0.1479 < 0$ & $f(3) = 16 > 0$

∴ Root lies b/w $a = 2.0812$ & $b = 3$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.0812(16) - 3(-0.1479)}{16 + 0.1479}$$

$x_3 = 2.0896$

$f(x_3) = f(2.0896) = (2.0896)^3 - 2(2.0896) - 5 = -0.055 < 0$

Approximate root of the equation $x^3 - 2x - 5$ using false Position method is 2.0896 (after 3 iterations), Ans

(2) $\sin(2x) - e^{x-1} = 0$, $[-2, -1]$ up to four iterations.

Solve:

$f(x) = \sin 2x - e^{x-1} = 0$

So, $f(-2) = \sin 2(-2) - e^{-2-1} = 0.707 > 0$

$f(-1) = \sin 2(-1) - e^{-1-1} = -1.0446 < 0$

∴ Root lies b/w $a = -2$ & $b = -1$

∵ $f(-2) \cdot f(-1) < 0$

1st iteration:

$$x_1 = \frac{a f(b) - bf(a)}{f(b) - f(a)} = \frac{-2(-1.0446) - (-1)(0.707)}{-1.0446 - 0.707}$$

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$x_1 = -1.5963 < 0$

$f(x_1) = f(-1.5963) = -0.0235 < 0$

2nd Iteration

Here $f(-1.5963) \cdot f(-2) < 0$

∴ Root lies b/w $\underset{a}{-1.5963}$ & $\underset{b}{-2}$

So,

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_2 = \frac{(-1.5963)(0.707) - (-2)(-0.0235)}{0.707 + 0.0235} = -1.6092 < 0$$

$f(x_2) = f(-1.6092) = \sin 2(-1.6092) - e^{-1.6092-1} = 0.00313 > 0$

3rd Iteration:

Here $f(-1.6092) \cdot f(-1.5963) < 0$

∴ Root lies b/w $a = -1.6092$ & $b = -1.5963$

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{-1.6092(-0.0235) - (-1.5963)(0.00313)}{-0.0235 - 0.00313}$$

$x_3 = -1.6076 < 0$

$f(x_3) = f(-1.6076) = \sin(2 \times -1.6076) - e^{-1.6076-1} = -0.00017 < 0$

4th Iteration:

Here $f(-1.6076) \cdot f(-1.6092) < 0$

∴ Root lies b/w $a = -1.6076$ & $b = -1.6092$

$$x_4 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{-1.6076(0.00313) - (-1.6092)(-0.00017)}{0.00313 + 0.00017}$$

$x_4 = -1.6076$

$f(x_4) = f(-1.6076) = \sin 2(-1.6076) - e^{-1.6076-1} = -0.00017 < 0$

Approximate root of the equation $\sin 2x - e^{x-1}$ using False position method is (-1.6076) (after 4 iterations)

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3) $x^4 - x - 10 = 0$, $[1, 2]$ up to two decimal places.

Solution:

Your Success is our Win

$f(x) = x^4 - x - 10 = 0$

So, $f(1) = (1)^4 - (1) - 10 = -10 < 0$

$f(2) = (2)^4 - 2 - 10 = 4 > 0$

∴ Root lies b/w $a = 1$ & $b = 2$

∵ $f(1) \cdot f(2) < 0$

Iteration: 01

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(4) - 2(-10)}{4 + 10} = 1.7142 > 0$$

>0

$f(x_1) = f(1.7142) = (1.7142)^4 - 1.7142 - 10 = -3.0795 < 0$

2nd Iteration:

Here $f(1.7142) \cdot f(2) < 0$ ∴ Root lies b/w

13) $a = 1.7142$ & $b = 2$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.7142(4) - 2(-3.0795)}{4 + 3.0795}$$

<0

$x_2 = 1.8385 > 0$

$f(x_2) = 1.8385^4 - 1.8385 - 10 = -0.4135 < 0$

3rd Iteration:

Here $f(1.8385) \cdot f(2) < 0$

∴ Root lies b/w $a = 1.8385$ & $b = 2$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.8385(4) - 2(-0.4135)}{4 + 0.4135} = 1.8536$$

$f(x_3) = f(1.8536) = 1.8536^4 - 1.8536 - 10 = -0.0486 < 0$

4th Iteration:

wing

Here $f(1.8536) \cdot f(2) < 0$

∴ Root lies b/w $a = 1.8536$ & $b = 2$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1.8536(4) - 2(-0.0486)}{4 + 0.0486} = 1.8553 > 0$$

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$f(x_4) = f(1.8553) = (1.8553)^4 - 1.8553 - 10 = -0.0069 < 0$

Approximate root of the equation $x^4 - x - 10$ using false position method is 1.8553.

4) $3x + \sin x - e^x = 0$, $[0,1]$ up to three decimal places.

Solve:

$f(x) = 3x - \sin x - e^x = 0$

So, $f(0) = 3(0) - \sin(0) - e^0 = -1 < 0$

$f(1) = 3(1) - \sin(1) - e = 1.1231 > 0$

$\therefore$ Root lies b/w $a = 0$ & $b = 1$

$\because f(0) \cdot f(1) < 0$

Iteration: 01

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0(1.1231) - 1(-1)}{1.1231 + 1} = 0.4711 > 0$$

$f(x_1) = f(0.471) = 3(0.471) + \sin(0.471) - e^{0.471} = 0.2651 > 0$

2nd Iteration:

$\because f(0.471) \cdot f(0) < 0$

$\therefore$ Root lies b/w 0.471 & 0

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0.471(-1) - 0(0.2651)}{-1 - 0.2651} = 0.3723 > 0$$

$f(x_2) = 3(0.3723) + \sin(0.3723) - e^{0.3723} = 0.0295 > 0$

3rd iteration:

$\because f(0.3723) \cdot f(0) < 0$

$\therefore$ Root lies b/w $a = 0.3723$ & $b = 0$

$$x_3 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_3 = \frac{0.3723(-1) - 0(0.0295)}{-1 - 0.0295} = 0.3616 > 0$$

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$f(x_3) = f(0.3616) = 3(0.3616) + \sin(0.3616) - e^{0.3616} = 0.002920$

4th iteration:

$\because f(0.3616) \cdot f(0) < 0$

$\therefore$ Root lies b/w $\underset{a}{0.3616}$ & $\underset{b}{0}$

$$x_4 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{0.3616(-1) - 0(0.0029)}{-1 - 0.0029} = 0.3605 > 0$$

$f(x_4) = f(0.3605) = 3(0.3605) + \sin(0.3605) - e^{0.3605} = 0.00019 > 0$

Approximate root of the equation $3x + \sin x - e^x$ using false position method is 0.3605.

(5) $f(x) = 2\cos x - x = 0$, $[1, 2]$ up to five decimal places.

Solve:-

$f(x) = 2\cos x - x = 0$

So, $f(1) = 2\cos(1) - 1 = 0.0806 > 0$

$f(2) = 2\cos(2) - 2 = -2.8322 < 0$

$\therefore$ Root lies b/w $a = 1$ & $b = 2$

$\therefore f(1) \cdot f(2) < 0$

Iteration: 01

$$x_1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{1(-2.8322) - 2(0.0806)}{-2.8322 - 0.0806}$$

$x_1 = 1.0276$

$f(x_1) = f(1.0276) = 2\cos(1.0276) - 1.0276 = 0.0061 > 0$

Iteration: 02

$f(1.0276) \cdot f(2) < 0$

$\therefore$ Root lies b/w $(a = 1.0276)$ & $b = 2$.

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$$x_2 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{1.0276(-2.8322) - 2(0.0061)}{-2.8322 - 0.0061}$$

$x_2 = 1.0296$

$f(x_2) = f(1.0296) = 2\cos(1.0296) - 1.0296 = 0.00072 > 0$

Iteration : 3

$\because f(1.0296) \cdot f(2) < 0$

$\therefore$ Root lies b/w $1.0296 = a$ & $b = 2$

$$x_3 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{1.0296(-2.8322) - 2(0.00072)}{-2.8322 - 0.00072}$$

$x_3 = 1.0298$

$f(x_3) = f(1.0298) = 2\cos(1.0298) - 1.0298 = 0.00018 > 0$

Iteration 4

$\because f(1.0298) \cdot f(2) < 0$

$\therefore$ Root lies b/w $a = 1.0298$ & $b = 2$

$$x_4 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{1.0298(-2.8322) - 2(0.00018)}{-2.8322 - 0.00018}$$

$x_4 = 1.02986$

$f(x_4) = f(1.02986) = 2\cos(1.02986) - 1.02986 = 0.000017$

Iteration : 5

$f(1.02986) \cdot f(2) < 0$

$\therefore$ Root lies b/w $a = 1.02986$ & $b = 2$.

$$x_5 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{1.02986(-2.8322) - 2(0.000017)}{-2.8322 - 0.000017}$$

$x_5 = 1.029865$

$f(x_5) = f(1.029865) = 2\cos(1.029865) - 1.029865 = 0.0001$

Approximate root of the equation $2\cos x - x$ using False Position method is 1.029865.

Exercise: 12-3

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Use Newton Raphson method to find a real root of following functions:

1. $f(x) = 2x^3 - 2x - 5$ up to three iterations with initial guess $x_0 = 2$.

Solve:

let $f(x) = 2x^3 - 2x - 5 = 0$

$f'(x) = 6x^2 - 2$

$x_0 = 2$

1st iteration:

$f(x_0) = f(2) = 2(2)^3 - 2(2) - 5 = 7$

$f'(x_0) = f'(2) = 6(2)^2 - 2 = 22$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{7}{22}$$

$x_1 = 1.6818$

2nd iteration:

$f(x_1) = f(1.6818) = 2(1.6818)^3 - 2(1.6818) - 5 = 1.1501$

$f'(x_1) = f'(1.6818) = 6(1.6818)^2 - 2 = 14.9707$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.6818 - \frac{1.1501}{14.9707}$$

$x_2 = 1.6049$

3rd iteration:

$f(x_2) = f(1.6049) = 2(1.6049)^3 - 2(1.6049) - 5$

$f'(x_2) = f'(1.6049) = 6(1.6049)^2 - 2 =$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6049 - \frac{0.0576}{13.4542} = 1.6006$$

Approximate ~~value~~ root of the equation $x^2 - 3 = 0$, using Newton's method is 1.6006 (After three iterations).

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2) $f(x) = x^3 - x - 1$ up to three iterations with initial guess $x_0 = 1$. 3

Solution:

Let $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$x_0 = 1$

1st iteration:

$f(x_0) = f(1) = (1)^3 - 1 - 1 = -1$

$f'(x_0) = 3(1)^2 - 1 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 + \frac{1}{2} = 1.5$$

2nd iteration:

$f(x_1) = f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875$

$f'(x_1) = 3(1.5)^2 - 1 = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{-0.875}{5.75} + 1.5$$

$x_2 = 1.3478$

3rd iteration:

$f(x_2) = f(1.3478) = (1.3478)^3 - 1.3478 - 1 = 0.1005$

$f'(x_2) = 3(1.3478)^2 - 1 = 4.4496$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3478 - \frac{0.1005}{4.4496} = 1.3252$$

Approximate ~~value~~ root of the equation $x^3 - x - 1$ using Newton's method is 1.3252 (after three iterations) Ans

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3: $f(x) = x^3 - 2x - 5$ up to two iterations with initial guess $x_0 = 1$

Solve:

Let $f(x) = x^3 - 2x - 5 = 0$

$f'(x) = 3x^2 - 2$

$x_0 = 1$

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1st iteration:

$f(x_0) = f(1) = (1)^3 - 2(1) - 5 = -6$

$f'(x_0) = f'(1) = 3(1)^2 - 2 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 + \frac{6}{1} = 7$$

2nd iteration:

$f(x_1) = f(7) = 7^3 - 2(7) - 5 = 324$

$f'(x_1) = f'(7) = 3(7)^2 - 2 = 145$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 7 - \frac{324}{145} = 4.7655$$

~~3rd iteration:~~

$\cancel{f(x_2) = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.7655}$

$\cancel{f(x_2) = f(4.7655) = (4.7655)^3 - 2(4.7655) - 5 = 93.6934}$

$\cancel{f'(x_2) = 3(4.7655)^2 - 2 = 66.1299}$

$\cancel{x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.7655 - \frac{93.6934}{66.1299}}$

$\cancel{x_3 = 3.34869}$

Approximate root ~~value~~ of the equation $x^3 - 2x - 5$ using Newton's method is ~~3.34869~~ 4.7655 (after two ~~three~~ iterations).

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Q: $f(x) = 2\cos x - x$, $x_0 = 0$

Solve:

Let $f(x) = 2\cos x - x = 0$

$f'(x) = -2\sin x - 1$

$x_0 = 0$

1st iteration:

$f(x_0) = 2\cos(0) - 0 = 2$

$f'(x_0) = -2\sin(0) - 1 = -1$

$x_1 = -\frac{f(x)}{f'(x)} + x_0 = 0 - \frac{2}{-1} = +2$

2nd iteration:

$f(x_1) = f(+2) = 2\cos(+2) - (+2) = \cancel{1.1677}\ -2.8322$

$f'(x_1) = f'(+2) = -2\sin(+2) - 1 = \cancel{0.8186}\ -2.8185$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \left[\frac{-2.8322}{-2.8185}\right] = 0.9951$

3rd iteration:

$f(x_2) = f(0.9951) = 2\cos(0.9951) - 0.9951 = 0.0937$

$f'(x_2) = f'(0.9951) = -2\sin(0.9951) - 1 = -2.6776$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.9951 - \frac{0.0937}{-2.6776} = 1.0301$

4th iteration:

$f(x_3) = f(1.0301) = 2\cos(1.0301) - 1.0301 = -0.00063$

$f'(x_3) = f'(1.0301) = -2\sin(1.0301) - 1 = -2.7147$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.02986$

Approximate root of the equation $2\cos x - x$ using Newton's method is 1.02986 (After 4 iterations)

5. $f(x) = 2^x - x - 1.7$, $x_0 = 1.5$ Pg23

Solve:-

Let $f(x) = 2^x - x - 1.7$

$f'(x) = 2^x \ln(2) - 1$

$x_0 = 1.5$

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1^{st} iteration:

$f(x_0) = f(1.5) = 2^{1.5} - 1.5 - 1.7 = -0.3715$

$f'(x_0) = f'(1.5) = 2^{1.5}\ln(2) - 1 = 0.9605$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 + \frac{0.3715}{0.9605} = 1.8867$

2^{nd} iteration:

$f(x_1) = f(1.8867) = 2^{1.8867} - 1.8867 - 1.7 = 0.11118$

$f'(x_1) = f'(1.8867) = 2^{1.8867}\ln(2) - 1 = 1.5631$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.8867 - \frac{0.11118}{1.5631} = 1.8155$

3^{rd} iteration:

$f(x_2) = f(1.8155) = 2^{1.8155} - 1.8155 - 1.7 = 0.0043$

$f'(x_2) = f'(1.8155) = 2^{1.8155}\ln(2) - 1 = 1.4397$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.8155 - \frac{0.0043}{1.4397} = 1.8125$

4^{th} iteration:

$f(x_3) = f(1.8125) = 2^{1.8125} - 1.8125 - 1.7 = 0.0000043$

$f'(x_3) = f'(1.8125) = 2^{1.8125}\ln(2) - 1 = 1.1944$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.8125 - \frac{0.0000043}{1.1944} = 1.8124$

using Newton method

Approximate root of the equation $2^x - x - 1.7$ is 1.8124 (After four iterations)

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(6) $f(x) = 3x - e^x$, $x_0 = 0$

Solution:-

let $f(x) = 3x - e^x = 0$

$f'(x) = 3 - e^x$

$x_0 = 0$

1st iteration:-

$f(x_0) = f(0) = 3(0) - e^0 = -1$

$f'(x_0) = f'(0) = 3 - e^0 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 + \frac{1}{2}$$

$x_1 = 0.5$

2nd iteration:-

$f(x_1) = f(0.5) = 3(0.5) - e^{0.5} = -0.1487$

$f'(x_1) = f'(0.5) = 3 - e^{0.5} = 1.3512$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.5 + \frac{0.1487}{1.3512}$$

$x_2 = 0.61$

3rd iteration:-

$f(x_2) = f(0.61) = 3(0.61) - e^{0.61} = -0.0104$

$f'(x_2) = f'(0.61) = 3 - e^{0.61} = 1.1595$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.61 + \frac{0.0104}{1.1595}$$

$x_3 = 0.6189$

4th iteration:-

$f(x_3) = f(0.6189) = 3(0.6189) - e^{0.6189} = -0.00018$

$f'(x_3) = f'(0.6189) = 3 - e^{0.6189} = 1.1431$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.6189 + \frac{0.00018}{1.1431} = 0.619$$

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Approximate root of the equation $3x - e^x$ using Newton's method is 0.619. (After 4 iterations).

(f) $f(x) = 3x - \sqrt{1+\sin x}$, $x_0 = 1$

Solve:

Let $f(x) = 3x - \sqrt{1+\sin x} = 0$

$$f'(x) = 3 - \frac{\cos x}{2\sqrt{1+\sin x}}$$

$x_0 = 1$

1st iteration:

$f(x_0) = f(1) = 3(1) - \sqrt{1+\sin(1)} = 1.6429$

$$f'(x_0) = f'(1) = 3 - \frac{\cos(1)}{2\sqrt{1+\sin(1)}} = 2.8009$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1.6429}{2.8009} = 0.4134$$

2nd iteration:

$f(x_1) = f(0.4134) = 3(0.4134) - \sqrt{1+\sin(0.4134)} = 0.0562$

$$f'(x_1) = f'(0.4134) = 3 - \frac{\cos(0.4134)}{2\sqrt{1+\sin(0.4134)}} = 2.6132$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.4134 - \frac{0.0562}{2.6132} = 0.3918$$

Approximate root of the equation $3x - \sqrt{1+\sin x}$ using Newton's method is 0.3918 (after 2 iterations)

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Exercise : 12.4

(1) Evaluate the following integrals by Trapezoidal rule.

(i) $\int_0^2 e^x dx$ with 6 intervals

Solve:-

$f(x) = e^x$

Here $a = 0$, $b = 2$ & subintervals $n = 6$

$h = \frac{b-a}{n} = \frac{2-0}{6} = \frac{1}{3} = 0.333$

The values of x & $f(x)$ are given in table

x	0	1/3	2/3	1	4/3	5/3	2
$f(x)$	1	1.3956	1.9477	2.718	3.7936	5.2944	7.389

Using Trapezoidal Rule

$$T_6 = \frac{h}{2}\left[f(x_0) + 2\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right] + f(x_6)\right]$$

$$T_6 = \frac{1}{3.2}\left[1 + 2\{1.3956+1.9477+2.718+3.7936+5.2944\} + 7.389\right]$$

$$T_6 = \frac{1}{6}(38.6876) = 6.4479 \text{ Ans}$$

(ii) $\int_1^3 1/\sqrt{x}\, dx$ with 5 intervals

Solution:-

$f(x) = 1/\sqrt{x}$

Here $a = 1$, $b = 3$ & $n = 5$

$h = \frac{b-a}{n} = \frac{3-1}{5} = 0.4$

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The values of x & $f(x)$ are given in table.

x	1	1.4	1.8	2.2	2.6	3
$f(x)$	1	0.8451	0.7453	0.6741	0.6201	0.5773

Using Trapezoidal Rule:

$$T_5 = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4)\} + f(x_5)\right]$$

$$T_5 = \frac{0.4}{2}\left[1 + 2(0.8451 + 0.7453 + 0.6741 + 0.6201) + 0.5773\right]$$

$$T_5 = 1.4693, \text{ Ans}$$

(iii) $\int_0^{\pi/2} \sin x \, dx$ with 7 intervals

Solve:

$f(x) = \sin x$

Here $a = 0$, $b = \pi/2$, $h = \frac{b-a}{n} = \frac{\pi/2 - 0}{7} = \pi/14$

$n = 7$

The values of x & $f(x)$ are given below

x	0	$\pi/14$	$\pi/7$	$3\pi/14$	$2\pi/7$	$5\pi/14$	$3\pi/7$
$f(x)$	0	0.2225	0.4338	0.6234	0.7818	0.9009	0.9749

Using Trapezoidal Rule:

$$T_7 = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\} + f(x_7)\right]$$

$$T_7 = \frac{\pi}{28}\left[0 + 2(0.2225 + 0.4338 + 0.6234 + 0.7818 + 0.9009 + 0.9749) + 1\right]$$

$$T_7 = 0.9957, \text{ Ans}$$

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(iv) $\int_0^{1.4} e^{-x^2} dx$ with 5 intervals

Solve:-

$f(x) = e^{-x^2}$

$a = 0,\ b = 1.4,\ n = 5,\ h = \frac{b-a}{n} = 0.28$

The values of x & $f(x)$ are given below.

x	0	0.28	0.56	0.84	~~1.12~~ 1.12	1.4
$f(x)$	1	0.9246	0.7308	0.4938	0.2852	0.1408

Using Trapezoidal Rule:

$T_5 = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4)\} + f(x_5)\right]$

$T_5 = \frac{0.28}{2}\left[1 + 2(0.9246 + 0.7308 + 0.4938 + 0.2852) + 0.1408\right]$

$T_5 = 0.8413$ Ans

2) Evaluate the following integrals by Simpson $1/3$ Rule.

i) $\int_2^3 (4x^2+6)\,dx$, with 4 intervals.

Solve:-

$f(x) = 4x^2+6$

$a = 2,\ b = 3,\ n = 4,\ h = \frac{b-a}{n} = 0.25$

The values of x & $f(x)$ are given below.

x	2	2.25	2.5	2.75	3
$f(x)$	22	26.25	31	36.25	42

Using Simpson's $1/3$ Rule by taking $y = f(x)$ for $n = 5$

$S_{1/3} = \frac{h}{3}\{(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\}$

$S_{1/3} = \frac{0.25}{3}\{(22+42) + 4(26.25+36.25) + 2(31)\}$

$S_{1/3} = 31.3333$ Ans

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(ii) $\int_0^2 1/e^x \, dx$, with 6 intervals

Solve:-

$f(x) = 1/e^x$

$a = 0,\ b = 2,\ n = 6,\ h = \frac{b-a}{n} = \frac{2}{6} = 0.333$

The values of x & $f(x)$ are given in following table.

x	0	1/3	2/3	1	4/3	5/3	2
$f(x)$	1	0.7165	0.5134	0.3678	0.2636	0.1888	0.1353

By using Simpson 1/3 rule. by taking $y = f(x)$ for $n = 5$.

$S_{1/3} = h/3 \left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \right]$

$S_{1/3} = 1/9 \left[(1 + 0.1353) + 4(0.7165 + 0.3678 + 0.1888) + 2(0.5134 + 0.2636) \right]$

$S_{1/3} = 0.8646$ Ans

(iii) $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ with 6 intervals

Solve:-

$f(x) = \sqrt{\sin x}$

$a = 0,\ b = \pi/3,\ n = 6,\ h = \frac{b-a}{n} = \pi/18$

The values of x & $f(x)$ are given below.

x	0	0.1745	0.349	0.5236	0.6981	0.8176	1.0471
$f(x)$	0	0.4167	0.5848	0.7071	0.8019	0.8752	0.9306

Using ~~Trapezoidal~~ Simpson 1/3 Rule by taking $y = f(x)$ for $n = 5$.

$S_{1/3} = h/3 \left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \right]$

$S_{1/3} = \frac{\pi}{54} \left[(0 + 0.9306) + 4(0.4167 + 0.7071 + 0.8752) + 2(0.5848 + 0.8019) \right]$

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$S_{1/3} = 0.6806$, Ans

(iv) $\int_0^1 \frac{2}{1+x^2}\,dx$ with 8 intervals

Solve:- $f(x) = \frac{2}{1+x^2}$

$a = 0,\ b = 1,\ n = 8,\ h = \frac{b-a}{n} = 0.125$

The values of x & f(x) are given below.

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
f(x)	2	1.9692	1.8823	1.7534	1.6	1.4382	1.28	1.1327	1

Using Simpson's 1/3 Rule by taking $y = f(x)$ for $n = 8$

$S_{1/3} = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$

$S_{1/3} = \frac{0.125}{3}\left[(2+1)+4(1.9692+1.7534+1.4382+1.1327)+2(1.8823+1.6+1.28)\right]$

$S_{1/3} = 1.5707$ Ans

3) Evaluate the following integrals by Simpson 3/8 rule.

(i) $\int_1^3 \sqrt{x}\,dx$, with 6 intervals

Solve:-

let $f(x) = \sqrt{x}$,

$a = 1,\ b = 3,\ n = 6,\ h = \frac{b-a}{n} = \frac{3-1}{6} = \frac{1}{3} = 0.3333$

The values of x & f(x) are given in the following table.

x	1	4/3	5/3	2	7/3	8/3	3
f(x)	1	1.1547	1.2909	1.4142	1.5275	1.6329	1.732

Using Simpson's 3/8 Rule.

$S_{3/8} = \frac{3}{8}h\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$

$$S_{3/8} = \frac{3}{8}\cdot\frac{1}{3}\left[(1+1.732)+2(1.4142)+3(1.1547+1.2909+1.5275+1.6329)\right]$$

$$S_{3/8} = 2.7973 \text{ , Ans}$$

iii) $\int_1^6 \ln x/x \, dx$, with three sub intervals.

Solve:-

let $f(x) = \ln x / x$

$a = 1,\ b = 6,\ n = 3,\ h = \frac{b-a}{n} = \frac{6-1}{3} = 1.6667$

The values of x & $f(x)$ are given in the following table

x	1	$8/3$	$13/3$	6
$f(x)$	0	0.3678	0.3383	0.2986

Using Simpson's 3/8 Rule:

$$S_{3/8} = \frac{3h}{8}\left[(y_0+y_3)+3(y_1+y_2)\right]$$

$$S_{3/8} = \frac{3}{8}\left(\frac{5}{3}\right)\left[(0+0.2986)+3(0.3678+0.3383)\right]$$

$$S_{3/8} = 1.5105 \text{ , Ans}$$

iii) $\int_0^2 e^{2x}/(1+x^2)\, dx$ with 9 sub intervals.

Solve:-

$f(x) = e^{2x}/1+x^2$

$a = 0,\ b = 2,\ n = 9,\ h = \frac{b-a}{n} = \frac{2-0}{9} = 0.2222$

The values of x & $f(x)$ are given below.

x	0	$2/9$	$4/9$	$2/3$	$8/9$	$10/9$	$4/3$	$14/9$	$16/9$
$f(x)$	1	1.4862	2.0312	2.6263	3.3051	4.1295	5.181	6.5636	8.4142

x	2
$f(x)$	10.919

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Using Simpson's 3/8 rules

$$S_{3/8} = \frac{3}{8}h\left[(y_0+y_9)+2(y_3+y_6)+3(y_1+y_2+y_4+y_5+y_7+y_8)\right]$$

$$S_{3/8} = \frac{3}{8}\left(\frac{2}{9}\right)\left[(1+10.919)+2(2.6263+5.181)+3(1.4862+2.0312+3.3051+4.1295+6.5636+8.4142)\right]$$

$S_{3/8} = 8.7769$, Ans

(iv) $\int_0^{\pi/4} \sin(x)\,dx$ with 6 sub intervals.

Solve:-

$f(x) = \sin x$

$a = 0,\ b = \pi/4,\ n = 6,\ h = \pi/24$

Table of values of x & $f(x)$

x	0	$\pi/24$	$\pi/12$	$\pi/8$	$\pi/6$	$5\pi/24$	$\pi/4$
$f(x)$	0	0.1305	0.2588	0.3827	0.5	0.6087	0.7071

Using Simpson's 3/8 rule,

$$S_{3/8} = \frac{3}{8}h\left[(y_0+y_6)+2(y_3)+3(y_1+y_2+y_4+y_5)\right]$$

$$S_{3/8} = \frac{3}{8}\left(\frac{\pi}{24}\right)\left[(0+0.7071)+2(0.3827)+3(0.1305+0.2588+0.5+0.6087)\right]$$

$S_{3/8} = 0.2928$, Ans

Review Exercise : "12" Pg 33

2. Using Bisection method find the root of $\cos x - xe^x$ with $a=0$ & $b=1$, by taking 5 iterations.

Solve:-

let $f(x) = \cos x - xe^x = 0$

Taking $a=0$, & $b=1$

$f(0) = \cos(0) - (0)e^0 = 1 > 0$

$f(1) = \cos(1) - (1)e^1 = -2.1779 < 0$

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$\therefore f(0) \cdot f(1) < 0$

So, root lies b/w 0 & 1.

1st iteration:

$c = \dfrac{a+b}{2} \Rightarrow c = 0.5$

$f(c) = f(0.5) = \cos(0.5) - (0.5)e^{0.5} = 0.05322 > 0$

$\therefore f(0.5) \cdot f(1) < 0$

$\therefore$ Root lies b/w 0.5 & 1.

2nd iteration:

$c = \dfrac{0.5+1}{2} = 0.75$

$f(c) = f(0.75) = \cos(0.75) - 0.75e^{0.75} = \cancel{-0.8048}\ \cancel{-0.856} < 0$ -0.856

$\therefore f(0.75) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w 0.75 & 0.5

3rd iteration:

$c = \dfrac{0.75+0.5}{2} = 0.625$

$f(c) = f(0.625) = \cos(0.625) - 0.625e^{0.625} = -0.3567 < 0$

$\therefore f(0.625) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w 0.625 & 0.5

4th iteration.

$c = \dfrac{0.625+0.5}{2} = 0.5625$

$f(c) = f(0.5625) = \cos(0.5625) - 0.5625e^{0.5625}$

$f(c) = -0.14129 < 0$

$\because f(0.5625) \cdot f(0.5) < 0$

$\therefore$ Root lies b/w

5th iteration:

$c = \dfrac{0.5625 + 0.5}{2} = 0.53125$

$f(c) = f(0.53125) = \cos(0.53125) - 0.53125e^{0.53125}$

$f(c) = -0.0413 < 0$

$\therefore f(0.53125) \cdot f(0.5) < 0$

Hence 0.53125 is the approximate root of $\cos x - xe^x$ after five iterations.

(3) Find a root for the equation $2e^x \sin x = 1$ using the false position method & correct it to three decimal places with three iterations, taking $[0, 1]$ as an interval.

Solution:

$f(x) = 2e^x \sin x - 1 = 0$

So, $f(1) = 2e^1 \sin(1) - 1 = 3.5747 > 0$

$f(0) = 2e^0 \sin(0) - 1 = \cancel{12.4377 > 0}\ -1 < 0$

$\therefore$ Root lies b/w $a = 0$ & $b = 1$.

$\because f(1) \cdot f(0) < 0$

1st iteration:

$c = 1/2 = 0.5$

$f(c) = f(0.5) = 2e^{0.5} \sin(0.5) - 1 = 0.5808 > 0$

$\Rightarrow f(0.5) \cdot f(0) < 0$

$\therefore$ Root lies b/w 0.5 & 0.

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$f(x) = 2e^{x}\sin x - 1 = 0$

So, $f(0) = 2e^{0}\sin(0) - 1 = -1 < 0$

$f(1) = 2e\sin(1) - 1 = 3.5747 > 0$

$\therefore$ root lies b/w $a = 0$ & $b = 1$

$\because f(0)\cdot f(1) < 0$

1st iteration:

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(3.5747) - 1(-1)}{3.5747 + 1}$$

$x_1 = 0.21859$

$f(x_1) = f(0.21859) = 2e^{0.21859}\sin(0.21859) - 1 = -0.4603 < 0$

2nd iteration:

$f(0.21859)\cdot f(1) < 0$

$\therefore$ Root lies b/w $a = 0.21859$ & $b = 1$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0.21859(3.5747) - 1(-0.4603)}{3.5747 + 0.4603}$$

$x_2 = 0.30773$

$f(x_2) = f(0.30773) = 2e^{0.30773}\sin(0.30773) - 1 = -0.17592 < 0$

3rd iteration:

$f(0.30773)\cdot f(1) < 0$

$\therefore$ Root lies b/w $a = 0.30773$ & $b = 1$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0.30773(3.5747) - 1(-0.17592)}{3.5747 + 0.17592}$$

$x_3 = 0.3402$

$f(x_3) = f(0.3402) = 2e^{0.3402}\sin(0.3402) - 1 = -0.06221$

Pg 36

(4) Find the cube root of 12 using the Newton Raphson method assuming $x_0 = 2.5$.

Solve:

$x^3 = 12$

$f(x) = x^3 - 12 = 0$

$f'(x) = 3x^2$

$x_0 = 2.5$

1st iteration:

$f(x_0) = f(2.5) = (2.5)^3 - 12 = 3.625$

$f'(x_0) = f'(2.5) = 3(2.5)^2 = 18.75$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2.5 - \dfrac{3.625}{18.75}$

$x_1 = 2.3066$

2nd iteration:

$f(x_1) = f(2.3066) = (2.3066)^3 - 12 = 0.272$

$f'(x_1) = f'(2.3066) = 3(2.3066)^2 = 15.9612$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 2.3066 - \dfrac{0.272}{15.9612} = 2.289$

Approximate root of the equation $x^3 - 12$ using Newton's method is 2.289. (after two iterations).

(5) Solve $\int_0^1 \cos x^2 \, dx$ using trapezoidal rule for $n = 5$.

Solve:

$f(x) = \cos x^2$

$a = 0, b = 1, n = 5, h = \frac{b-a}{n} = \frac{1}{5} = 0.2$

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The values of x & $f(x)$ are given in table.

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1	0.9992	0.9872	0.9358	0.802	0.5403

By using Trapezoidal Rule

$$T_5 = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4)\} + f(x_5)\right]$$

$$T_5 = \frac{0.2}{2}\left[1 + 2(0.9992 + 0.9872 + 0.9358 + 0.802) + 0.5403\right]$$

$T_5 = 0.89887$, Ans

(6) $\int_{-2}^{4} e^{x^2}\, dx$ using Simpson one-third as well as Simpson Three eight rule for $n=6$.

Solve:-

let $f(x) = e^{x^2}$

$a = -2$, $b = 4$, $n = 6$, $h = \frac{b-a}{n} = \frac{4-(-2)}{6} = 1$

x	-2	-1	0	1	2	3	4
$f(x)$	54.598	2.7182	1	2.7182	54.598	8103.08	8886110.521

By using Simpson 1/3 Rule.

$$S_{1/3} = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$S_{1/3} = \frac{1}{3}\left[(54.598 + 8886110.521) + 4(2.7182 + 2.7182 + 8103.08) + 2(1 + 54.598)\right]$$

$S_{1/3} = 2972903.46$

By using Simpson 3/8 Rule.

$$S_{3/8} = \frac{3}{8}h\left[(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)\right]$$

$$S_{3/8} = \frac{3}{8}\left[(54.598 + 8886110.521) + 2(2.7182) + 3(2.7182 + 1 + 54.598 + 8103.08)\right]$$

$S_{3/8} = 3341495.529$, Ans