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Mathematics XII Unit 10 Differential Equation.

Ordinary Differential Equation D.E

An equation involving derivatives (Ordinary derivatives) of one single independed variable is called Ordinary differential equation.

Order of the differential equation.

The order of a differential equation is the order of the highest derivative appearing in it.

Degree of the differential equation.

The degree of the differential equation is the Power of the highest order derivative Occuring in it.

Solution of differential equation.

A solution of a differential equation is a relation between the variable free from derivatives, such that this relation and the derivatives obtained from it satisfies the given differential equation.

General and Particular Solution.

A general solution of a differential equation is the one in which the number of arbitrary constants is equal

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To the order of the differential equation.

Particular Solution

A Solution obtained from the general solution by giving particular values to the arbitrary Constants is called Particular Solution.

Exercise 10.1

Q#1. Find the order and degree of each of the following differential equation.

(i) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+xy=0$ Order = 2, Degree = 1

(ii) $x^2dx+y^3dy=0$

$x^2+y^3\frac{dy}{dx}=0$ Order = 1, Degree = 1

(iii) $\frac{d^2y}{dx^2}=\sqrt{\left(\frac{dy}{dx}\right)^2+3}$ Order = 2, Degree = 2

(iv) $\frac{d^3y}{dx^3}-5\left(\frac{d^2y}{dx^2}\right)+7\left(\frac{dy}{dx}\right)^8=0$ Order = 3, Degree = 1

(v) $\frac{d^4y}{dx^4}-\left(\frac{dy}{dx}\right)^{\frac{1}{2}}=0$ Order = 4, Degree = 2

$\left(\frac{d^4y}{dx^4}\right)^2=\frac{dy}{dx}$

Q#2. Show that $y=x-x\ln x$ is the Solution of the differential equation

$x\frac{dy}{dx}+x-y=0$

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$y=x-x\ln x$

$\Rightarrow \frac{dy}{dx}=\frac{d(x-x\ln x)}{dx}=1-\frac{d(x\ln x)}{dx}=1-\left[\ln x+x\cdot\frac{1}{x}\right]$

$\Rightarrow \frac{dy}{dx}=1-\ln x-1=-\ln x$

$x(-\ln x)+x-[x-x\ln x]=0$

$-x\ln x+x-x+x\ln x=0$

$0=0$

Q#3. Show that $y=Ae^{2x}+Be^{3x}$ is the General Solution of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$

$y=Ae^{2x}+Be^{3x}$

$\frac{dy}{dx}=\frac{d}{dx}(Ae^{2x}+Be^{3x})=2Ae^{2x}+3Be^{3x}$

$\frac{d^2y}{dx^2}=\frac{d}{dx}(2Ae^{2x}+3Be^{3x})=4Ae^{2x}+9Be^{3x}$

$\Rightarrow (4Ae^{2x}+9Be^{3x})-5(2Ae^{2x}+3Be^{3x})+6(Ae^{2x}+Be^{3x})=0$

$\Rightarrow 4Ae^{2x}+9Be^{3x}-10Ae^{2x}-15Be^{3x}+6Ae^{2x}+6Be^{3x}=0$

$\Rightarrow 10Ae^{2x}-10Ae^{2x}+15Be^{3x}-15Be^{3x}=0$

$0+0=0$

$0=0$

Q#4. Obtain the differential equation by eliminating arbitrary Constant from the solution.

i) $y=A\cos x+B\sin x$

$\frac{dy}{dx}=\frac{d}{dx}(A\cos x+B\sin x)=A(-\sin x)+B\cos x=-A\sin x+B\cos x$

$\frac{d^2y}{dx^2}=\frac{d}{dx}(-A\sin x+B\cos x)=-A\cos x-B\sin x=-A\cos x-B\sin x$

$\frac{d^2y}{dx^2}+y=-A\cos x-B\sin x+A\cos x+B\sin x$

$\frac{d^2y}{dx^2}+y=0$

ii) $y=A\sin(x+1)$

$\frac{dy}{dx}=\frac{d}{dx}A\sin(x+1)=A\cos(x+1)$

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$\frac{d^2y}{dx^2} = \frac{d}{dx}(A\cos(x+1)) = -A\sin(x+1)$

$y = A\sin(x+1)$, $\frac{dy}{dx} = A\cos(x+1)$, $\frac{d^2y}{dx^2} = -A\sin(x+1)$

$\frac{y}{\sin(x+1)} = A$ $\Rightarrow$ $\frac{dy}{dx} = \frac{y}{\sin(x+1)}\cos(x+1)$

$\frac{dy}{dx} = y\cot(x+1)$

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(iii) $y = ax^2 + bx$

$y' = \frac{dy}{dx} = \frac{d}{dx}(ax^2+bx) = 2ax+b$

$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(2ax+b) = 2a$

$x^2y'' = x^2(2a) = 2ax^2$

$-2xy' = -2x(2ax+b) = -4ax^2 - 2bx$

$2y = 2(ax^2+bx) = 2ax^2 + 2bx$

$x^2y'' - 2xy' + 2y = 0$

$2ax^2 - 4ax^2 - \cancel{2bx} + 2ax^2 + \cancel{2bx} = 0$

$0 = 0$

(iv) $y = c_1e^{x} + c_2e^{-2x}$

$\frac{dy}{dx} = \frac{d}{dx}(c_1e^{x} + c_2e^{-2x}) = c_1e^{x} - 2c_2e^{-2x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(c_1e^{x} - 2c_2e^{-2x}) = c_1e^{x} + 4c_2e^{-2x}$

$\frac{d^2y}{dx^2} = c_1e^{x} + 4c_2e^{-2x}$

$\frac{dy}{dx} = c_1e^{x} - 2c_2e^{-2x}$

$y = c_1e^{x} + c_2e^{-2x}$

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

$[c_1e^{x} + 4c_2e^{-2x}] + [c_1e^{x} - 2c_2e^{-2x}] - 2[c_1e^{x} + c_2e^{-2x}]$

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Q#5 Find the particular solution of

i) $\frac{dy}{dx} = \frac{\sqrt{1+\cos y}}{\sin y}$, $y(3) = \frac{\pi}{2}$, given that $x + 2\sqrt{1+\cos y} + c = 0$ is the general solution of the differential equation.

$x = 3$, $y = \frac{\pi}{2}$

$3 + 2\sqrt{1+\cos(\frac{\pi}{2})} + c = 0$

$3 + 2\sqrt{1+0} + c = 0$

$3 + 2 + c = 0$ $\Rightarrow$ $c = -5$

$x + 2\sqrt{1+\cos y} - 5 = 0$

$x = 5 - 2\sqrt{1+\cos y}$

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ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$, $y(0) = 1$, $y'(0) = 3$, given that $y = Ae^{2x} + Be^{-x}$ is the general solution of the differential equation.

$y(0) = 1$, $x = 0$, $y = 1$

$1 = Ae^{2(0)} + Be^{-(0)} = Ae^{0} + Be^{0} = A + B$

$1 = A + B$

$y'(0) = 3$

$\frac{dy}{dx} = \frac{d}{dx}(Ae^{2x} + Be^{-x}) = 2Ae^{2x} - Be^{-x}$

$3 = 2Ae^{2(0)} - Be^{0} = 2Ae^{0} - Be^{0} = 2A - B$

$3 = 2A - B$

Now By adding $1 = A + B$, $3 = 2A - B$

$4 = 3A$ $\boxed{A = 4/3}$

Put in $1 = A + B$

$1 = \frac{4}{3} + B$

Pg 06

$B = 1 - \frac{4}{3} = \frac{3-4}{3} = -\frac{1}{3}$

$y = \frac{4}{3}e^{2x} - \frac{1}{3}e^{-x}$

$y = \frac{4e^{2x} - e^{-x}}{3} \Rightarrow \boxed{3y = 4e^{2x} - e^{-x}}$

Unit 10, Ex 10.2 Solution of Differential Equation

i) Separable Variables.

If the differential equation $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ where $f(x)$ is the function of x only and $g(y)$ is the function of y only, then

$g(y)dy = f(x)dx$

For solution, find integral both side.

$\int g(y)dy = \int f(x)dx + c$

ii) Homogenous Differential Equation.

Homogenous Function:-

A function is said to be homogeneous of degree n if

$f(\lambda x, \lambda y) = \lambda^n f(x,y)$

Homogeneous differential Equation.

A differential equation is said to be homogeneous differential equation if $f(x,y)$ and $g(x,y)$ are the homogenous function of the same degree in x and y.

To solve the homogeneous differential equation we reduce it into the separable variable form by putting

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Unit 10, Ex. 10.2 Solution of Differential Equation Pg (7)

(i) Separable Variables.

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$$g(y)dy = f(x)dx$$

For solution, find integral both side

$$\int g(y)dy = \int f(x)dx + c$$

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(ii) Homogenous Differential Equation.

iia) Homogenous Function:-

A function is said to be homogeneous of degree n if

$$f(\lambda x, \lambda y) = \lambda^n f(x,y)$$

or $$f(x,y) = x^n f\left(\frac{y}{x}\right)$$

iib) Homogeneous differential equation.

A differential equation is said to be homogeneous differential equation, if $f(x,y)$ and $g(x,y)$ are the homogenous function of the same degree in x and y.

To solve the homogeneous differential equation, we reduce it into the separable variable form by putting

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Pg 08

(iii) Equations reducible to homogeneous form:

The differential equation of the form $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$ is not homogeneous but can be reduced to the homogeneous form, when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

we put $x = X + h$, $y = Y + K$

$dx = dX$ $dy = dY$

The given differential equation become homogeneous and then we reduce it into seprable variable form.

Q#11. Solve the following differential

Pg 09

Q#1) Solve the following differential equation by separating the variables.

(i) $x \tan y \, dy = dx$

$\tan y \, dy = \frac{1}{x} dx$

By integration $\int \tan y \, dy = \int \frac{1}{x} dx$

$\ln \sec x = \ln x + \ln c$ $\because \int \tan x dx = \ln \sec x$

$\Rightarrow \ln \sec x = \ln xc$

$\boxed{\sec x = cx}$

(ii) $x \sin y \, dx + (x^2+1) \cos y \, dy = 0$

$x \sin y \, dx = -(x^2+1) \cos y \, dy = 0$

$\frac{x}{x^2+1} dx = \frac{\cos y}{\sin y} dy$ By separating the variable.

By integration

$\int \frac{x}{x^2+1} dx = -\int \frac{\cos y}{\sin y} dy$

$\Rightarrow -\frac{1}{2} \int \frac{2x}{x^2+1} dx = \int \cot y \, dy$

$\Rightarrow -\frac{1}{2} \ln(x^2+1) = \ln \sin y + \ln c$

$\Rightarrow -\ln(x^2+1)^{1/2} = \ln \sin y + \ln c$

$\Rightarrow -\ln(x^2+1)^{1/2} + \ln c = \ln \sin y$

$\Rightarrow \cancel{\ln} (x^2+1)^{-1/2} \cdot c = \cancel{\ln} \sin y$

$\Rightarrow \frac{c}{(x^2+1)^{1/2}} = \sin y$

$c = (x^2+1)^{\frac{1}{2}} \sin y$

(iii) $y(1+x) dx + x(1+y) dy = 0$

$y(1+x) dx = -x(1+y) dy$

$\Rightarrow \frac{1+x}{x} dx = -\frac{1+y}{y} dy$

$\Rightarrow \int \frac{1}{x} + 1 \, dx = -\int \frac{1}{y} + 1 \, dy$

$\Rightarrow \ln x + x = -[\ln y + y] + c$

$\Rightarrow \ln x + \ln y + x + y = c \Rightarrow (x+y) + \ln xy = c$

P(10)

(iv) $(1+x^3)dy - x^2dx = 0$

$(1+x^3)dy = x^2dx$

$\Rightarrow (1+x^3)dy = x^2dx$

$\Rightarrow dy = \frac{x^2}{1+x^3}dx$

$\Rightarrow dy = \frac{3x^2}{3(1+x^3)}dx$

$\Rightarrow \int dy = \frac{1}{3}\int \frac{3x^2}{(1+x^3)}dx$

$\Rightarrow y = \frac{1}{3}\ln(1+x^3) + c$.

(v) $xydy = (y+1)(1-x)dx$

$\frac{y\,dy}{y+1} = \frac{1-x}{x}dx$

$\int \frac{y}{y+1}dy = \int \frac{1}{x} - 1\,dx$.

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Now

$\int \frac{y}{y+1}dy$

$u = y+1 \Rightarrow y = u-1$

$du = dy$

$\Rightarrow \int \frac{u-1}{u}du = \int \frac{u}{u} - \frac{1}{u}du$

$\Rightarrow \int (1 - \frac{1}{u})du = u - \ln u$

$= (y+1) - \ln(y+1)$

For $\int \frac{1}{x} - 1\,dx$

$\Rightarrow \ln x - x +$

Pg (11)

$\Rightarrow (y+1) - \ln(y+1) = \ln x - x$

$\Rightarrow x+y+1-\ln(y+1)-\ln x = c$

$\Rightarrow x+y-[\ln(y+1)+\ln x] = c-1$

$\Rightarrow x+y-\ln[x(y+1)] = c$

$\Rightarrow x+y-\ln(xy+x) = c$

(vi) $\frac{dy}{dx} = 3y^2 - y^2\sin x$

$\frac{dy}{dx} = y^2(3-\sin x)$

$\frac{dy}{y^2} = (3-\sin x)dx$

By Applying Integration

$\Rightarrow \int y^{-2}dy = \int (3-\sin x)dx$

$\Rightarrow \frac{y^{-2+1}}{-2+1} = 3x+\cos x+c$

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$\Rightarrow \frac{y^{-1}}{-1} = 3x+\cos x+c$

$\Rightarrow -\frac{1}{y} = 3x+\cos x+c$

(vii) $(y^2+1)\frac{dy}{dx} = 4xy^2$

$\frac{y^2-1}{y^2}dy = 4x\,dx$

$\Rightarrow 1-y^{-2}dy = 4x\,dx$

Applying Integration

$\Rightarrow \int 1-y^{-2}dy = \int 4x\,dx$

$\Rightarrow y - \frac{y^{-1}}{-1} = \frac{4x^2}{2}+c$

$\Rightarrow y+\frac{1}{y} = 2x^2+c$

(viii) $x\cos^2 y\,dx + \tan y\,dy = 0$

$\cos^2 y\; x\,dx = -\tan y\,dy$

$x\,dx = \dfrac{-\tan y}{\cos^2 y}\,dy$

$\dfrac{\tan y}{\cos^2 y}\,dy = -x\,dx$

$\dfrac{\sin y}{\cos^3 y}\,dy = -x\,dx$

Applying Integration

$\int \dfrac{\sin y}{\cos^3 y}\,dy = \int -x\,dx$

$\int \dfrac{\sin y}{\cos^3 y}\,dy$

let $u = \cos y$

$du = -\sin y\,dy$

$-du = \sin y\,dy$

$\int \dfrac{-du}{u^3} = -\int u^{-3}\,du = -\dfrac{u^{-3+1}}{-3+1} = \dfrac{+u^{-2}}{+2}$

$\Rightarrow \int \dfrac{\sin y}{\cos^3 y}\,dy = \dfrac{u^{-2}}{2} =$

$\int \dfrac{\sin y}{\cos^3 y}\,dy = \cos^{-2} y = \dfrac{1}{\cos^2 y}$

$x\,dx = -\tan y\,\sec^2 y\,dy$

Applying Integration

$\Rightarrow \int x\,dx = \int -\tan y\,\sec^2 y\,dy$

$\Rightarrow \dfrac{x^2}{2} = -\dfrac{\tan^2 y}{2} + c$

$\Rightarrow -\dfrac{\tan^2 y}{2} = \dfrac{x^2}{2} + c$

$$+\frac{\tan^2 y}{2} = -\frac{x^2}{2} - c$$ Pg 13

$$\tan^2 y = -x^2 + c$$

Q#2 Solve the following homogenous differental equation.

(i) $(x+y)dy - (x-y)dx = 0$

$$(x+y)dy = (x-y)dx$$

$$\frac{dy}{dx} = \frac{x-y}{x+y}$$

First we check the given differential equation is homogeneous or not

$$f(x,y) = \frac{x-y}{x+y}$$

$$f(tx,ty) = \frac{tx-ty}{tx+ty} = \frac{t(x-y)}{t(x+y)} = \frac{x-y}{x+y}$$ is homogeneous differential eq.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Put in D.E

$$v + x\frac{dv}{dx} = \frac{x - vx}{x + vx} = \frac{1-v}{1+v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v}{1+v} - v = \frac{(1-v) - v(1+v)}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v-v-v^2}{1+v} = \frac{1-2v-v^2}{1+v}$$

$$\Rightarrow \frac{1+v}{1-2v-v^2}dv = \frac{1}{x}dx$$

Now it is separable differential equation and integrating

$$\Rightarrow \int \frac{1+v}{1-2v-v^2}dv = \int \frac{1}{x}dx$$

Pg

$$-\int \frac{-(1+v)}{1-2v-v^2}dv = \int \frac{1}{x}dx$$

$$\Rightarrow -\int \frac{-2(1+v)}{2(1-2v-v^2)}dv = \int \frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{2}\int \frac{-2(1+v)}{(1-2v-v^2)}dv = \int \frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{2}\ln(1-2v-v^2) = \ln x + \ln c$$

$$\Rightarrow \ln(1-2v-v^2)^{-\frac{1}{2}} = \ln(xc)$$

$$\Rightarrow (1-2v-v^2)^{-\frac{1}{2}} = xc$$

$$\Rightarrow \left(1-2\left(\frac{y}{x}\right)-\left(\frac{y}{x}\right)^2\right)^{-\frac{1}{2}} = xc$$

$$\Rightarrow \left(1-\frac{2y}{x}-\frac{y^2}{x^2}\right)^{-\frac{1}{2}} = xc$$

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$$\Rightarrow \left(\frac{x^2-2xy-y^2}{x^2}\right)^{-\frac{1}{2}} = xc$$

$$\Rightarrow \left(\frac{x^2}{x^2-2xy-y^2}\right)^{\frac{1}{2}} = xc$$

$$\frac{\cancel{x}}{\sqrt{x^2-2xy-y^2}} = \cancel{x}c$$

$$\sqrt{x^2-2xy-y^2} = \frac{1}{c}$$

$$\sqrt{x^2-2xy-y^2} = c$$

Q#2 (ii) $(6x^2+2y^2)dx - (x^2+4xy)dy = 0$

$$(6x^2+2y^2)dx = (x^2+4xy)dy$$

$$\frac{dy}{dx} = \frac{6x^2+2y^2}{x^2+4xy}$$

First we check the given differential equation is homogeneous or not

Pg (15)

$$f(x,y)=\frac{6x^2+2y^2}{x^2+4xy}$$

Put $x = tx$ and $y = ty$

$$\Rightarrow f(tx,ty)=\frac{6(tx)^2+2(ty)^2}{(tx)^2+4tx.ty}=\frac{6t^2x^2+2t^2y^2}{t^2x^2+4t^2xy}=\frac{t^2(6x^2+2y^2)}{t^2(x^2+4xy)}$$

$$\Rightarrow f(tx,ty)=t^{\circ}\left(\frac{6x^2+2y^2}{x^2+4xy}\right)$$ it is homogenous.

Put $y = vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{6x^2+2(vx)^2}{x^2+4x(vx)}=\frac{6x^2+2v^2x^2}{x^2+4x^2v}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{x^2(6+2v^2)}{x^2(1+4v)}=\frac{6+2v^2}{1+4v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{6+2v^2}{1+4v}-v=\frac{(6+2v^2)-v(1+4v)}{1+4v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{6+2v^2-v-4v^2}{1+4v}=\frac{-2v^2-v+6}{1+4v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{-(2v^2+v-6)}{4v+1}$$

$$\Rightarrow -\frac{4v+1}{2v^2+v-6}dv=\frac{dx}{x}$$

$$\Rightarrow -\int\frac{4v+1}{2v^2+v-6}dv=\int\frac{1}{x}dx$$

$$\Rightarrow -\ln(2v^2+v-6)=\ln x+\ln c$$

$$\Rightarrow \ln(2v^2+v-6)^{-1}=\ln cx$$

$$\Rightarrow (2v^2+v-6)^{-1}=cx$$

$$\Rightarrow \frac{1}{2v^2+v-6}=cx$$

$$\Rightarrow c=x(2v^2+v-6)$$

$$\cancel{c=x(2(\tfrac{y}{x})^2+\tfrac{y}{x}-6)}$$

$$\cancel{c=x(2\tfrac{y^2}{x^2}+\tfrac{y}{x}-6)}$$

$$\cancel{c=xy}$$

$$\Rightarrow c=x(2v^2+4v-3v-6)$$

$$\Rightarrow c=x[2v(v+2)-3(v+2)]$$

$$\Rightarrow c=x(v+2)(2v-3)$$

$$\Rightarrow c=x(\tfrac{y}{x}-2)(2\tfrac{y}{x}-3)$$

$$\Rightarrow c=x\left(\frac{y-2x}{x}\right)\left(\frac{2y-3x}{x}\right)$$

$$\Rightarrow cx=(y-2x)(2y-3x)$$

$$\Rightarrow cx=(y-2x)(2y-3x)$$

(iii) Pg (16)

Q#2 $(x^2+3y^2)dx - 2xydy = 0$

$(x^2+3y^2)dx = 2xydy$

$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

let $y = vx$

$v + x\frac{dv}{dx} = \frac{x^2+3(vx)^2}{2xvx}$

$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+3v^2}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v} = \frac{1+v^2}{2v}$

$\Rightarrow \frac{2vdv}{1+v^2} = \frac{dx}{x}$

$\Rightarrow \int\frac{2vdv}{1+v^2} = \int\frac{dx}{x}$

$\Rightarrow \ln(1+v^2) = \ln(x) + \ln c$

$\Rightarrow \ln(1+v^2) = \ln(xc)$

$\Rightarrow 1+v^2 = xc$

$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = xc$

$\Rightarrow 1+\frac{y^2}{x^2} = xc$

$\Rightarrow \frac{x^2+y^2}{x^2} = xc$

$\Rightarrow \boxed{x^2+y^2 = x^3C}$

First we check The given differential equation is homogeneous or not

$x = xt,\ y = yt$

$f(x,y) = \frac{x^2+3y^2}{2xy}$

$f(tx,ty) = \frac{(tx)^2+3(ty)^2}{2(tx)(ty)}$

$f(tx,ty) = \frac{t^2(x^2+3y^2)}{t^2(2xy)}$

$f(tx,ty) = t^0 f(x,y)$

Homogeneous.

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(iv) $(x^2+y^2)dx - 2xydy = 0$

First, we check The given differential equation is homogeneous or not

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Pg (17)

$$2xy\,dy = (x^2+y^2)\,dx$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

Put $y = ty$, $x = tx$

$$f(tx, ty) = \frac{(tx)^2+(ty)^2}{2(tx)(ty)}$$

$$f(tx, ty) = \frac{t^2(x^2+y^2)}{t^2(2xy)} = t^0\,\frac{(x^2+y^2)}{2xy} = t^0 f(x,y)$$

This is homogeneous diff.eq.

For Solution Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{2xvx} = \frac{x^2(1+v^2)}{x^2(2v)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

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$$\Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x}$$

$$\Rightarrow -\int \frac{-2v\,dv}{1-v^2} = \int \frac{dx}{x}$$

$$\Rightarrow -\ln(1-v^2) = \ln x + \ln c$$

$$\Rightarrow \ln(1-v^2)^{-1} = \ln xc$$

$$\Rightarrow \frac{1}{1-v^2} = xc$$

$$\Rightarrow x(1-v^2)\,c = 1$$

$$x\left(1-\frac{y^2}{x^2}\right) = \frac{1}{c}$$

$$x\left(\frac{x^2-y^2}{x^2}\right) = c$$

$$\boxed{x^2-y^2 = cx}$$

Pg 33

$\Rightarrow -\frac{x^2}{2} + c = \frac{2y^2}{2}$

$\boxed{y^2 = -\frac{x^2}{2} + C}$

Q#2. Find the orthogonal trajectories of the hyperbola $xy = c$

Step 1: $xy = c$

Step 2: Form the differential equations.

$\frac{d(xy)}{dx} = \frac{d(c)}{dx}$

$\Rightarrow x\frac{dy}{dx} + y\frac{dx}{dx} = 0$

$\Rightarrow -x\frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

Step 3: Substitute $-\frac{dx}{dy}$ for $\frac{dy}{dx}$

$-\frac{dx}{dy} = -\frac{y}{x}$

$\Rightarrow x\,dx = y\,dy$

Step 4: Solve

$\int x\,dx = \int y\,dy$

$c + \frac{x^2}{2} = \frac{y^2}{2}$

$\Rightarrow c = \frac{y^2}{2} - \frac{x^2}{2}$

$\Rightarrow y^2 - x^2 = 2c$

$\boxed{y^2 - x^2 = C}$

Q#3. Find the orthogonal trajectories of the family of Parabolas $y^2 = 4ax$

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Step 1:

$$y^2 = 4ax \quad \text{---(1)}$$

Step 2: Form the differential equation

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow y\frac{dy}{dx} = 2a \qquad (1) \Rightarrow a = \frac{y^2}{4x}$$

$$\Rightarrow y\frac{dy}{dx} = 2\frac{y^2}{4x}$$

$$\Rightarrow y\frac{dy}{dx} = \frac{y^2}{2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x}$$

Step 3:

Substitute $-\frac{dx}{dy}$ for $\frac{dy}{dx}$

$$-\frac{dx}{dy} = \frac{y}{2x}$$

Step 4:

$$\Rightarrow -2x\,dx = y\,dy$$

$$\Rightarrow \int -2x\,dx = \int y\,dy$$

$$\Rightarrow -\frac{2x^2}{2} + c = \frac{y^2}{2}$$

$$\Rightarrow \frac{y^2}{2} = -x^2 + c$$

$$\Rightarrow y^2 = -2x^2 + 2c$$

$$y^2 = -2x^2 + C$$

$$\boxed{y^2 + 2x^2 = C}$$

Q#(4) Find the orthogonal trajectories of the family of curves $y = \frac{x}{1+c_1x}$

Step 1: $y = \frac{x}{1+c_1x}$

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Step 2: Form the differential equation

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{1+c_1x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+c_1x)\frac{dx}{dx} - (x)(c_1)}{(1+c_1x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+c_1x - c_1x}{(1+c_1x)^2} = \frac{1}{(1+c_1x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1+c_1x)^2} \qquad \Rightarrow 1+c_1x = \frac{x}{y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{x}{y}\right)^2} = \frac{y^2}{x^2}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{y^2}{x^2}}$$

Step 3: Substitute $-\frac{dx}{dy}$ for $\frac{dy}{dx}$

$$\Rightarrow -\frac{dx}{dy} = \frac{y^2}{x^2}$$

Step 4: Solve

$$\Rightarrow -x^2dx = y^2dy$$

$$\Rightarrow -\frac{x^3}{3} = \frac{y^3}{3} + c$$

$$\boxed{\frac{x^3}{3} + \frac{y^3}{3} = c}$$

Q#05 Find the general equation of family of curves perpendicular to the $y = c_1 \sin x$

Step 1: $y = c_1 \sin x$ — (1)

Step 2: Form the differential equation

$$\frac{dy}{dx} = c_1 \cos x$$

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(1) $\Rightarrow$ $c_1 = \frac{y}{\sin x}$

$$\frac{dy}{dx} = \frac{y}{\sin x} \cdot \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} \cdot y$$

Step 3: substitute $\frac{dy}{dx} = -\frac{dx}{dy}$

$$-\frac{dx}{dy} = \frac{\cos x}{\sin x} \cdot y$$

Step 4: Solve

$$-\frac{\sin x}{\cos x} dx = y dy$$

$$\Rightarrow -\tan x \, dx = y \, dy$$

Integration on both side

$$\Rightarrow \int -\tan x \, dx = \int y \, dy$$

$$\Rightarrow -\ln \sec x + \ln c = \frac{y^2}{2}$$

$$\Rightarrow -2\ln \sec x + 2\ln c = y^2$$

$$\Rightarrow \ln \sec^{-2} x + \ln c^2 = y^2$$

$$\Rightarrow \ln c^2 \cos^2 x = y^2$$

$$\Rightarrow c \cos^2 x = e^{y^2}$$

$$\boxed{c \cos^2 x = e^{y^2}}$$

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Q#6 Find the general equation of family of Curves Perpendicular to the $x^{\frac{1}{3}} + y^{\frac{1}{3}} = c$

Step 1 : $x^{\frac{1}{3}} + y^{\frac{1}{3}} = c$

Step 2: Form the differential equation.

$\Rightarrow \frac{d}{dx}(x^{\frac{1}{3}} + y^{\frac{1}{3}}) = \frac{dc}{dx}$

$\Rightarrow \frac{1}{3} x^{-\frac{2}{3}} + \frac{1}{3} y^{-\frac{2}{3}} \frac{dy}{dx} = 0$

$\Rightarrow x^{-2/3} + y^{-2/3} \frac{dy}{dx} = 0$

$x^{-2/3} = -y^{-2/3} \frac{dy}{dx}$

$-\frac{x^{-2/3}}{y^{-2/3}} = \frac{dy}{dx}$

Step 3: Substitute $\frac{dy}{dx}$ as $-\frac{dx}{dy}$

$\frac{+x^{-2/3}}{y^{-2/3}} = \frac{+dx}{dy}$

$y^{+2/3} dy = x^{2/3} dx$

Step 4: Integration on both side

Solve $\int y^{2/3} dy = \int x^{2/3} dx$

$\frac{y^{5/3}}{5/3} = \frac{x^{5/3}}{5/3} + c$

$y^{5/3} = x^{5/3} + \frac{5}{3} c$

$\boxed{y^{5/3} - x^{5/3} = c}$