Chapter-6:

Fluid Dynamics

Fluid:

"Anything that can flow from one place to another place is called fluid."

Example:

Liquids and gases are fluids.

Fluid dynamics:

The branch of physics in which we deal with fluid when it is in motion is known as fluid dynamics.

Viscosity:

The frictional effect between two different layers of the flowing fluid.

The coefficient of viscosity is "η".

Drag Force:

When the object moves through fluid then the retarding force is known as drag force.

$$F = 6\pi\eta r v$$

Stoke's law:

It states that the drag

Force "F" on a sphere of radius "r" moving with a velocity "v" having "η" co-efficient of viscosity.

$$F_d = 6\pi\eta r v$$

Dependence:

It depends upon three factors η increases F_D also increases similarly all of these.

(i) "η" co-efficient of viscosity
(ii) "r" radius of sphere
(iii) "v" velocity of fluid

Limitations of Stoke's law:

The following are the limitations.

- Object must be sphere.
- There is no larger speed of object.
- At high speed, the drag force is not proportional to speed.

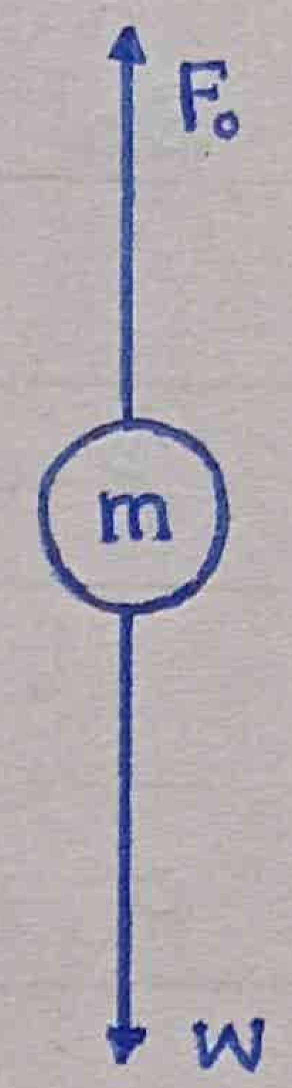
Terminal Velocity:

The maximum constant velocity of an object by which the object moves vertically downward when the drag force becomes equal to its weight.

Example:

Fog is suspended.

Explanation:



Consider a rain droplet of mass in the form of sphere moving vertically downward due to its own weight. The air resistance act vertically upward on the droplet.

At any instant when the drag force becomes equal to its weight. Then the droplet moves with constant velocity which is known as terminal velocity.

Condition of terminal velocity:

$$F_D = W$$

$\therefore F = W = mg$

$$6\pi\eta r v_t = mg$$

$\because F_D = 6\pi\eta r v$

$$V_t = \frac{mg}{6\pi\eta r}$$

For mass:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$m = \rho \times V \quad \text{——— (1)}$$

As droplet is sphere.

So

$$V = \frac{4}{3}\pi r^3$$

putting the value of V in (1)

$$m = \rho \times \frac{4}{3}\pi r^3$$

Putting the value of "m" in

$$V_t = \left(\frac{4}{3}\rho\pi r^3\right)\left(\frac{g}{6\pi r\eta}\right)$$

$$V_t = \frac{4\rho r^2 g}{18\eta}$$

$$V_t = \frac{2\rho r^2 g}{9\eta}$$

$$V_t = \left(\frac{2\rho g}{9\eta}\right) r^2$$

$$\therefore \frac{2g\rho}{9\eta} = \text{Constant}$$

$$V_t \propto r^2$$

Conclusion:

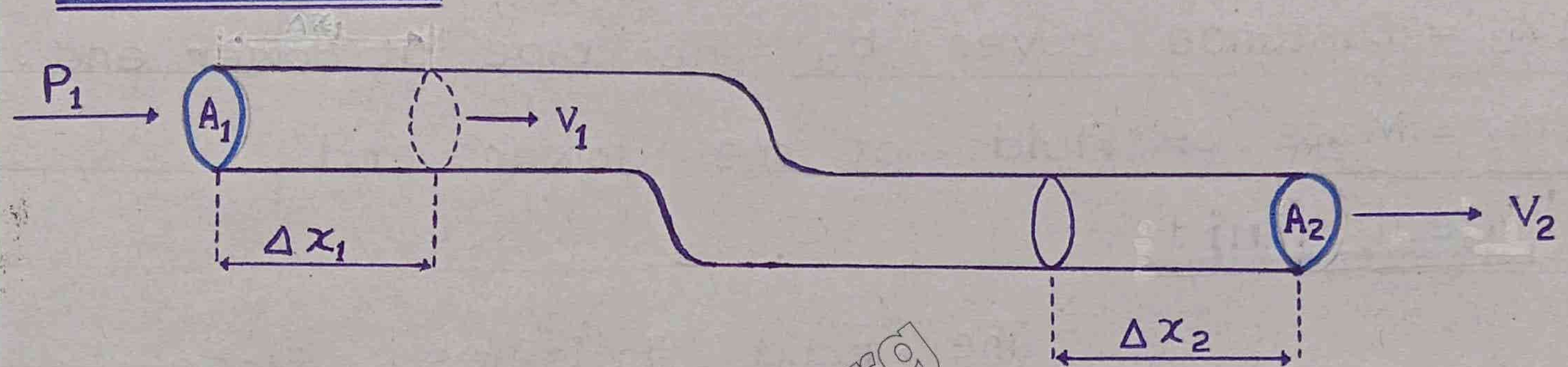
The terminal velocity is directly proportional to the square of radius of droplet.

Equation of continuity:

"For an ideal fluid, the product of cross sectional area of the pipe and the speed of the fluid at any point of the pipe remains constant."

$$Av = \text{constant}$$

Explanation:



Consider a fluid moving through the horizontal pipe along stream line flow.

At Upper end of pipe:

A_1 = Area of pipe at upper end.

v_1 = Speed of fluid at upper end.

Δx_1 = Distance covered by the fluid at upper end.

Δm_1 = Mass of fluid at upper end.

For mass (Δm_1):

$$\rho = \frac{m}{V} \qquad \therefore V = A \times L$$

$$\Delta m_1 = \rho_1 V_1$$

$\therefore V_1 = A \times \Delta x_1$

$$\Delta m_1 = \rho_1 A_1 \Delta x_1$$

$\therefore \Delta x_1 = v_1 t$

$$\Delta m_1 = \rho_1 A_1 v_1 t \quad \text{——— (1)}$$

$\therefore S = vt$

At Lower end:

A_2 = Area of pipe at lower end.

v_2 = Speed of pipe at lower end.

Δx_2 = Distance cover by the pipe at lower end.

Δm_2 = Mass of fluid at the lower end.

Ideal Fluid:

The Fluid satisfies these conditions:-

(i) Non - viscous

(ii) The Fluid is incopressible

(iii) The Fluid motion is steady.

(iv) Irrotational Flow of fluid.

For mass (Δm_2):

$$\rho = \frac{m}{V}$$

Volume

$\therefore V = A \times L$

$$\Delta m_2 = \rho_2 V_2$$

$$\Delta m_2 = \rho_2 A_2 \Delta x_2$$

$$\Delta m_2 = \rho_2 A_2 v_2 t$$

As Fluid is non-viscous:

$$\Delta m_1 = \Delta m_2$$

$$\boxed{\rho_1 A_1 v_1 t = \rho_2 A_2 v_2 t} \quad \text{——— (2)}$$

As Fluid is incompressible:

$$\rho_1 = \rho_2 = \rho$$

The equation (2) becomes

$$\cancel{\rho} A_1 v_1 \cancel{t} = \cancel{\rho} A_2 v_2 \cancel{t}$$

$$A_1 v_1 = A_2 v_2$$

$$\boxed{Av = \text{constant}}$$

Bernouli's Equation

The sum of pressure, kinetic energy per unit volume and potential energy per unit volume for an ideal fluid flowing through a non-uniform size pipe remains constant.

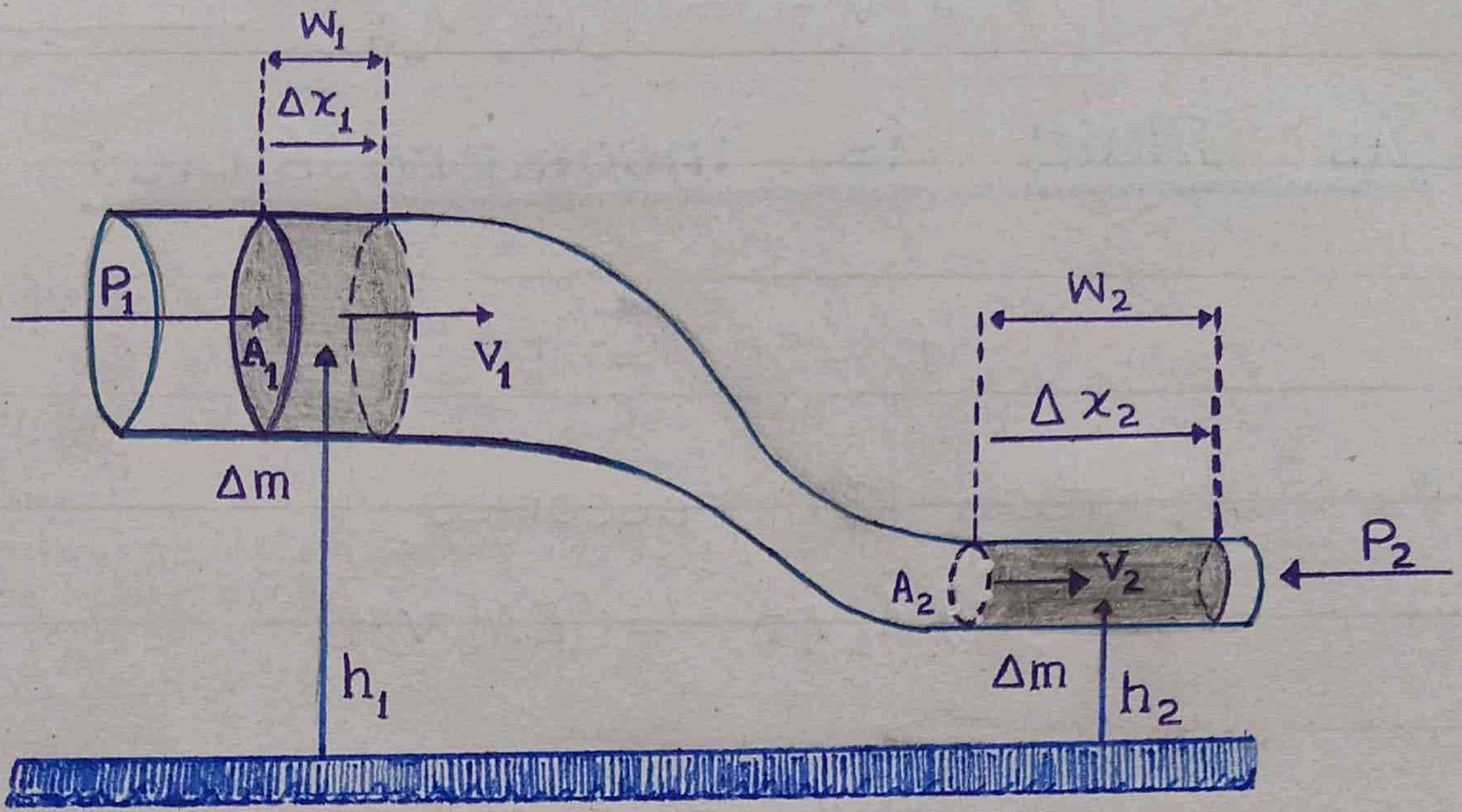
$$P + \frac{K.E}{V} + \frac{P.E}{V} = \text{constant}$$

Proof:

The fluid is moving by the

work done on it which it is converted into change of kinetic energy and potential energy.

Figure:



Work done at upper end:

A_1 = Area of fluid at upper end

V_1 = Speed of fluid at upper end

Δx_1 = Distance covered by fluid at upper end

P = Pressure at upper end

W_1 = Work done of fluid at upper end

As work will be:

$$W_1 = \vec{F_1} \cdot \Delta \vec{x_1}$$

$$W_1 = F_1 \Delta x_1 \cos\theta$$

$$W_1 = F_1 \Delta x_1 \cos 0^\circ$$

$$W_1 = F_1 \Delta x_1$$

$$W_1 = P_1 A_1 \Delta x_1$$

$$\boxed{W_1 = P_1 A_1 V_1 t} \quad \text{———— (i)}$$

Work done at Lower end:

A_1 = Area of Fluid at lower end

V_1 = Speed of Fluid at lower end

Δx_1 = Distance covered by Fluid

P_2 = Pressure at lower end

W_2 = Work done by Fluid at lower end.

As work will be:

$\therefore \theta = 180^\circ$

$\cos 180^\circ = -1$

$$W_2 = \vec{F_2} \cdot \vec{\Delta x_2}$$

$$W_2 = F_2 \Delta x_2 \cos\theta$$

$$W_2 = F_2 \Delta x_2 \cos 180^\circ$$

$$W_2 = F_2 \Delta x_2 (-1)$$

$$W_2 = -F_2 \Delta x_2$$

$$W_2 = -P_2 A_2 \Delta x_2$$

$$\boxed{W_2 = -P_2 A_2 V_2 t} \quad \text{———— (ii)}$$

Total Work:

$$W = W_1 + W_2$$

$$W = P_1 A_1 V_1 t + P_2 A_2 V_2 t \quad \text{———— (iii)}$$

As equation of continuity:

$$A_1 V_1 t = A_2 V_2 t$$

$$A V_t = \text{constaint}$$

By equation (iii)

$$W = P_1 A V t - P_2 A V t \qquad \because A_1 V_1 = A_2 V_2 = AV$$

$$W = P_1 A \Delta x - P_2 A \Delta x_2 \qquad \Delta x = vt$$

$$W = P_1 V - P_2 V \qquad A \times \Delta x = V$$

$$W = (P_1 - P_2) V \qquad \therefore \begin{bmatrix} A_2 V_2 t = V \\ A_1 V_1 t = V \end{bmatrix} \text{constont}$$

Putting the value of "w" in equation

$$W = \Delta K.E + \Delta P.E$$

$$(P_1 - P_2) V = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2) \frac{m}{\rho} = m\left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2 + gh_2 - gh_1\right)$$

$$(P_1 - P_2) = \rho\left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2 + gh_2 - gh_1\right)$$

$$(P_1 - P_2) = \frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2 + g\rho h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$$

$$\boxed{P + \frac{K.E}{V} + \frac{P.E}{V} = \text{constant}}$$

Hint:

$$= \frac{1}{2} \rho V_1^2$$

$$= \frac{1}{2} \frac{m}{V} V_1^2$$

$$= \frac{1}{2}\left(\frac{1}{2} m V_1^2 \right)$$

$$= \frac{K.E}{V}$$

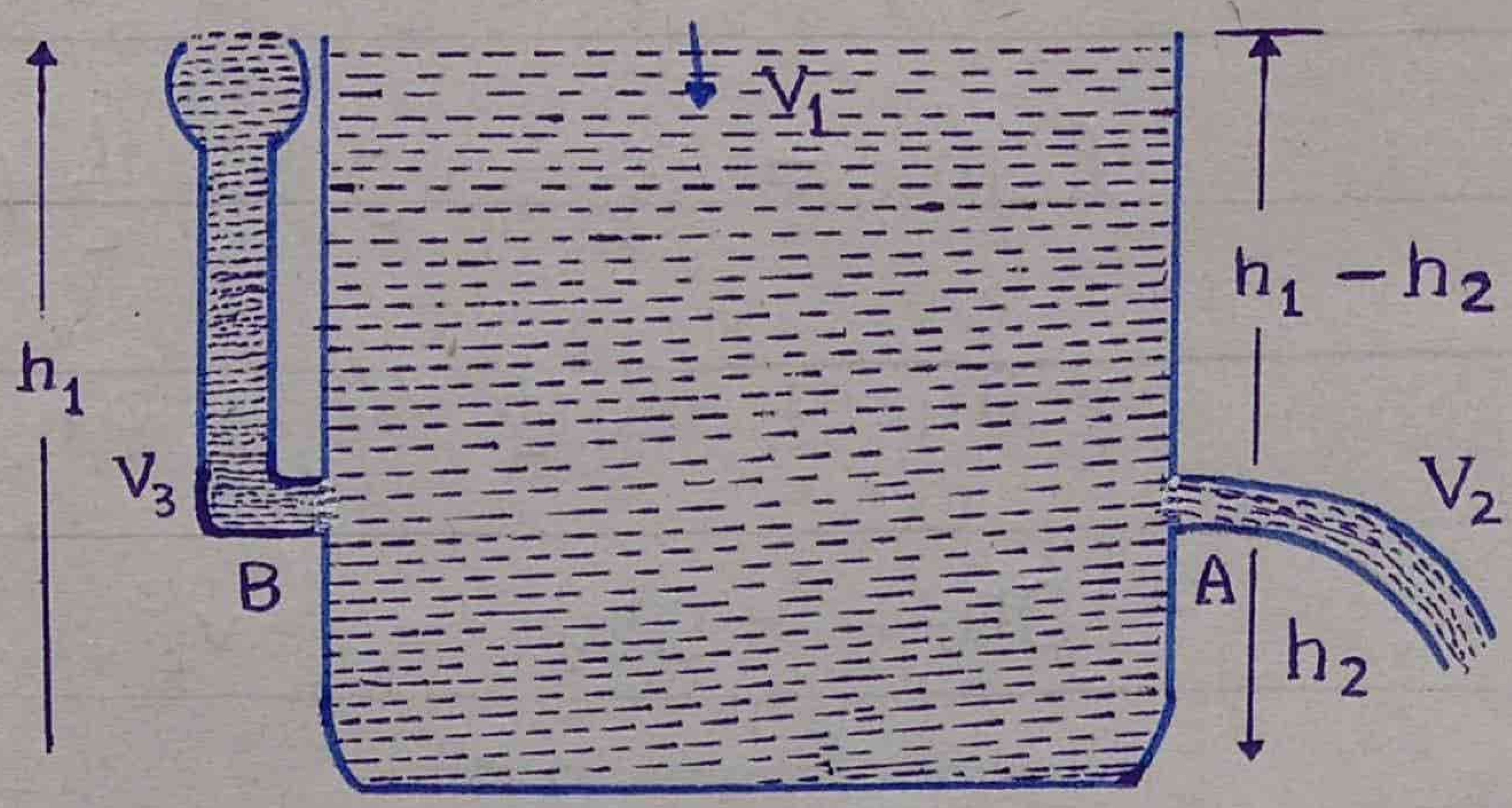
∵ It means that how the volume per unit becomes.

Application of Bernouli's equation:

Torricelli's Theorem:

The speed of efflux at any point is equal to the velocity gained by the fluid of height $(h_1 - h_2)$ under the action of gravity.

Figure:



Formula:

$$v = \sqrt{2g(h_1 - h_2)}$$

Proof:

Consider we take a tank having two holes at point "A" and "B". The Fluid is moving through height $(h_1 - h_2)$.

V_A = Speed of efflux at point A.

V_B = Speed of efflux at point B.

By using equation of Bernoulli's:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2 \quad \text{——— (A)}$$

$v_1 \cong 0$ and the atmospheric pressure is also Same.

$$\therefore \; P_1 = P_2 = P$$

Equation A becomes

$$P + \frac{1}{2}\rho(0)^2 + \rho g h_1 = P + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\rho g h_1 = \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\rho g h_1 = \rho\left(\frac{1}{2}V_2^2 + g h_2\right)$$

$$g h_1 = -g h_2 = \frac{1}{2}V_2^2$$

$$\sqrt{2(g h_1 - g h_2)} = \sqrt{V_2^2}$$

$$V_2 = \sqrt{2g(h_1 - h_2)}$$

Relation of Speed with

Pressure:

When the speed of fluid is high then pressure will be low.

Explanation:

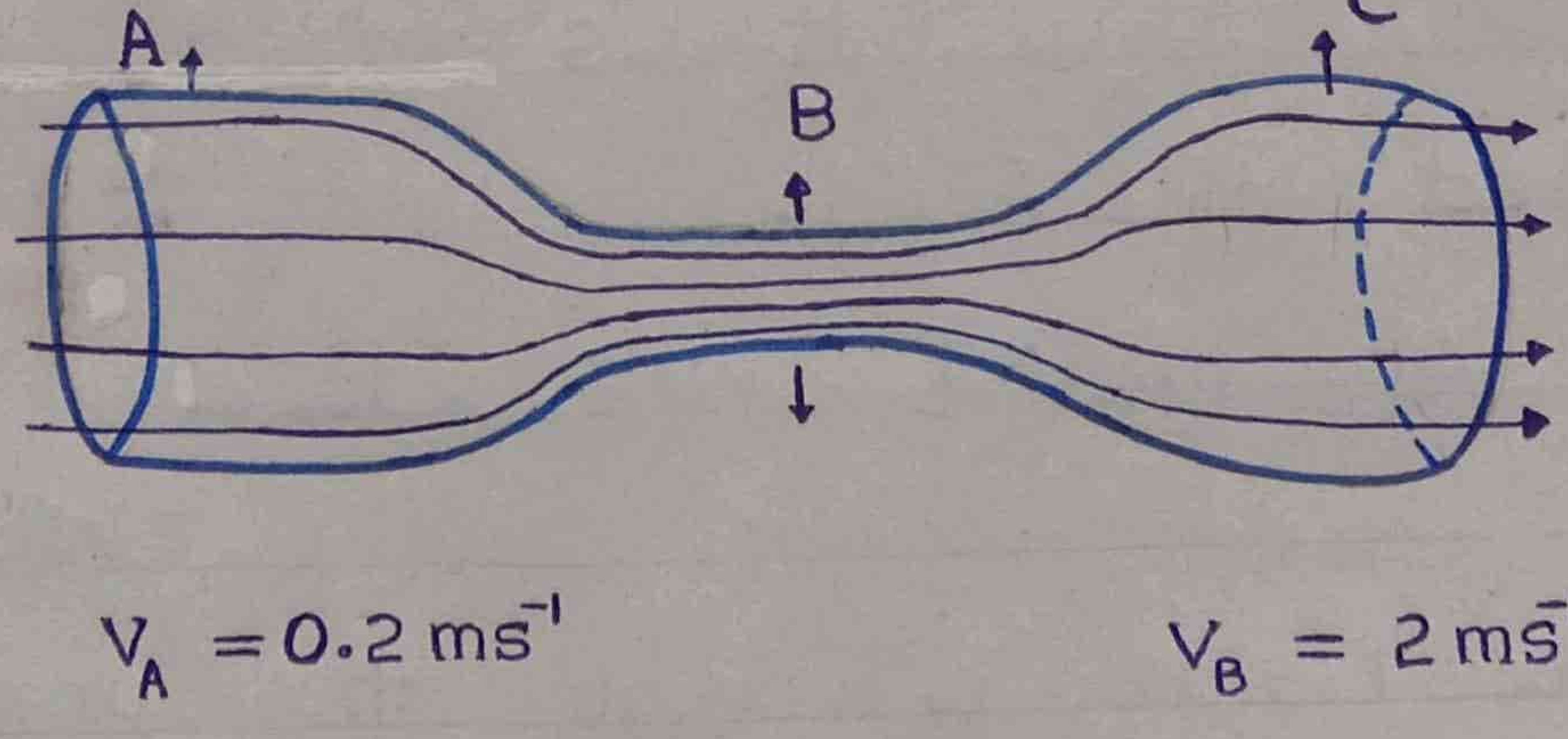
Consider we take a non uniform size horizontal pipe. The pipe of fluid at point "A" is smaller then "B" due to larger area at point "A".

Speed of fluid at point "A"

Speed of fluid at point "B"

Suppose $V_A = 0.2\ ms^{-1}$

$V_B = 2\ ms^{-1}$



By using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_1 = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_2 \quad \text{——— (A)}$$

As height of all point is same

$$h_1 = h_2 = h$$

$$P_A + \frac{1}{2}\rho V_A^2 + \cancel{\rho g h} = P_B + \frac{1}{2}\rho V_B^2 + \cancel{\rho g h}$$

$$P_A - P_B = \frac{1}{2}\rho V_B^2 - \frac{1}{2}\rho V_A^2$$

$$P_A - P_B = \frac{1}{2}\rho (V_B^2 - V_A^2)$$

$$P_A - P_B = \frac{1}{\cancel{2}}(\cancel{1000}^{500})[(2)^2 - (0.2)^2]$$

$$P_A - P_B = 500(4 - 0.04)$$

$$P_A - P_B = 500(3.96)$$

$$P_A - P_B = 1980\ Nm^{-2}$$

$$P_A > P_B$$

Venturi Relation

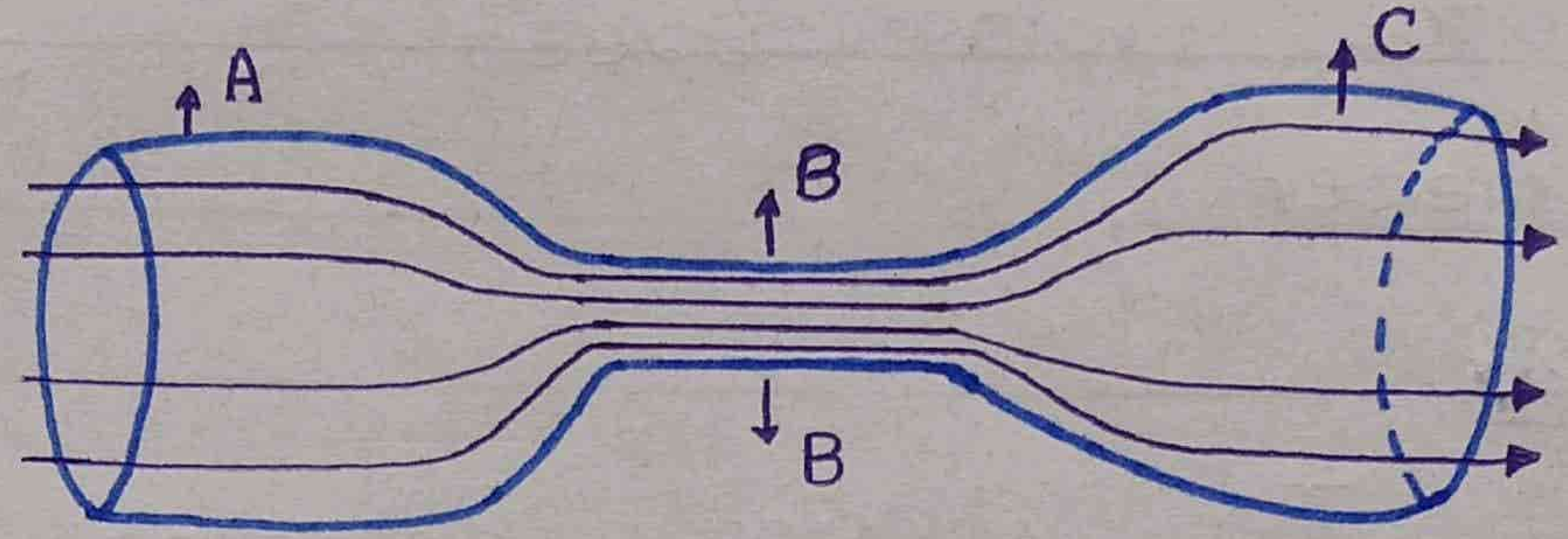
It is used to find the speed of fluid.

Relation of speed with "P":

When the speed of fluid is high the pressure will be low.

Explanation:

Consider we take a non-uniform size horizontal pipe. The speed of fluid at point "A" is smaller than "B" due to larger area at a point "A"



$V_A = 0.2\,ms^{-1}$ $V_B = 2\,ms^{-1}$

V_A = Speed of fluid at point A

V_B = Speed of fluid at point B.

So

$$V_A < V_B$$

$$V_A \cong 0$$

By using Bernouli's equation:

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_1 = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_2 \quad \text{——— (A)}$$

As the height of all the point same.

$$h_1 = h = h_2$$

$$P_A + \frac{1}{2}\rho V_A^2 + \cancel{\rho g h} = P_B + \frac{1}{2}\rho V_B^2 + \cancel{\rho g h}$$

$$P_A - P_B = \frac{1}{2}\rho V_B^2 - \frac{1}{2}\rho V_A^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$P_A - P_B = \frac{1}{2} \rho (v_B^2 - 0)$$

$$\boxed{P_A - P_B = \frac{1}{2} \rho V_B}$$

Venturi meter:

The device which measures the fluid speed is called venturi meter.

Principal:

The working principal of venturi meter is venturi reaction.

Blood Flow:

Blood:

- Blood is incompressible fluid.
- Density of blood is nearly equal to that of water.
- Viscosity of blood increases three to five times that of water due to high concentration of red blood cells (-50%).

Blood Pressure:

"The pressure exerting by the circulating blood on the walls of blood vessels is called blood pressure."

Example:

A person has blood pressure is 120/80 at normal.

Unit:

It is measured in torr or mm of Hg.

Systolic Pressure:

It represents the maximum pressure exerted when the heart contracts.

The value of blood pressure is 120 torr.

Diastolic Pressure:

It represents the maximum pressure in artries when the heart is at rest.

The value of Blood pressure is 75 - 80 torr.

Sphygmomanometer is an instrument which is used to measure blood pressure.

Relation between torr and Pascal:

$$1\,\text{torr} = 133.3\ \text{Pa}$$

$$1\,\text{torr} = 133.3\ \text{N/m}^2$$

Steps to measure Blood Pressure:

- An inflatable bag is wound around arm.
- The bag is inflated to increase the external pressure on arm to compress the blood vessels inside.
- When the external pressure becomes larger than systolic pressure, the vessels falls down.
- The flow of Blood is cutoff.
- Head of Stethoscope is placed over artery.

Laminar Flow:

The regular flow of fluid.

Turbulent:

The irregular flow of fluid.