Chapter 17

ADVENT OF MODERN PHYSICS

Frame of reference:

A set of coordinate axes by which the position of any object may be specified as it changes with time.

For example: Position of a bird in a cage is defined by the reference of the cage, hence cage is frame of reference for the bird.

Inertial frame of reference:

A frame of reference moving with constant velocity is called Inertial frame of reference.

For example: A car moving on a road.

Transformation:

The mathematical relation that relates a measurement made in one reference frame to another is called Transformation.

Galilean Transformation:

It is mathematical relation satisfying Newton's relativity, named after its inventor Galileo.

[Galilean transformation equations are based on assumption that all laws of mechanics are same in all inertial reference frames]

Mathematical relations:

$$X' = X - vt$$

$$Y' = Y$$

$$Z' = Z$$

Einstein's Special Theory of Relativity:

In 1905 Albert Einstein put forward his special theory of relativity which revolutionize the word of physics this theory consists of two postulates:

1. **The principle of relativity:** All the laws of physics are the same in all inertial frames (i.e., there is no absolute frame of reference).

2. **The constancy of the speed of light:** The speed of light in a vacuum has the same value, **$c = 2.997\,924\,58 \times 10^8$ m/s**, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

CONSEQUENCES OF SPECIAL RELATIVITY:

Einstein show that Galilean transformation equation are failed when velocity of the object become comparable to velocity of light. Due to this reason he used Lorentz transformation equation (containing Lorentz factor) instead of Galilean transformation equation and found the mathematical result for following phenomenon.

Einstein's special theory of relativity gives the following results.

1. Mass Variation:

If "m_o" is mass of a body at rest in observer's frame of reference, then it's mass "m" as measured by an observer from another frame of reference moving with uniform velocity "v" with respect to the body's frame is given by:

$$m = \frac{m_o}{\sqrt{1 - \frac{V^2}{C^2}}}$$

The above relation shows that mass "m" of the body appears to increase to an observer moving with velocity "v" with respect to the body. Hence mass of a body depends upon whether the body is at rest or is in motion relative to the observer. This effect takes place only if the relative velocity between the object and the observer is comparable to the speed of light.

2. Length Contraction:

If "L_0' is the length of a rod when it is at rest relative to an observer then its new length "L" when it is in motion with velocity "v" relative to the same observer, is given by;

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

Where,

L= Relativistic length.

L_o= Proper length.

Hence length of the rod appears to reduce when there is relative motion between an observer and the rod, provided the relative velocity is comparable to speed of light. This effect is known as **"Length Contraction".**

Length contraction takes place only along the direction of motion of the body. There no change in length of the body perpendicular to the direction of its motion, hence change appears when the length is parallel to direction of motion.

3. Time Dilation:

Let "t_o" be the time interval between two events at some point in space as recorded by an observer at rest with respect to that point. Then the time interval recorded between the same two events by another observer moving with velocity "v" relative to that point is given by;

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

t= Relativistic time.

t_o =Proper time.

Factor $\sqrt{1-\frac{v^2}{c^2}}$ appears in these equations is called "Lorentz Factor". For ordinary relative velocities this factor is practically unity. Hence relativistic effects cannot be detected at ordinary velocities. These effects cannot be neglected if the relative speed is comparable to the speed of light.

4. Mass Energy Relation:

Einstein concluded that energy possesses inertia and since the inertia is the property of mass hence the energy is directly related to the mass of a body.

Hence Einstein gave the idea that mass and energy are interconvert able to one and other by the relation,

$$E = mc^2$$

Where

E = Energy.

m = Relativistic mass.

C = Velocity of light

Definitions:

Rest mass:

The mass of an object as measured by an observer moving along with object in a frame of reference in which object is moving it is denoted by m_o and give by

$$m_o = m\sqrt{1 - \frac{V^2}{C^2}}$$

Relativistic mass:

The mass of an object as measured by an observer at rest in a frame of reference in which object is moving it is denoted by m and given by

$$m = \frac{m_o}{\sqrt{1 - \frac{V^2}{C^2}}}$$

Proper length:

Length of an object measured by an observer which in rest with respect to the object. It is given by

$$L_o = \frac{L}{\sqrt{1 - \frac{V^2}{C^2}}}$$

Relativistic length:

It is Length of an object measured by an observer moving with respect to the object. It is given by

$$L = L_o\sqrt{1-\frac{V^2}{C^2}}$$

Proper time:

Time measured by an observer moving along with clock. It is given by

$$t_o = t\sqrt{1-\frac{V^2}{C^2}}$$

Relativistic time:

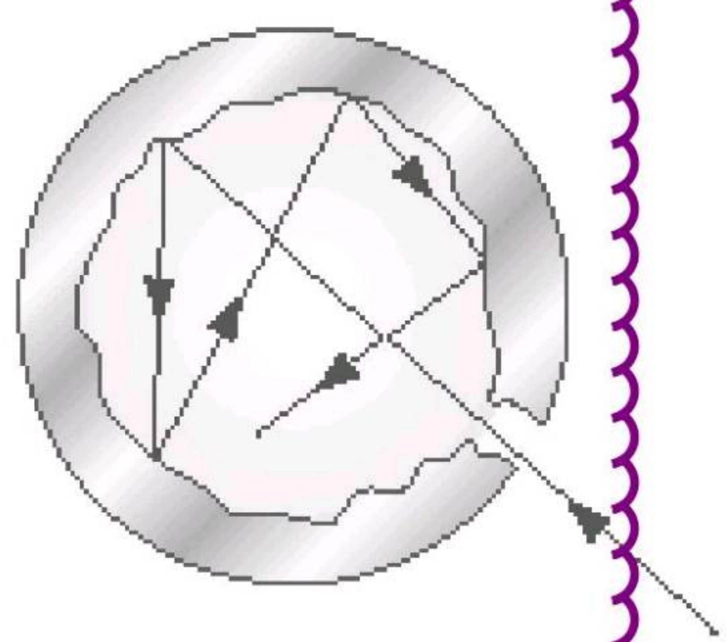
Time measured by an observer which is at rest with respect to a moving clock.

It is given by

$$t = \frac{t_o}{\sqrt{1-\frac{V^2}{C^2}}}$$

Black Body Radiation and Quantum Theory

Black body:



A body that absorb all the radiation falling on it and emit all the absorb radiation on heating is called black body.

The absorptance and emissivity of the blackbody is equal to 1.

Construction:

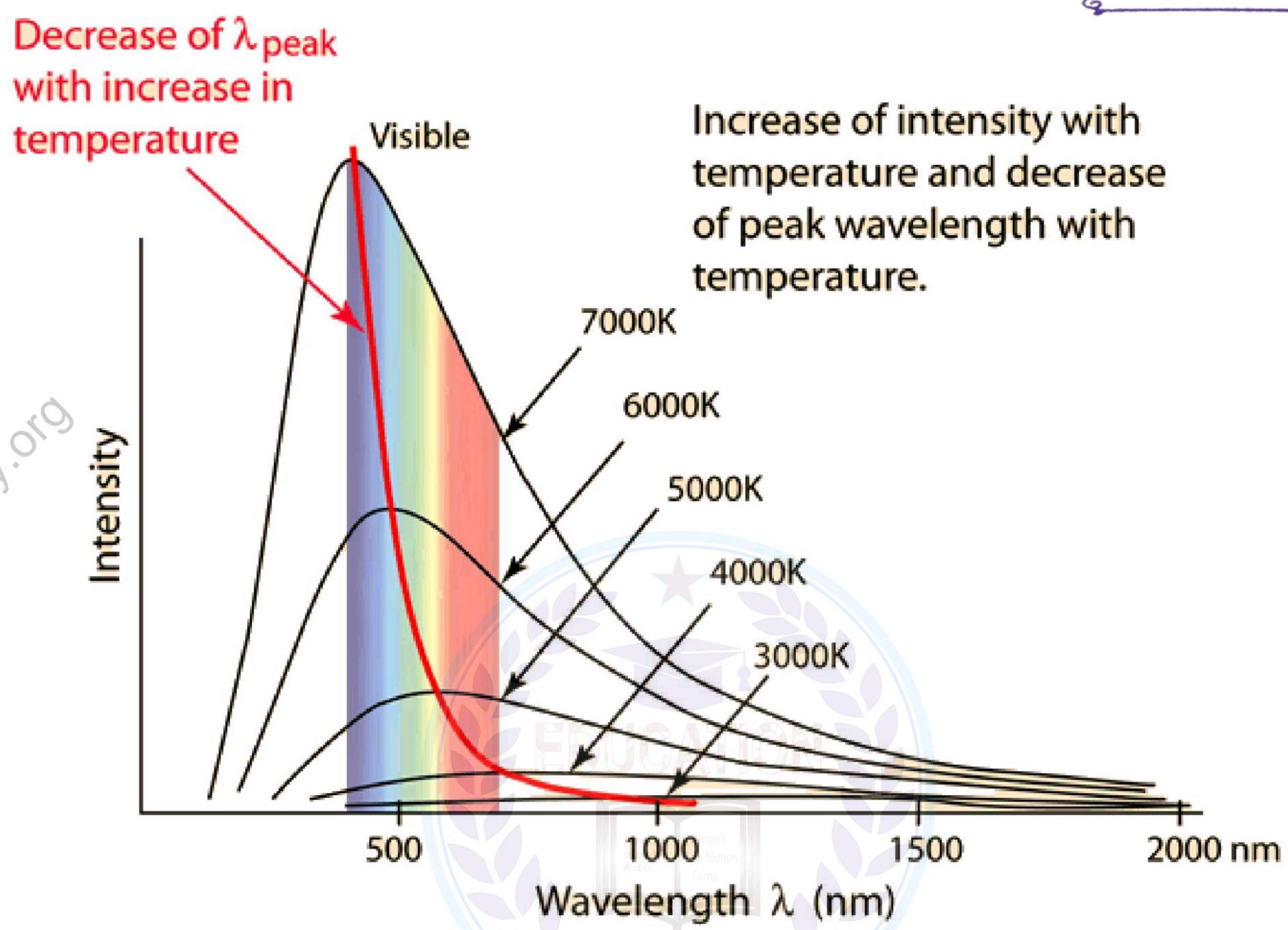
In reality there is no perfect black body but an opening in the cavity of a body is a good approximation of a black body.

Black body radiation:

Radiation emitted by black body on heating is called black body radiation or thermal radiation or thermal radiations.

Intensity versus wave length graph:

Intensity versus wave length graph of black body radiations observed at different temperature is given below:



Properties of black body radiation:

- Black body radiations are independent of material of black body.
- Black body radiations are only dependent of temperature.
- Black body radiations are in the region of infrared and visible light only.
- Black body radiation is group of different wave length at a particular temperature.

Laws of Black body radiations:

There are following laws of black body radiation.

Stefan-Boltzmann law:

Intensity of radiation is directly proportional to the **4th** power of the temperature.

$$E \propto T^4$$

$$E = \sigma T^4$$

Where σ is Stefan-Boltzmann and its value is $5.67 \times 10^4 watt/m^2k^4$

Wien's displacement law:

Wave length corresponding to the maximum intensity is inversely proportional to the absolute temperature.

$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m T = (constant)$$

Rayleigh jeans law

Energy corresponding to a particular wave length is inversely proportional to the 4th power of the wave length

$$\boxed{E \propto \frac{1}{\lambda^4}}$$

Planks law:

Energy emitted is directly proportional to frequency of vibration of the atoms of black body

$$E \propto f$$

$$E = hf$$

Where

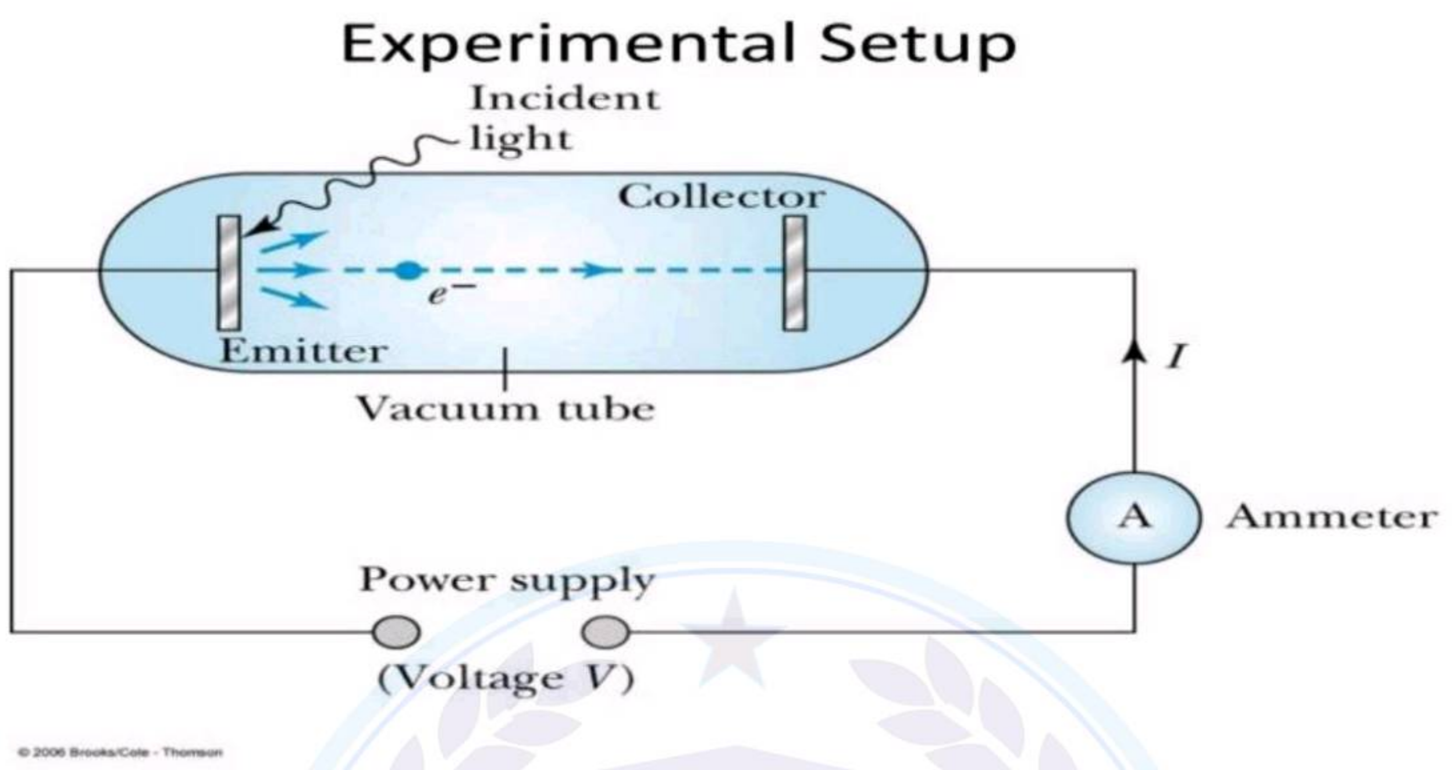
h= Plank's constant=**6.63 x 10^{-34} J.s**

Photo Electric Effect

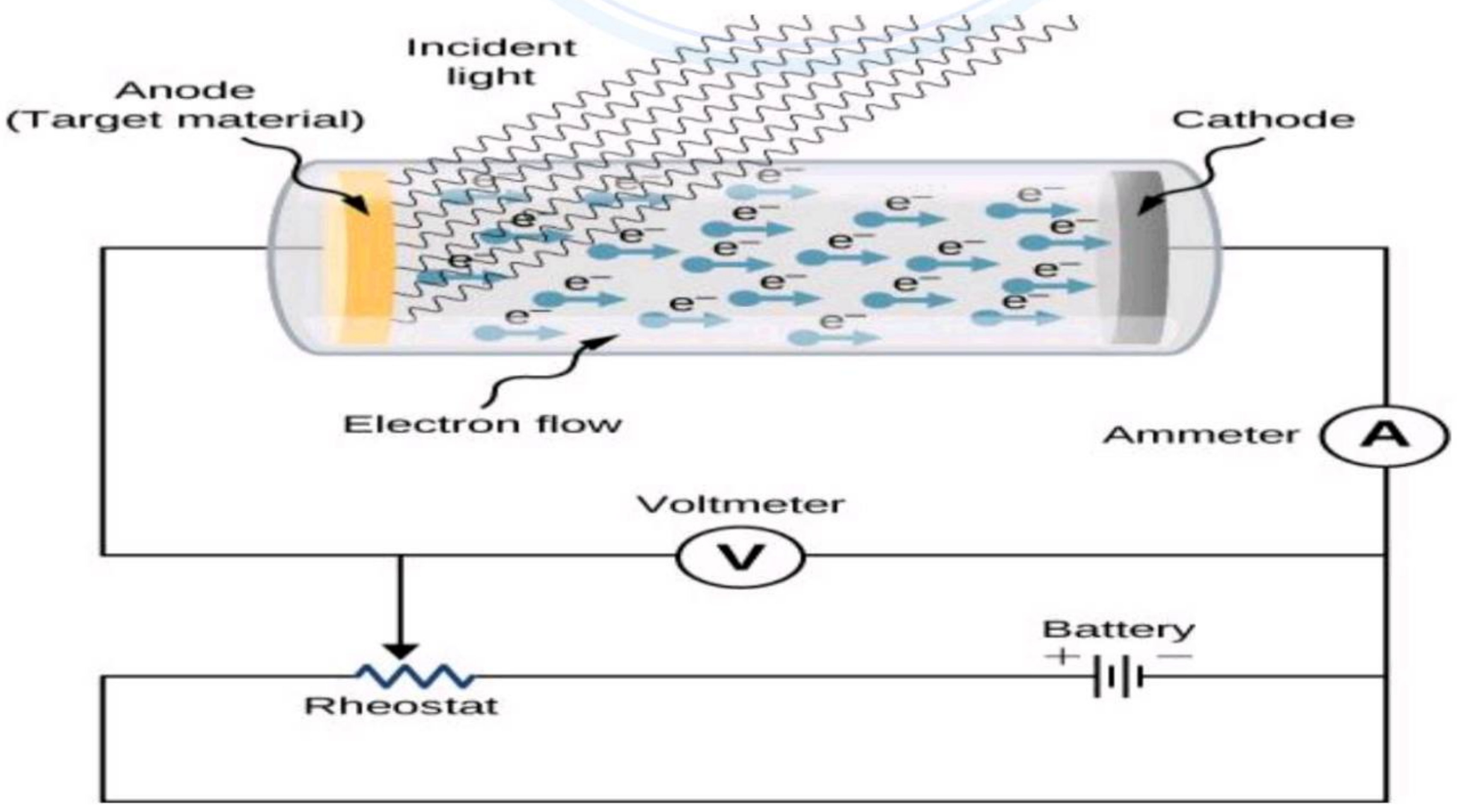
The phenomenon in which electrons (called photoelectrons) are ejected out of the Metallic surface when they are exposed to the electromagnetic radiation is known as photoelectric effect.

Explanation:

Consider the experimental apparatus as shown in figure two metallic plates connected to a voltage source in which one is connected to the positive terminal of the battery and other is connected to the negative terminal of the battery. When light strikes the plate the electrons are ejected out of the plate. The electrons are attracted toward the other plate by applying sufficient positive voltage it. If the light source is removed there will be no current in the circuit.

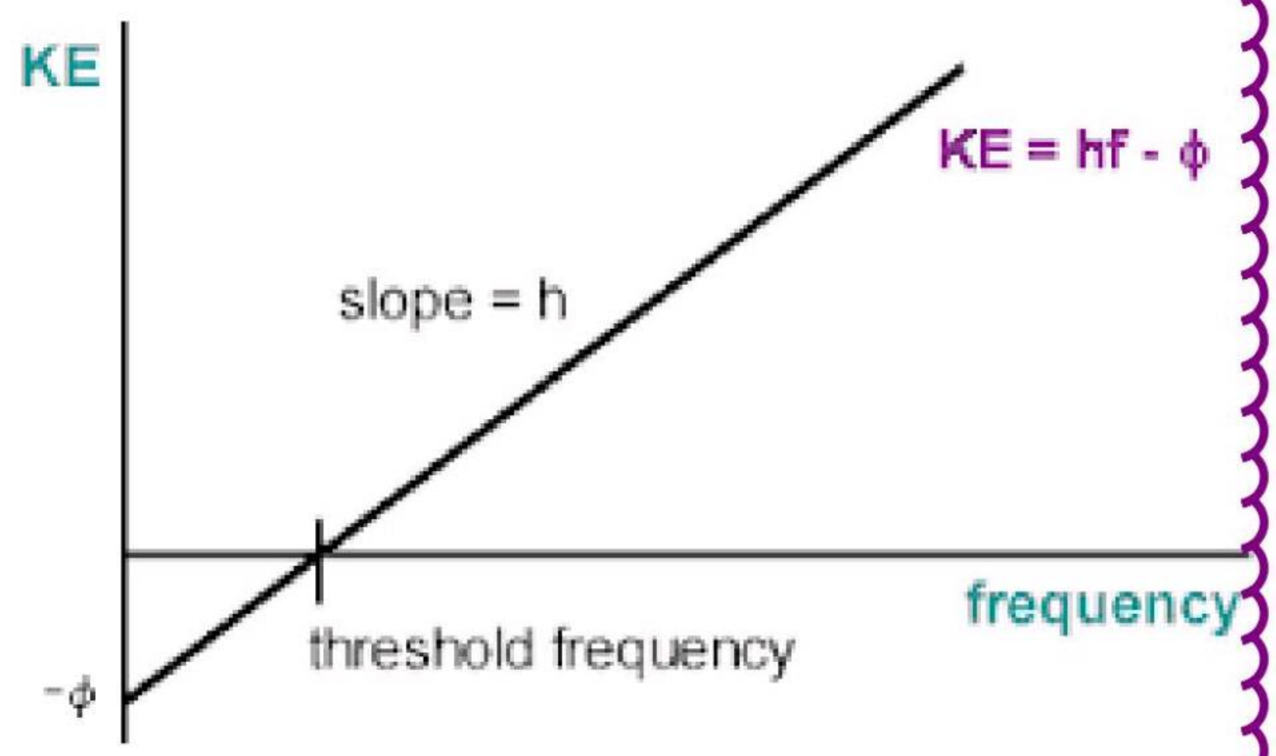


Suppose that we now reverse the potential difference between the electrodes so that the target material now connects with the positive terminal of a battery, and then we slowly increase the voltage. The photocurrent gradually dies out and eventually stops flowing completely at some value of this reversed voltage. The potential difference at which the photocurrent stops flowing is called the **stopping potential**.



Properties of photoelectrons:

The graph shows that No electrons are emitted out unless the frequency of the incident light is not equal to or greater than a certain frequency is called **"threshold frequency"**

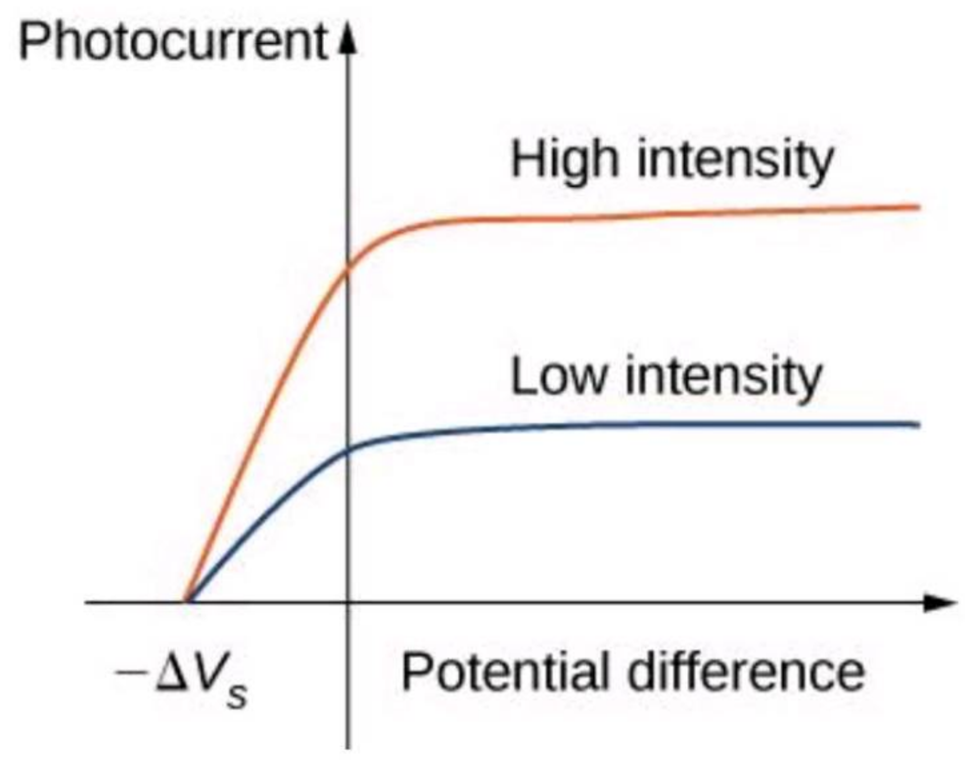


I. **Effect Of Intensity**

By increasing the intensity (number of photons)of the incident light, the number of the emitted electrons and current also increases.

(Keeping frequency = Constant)

But the kinetic energy of the emitted electrons, and the stopping potential, remains almost the same.

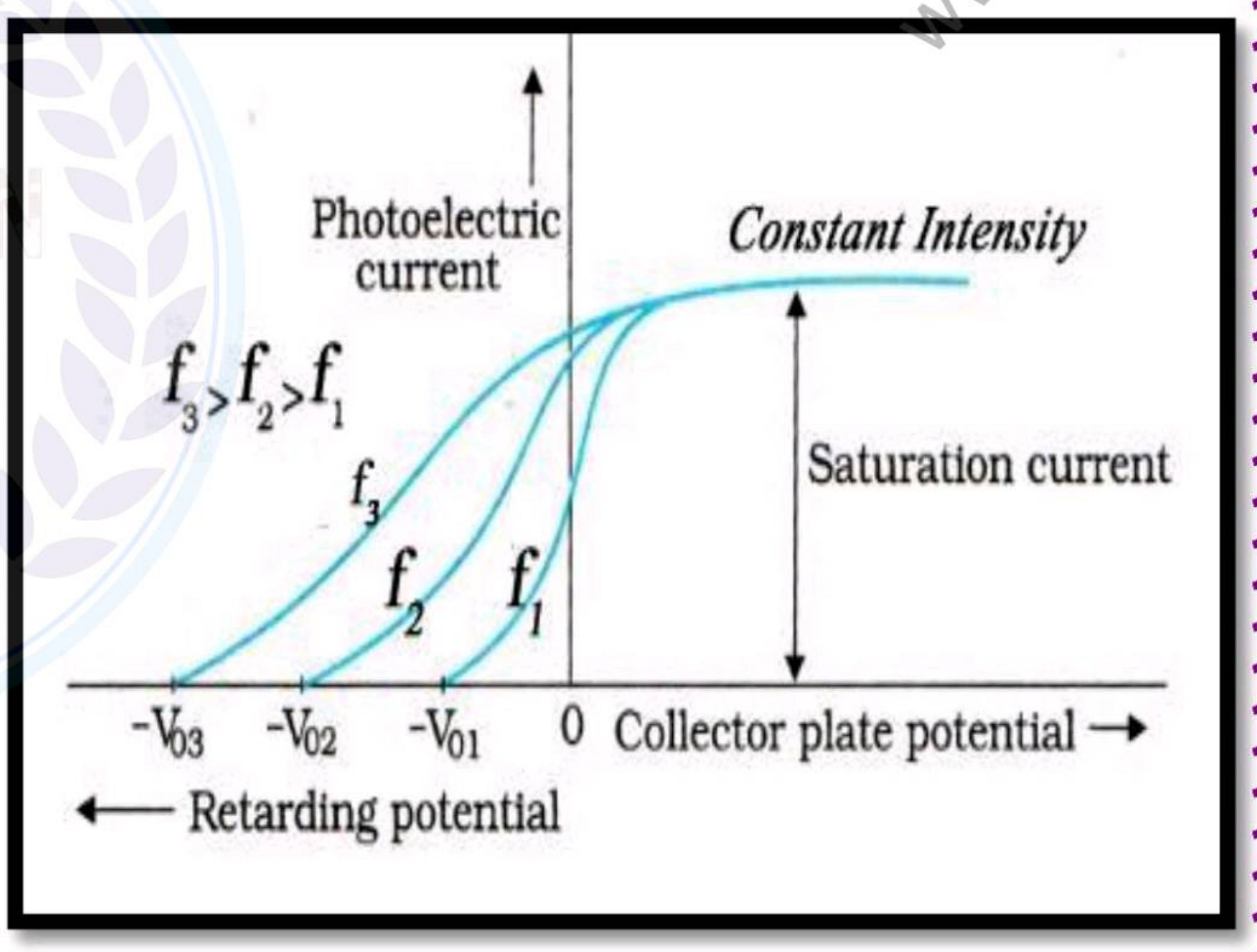


II. **Effect Of Frequency**

By increasing the frequency of the incident light, the kinetic energy of the emitted electrons, and hence the stopping potential alsoincreases.

(Keeping Intensity = Constant)

But the number of the emitted electrons, andhence the current, remain almost the same.



Laws of Photoelectric Effect:

After the number of experiments performed by the scientists some fundamental laws were formulated about the emission of photoelectrons. These laws are:

i) To every metal surface there must needed radiations in a particular frequency range, below which no photoelectric emission takes place. The minimum frequency needed to emit photoelectrons from a metal surface, is referred as **"Threshold Frequency".** It is symbolized by **"ν_o".** Its value depends on, nature of material of the metal surface.

ii) The number of photoelectrons is directly depending on intensity of radiation, provided $\nu > \nu_o$.

iii) The velocity and hence kinetic energy of photoelectrons emitted out of metal surface directly proportional to the frequency of incident radiations, provided $\nu > \nu_o$.

Einstein Explanation of photo electric Effect

Einstein solved the problem regarding photo electric effect by adopting a quantized model of light. According to Einstein the energy of light is not distributed evenly over the classical wave front, but is concentrated in discrete regions (or in "bundles"), called quanta called photons , each containing energy, hf.These photons strike electrons in the metals and if the energy of photon E is equal to the Work Function of Metals ϕ_o (Minimum energy required for dislodging the electron) and the kinetic energy of the escaping electron.

The magnitude of photoelectric work function directly depends on the threshold frequency needed for metal surface.
Mathematically;

$$\phi_o \propto \nu_o$$

$$\phi_o = h\nu_o$$

Einstein's Equation for Photoelectric Effect:

Let us assume that;
i. The frequency of incident light ν
ii. The energy of incident photon = h ν
iii. The threshold frequency for metal surface = ν_o
iv. The photoelectric work function needed for the metal surface = ($\phi_o = h\nu$)
v. The K.E gained by the liberated *electron.*

$$K.E_{max} = \frac{1}{2} m v^2_{max}$$

Now, applying law of conservation of energy,

$$E = \phi_o + K.E_{max}$$

$$h\nu = \phi_o + K.E_{max}$$

$$h\nu = h\nu_o + \frac{1}{2} m v^2_{max}$$

$$h\nu - h\nu_o = \frac{1}{2} m v^2_{max}$$

$$h(\nu - \nu_o) = \frac{1}{2} m v^2_{max} \quad \left(\because \frac{1}{2} m v^2_{max} = V_o e\right)$$

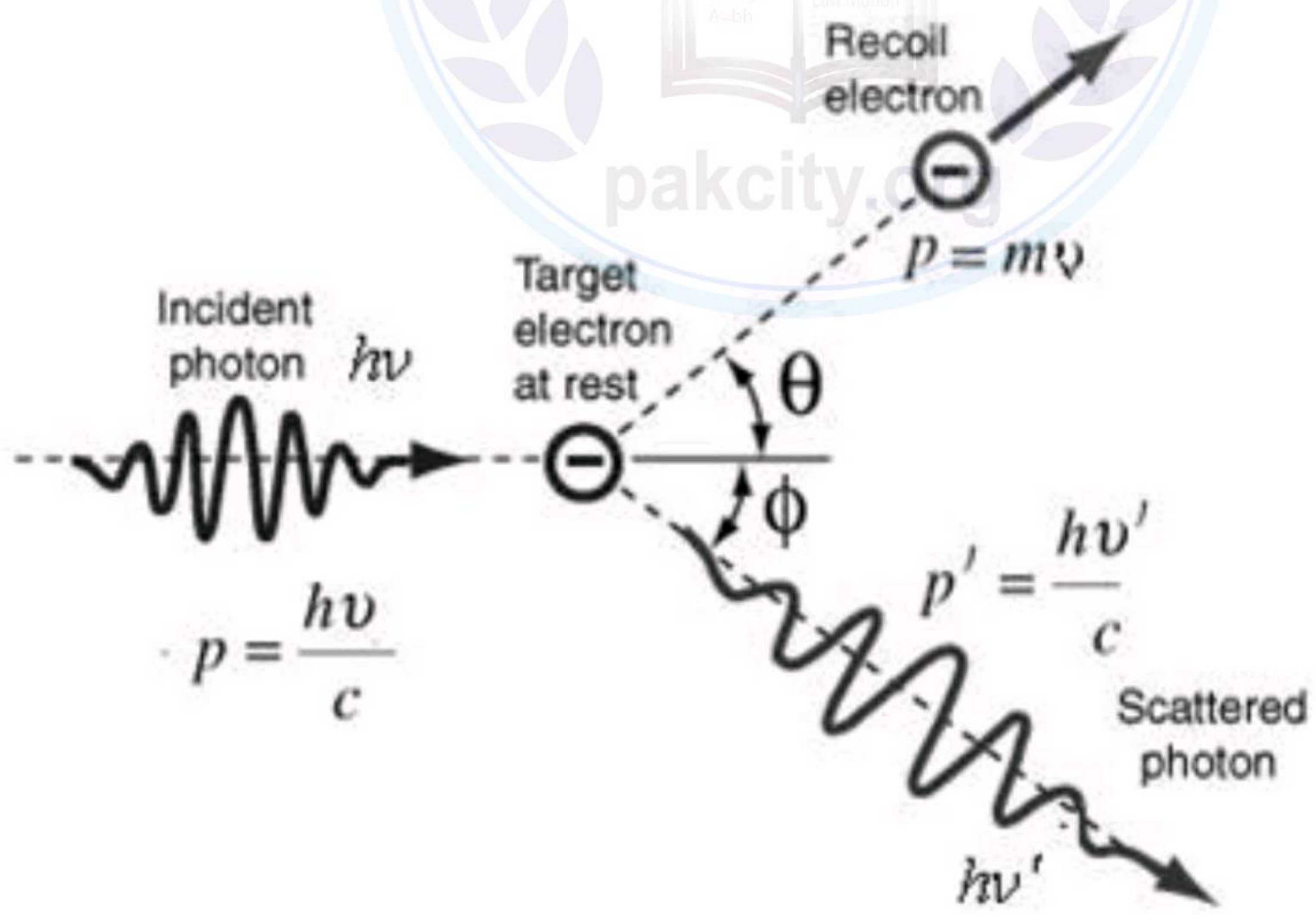
$$h(\nu - \nu_o) = V_o e \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$\boxed{h\left(\frac{c}{\lambda} - \frac{c}{\lambda_o}\right) = V_o e}$$

Compton's Effect:

In 1922 Arthur Holly Compton experimentally that x-ray photons behave like particles with momentum.

When X-ray photon strikes a crystal and catered at an angle intensity of scattered radiation is peaked at two different wave lengths and scattered wavelength varies with scattering angle which contradicts the Bohr Theory. According to Bohr Theory scattered X-ray should be of same wave length as incident X-ray. The change in wavelengths of scattered photon and incident photon is called "Compton Shift" in wavelength and symbolized by "$\Delta\lambda$".



Applying law of conservation of energy to the phenomenon

$$\textbf{Total energy before collision} = \textbf{Total energy after collision}$$

$$E_1 + E_e = E_2 + E_e'$$

$$\boxed{hf_1 + m_e c^2 = hf_2 + mc^2} --- (a)$$

Applying Law of conservation of momentum

$$\textbf{Total momentum before collision} = \textbf{Total monetum after collision}$$

In X-direction:

$$P_1 + P_e = P_2 cos\theta + P_e' cos\varphi$$

$$\frac{hf_1}{c} + 0 = \frac{hf_2}{c} cos\theta + m_e v cos\varphi$$

$$\boxed{\frac{hf_1}{c} = \frac{hf_2}{c} cos\theta + m_e v cos\varphi} --- (b)$$

In Y-direction:

$$\boxed{0 = \frac{hf_2}{c} sin\theta - m_e v sin\varphi} --- (c)$$

Solving Equation (a) (b) and (c) we get

$$\frac{1}{f_2} - \frac{1}{f_1} = \frac{h}{m_e c^2}(1 - cos\theta)$$

Since

$$c = \lambda f$$

$$\frac{1}{f} = \frac{\lambda}{c}$$

$$\frac{\lambda_2}{c} - \frac{\lambda_1}{c} = \frac{h}{m_e c^2}(1 - cos\theta)$$

$$\frac{\lambda_2 - \lambda_1}{\cancel{c}} = \frac{h}{m_e c^{\cancel{2}}}(1 - cos\theta)$$

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - cos\theta)$$

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - cos\theta)$$

$$\lambda_2 - \lambda_1 = \lambda_c(1 - cos\theta)$$

$$\boxed{\Delta\lambda = \lambda_c(1 - cos\theta)}$$

Here

$$\lambda_c = \frac{h}{m_e c} = \frac{6.63 \times 10^{-34 J}.s}{9.11 \times 10^{-31} \times 3 \times 10^8} = \mathbf{2.42 \times 10^{-12} m}$$

And λ_c is called Compton Shift.

De-Broglie Hypotheses:

According to De-Broglie

"If Electromagnetic radiation can act as particle than particles

like electron and proton can also behaves like wave "

Mathematically:

According to De-Broglie the wave length of a particle can be calculated by using relation

$$P = \frac{h}{\lambda} = mv$$

$$\boxed{\lambda = \frac{h}{mv}}$$

Heisenberg Uncertainty Principle:

Statement:

"It is impossible to determine Momentum and position of a particle simultaneously."

$$\Delta P \Delta x \geq h$$

OR

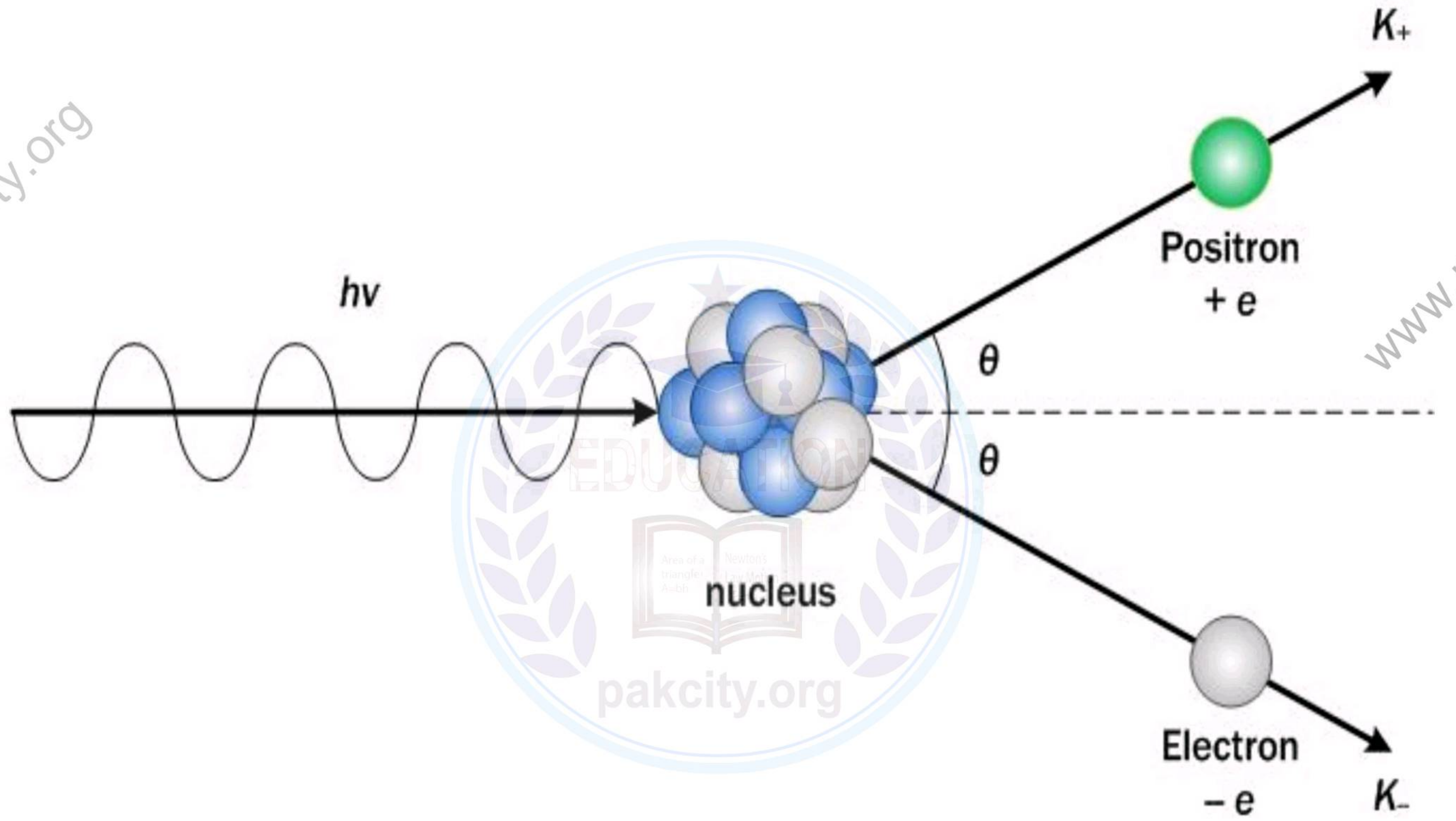
"It is impossible to determine energy and time simultaneously."

$$\Delta E \Delta t \geq h$$

Pair production

Pair production is a phenomenon of nature where energy is converted to mass. In this phenomenon a wave of energy or a photon (a packet of energy) interacts with a heavy nucleus to form an electron - positron pair. Pair production is observed to occur in nature when a photon or an energy wave packet, of greater 1.02 **MeV** passes near the electric field of a large atom such as lead, uranium or other heavy material with a large number of protons (around an atomic number of 80 or 90). The photon is literally split into an electron and its anti-particle, called a positron. Both have a rest mass energy equivalent of **0.511 MeV.** So, it is can be represented by an equation that shows the conservation of total energy:

$$h\nu = E_- + E_+ = (m_0c^2 + K_-) + (m_0c^2 + K_+) = K_- + K_+ + 2m_0c^2$$

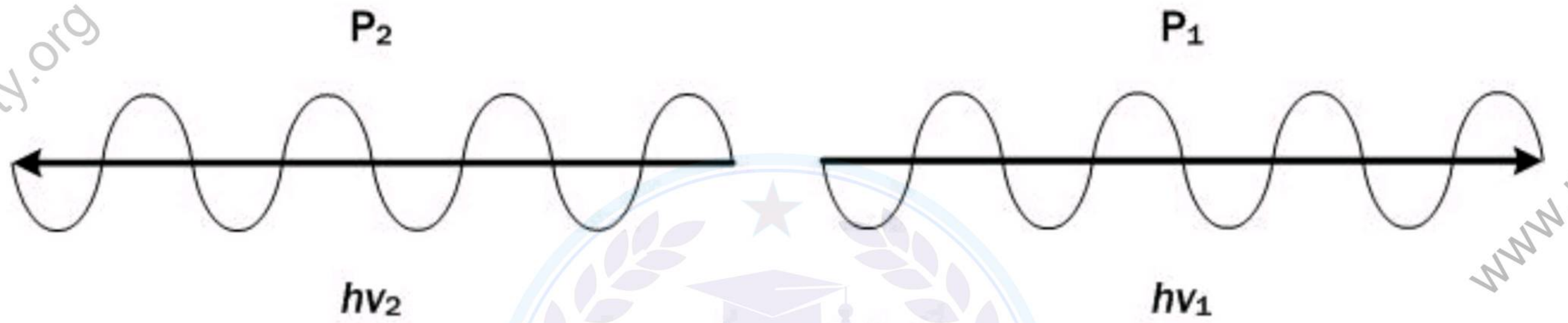


Pair Annihilation

Pair Annihilation means the reverse process of pair production. In the pair annihilation, the electron and positron combine with each other and annihilate. Surely, the particles are disappeared and radiation energy will occur instead of two particles. For the momentum conservation, the most frequent process in pair annihilation is making two photons that have exactly opposite direction and the same amount of momentum.

$$K_- + K_+ + 2m_0c^2 = 2\,h\nu$$

before pair annihilation



after pair annihilation

CHAPTER-17 **NUMERICALS from Past Papers**

1987

Q.7. (c) Find the cut off wavelength for a given metal whose work function is 4.14eV. **(3002.7A°)**

1989

Q.7. (c) The work function of certain metal is 3.03eV. When this metal is illuminated by the infrared light of 1.2×10^{15} Hz. Find the maximum kinetic energy of the emitted photoelectrons. **(1.9725eV)**

1991

Q.7. (c) A 50 m trailer is moving with relativistic speed. It passes over a bridge of length 40m. To an observer at rest with respect to the bridge at one instant, the trailer seems to overlap the bridge i.e. the ends of the trailer seem to coincide with the ends of bridge. Find the speed of the trailer. **(1.8×10^{8}m/s)**

1992

Q.7. (c) The work function of a photo emissive surface is 4.0eV. What will be the velocity of fastest photoelectrons emitted from it by an accident light of frequency 3.0×10^{15} Hz. **(1.722×10^{6} m/s)**

1994

Q.7. (c) The work function of metal is 2eV. The light of wavelength 3000 A° is made to fall on it. Find the kinetic energy of the fastest emitted photoelectrons. **(2.144eV)**

1996

Q.7. (c) Find the relativistic speed at which the kinetic energy of a particle of rest mass m_o becomes doubles its rest mass energy. Given $m_o = 1.67\times10^{-27}$ Kg. Also calculate:

1) Rest mass energy
2) Kinetic energy
3) Total energy **(939.375MeV, 1878.75MeV, 2818.125 MeV)**

1998

Q.7. (c) The range of visible light is 4000A° to 7000A°. Will photoelectrons be emitted by a copper surface of work function 4.4eV, when illuminated by visible light? Give the mathematical proof of your answer.

2001

Q.7. (c) When the light of the wavelength 4000°A falls on a metal surface, stopping potential is 0.6 volt. Find the value of the work function of the metal. **(2.5eV)**

2002 (Pre Med. group)

Q.7. (d) Find the speed at which the mass of a particle will be doubled. **(2.56×10^{8} m/s)**

2002 (Pre Engg. group)

Q.8. (d) Given $m_oc^2 = 0.511$MeV. Find the total energy E and the kinetic energy K of an electron moving with a speed v = 0.85c. **(0.979MeV, 0.459MeV)**

2003 (Pre Med. group)

Q.7. (d) If a neutron is converted entirely into energy, how much energy is produced? Express your answer in joule and electron – volt. **(9.39×10^8eV)**

Q.8. (d) Sodium surface is shined with the light of wavelength 3×10^{-7}m. If the work function of Na=2.46eV, find the kinetic energy of the photoelectrons. **(1.68eV)**

2003 (Pre Engg. group)

Q.8. (d) A sodium surface is shined with the light of wavelength 3×10^{-7}m, if the work function of sodium is 2.46 eV, find the kinetic energy of the photoelectron. **(1.68eV)**

2004

Q.7. (d) What minimum energy is required in an X-ray tube to produce X-rays with a wavelength of 0.1×10^{-10}m. **(12.43 eV)**

Q.8. (d) Compare the energy of a photon of wavelength 2×10^{-6}m with the energy of X-ray photon of wavelength 2×10^{-10}m. **(10^{-4})**

2005

Q.8. (d) Estimate the relativistic mass and the wavelength associated with an electron moving at 0.9c. **(2.087×10^{-30}kg, 1.176×10^{-12} m)**

2006

Q.7.(d) An electron exists within a region of 10^{-10}m, find its momentum uncertainty and approximate kinetic energy. **(1.05×10^{-24} Ns, 3.78eV)**

2007

Q.8. (d) In Compton Scattering process the fractional change in wavelength of X-Rays Photon is 1% at an angle 120°; find the wavelength of X – rays used in this experiment.

(3.63×10^{-10} m)

2008

Q.7. (d) Calculate the relativistic speed at which the mass of a particle becomes double its rest mass **(2.59×10^8 m/s)**

2009

Q.7. (d) A sodium surface is shined with the light of wavelength 3×10^{-7}m, if the work function of sodium is 2.46 eV, find the kinetic energy of the photoelectron and cutoff wavelength. **(1.68eV, 5061A°)**

2010

Q.2. (xiv) If the electron beam in a television picture tube is accelerated by 10,000 V what will be the de Broglie's wavelength? (h = 6.63×10^{-34} J.s., m = 9.1×10^{-31} kg). **(1.28×10^{-11} m)**

2011

Q.2. (xii) What will be the relative velocity and momentum of a particle whose rest mass is m_o and kinetic energy is equal to twice of its rest mass energy. ($\frac{2\sqrt{2}}{3}c$, $2\sqrt{2}\,m_o\,c$)

Q.2. (xv) If the electron beam in a television picture tube is accelerated by 10 kV. What will be the de Broglie wavelength of an electron? ($h = 6.63 \times 10^{-34}$Js, $m = 9.1 \times 10^{-31}$ kg).

(1.28×10^{-11} m)

2012

Q.2 (iv) Given m_oc^2=0.511 MeV. Find the total energy "E" and the kinetic energy K of an electron moving with speed ***v=0.85c*.m_o= 9.1 x 10^{-31} kg, c=3 x 10^8 m/s.**

Q.2 (xv) A sodium surface is shined with the light of wavelength 3 x 10^{-7}m, if the work function of sodium is 2.46 eV, find the kinetic energy of the photoelectron and cutoff wavelength. ***h=6.63x10^{-34} J-s, c =3 x 10^8m/s***

2013

Pair annihilation occurred due to a head-on-collision of an electron and positron having the same kinetic energy, produce pair of photons each having energy of 2.5 MeV. What were their kinetic energies before collision? **Given *m_oc^2=0.511MeV.***

2014

Q2. (iv) What will be the velocity and momentum of a particle whose rest mass is mo and kinetic energy is equal to twice of its rest mass energy.

Q2. (ix) In a TV picture tube, an electron is accelerated by a potential difference of 12000V. Determine de-Broglie wavelength.

Given that **(h = 6.63x10^{-34} J.s e=1.6x10^{-19} C, m_e=9.11x10^{-31} kg).**