

Chapter = 17

"The Advent of Modern Physics"



Relative Motion:

The rest position or the motion of an object is not same for different observers. Therefore we can't say whether an object is absolutely at rest or absolutely in motion. All motions are relative to a person or instrument observing it.

Examples:

1. The walls of the cabin of a moving train are stationary with respect to the passengers sitting inside it but are in motion to a person stationary on the ground.
2. If an observer sitting in a vehicle moving with a uniform speed throw an object vertically upward then the observer will see the object falls along the same vertical path but an observer at rest on the ground the path of the object will appear to be a curve.

Frame of Reference:

A frame of reference is any coordinate system relative to which measurement are taken.

Examples:

1. Position of a table in a room can be located relative to the walls of the room. The room is then the frame of reference.
2. The position of a spaceship can be described relative to the positions of the distant stars. A coordinate system based on these stars is then the frame of reference.

Inertial Frame of Reference:

An inertial frame of reference is a coordinate system in which the law of inertia is valid i.e. a body at rest remains at rest unless an unbalanced force produces acceleration in it. It is also called non-accelerated frame of reference.

Any frame of reference which is moving with uniform velocity relative to an inertial frame is also an inertial frame.

Example:

Earth can be considered as inertial frame of reference because of very small acceleration which arises due to spin and orbital rotation.

Special Theory of Relativity:

The special theory of relativity is concerned with the way in which observers who are in a state of motion describe physical phenomenon. The two postulates of special theory of relativity are given below:

1. The laws of physics are the same in all inertial frames.
2. The speed of light in vacuum is the same for all observers in uniform translational relative motion and is independent of the motion of the observer and the source.

Consequences of Special Theory of Relativity:

Time Dilation:

According to special theory of relativity, time is not absolute quantity. It depends upon the motion of the frame of reference.

Mathematically:

Suppose an observer is stationary in an inertial frame and measures the time interval " t_0 " between two events in his frame. This is known as proper time. If observer is moving with respect to the frame of events with speed " v " or if the frame of events is moving with respect to observer with a uniform speed " v ", the time interval measured by the observer would be " t " which is known as relativist time and is given by the following relation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Conclusion:

Therefore time has dilated or stretched due to relative motion and we can say that clocks in moving frame run slowly.

Mass Variation:

According to special theory of relativity, mass of an object is a varying quantity and depends upon the speed of the object.

Mathematically:

An object whose mass when measured at rest is " m_0 " will have increased to be mass " m " when observed to be moving at speed v . They are related by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Conclusion:

The increase in mass indicates the increase in inertia the object has at high speed. As " v " approaches " c ", it requires larger and larger force to change the speed of the object.

Length Contraction:

The length of an object in a frame of reference stationary with respect to the observer is not same when it is measured in the frame of reference moving with velocity with respect to the observer. If you are in motion relative to two points that are a fixed distance apart, the distance between the two points appear shorter than if you were at rest relative to them.

Mathematically:

If " l_0 " is the length of an object in a stationary frame of reference and " l " is the length of the same object when measured by the observer in the moving frame of reference with a relative velocity " v " then

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$



Conclusion:

The distance become shorter along the direction of motion e.g. the distance from earth to a star measured by an observer in a moving spaceship would seem smaller than the distance measured by an observer on earth.

Energy - Mass Relation:

According to special theory of relativity mass and energy are different entities but are inter convertible.

Mathematically:

The total energy " E " and mass " m " of an object are related by expression

$$E = mc^2$$

Where m depend upon the speed of the object. At rest, the energy equivalent of an object's mass " m_0 " is called rest mass energy " E_0 ".

$$E_0 = m_0c^2$$

As mc^2 is greater than m_0c^2 , the difference of energy is due to motion as such it represents the kinetic energy of the mass, hence

$$K.E = mc^2 - m_0c^2$$

$$K.E = (m - m_0)c^2$$

Conclusion:

Any process that changed the mass by detectable amount would involve huge amount of energy.

Blackbody Radiations :

The radiations emitted by a body due to its temperature is called black body radiations. An object which can absorb all the radiations that fall on it is called black body.

Explanation:

A hollow sphere of metal with a fine hole in it is called cavity as shown in figure. This cavity is approximately a black body because any radiation entering the hole is trapped by multiple reflections inside and very little of it is able to escape. Just as a black body is nearly a perfect absorber, so is the most effective emitter of radiation when heated.

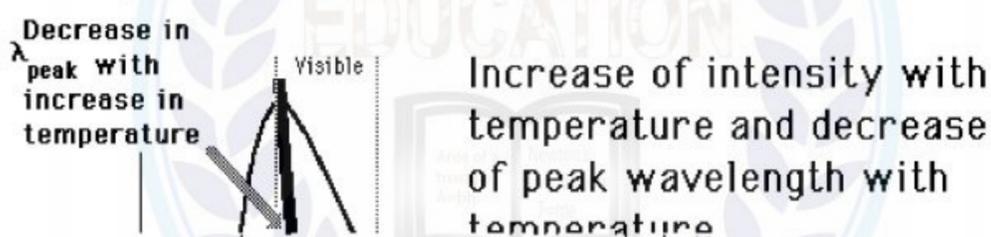


Laws of Black Body Radiations:

Wien's Displacement law:

Statement:

"Wavelength at which maximum energy of radiation occurs is shifted towards the shorter wave length with the rise of temperature."



Mathematically:

The wavelength for maximum radiation λ_{max} is related to the absolute temperature T of the black body by following expression

$$\lambda_{\text{max}} \times T = \text{constant}$$

The Wien's law is in agreement with the common observation that is a white hot furnace is hotter than one which is red hot.

Stefan's Boltzmann's law:

Statement:

"The total energy radiated per second per unit surface area is proportional to the fourth power of absolute temperature."

Mathematically:



$$E \propto T^4$$

$$E = \sigma T^4$$

$\sigma = \text{Stefan's Boltzmann's constant}$

Rayleigh and Jeans law:

Statement:

"Energy associated with a particular wavelength is inversely proportional to the fourth power of wavelength."

Mathematically:

$$E = \frac{\text{constant}}{\lambda^4}$$

Plank's law (Quantum Theory):

Statement:

"Radiant energy comes out in discrete amount or quanta of energy . The energy content of each quantum was directly proportional to the frequency ν ."

Mathematically:

$$E \propto \nu$$

$$E = h\nu$$

$$E = \frac{hc}{\lambda} \quad [\text{since } \nu = \frac{c}{\lambda}]$$

The constant, h is called plank's constant and the values of h was determined to be 6.634×10^{-34} J-s. On the basis of assumption that energy could only be emitted or absorbed by atomic oscillator in discrete quanta, the Plank's law would be

$$E = nh\nu$$

Where $n = 0, 1, 2, 3$ and so on.

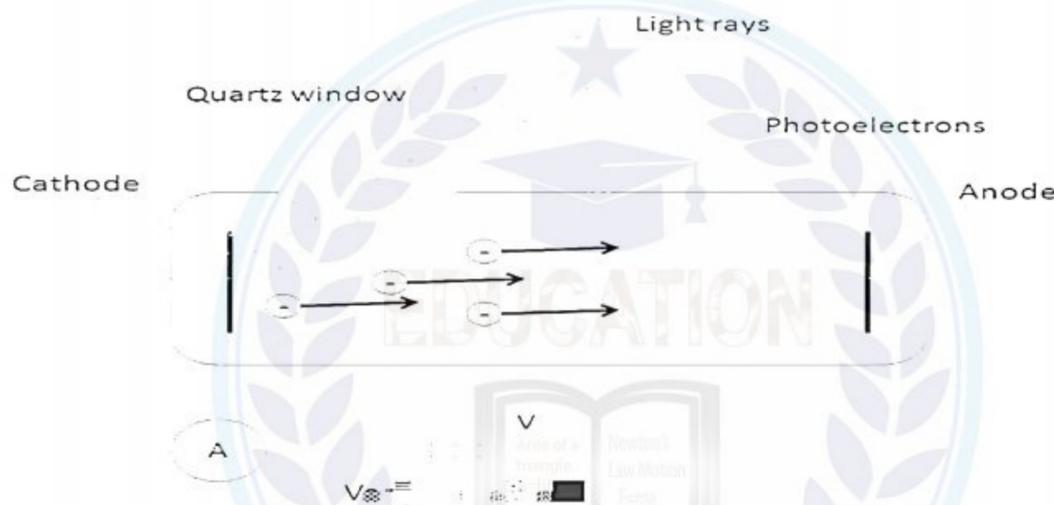
Photoelectric Effect:

Hertz in 1887 discovered that when ultraviolet light falls on certain metals, electrons are emitted. The phenomenon in which certain metals emit electrons when exposed to high frequency light is called photoelectric current.



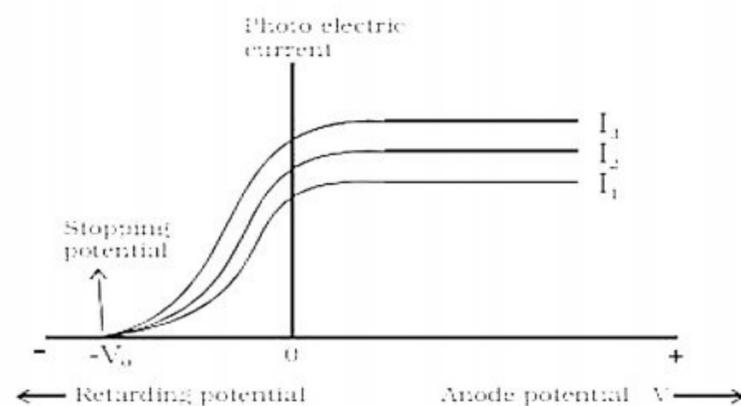
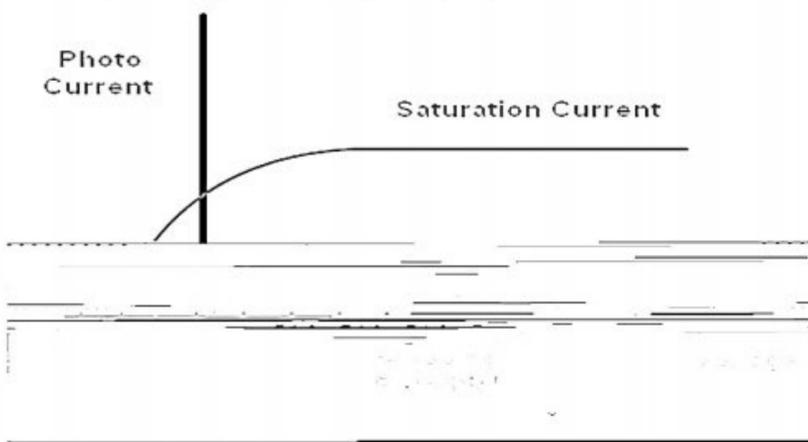
Explanation:

An evacuated glass tube contains two electrodes. The electrode connected to the positive terminal of the battery is known as anode. The electrode connected to negative terminal is known as cathode. When monochromatic light is allowed to shine on cathode, it begins to emit electrons. These photoelectrons are attracted by the positive terminal anode and the resulting current is measured by an ammeter. The current stops when light is cut off.

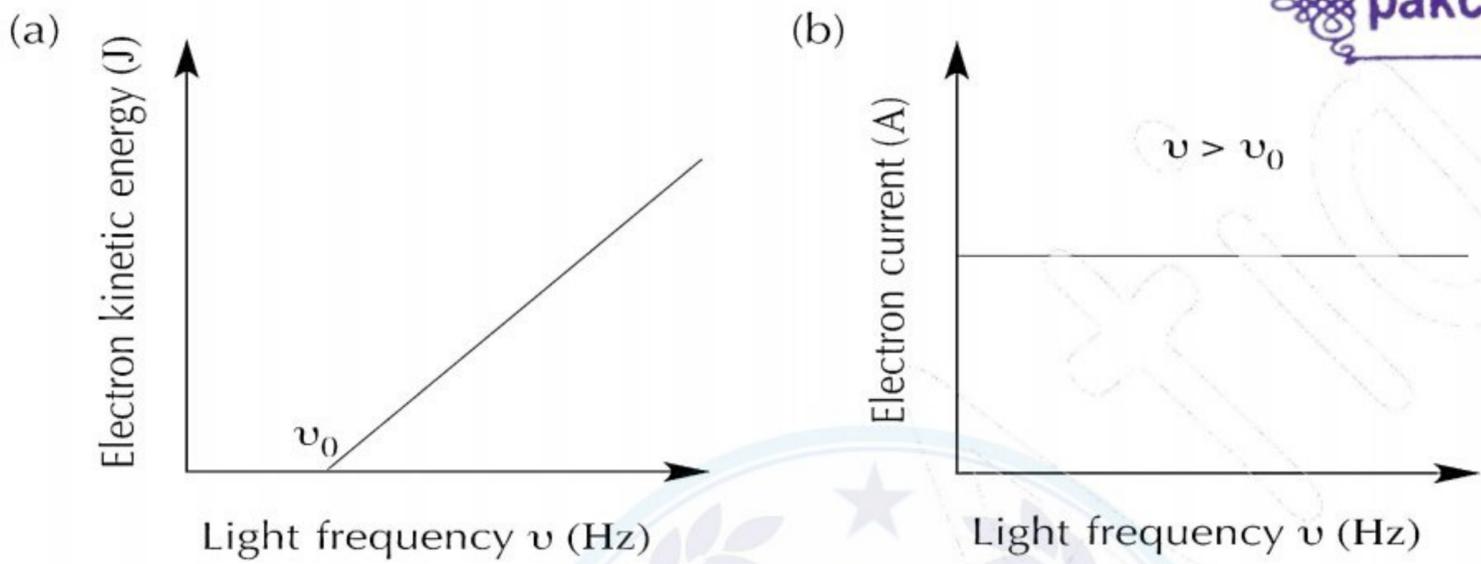


Experimental results of Photoelectric Effect:

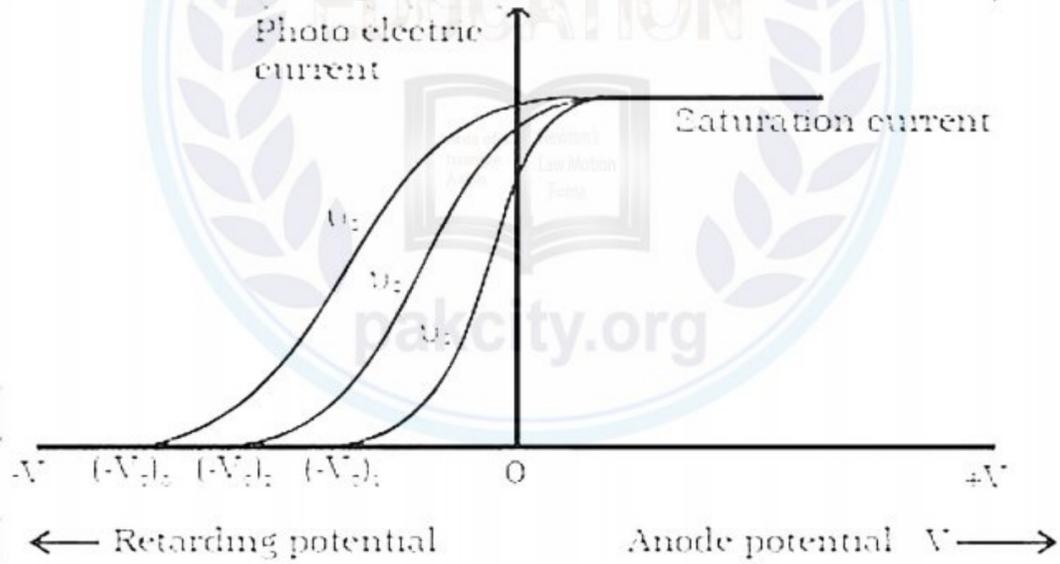
1. Increasing the intensity of the source of light, increases the number of photoelectrons i.e. electric current increases. It follows that there is a saturation current for different intensities.



2. For each substance there is a certain frequency called the threshold frequency below which the effect does not occur. The graph shows a threshold frequency " ν_0 " is a minimum frequency below which no electrons escape the metal surface. If the frequency increases beyond the threshold frequency the photo electric current remains constant.



3. The higher the frequency of the incident light, the greater the kinetic energy of photo electrons. The saturation current depends upon intensity and not on the frequency. However, the stopping potential becomes more negative with the increase in frequency.



Stopping Potential:

If the connection of the battery is reversed then the negative potential difference at which the photoelectric current becomes zero is called stopping potential. At this potential even the electrons of maximum energy are not able to reach the collector (anode plate).

Mathematically:

$$\text{Kinetic energy} = \text{Electric potential energy}$$

$$\frac{1}{2}mv_{max}^2 = V_0e$$

v_{max} = maximum energy of electrons

e = charge on electron

V_0 = stopping potential



Einstein Explanation of Photoelectric Effect on the basis of Quantum Theory:

An electron absorbs a photon of energy $h\nu$ and electron may lose some of this energy before leaving the metal surface and is ejected with a kinetic energy less than $h\nu$. The electron use some of its energy to overcome the force of attraction which holds the electrons within the metal. The energy required to overcome this binding force is called the work function of the particular metal and is denoted by ϕ which is a constant of the metal. Hence the Einstein's equation for photoelectric effect will be written as:

$$K.E_{max} = \frac{1}{2}mv_{max}^2 = h\nu - \phi$$

In this equation ϕ is a work function which directly depends on the threshold frequency ν_0

$$\phi = h\nu_0$$

Therefore,

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$$

Since we know that the frequency and wavelength are related by

$$\nu = \frac{c}{\lambda}$$

Therefore,

$$\frac{1}{2}mv_{max}^2 = h\frac{c}{\lambda} - h\frac{c}{\lambda_0}$$

$$\frac{1}{2}mv_{max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Since

$$\frac{1}{2}mv_{max}^2 = V_0e$$

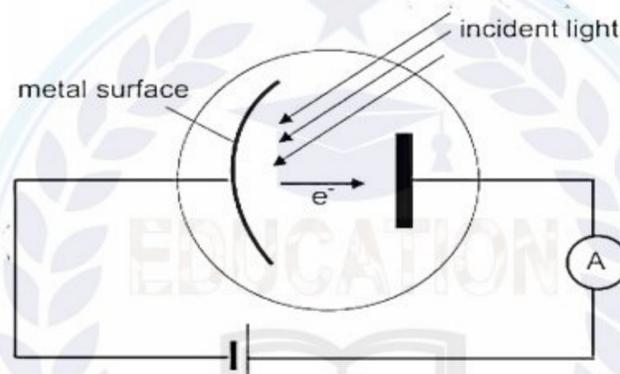
Therefore Einstein's photoelectric equation can also be written as

$$V_0e = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

Photocell:



The photocell or phototube consists of a evacuated glass tube fitted with an anode plate and a concave metallic cathode of a appropriate surface. The material of the cathode can be chosen to respond to the frequency range over which the photocell operates. The response can be made proportional to the intensity of light source.



Applications of photocell:

A photocell can be used in any situation where beam of light falling on a cell is interrupted or broken.

1. To count vehicles passing a road or items on a conveyer belt.
2. To open doors automatically in a building.
3. To operate burglar alarms.

Photoconductors:

Photoconductors are materials in which an internal photoelectric effect may liberate free charge carriers in a material that is otherwise an insulator, and thereby increase its electrical conductivity by as much as 10000 times when it is illuminated by a light source. Such materials are called photoconductors.

Applications:

Photoconductive cells may be used in



1. Detection and measurements of infra-red radiation where the wavelength is of the order of 10^{-6} m.
2. As relays for switching on artificial lighting, such as street lights.

Photovoltaic cell:

A photovoltaic cell which may consist of a sandwich of copper oxide and a thin film (layer) of translucent gold. An emf capable of giving a current of 1mA can be generated when the film is illuminated. Such cells need no source of e.m.f.



Compton's Effect:

When x-rays are scattered due to interaction with a light body such as an electron, the scattered rays exhibit lower frequencies (i.e. higher wavelengths) than the incident radiations. This phenomenon is called Compton's Effect.



Explanation:

The scattering is treated to be a two body collision between a photon and an electron. Compton's effect provided a solid support for photon theory of light. The figure below shows the event of scattering "before" and "after" the collision. The electron is treated at rest because for high energy photons, the initial motion of the electrons may be neglected.

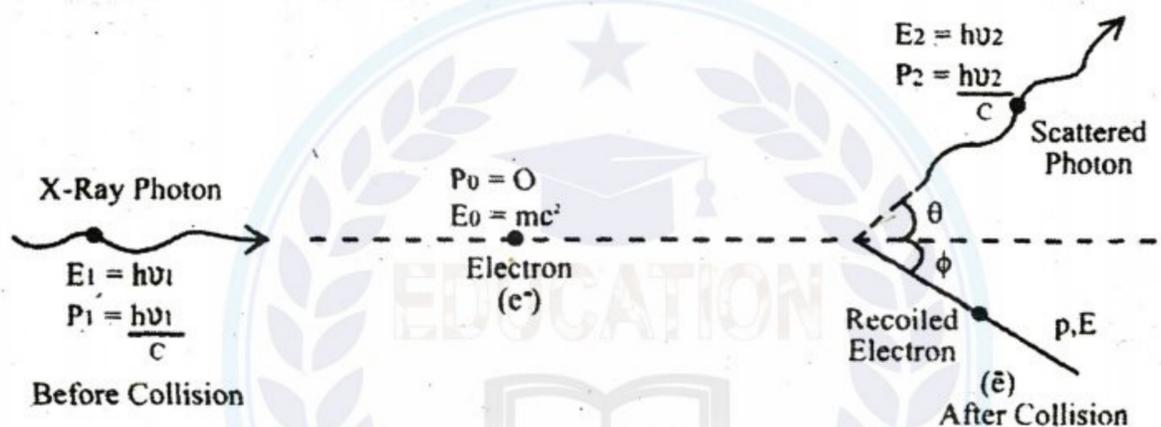


Fig. (17.15) The Compton scattering in lab-frame.

Derivation for Compton's Shift:

The photon approaches towards the electron with a frequency ν_1 and is scattered at an angle θ with a lower frequency ν_2 . The photon energies before and after collision are $h\nu_1$ and $h\nu_2$. Whereas the corresponding momentum are $\frac{h\nu_1}{c}$ and $\frac{h\nu_2}{c}$. The energy and momentum of the recoiled electron are E and P respectively.

A/c to law of conservation of linear momentum

$$\text{Total momentum before collision} = \text{Total momentum after collision}$$

Along x-direction

$$0 + \frac{h\nu_1}{c} = p\cos\phi + \frac{h\nu_2}{c}\cos\theta$$

$$\frac{hv_1}{c} = p \cos \varphi + \frac{hv_2}{c} \cos \theta \text{ --- (1)}$$

Along y-direction



$$0 + 0 = \frac{hv_2}{c} \sin \theta - p \sin \varphi$$

$$0 = \frac{hv_2}{c} \sin \theta - p \sin \varphi \text{ --- (2)}$$

A/c to law of conservation of energy

Total energy before collision = Total energy after collision

$$m_0c^2 + hv_1 = E + hv_2 \text{ --- (3)}$$

To obtain an expression for the final frequency ν_2 as a function of initial frequency ν_1 and scattering angle θ we have to eliminate p and E from the above three equations. After solving these three equations, we get

$$\frac{1}{\nu_2} - \frac{1}{\nu_1} = \frac{h}{m_0c^2} (1 - \cos \theta)$$

since we know that, $\nu = \frac{c}{\lambda}$, therefore we get

$$\frac{1}{\frac{c}{\lambda_2}} - \frac{1}{\frac{c}{\lambda_1}} = \frac{h}{m_0c^2} (1 - \cos \theta)$$

$$\frac{\lambda_2}{c} - \frac{\lambda_1}{c} = \frac{h}{m_0c^2} (1 - \cos \theta)$$

$$\lambda_2 - \lambda_1 = \frac{h}{m_0c} (1 - \cos \theta)$$

The above equation is known as the Compton's formula for the increase in wavelength for the scattered photon. In the above formula $\Delta\lambda = \lambda_2 - \lambda_1$ is called the Compton's shift and $\lambda_c = \frac{h}{m_0c}$ is called the Compton's wavelength.

Pair Production or Materialization of Energy:

The phenomenon in which photon in the vicinity of a nucleus disappear with the production of an electron-positron pair is called pair production or materialization of energy.

Explanation:



The pair production requires that the two particles created by the interaction of photon with matter must have equal and opposite charges and the photon from which the pair was produced must have an energy at least $2m_0c^2$.

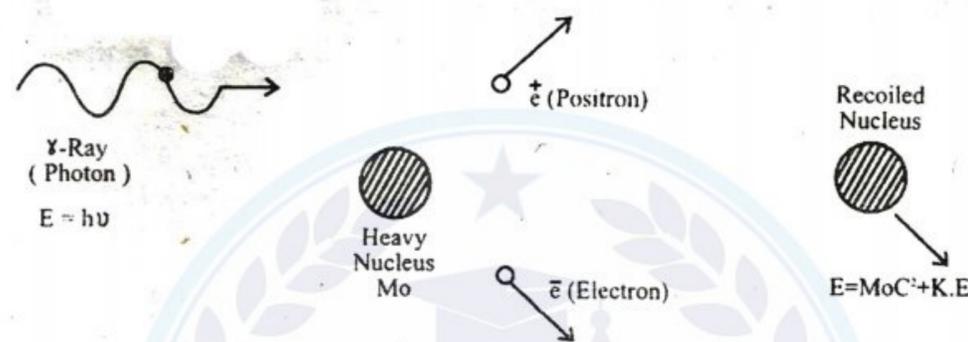


Fig. (17.16) The Pair Production

The positron has been identified to be identical with an electron in mass and carries an equal positive charge and is called the anti-particle of electron. Due to large mass of the heavy nucleus the recoiled kinetic energy of the nucleus is negligible as compared with the kinetic energies of the pair. A/C to the law of conservation of energy

$$h\nu = 2m_0c^2 + K.E_{e^-} + K.E_{e^+}$$

Annihilation of Matter:

The destruction or annihilation of electron-positron pair with the creation of at least two or more photons is called annihilation of matter.

Explanation:

In annihilation of matter a particle and its anti particle come close enough to be converted completely into radiation energy of the two photons moving in opposite direction conserving the total momentum of the creation and annihilation process. Each photon will have energy equal to rest mass energy m_0c of an electron.

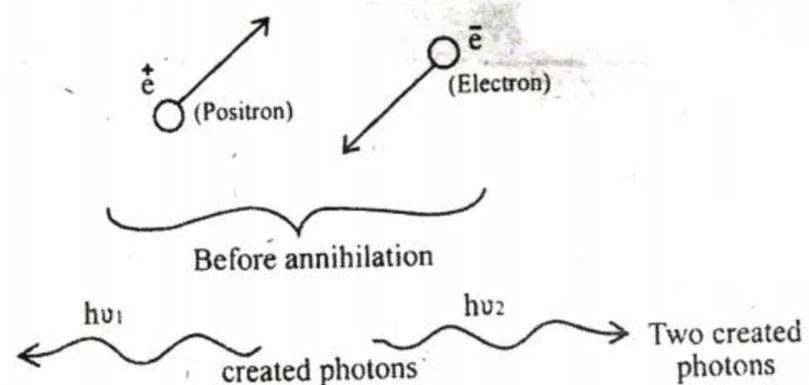


Fig. (17.17) Annihilation of electron positron pair in collision



Since the rest mass energy of electron-positron pair is 1.02 Mev, each photon created in the annihilation process will have an energy of 0.51 MeV. The energy conservation equation for the process will be

$$m_0c^2_{e^-} + K.E_{e^-} + m_0c^2_{e^+} + K.E_{e^+} = 2hv$$

De-Broglie Hypothesis:

"If light (electromagnetic radiation) can have particle behavior, then material particles, such as electron and proton etc. can also behave in a wave like manner."

Thus a particle like electron can possess a momentum given by

$$p = mv = \frac{h}{\lambda}$$

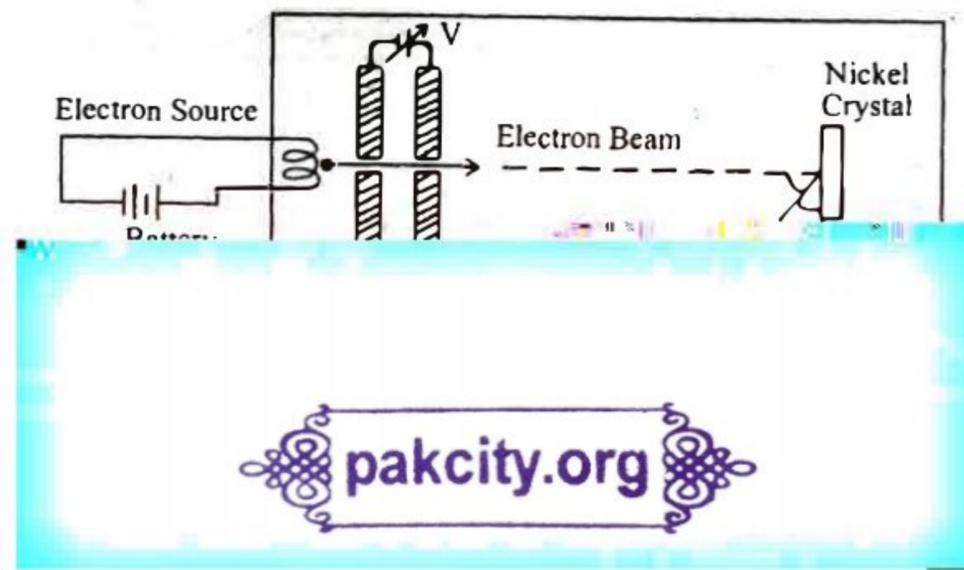
Where m is the mass of the particle. This relation called deBroglie's relation has related the electron (particle) and the wave character of a frequency. Therefore we can write the wavelength of a particle

$$\lambda = \frac{h}{mv}$$

Davisson and Germer Experiment:

Introduction:

The theoretical prediction of deBroglie's hypothesis was experimentally confirmed by Davisson and Germer. They were investigating the scattering of an electron beam by the metallic crystal of Nickel. The experiment setup is shown below



Construction and Working:

a beam of electron accelerated through the potential V were allowed to strike the nickel crystal. Measurements were made to count the number of electrons scattered by the crystal. It was observed that electrons reflected very strongly at certain angles only and not at other direction. They concluded that electron behaves as waves of wavelength λ . They calculated the wavelength of electron from the known accelerating potential V by applying the relation for kinetic energy of the electron i.e.

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

Since we know that the deBroglie's relation

$$\lambda = \frac{h}{mv}$$

Substituting the value of v in deBroglie's relation, we get

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2emV}}$$



Heisenberg's Principle of Uncertainty:

"It is in principle impossible to measure with accuracy both position and momentum of a particle simultaneously."

Mathematically:

If we denote Δx and Δp_x the corresponding uncertainties in position: then according to Heisenberg's principle of uncertainty, the product of these uncertainties must always satisfy the inequality:

$$(\Delta x) (\Delta p_x) \geq \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J-s}$$

Similarly if ΔE and Δt represent the uncertainties in the energy and time respectively then we have an expression

$$(\Delta E) (\Delta t) \geq \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J-s}$$

