

Chaper-05:

Circular Motion

Circular Motion:

"The motion of an object in a circular path is called the circular motion."

Example:

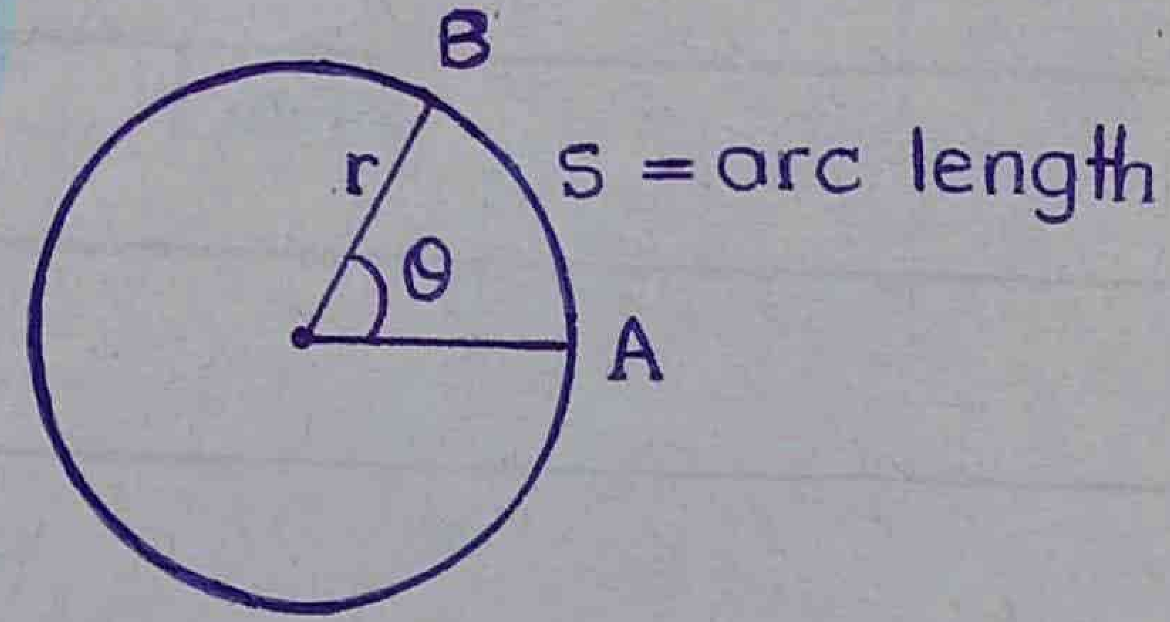
A stone whitled by string motion of ceiling fan.

Angular displacement:

"The angle subtended by the object at the centre of circle at any time."

Formula:

$$\theta = \frac{S}{r} \text{ rad}$$



Unit:

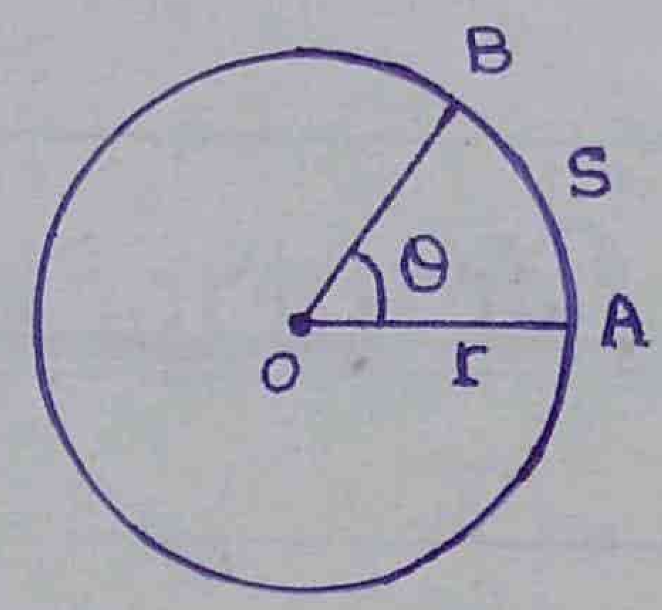
Radian

Sign Convention:

For anticlockwise direction the angular displacement $\Delta\theta$ is positive while for clockwise direction the angular displacement is negative.

Radian:

The angle subtended at the centre of circle when arc length is equal to radius of circle.



$$S = r\theta$$

Relation between Radion and degree.

Prove that 1 rad = 57.3°.

As

$$S = 2\pi r$$

$$\theta = \frac{S}{r} \text{ rad}$$

$$\theta = \frac{2\pi r}{r} \text{ rad}$$

$$\theta = 2\pi \text{ rad}$$

$$1 \text{ revolution} = 2\pi \text{ rad} = 360°$$

$$1 \text{rad} = \frac{360°}{2\pi} \qquad \therefore \pi = 3.1415$$

$$= \frac{360°}{2 \times 3.1415}$$

$$1 \text{ rad} = 57.3°$$ □

Angular Velocity:

"The rate of change of angular displacement is called angular velocity."

Formula: $$\omega_{av} = \frac{\Delta \theta}{\Delta t}$$

Instantaneous angular velocity:

The intantaneous angular velocity is defined as the limiting value of $\Delta\theta/\Delta t$ as the time interval Δt, Following the time t approches to zero.

Formula:

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

Unit:

The SI unit of angular velocity is rad/s.

Direction:

It is a vector quantity.

Angular acceleration:

"The time rate of change of angular velocity is called angular acceleration."

Formula:

$$a_{av} = \frac{\Delta \omega}{\Delta t}$$

Unit:

The SI unit of angular acceleration is rad/s^2.

Direction:

It is a vector quantity.

Average Angular Acceleration:

The ratio of total change of angular velocity to total time.

Formula:

$$a_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

The instantaneous angular acceleration can be defined as limiting value of as the total interval $\frac{\Delta\omega}{\Delta t}$ as the time interval "t" approches to zero is called instantaneous angular acceleration.

Formula:

$$a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Relation between angular and linear acceleration: or $a = r\alpha$:

As we know

$$v = r\omega$$

dividing by time t on both sides

$$\frac{v}{t} = \frac{r\omega}{t}$$

$$\therefore a = \frac{v}{t}$$

$$\therefore \frac{\omega}{t} = \alpha$$

$$\boxed{a = r\alpha}$$

Relation between angular and linear velocity OR $v = r\omega$:

As we know

$$S = r\theta$$

dividing time t on both sides

$$\frac{S}{t} = \frac{r\theta}{t}$$

$\therefore v = \frac{S}{t}$

$\therefore \omega = \frac{\theta}{t}$

$$\boxed{v = r\omega}$$

Equations of angular motion:

Linear	into	Angular
(i) $v_f = v_i + at$		$\omega_f = \omega_i + at$
(ii) $S = v_i t + \frac{1}{2}at^2$		$\theta = \omega_i t + \frac{1}{2}at^2$
(iii) $2aS = v_f^2 - v_i^2$		$2a\theta = \omega_f^2 - \omega_i^2$

Centripetal Force:

"A force which is required to move an object in a circular path is called centripetal force."

or

"The force which compells the object to

move in a circular path is called centripetal force."

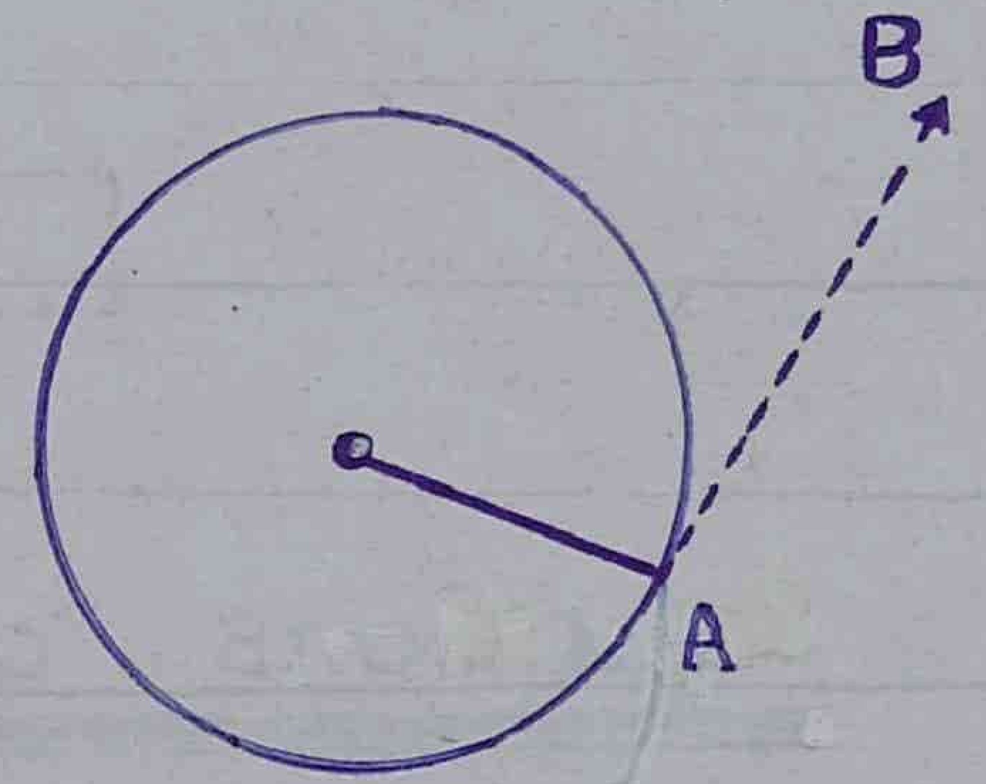
Formula: $F_c = \frac{mv^2}{r}$

Example:

(i) Rotation of earth around the sun.

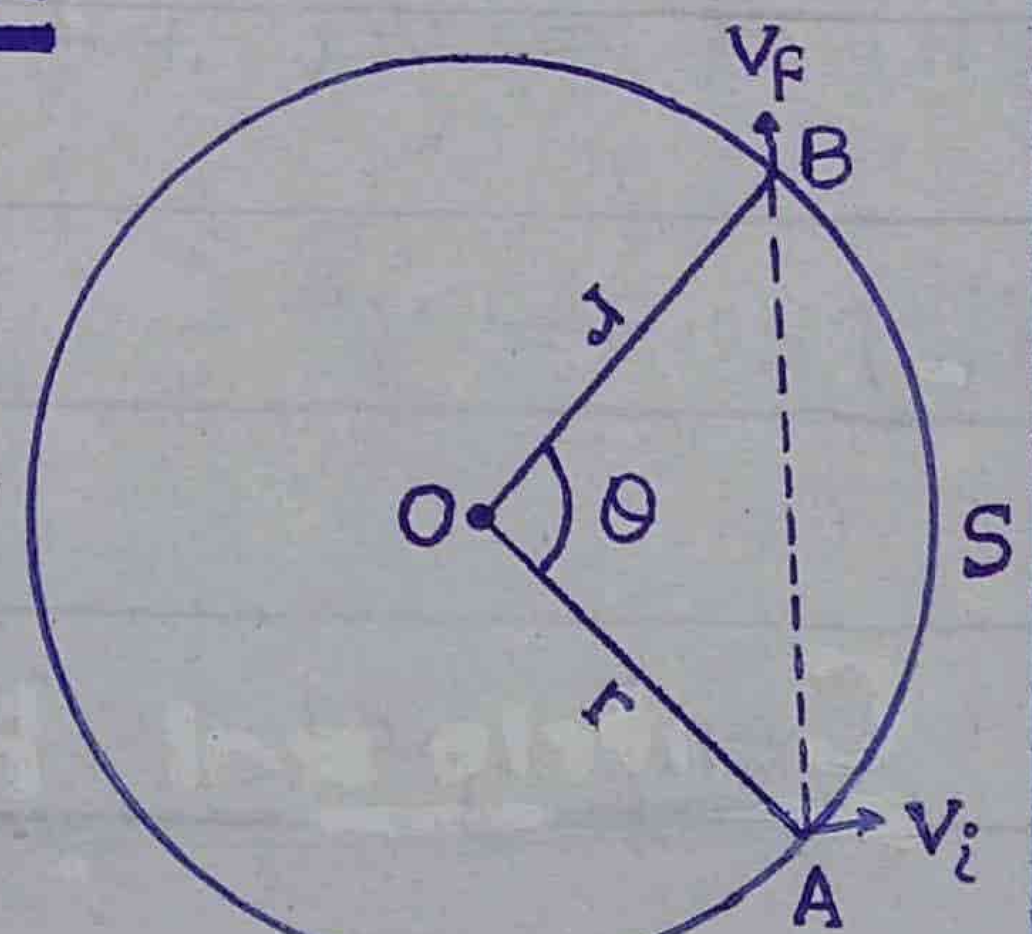
(ii) Rotation of setellites around the earth.

Explanation:



Consider a body attached with spring. If a string is broken then it would not continue to move in a circle. Observation shows that if the spring broken, that it moves and follow straight path AB.

For centripetal acceleration:



As body is point "A" having initial velocity "v_i" then it moves from point "B". So, its velocity becomes "v_f" at point "B".

As velocity is changing from "A" to "B". So the acceleration will be produced.

$$a_c = \frac{\Delta v}{\Delta t} \quad \text{———} \quad (i)$$

As
$$S = V\Delta t$$
$$\Delta t = \frac{S}{V} \quad \text{———— (ii)}$$

By putting (i) in (ii)

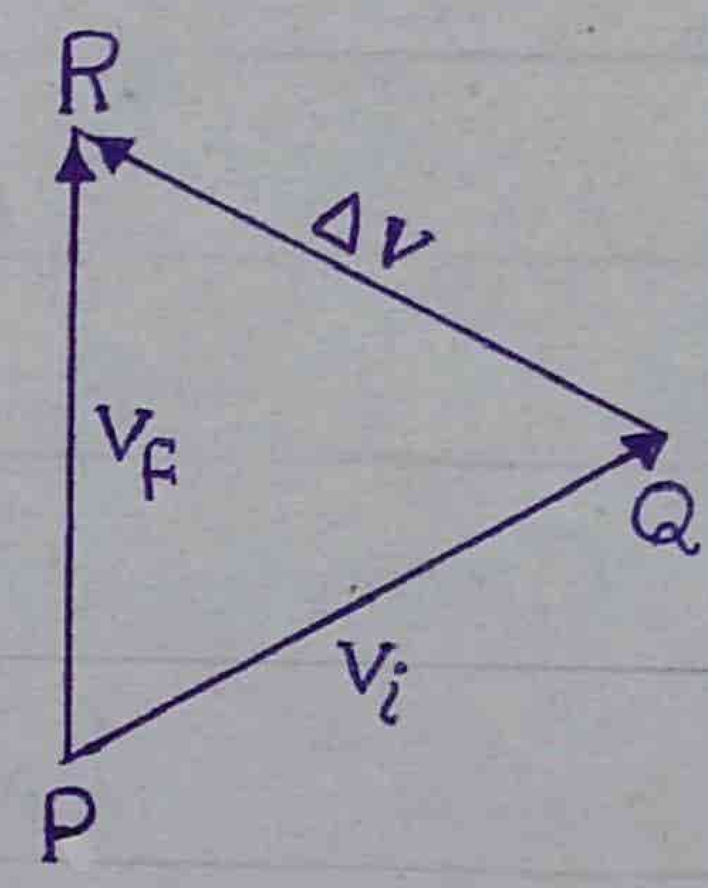
$$a_c = \frac{\Delta V}{\frac{S}{V}}$$

$$a_c = \frac{V\Delta V}{S} \quad \text{———— (A)}$$

Now, we draw the triangle ΔPQR

PR is parallel and equal to V_F.

QR is parallel and equal to ΔV.



As ΔOAB and ΔPQR are isoceles triangle.

$$\frac{\overline{AB}}{\overline{OA}} = \frac{\overline{QR}}{\overline{PQ}}$$

$\therefore V_i = V_F = V$

$\therefore \theta_1 = \theta_2$

$\theta = \tan\theta$

$$\frac{S}{r} = \frac{\Delta V}{V}$$

$$\Delta V = \frac{S}{r} V$$

By putting in (A)

$$a_c = \frac{V}{\cancel{S}} \cdot \frac{\cancel{S}}{r} V$$

$$a_c = \frac{V^2}{r}$$

For centripetal Force:

$$\therefore \quad F = ma$$

$$F_c = ma_c \qquad \because a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

In angular Form:

$$F_c = \frac{m(r\omega)^2}{r} \qquad \because v = r\omega$$

$$F_c = \frac{mr^2\omega^2}{r}$$

$$F_c = mr\omega^2$$

Centripetal Force is newton and dimension are $[MLT^{-2}]$.

Moment of Inertia

The product of mass of an object and square of its distance from the axis of rotation.

Formula:

$$I = mr^2$$

Unit:

Its unit is $Kg\,m^2$.

Dependence:

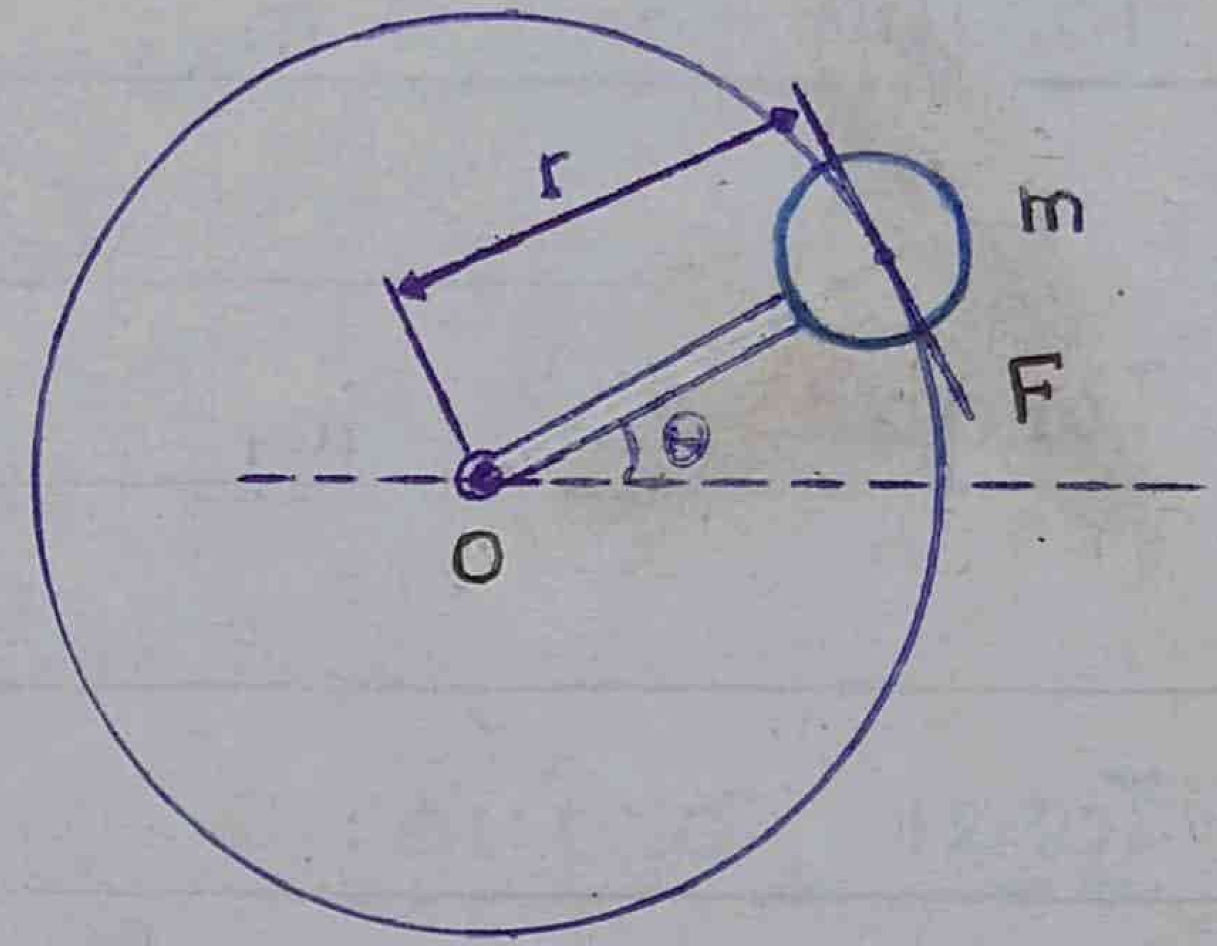
(1) Mass.

(2) Square of distance.

(3) Axis of rotation.

Explanation:

Consider a body having mass m which is attached with a massless rod then a force F applied and it moves in a circular path.



Force acting on an object:

$$F = ma \quad \text{———} \quad (1)$$

As the motion is angular. So, "a" is converted in "α".

$\because a = r\alpha$

$$F = mr\alpha \quad \text{———} \quad (2)$$

Multiply equation (2) by "r".

$\because rF = \tau$

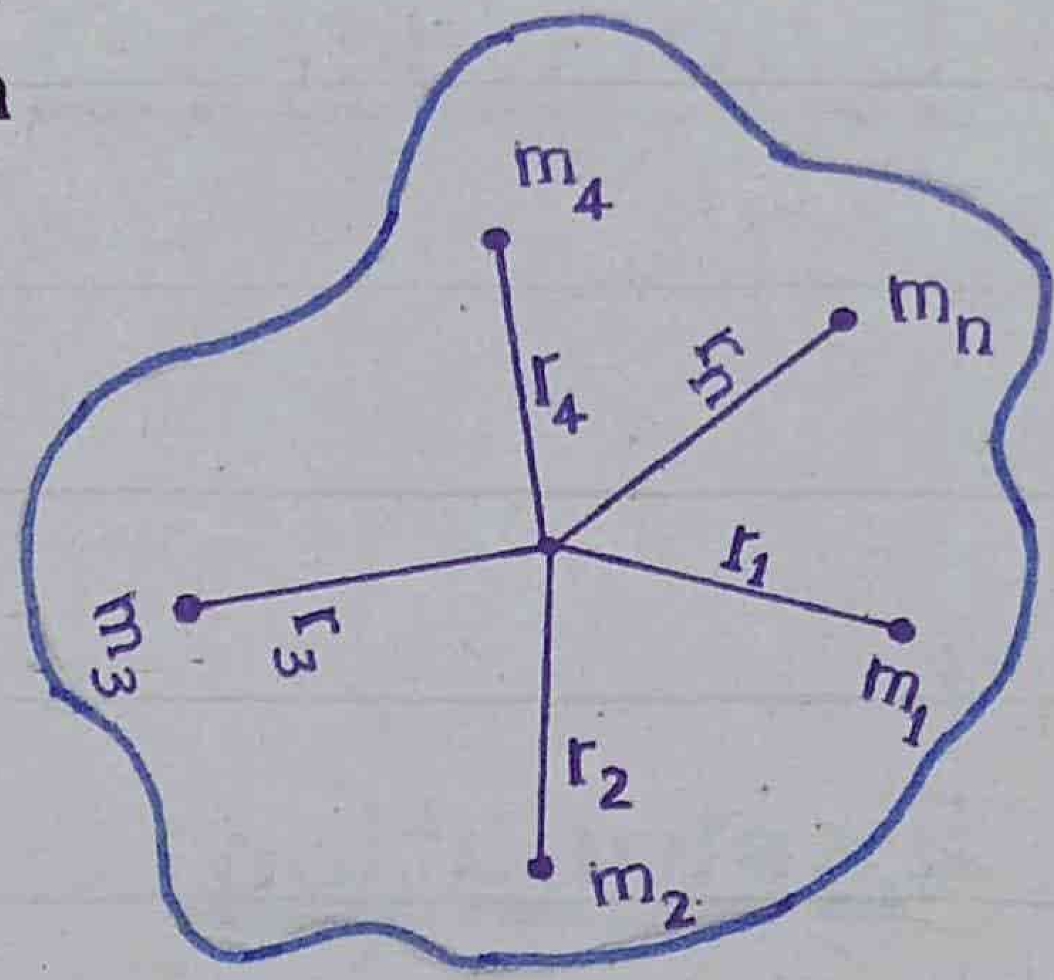
$\because mr^2 = I$

$$rF = mr^2\alpha$$

$$\boxed{\tau = I\alpha}$$

Torque on Rigid Body:

Consider a rigid body consist of large number of masses $m_1, m_2, m_3, m_4, \cdots m_n$ having distance $r_1, r_2, r_3, r_4, \cdots, r_n$ From the axis of rotation.



Torque on m_1:

$$\tau_1 = m_1 r_1^2 a_1$$

Torque on m_2:

$$\tau_2 = m_2 r_2^2 a_2$$

Torque on m_3:

$$\tau_3 = m_3 r_3^2 a_3$$

Total torque:

$$\tau_{Total} = \tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n$$

$$\tau_{Total} = m_1 r_1^2 a_1 + m_2 r_2^2 a_2 + m_3 r_3^2 a_3 + \cdots + m_n r_n^2 a_n$$

An axis of rotation of with same angular acceleration a:

$$\because \quad a_1 = a_2 = a_3 = a_n = a$$

$$\tau = m_1 r_1^2 a + m_2 r_2^2 a + m_3 r_3^2 a + \cdots + m_n r_n^2 a$$

$$\tau = \left(\sum_{i=1}^{n} m_i r_i^2 \right) a \qquad \because \sum_{i=1}^{n} m_i r_i^2 = I$$

$$\boxed{\tau = Ia}$$

Angular momentum:

The cross product of position vector $\vec{r}$ and linear momentum $\vec{p}$ about the axis of rotation.

Formula:

$$\vec{L} = \vec{r} \times \vec{p}$$

Unit:

The SI unit of angular momentum is kgm^2s^{-1} or Js.

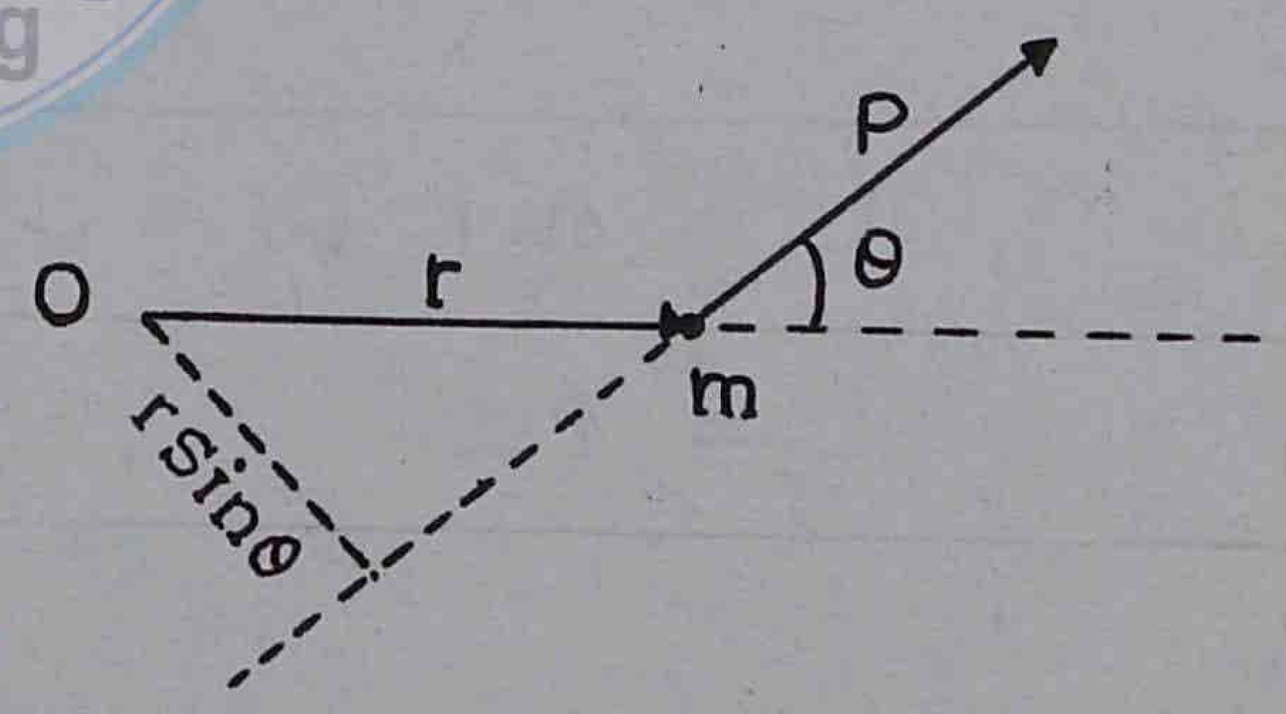
Dimension:

The dimension of angular momentum is $[ML^2T^{-1}]$.

Explanation:

Consider a body having mass mass m, $\vec{r}$ is the position vector from axis of rotation and is the linear momentum $\vec{p}$ then angular momentum will be

$$\vec{L} = \vec{r} \times \vec{p}$$



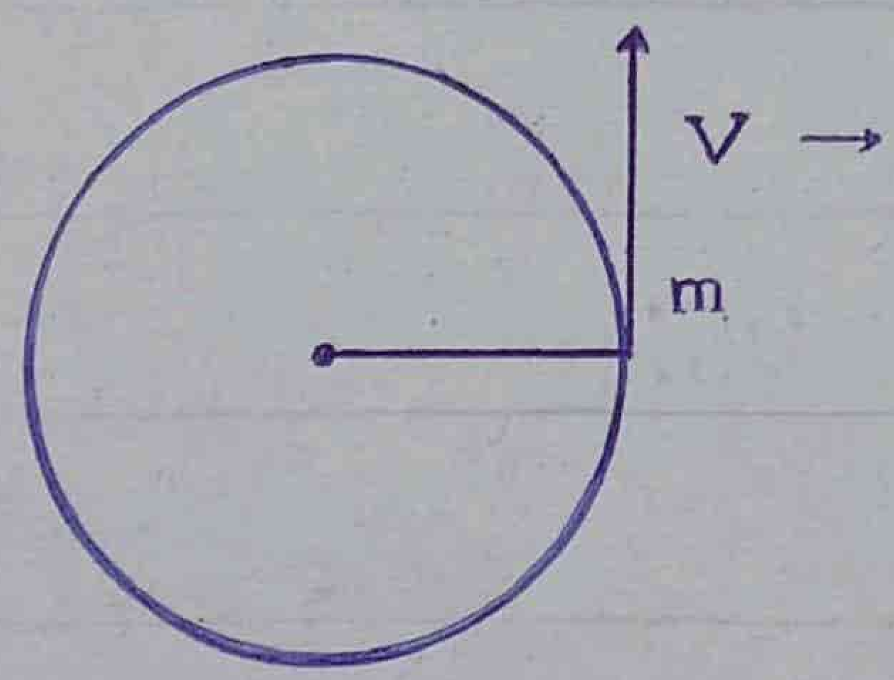
For magnitude:

$$L = rp\sin\theta$$

Direction:

We can Find the direction of "L" by Right hand Rule.

IF a body is moving in a circle:



If a body is moving in a circle then velocity is tangent to the circle which is perpendicular to "r".

$$\theta = 90^\circ$$

$$L = rp\sin\theta \qquad \because \sin 90^\circ = 1$$

$$L = rp\sin 90^\circ$$

$$L = rp(1) \qquad \because P = mv$$

$$L = rp \qquad \because v = r\omega$$

$$L = rmv \qquad \because mr^2 = I$$

$$L = mvr$$

$$L = m(r\omega)r$$

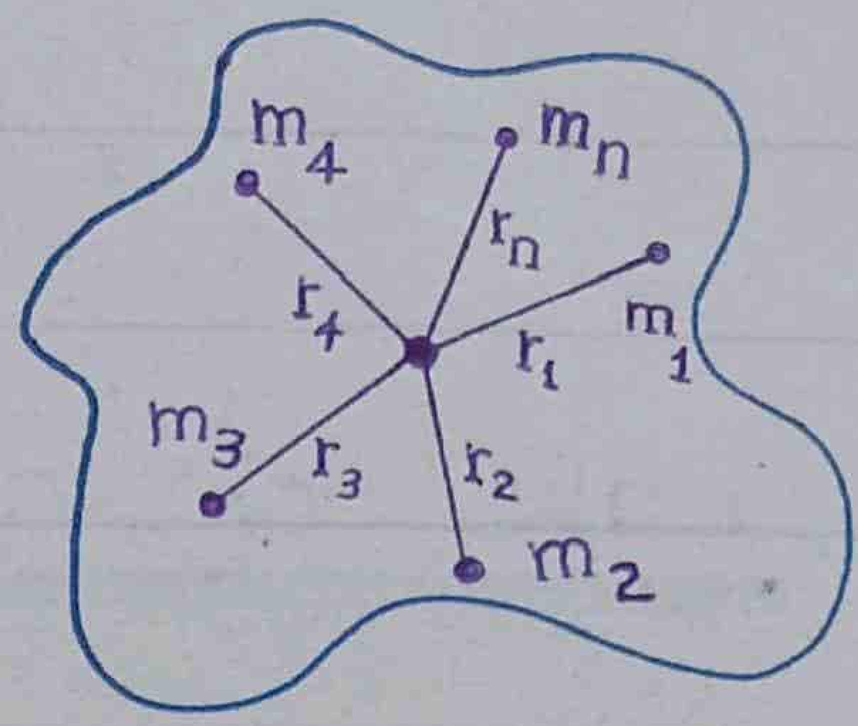
$$L = mr^2\omega$$

$$L = I\omega$$

Angular momentum of Rigid Body:

"A body in which the

distance between the molecules remains constant."



Explanation:

Consider a rigid body consist of large number of masses $m_1, m_2, m_3, \dots, m_n$ having distances $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation.

Angular momentum on m_1:

$$L_1 = m_1 r_1^2 \omega_1$$

Angular momentum on m_2:

$$L_2 = m_2 r_2^2 \omega_2$$

Angular momentum on m_n:

$$L_n = m_n r_n^2 \omega_n$$

Tolal angular momentum:

$$L = L_1 + L_2 + L_3 + \dots + L_n$$

$$L = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + m_3 r_3^2 \omega_3 + \dots + m_n r_n^2 \omega_n$$

As axis of rotation is same on mass.

$$\omega_1 = \omega_2 = \omega_3 = \omega_n = \omega$$

Now

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \dots + m_n r_n^2 \omega$$

$$L = \left(\sum_{i=1}^{n} m_i r_i \right) \omega$$

$$\boxed{L = I\omega} \qquad \because \sum_{i=0}^{n} = m_i r_i = I$$

Law of conservation of angular momentum:

"If no torque act on an object then the total angular momentum remains constant."

$$\vec{L_1} + \vec{L_2} + \cdots + \vec{L_n}$$

$$I_1\omega_1 + I_2\omega_2 + \cdots + I_n\omega_n$$

$$\Sigma I\omega = \text{Constant}$$

Rotantional Kinetic Energy:

"The energy possessed by a body due to its motion in a circular path is called rotantional kinetic energy."

$$K.E_{rot} = \frac{1}{2} mv^2 \qquad \therefore v = r\omega$$

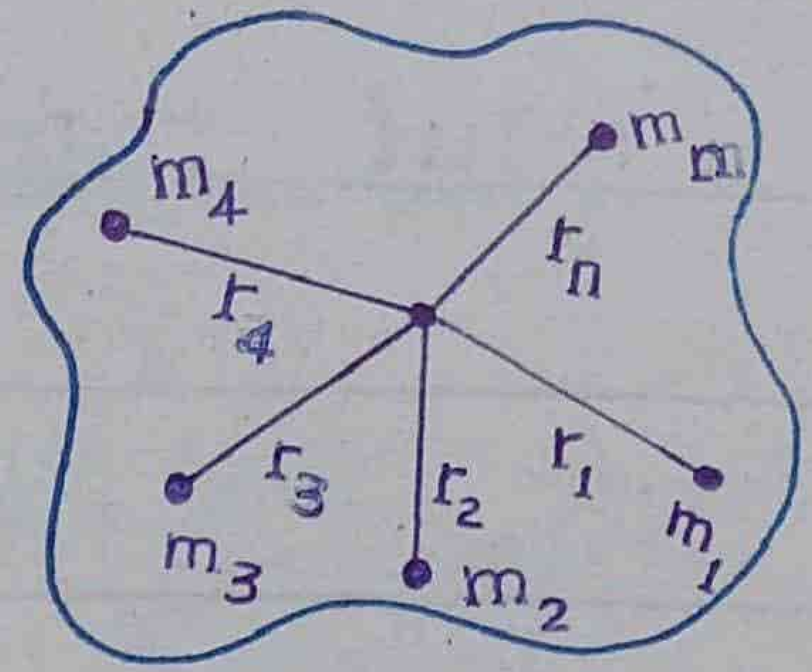
$$= \frac{1}{2} m(r\omega)^2$$

$$= \frac{1}{2} mr^2\omega^2$$

$$\boxed{K.E_{rot} = \frac{1}{2} I\omega^2}$$

In moment of inertia.

Rotational Kinetic energy of Rigid Body:



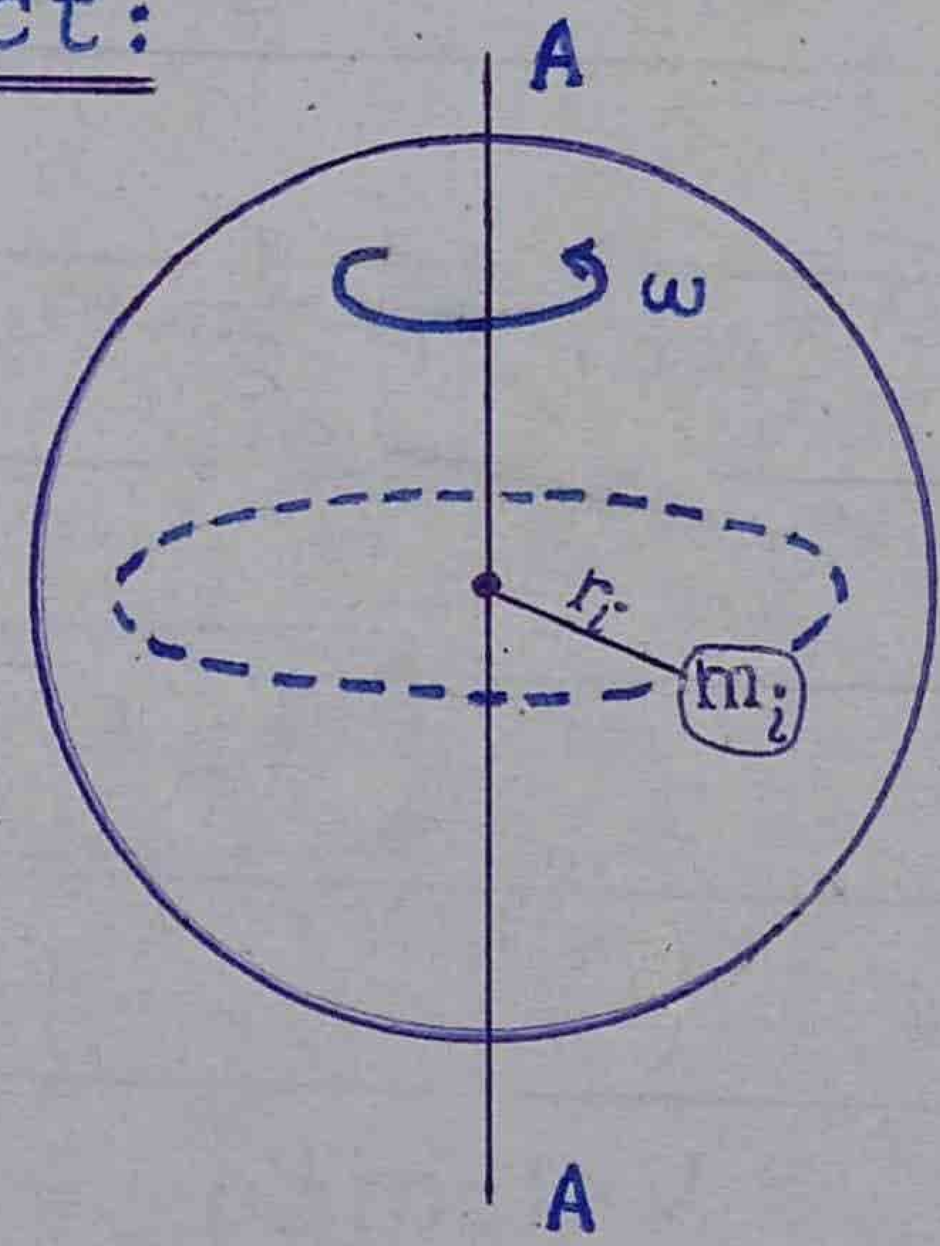
Rigid body:

"A body in which distance of molecules remains constant."

Explanation:

Consider a body having masses $m_1, m_2, m_3, \cdots, m_n$ having distances $r_1, r_2, r_3, \cdots, r_n$ from the axis of rotation.

Rotational K.E of single object:



Consider a body having mass m which is moving in circular path by rotantional K.E. ω is the angular velocity of the object.

Rotational K.E of m_1:

$$K.E_{rot} = \frac{1}{2} m_1 r_1^2 \omega_1^2$$

Rotational K.E of m_2:

$$K.E = \frac{1}{2} m_2 r_2^2 \omega_2^2$$

Rotational K.E of m_3:

$$K.E = \frac{1}{2} m_3 r_3^2 \omega_3^2$$

Rotational K.E of m_n:

$$K.E = \frac{1}{2} m_n r_n^2 \omega_n^2$$

Total rotational K.E of all masses:

$$K.E_{rot} = \frac{1}{2} m_1 r_1^2 \omega_1^2 + \frac{1}{2} m_2 r_2^2 \omega_2^2 + \frac{1}{2} m_3 r_3^2 \omega_3^2 + \cdots + \frac{1}{2} m_n r_n^2 \omega_n^2$$

As axis of rotation same.

$$\omega_1^2 = \omega_2^2 = \omega_3^2 = \omega_n^2 = \omega^2$$

$$K.E_{rot} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \cdots + \frac{1}{2} m_n r_n^2 \omega^2$$

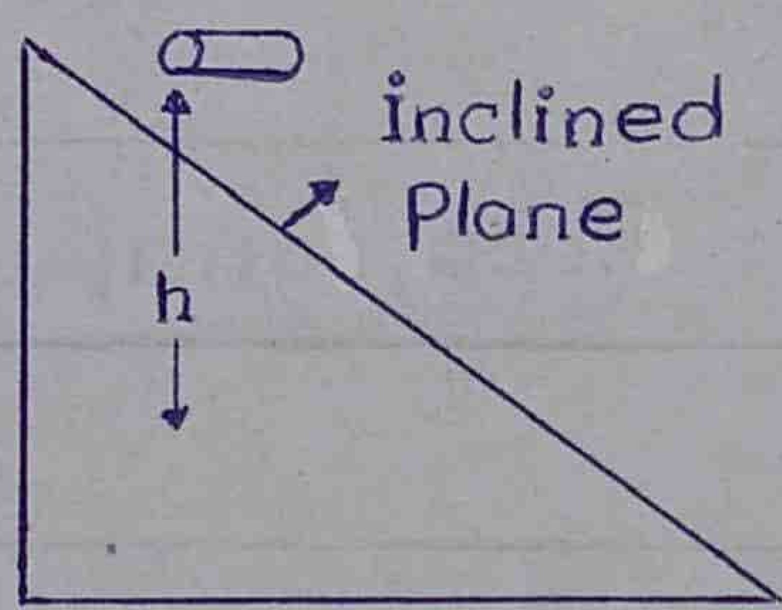
$$K.E_{rot} = \frac{1}{2} \left(\sum_{i=1}^{n} m_i r_i \right) \omega^2$$

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

$$\boxed{K.E_{rot} = \frac{1}{2} I \omega^2}$$

Velocity of Disc and Hoop:

Consider a disk and hoop at the top of inclined plane then they moves and change potential energy will be



$$P.E = K.E_T + K.E_{rot} \quad \text{———} \quad (i)$$

For Disc:

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

$\because I_o = \frac{1}{2} mr^2$

$\therefore v = r\omega$

$\because \omega = \frac{v}{r}$

$\because K.E_T = \frac{1}{2} mv^2$

$$K.E_{rot} = \frac{1}{2}\left(\frac{1}{2} m\cancel{r^2}\right)\frac{v^2}{\cancel{r^2}}$$

$$K.E_{rot} = \frac{1}{4} mv^2$$

As

$$P.E = K.E_T + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\cancel{m}gh = \left(\frac{1}{2} + \frac{1}{4}\right)\cancel{m}v^2$$

$$gh = \left(\frac{2+1}{4}\right)v^2$$

$$gh = \left(\frac{3}{4}\right)v^2$$

Taking square root on both sides

$$\sqrt{gh} = \sqrt{\frac{3}{4}v^2}$$

$$\sqrt{\frac{4}{3}gh} = \sqrt{v^2} \quad \Rightarrow \quad \sqrt{\frac{4}{3}gh} = v_D$$

or

$$\boxed{v_D = \sqrt{\frac{4}{3}gh}}$$

$\therefore \sqrt{\frac{4}{3}} = 1.15$

$$\boxed{v_D = 1.15\sqrt{gh}}$$

For Hoop:

Moment of inertia For Hoop

$\because I_H = mr^2$

$\omega = \frac{v}{r}$

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}(m\cancel{r^2}) \cdot \frac{v^2}{\cancel{r^2}}$$

$$K.E_{rot} = \frac{1}{2} mv^2$$

As

$$P.E = K.E_T + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\cancel{m}gh = \left(\frac{1}{2} + \frac{1}{2}\right)\cancel{m}v^2$$

$$gh = \left(\frac{\cancel{2}}{\cancel{2}}\right)v^2$$

$$gh = v^2$$

$$\sqrt{gh} = v$$

or

$$v_H = \sqrt{gh}$$

Result:

As $v_D = 1.15\sqrt{gh}$, so velocity of disk is 1.15 times greater than the velocity of Hoop ($v_H = \sqrt{gh}$), so disk reached first at bottom.

Satellite:

"Anything that revolves around the Earth is called satellite."

Artificial Satellites:

"Manmade satellite that revolves around the earth is called artificial satellite."

Example:

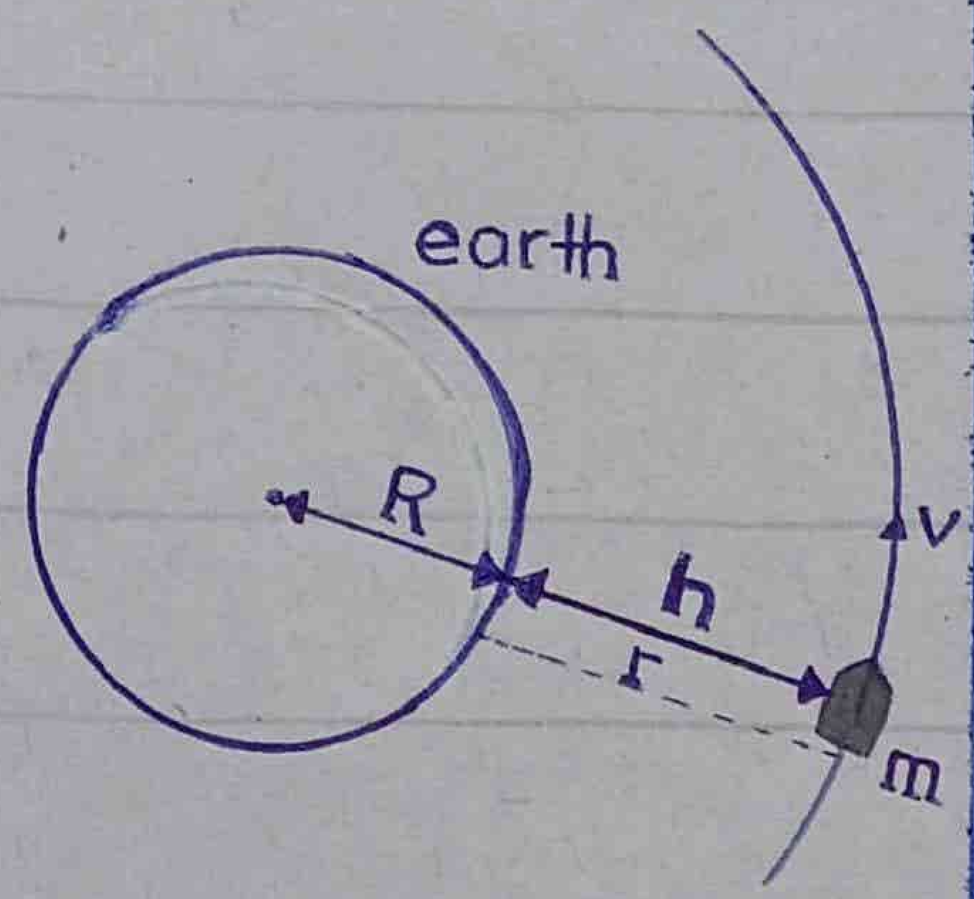
The most common examples are military satellite, communication satellite.

Critical Orbital Velocity:

"The minimum velocity which is required to move the satellite in a paticular orbit around Earth".

Explanation:

Consider a satellite having mass "m" that revolves around the earth in circular path.



R = Radius of earth.

h = Height of satellite from surface of Earth.

r = Radius of satellite from centre of

Earth.

As the body moves in a circular path then the body move due to centripetal force.

$$F = \frac{mv^2}{R} \quad \text{———} \quad (i)$$

The force is applied by its weight.

$$F = w = mg$$

Comparing (i) and (ii)

$$\cancel{m}g = \frac{\cancel{m}v^2}{R}$$

$$g = \frac{v^2}{R}$$

$$v^2 = gR$$

Taking square root on both sides

$$\sqrt{v^2} = \sqrt{gR}$$

$\because R = 6.4 \times 10^6$

$\because g = 9.8\ ms^{-1}$

$$v = \sqrt{gR}$$

$$v = \sqrt{(9.8)(6.4 \times 10^6)}$$

$$v = 7900\ ms^{-1}$$

$$v = 7.9 \times 10^3\ ms^{-1}$$

$$\boxed{v = 7.9\ kms^{-1}}$$

Time period:

The time required to complete one rotation around the Earth.

$$S = vt$$

$$T = \frac{S}{V} = \frac{\text{circumference}}{\text{velocity}} = \frac{2\pi R}{7900}$$

$$T = \frac{2(3.14)(6.4 \times 10^6)}{7900}$$

$$T = 5090\,s$$

$$T = \frac{5090}{60}$$

$$\boxed{T = 84\ \text{min}}$$

Real Weight:

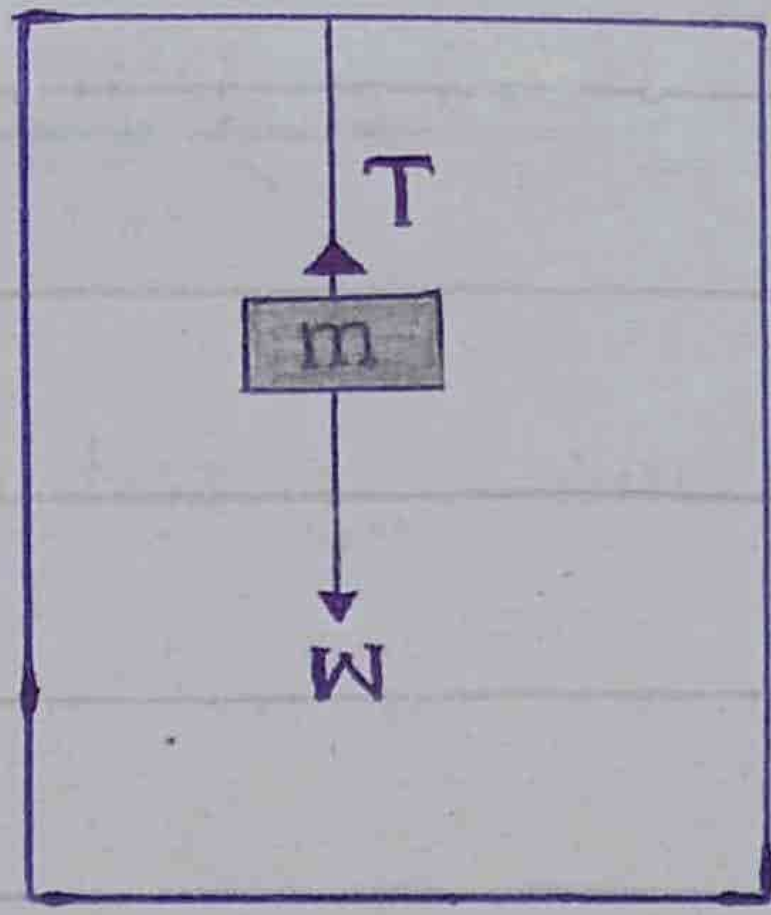
"It is the gravitational pull of the Earth on the object."

Apparent Weight:

"The measureable weight by spring balance when a body accelerate upward and downward is called Apparent weight."

Appeorent weight is equall and opposite to the force required to stop it from falling in the frame of referance.

Explanation:



Consider a body having mass "m" which is attached to a string and it is lifted in a lift.

Case-I: When lift is at rest:

$$F_{not} = ma$$

As tension high.

$$T - w = ma$$

$$T = w + ma \qquad \because a = 0$$

$$T = w + m(0)$$

$$T = w$$

Result

The weight is equal to the appearent weight.

Case-II: When lift is moving upward:

$$F_{net} = ma$$

As tension high.

$$T - w = ma$$

$$T = w + ma$$

Result: The apparent weight is increases by the amount of ma.

Case-III: When lift is moving downward:

$$F_{net} = ma$$

$\because$ As tension is low, weight is high.

$$W - T = ma$$

$$W - ma = T$$

$$T = W - ma$$

Result:

The apparent weight is decreases by amount of ma.

Case-IV: When lift is Freely Falling:

$$F_{net} = ma$$

$$W - T = ma$$

As body is freely falling into g

$$mg - T = mg$$

$$mg - mg = T \qquad \therefore W = mg$$

$$0 = T$$

$$T = 0$$

Result:

The apparent weight becomes zero.

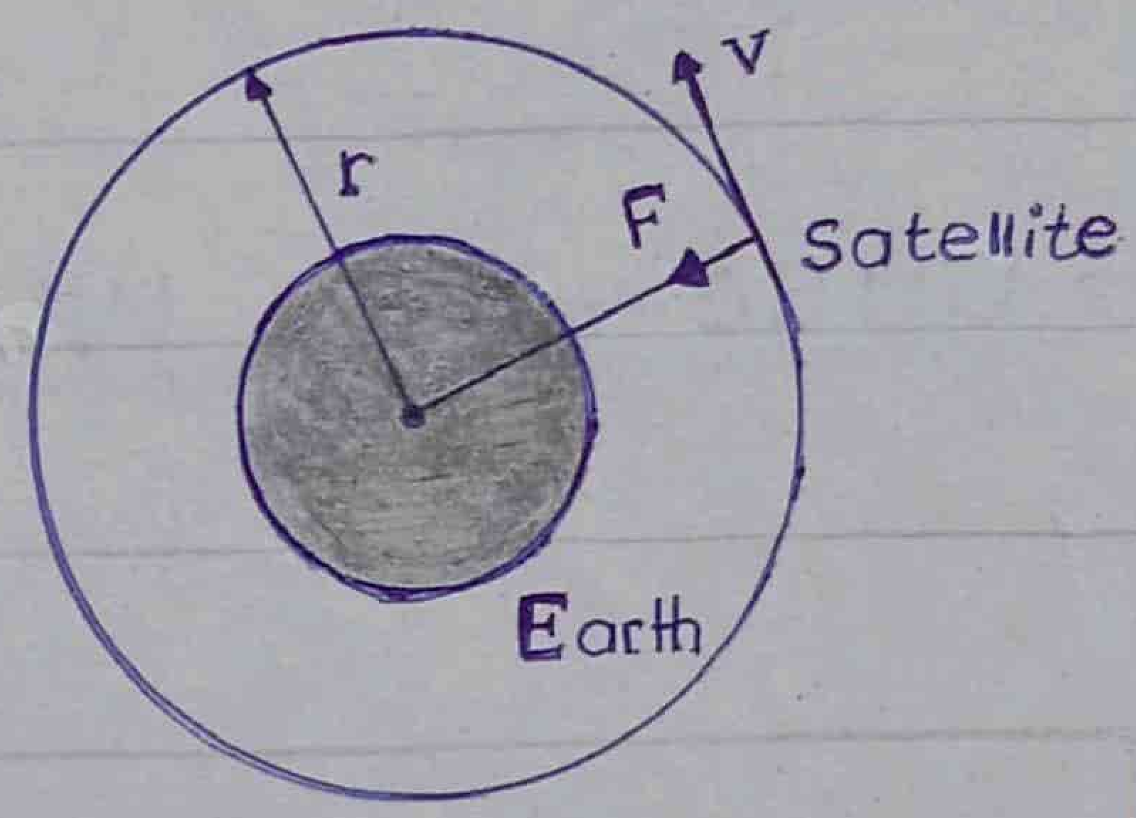
Weightlessness:

"When the apparent weight becomes zero this state is called weightlessness."

A free falling body moves under the action of gravitational force. So that the object is said to be in weightlessness.

Orbital Velocity:

The velocity of satellite by which it rotate around the Earth in circular path.



Explanation:

Consider a satellite moving around the Earth having mass "m" and its radius of Earth R and the radius of orbital path "r".

The satellite rotate around the Earth due to centripetal Force.

$$F = \frac{mv^2}{r} \qquad \text{—— (i)}$$

The Force is provided by gravitational force of Earth.

$$F = \frac{GmM}{r^2} \quad \text{———} \quad (ii)$$

Compare (i) and (ii), we have

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{G\cancel{m}M}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$

Taking square root on both sides

$$\sqrt{v^{\cancel{2}}} = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{GM}{r}}$$

Result:

The formula shows that orbital velocity does not depand upon the mass of satellite.

Artificial Gravity:

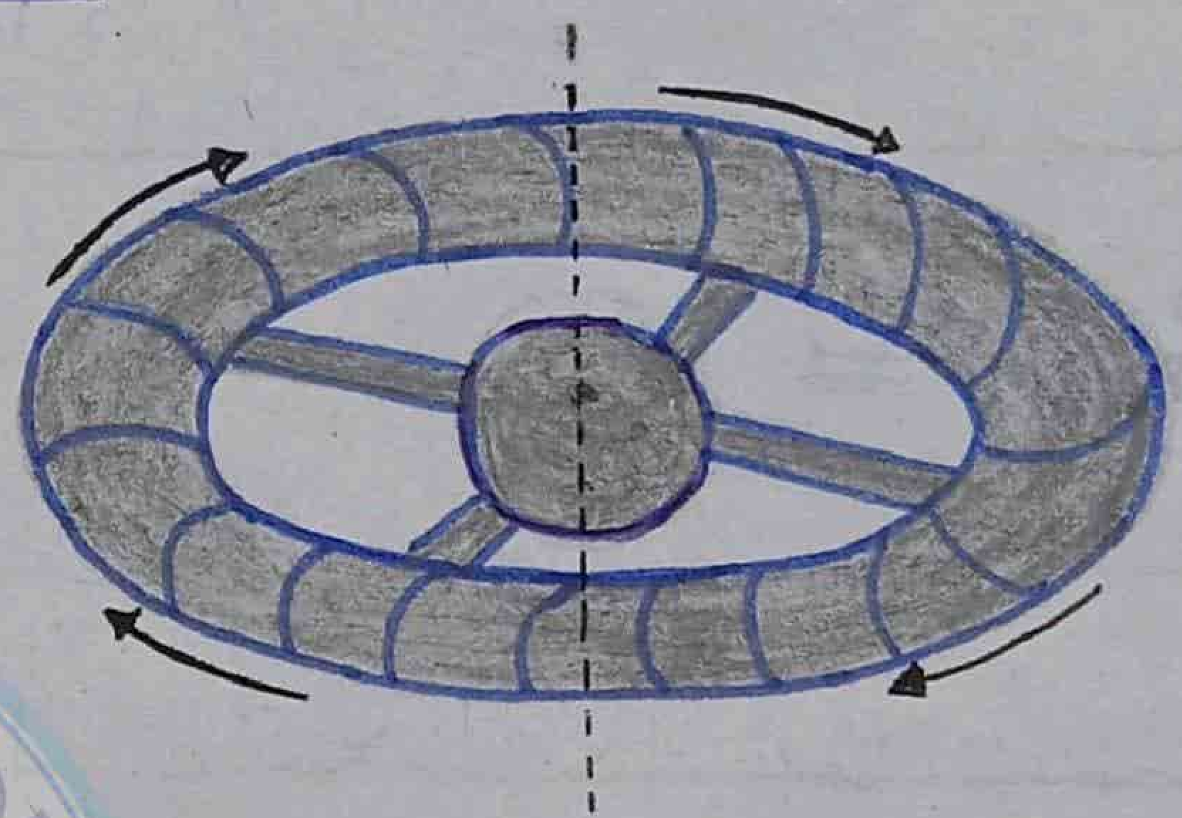
"The force which is produced by the object when the object moves its own an axis then the force is called artificial gravity."

Explanation:

In a gravity free space

satellite there will be no force that will push any body due to any side of the spacecraft. If this satellite is to stay in orbit ever an extend and period of time. This weightlessness may affect the performance of the astronauts present in the spacecraft. To overcome the difficulty, an artificial gravity is created in spacecraft.

Expression For Frequency:



Consider a spaceship which is rotating around its own an axis due to angular velocity ω then the centripetal acceleration will be

$$a_c = \frac{v^2}{r}$$

As

$$v = r\omega$$

$$a_c = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r}$$

$$a_c = r\omega^2$$

$$a_c = r\left(\frac{2\pi}{t}\right)^2$$

$\because$ As $\omega = \frac{\theta}{T}$

$\because$ $\theta = 2\pi$

$$a_c = r \cdot \frac{4\pi^2}{t^2}$$

$\because$ $f^2 = \frac{1}{t^2}$

$$a_c = 4\pi^2 r \cdot \frac{1}{t^2}$$

$$a_c = 4\pi^2 r \cdot f^2$$

$$\frac{a_c}{4\pi^2 r} = f^2$$

Taking squareroot on both sides

$$\sqrt{\frac{a_c}{4\pi^2 r}} = \sqrt{f^2} \Rightarrow \sqrt{\frac{a_c}{4\pi^2 r}} = f$$

or

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}}$$

$a_c = g$

Geostationary Orbit:

The orbit which has time period of rotation exactly equal to the time period of rotation of Earth.

Geostationary Satellite:

The Satellite which has time period of rotation exactly equal to the time period of rotation of Earth.

OR

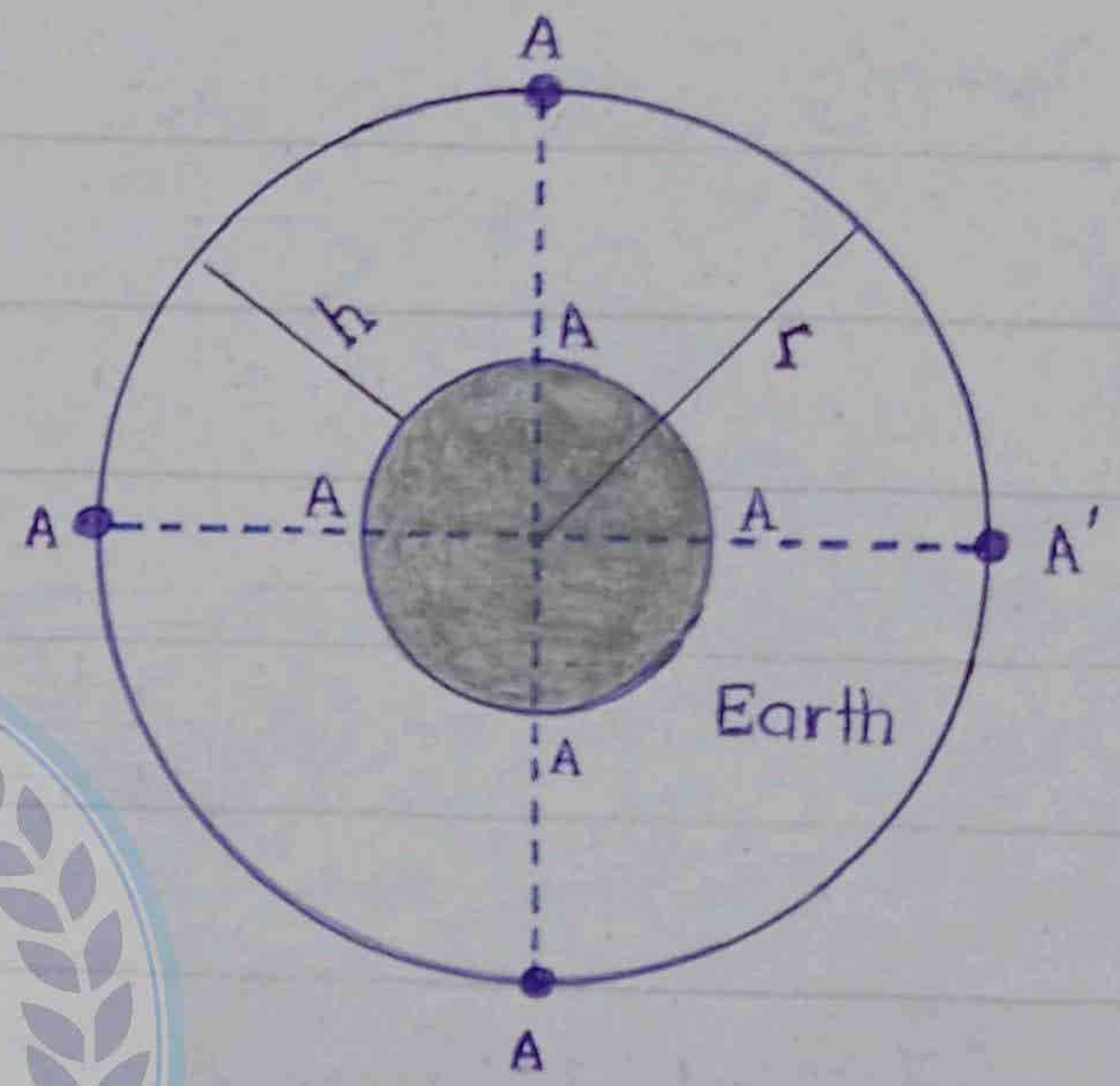
"The satellite which has time rotation hours is called Geostationary satellite."

Application of geostationary Satellite:

Communication Satellite which are used for weather report or use of military perpose.

Explanation:

Consider a satellite which revolves around the Earth in 24 hours. So at some time Earth complete its one revolution around its own axis in 24 hours. So, the satellite is geo-stationary satellite and orbit in which geo-stationary satellite revolves is called geo-stationary orbit.



Radius of Geostationary Satellite:

As a satellite moves around the Earth due to orbital velocity.

$$v = \sqrt{\frac{GM}{r}} \quad \text{———— (i)}$$

As, the velocity is same at that of Earth.

$$S = vt$$

$$v = \frac{S}{t} \qquad \because S = \text{circumference} = 2\pi r$$

$$v = \frac{2\pi r}{t} \quad \text{———— (ii)}$$

By comparing (i) and (ii)

$$\frac{2\pi r}{t} = \sqrt{\frac{GM}{r}}$$

By squaring on both sides

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMt^2}{4\pi^2}$$

Taking Cube on both sides

$$\left(r^3\right)^{\frac{1}{3}} = \left(\frac{GMt^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$r = \left(\frac{GMt^2}{4\pi^2}\right)^{\frac{1}{3}} \quad \text{———— (iii)}$$

As

$$G = 6.67 \times 10^{-11}$$

$$M = 6 \times 10^{24}$$

$$t = 24 \times 60 \times 60 = 86400\,s$$

$$\pi = 3.1415$$

Putting values in equation (iii), we have

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (86400)^2}{4(3.1415)^2}\right]^{\frac{1}{3}}$$

$$r = 42298355.53\,m$$

$$r = 4.23 \times 10^{7}\,m$$

$\because 10^3 = 1$ Kilo

$$r = 4.23 \times 10^{4} \times 10^{3}\,m$$

$$\boxed{r = 4.23 \times 10^{4}\,Km}$$

ANSWERS OF THE QUESTIONS

Question 5.1:

Explain the difference between tangential velocity and the angular velocity, if one of these is given for a wheel of known radius, who will you find the other?

Answer:

Tangential velocity:

When a body is moving in a curved or in a circular path, its linear velocity is called agents in velocity v. It is along the tangent to the curve of circle.

Angular velocity:

Rate of change of angular displacement of a particle moving along a curved path is called angle angular velocity. It is denoted by ω.

Relation between them:

$$V = r\omega$$

Hence if one of these is given for a wheel of known radius, the other can be found from above relation.

Question 5.2:

Explain what is the meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

Answer:

Centripetal Force:

The force which is required to move an object in a circular path is called centripetal force.

Mathematically:

$$F_c = \frac{mv^2}{r}$$

Reason for providing centripetal force:

If we want to move an object in a circular path then centripetal force is required. If centripetal force is not provided than object does not move in circular path it moves in a straight path.

Question 5.3:

What is meant by moment of inertia? Explain its significance.

Answer:

Moment of inertia:

The product of the mass of the object and square of its distance from the axis of rotation is called moment of inertia. It is represented by I.

$$I = mr^2$$

Where m is the mass of an object and r is the distance from the axis of rotation.

Significance:

It plays the same rule in angular motion as mass plays its role in a linear motion. As mass is a scalar quantity moment of inertia is also a scalar quantity. Inertia depends upon mass but moment of inertia depends upon both mass and square of distance from the axis of rotation.

Question 5.4:

What is meant by angular momentum? Explain the law of conservation of angular momentum.

Answer:

Angular Momentum:

Cross product of position vector $\vec{r}$ and linear momentum $\vec{p}$ is called angular momentum or moment of momentum.

$$\vec{L} = \vec{r} \times \vec{p}$$

Or $$L = I\omega$$

Units:

(i) Kg m^2 s^{-1} (ii) Js

Law of Conservation of angular momentum:

If no external torque acts on the system then the total angular momentum of a system of bodies remains constant.

$$I_1\omega_1 = I_2\omega_2$$

$$\vec{L} = \text{constant}$$

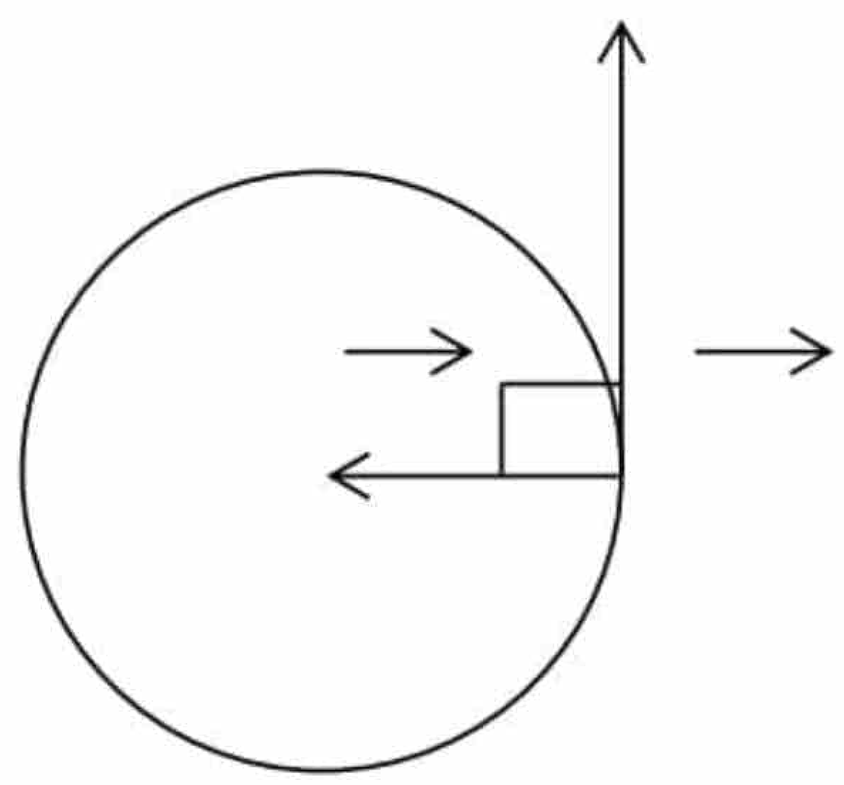
Question 5.5:

Show that orbital angular momentum L_0 = mvr.

Answer:

$$L_0 = mvr$$

Proof:



$$\vec{L}_0 = \vec{r} \times \vec{p}$$

$$L_0 = r\,p\,\sin\theta\;\hat{n}$$

$$L_0 = r\,p\,\sin 90^0 \quad \text{for circular motion } \theta = 90^0$$

$$L_0 = r\,p \quad (1)$$

$$L_0 = r\,p \qquad \text{but } p = mv$$

$$L_0 = rmv$$

$$L_0 = mvr$$

Question 5.6:

Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Answer: The minimum velocity for a satellite to orbit around the earth is called orbital velocity. For satellite orbiting close to the earth is given by:

Units:

(i) $Kg\ m^2\ s^{-1}$ (ii) Js

Law of Conservation of angular momentum:

If no external torque acts on the system then the total angular momentum of a system of bodies remains constant.

$$I_1\omega_1 = I_2\omega_2$$

$$\vec{L} = \text{constant}$$

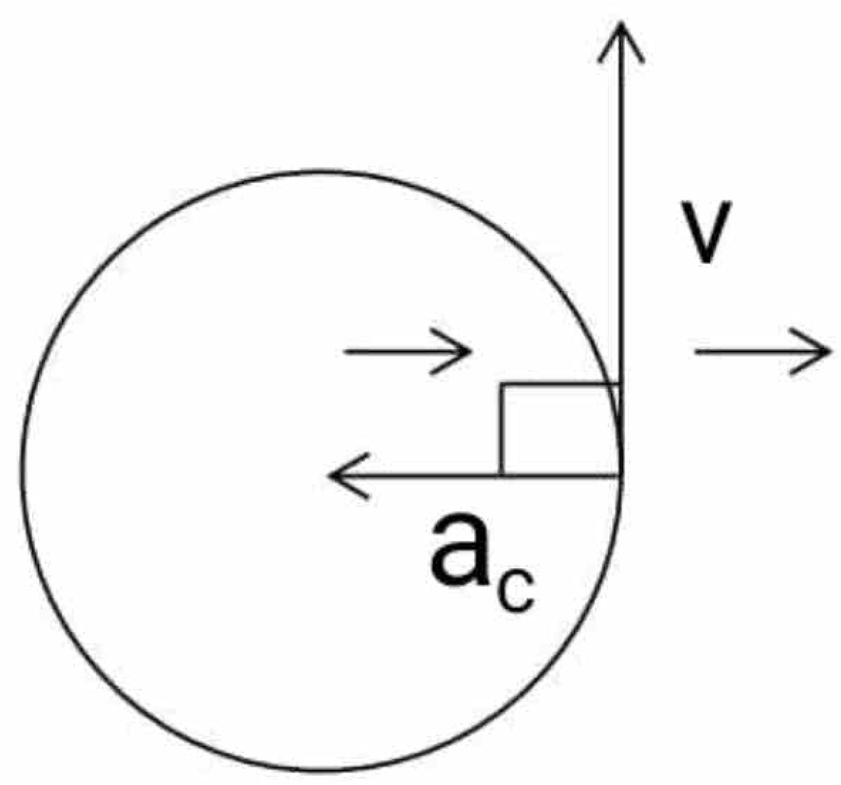
Question 5.5:

Show that orbital angular momentum $L_0 = mvr$.

Answer:

$$L_0 = mvr$$

Proof:



$$\vec{L}_0 = \vec{r} \times \vec{p}$$

$$L_0 = r\,p\,\sin\theta\ \hat{n}$$

$$L_0 = r\,p\,\sin 90^0 \quad \text{for circular motion } \theta = 90^0$$

$$L_0 = r\,p \quad (1)$$

$$L_0 = r\,p \qquad \text{but } p = mv$$

$$L_0 = rmv$$

$$L_0 = mvr$$

Question 5.6:

Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Answer: The minimum velocity for a satellite to orbit around the earth is called orbital velocity. For satellite orbiting close to the earth is given by:

$$v = \sqrt{gR}$$

$$v = \sqrt{9.8ms^{-2} \times 6.4 \times 10^{6} m}$$

$$v = 7.9\ kms^{-1}$$

This velocity is also called **critical velocity.**

Question 5.7:

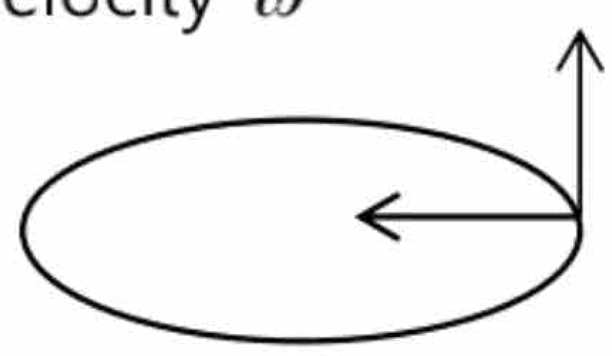
State the direction of the following vectors in simple situations; the angular momentum and angular velocity.

Answer:

In simple situation the direction of angular momentum $\vec{L}$ and angular velocity $\vec{\omega}$ are same. Their directions are along the axis of rotation and given by the right hand rule.

Right Hand Rule:

Grasp the axis of rotation in the right hand with figures curling in the direction of rotation. The erect thumb points in the direction of angular velocity $\vec{\omega}$ and angular momentum $\vec{L}$.



Question 5.8:

Explain why an object, orbiting the earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under the certain circumstances.

Answer:

It is said to be freely falling under the action of gravity.

Explanation:

An object orbiting around the earth is said to be freely falling because it is moving freely under the action of gravity. It does not fall on the center of the Earth due to curvature of Earth.

In this case: $F_{net} = ma$

$$W - T = ma \qquad \because a = g$$

$$T = W - mg$$

$$T = mg - mg$$

$$T = 0$$

As apparent weight becomes zero. It is called weightless and this state is called weightlessness.

Question 5.9:

Why mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Answer:

Mud flies off the tyre along the tangent.

Explanation:

Mud is attached to the tyre due to sticking force between mud and tyre, which is provided by the centripetal force. When speed of the tyre is increases then sticking force is decreases due to decreasing of centripetal force. Hence mud flies off the tyre along the tangent.

Question 5.10:

A disk and a hope start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

Answer:

Velocity of disc is:

$$V_D = 1.15\sqrt{gh}$$

Velocity of hoop is:

$$V_H = \sqrt{gh}$$

This shows that velocity of the disc is 1.15 times greater than the velocity of hoop, so disc reach first at the bottom.

Question 5.11:

Why does a diver change his body positions before and after diving in the pool?

Answer:

The driver change his body position to change the moment of inertia.

Explanation:

Before diving: For stretched position of the diver the moment of inertia (I_1) is increase and ω_1 is decreases.

After diving: When a diver close his arms and legs into tuck position, then his moment of inertia I_2 decreases and ω_2 increases. But the total angular momentum remains constant.

$$I_1\omega_1 = I_2\omega_2$$

Question 5.12:

A student holds to dump-bells with stretched arm while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards his chest. What will be the effect on the rate of rotation?

Answer:

Using the law of conservation of angular momentum

If
$$\tau_{ext} = 0$$

$$\vec{L}_f = \text{constant}$$

$$I_1\omega_1 = I_2\omega_2$$

Or

$$I\omega = \text{constant}$$

$$\omega = \frac{const}{I}$$

$$\omega \propto \frac{1}{I}$$

Where $$I = \sum_{i=1}^{n} m_i r_i^2$$

In Figure (a):

(i) Moment of inertia is greater

(ii) Angular velocity is smaller

In Figure (b):

(i) Moment of inertia becomes smaller

(ii) Angular velocity increases.

Hence the product; $I\omega$ = constant.

Question 5.13:

Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

Answer:

The whole Earth can be covered by three geo-stationary satellites.

Explanation:

As each geo-stationary satellite can cover 120° of longitude. Hence

$$120° + 120° + 120° = 360°$$

So only three geo-stationary satellites are required for the whole coverage of the earth.

Numerical Problems Coming Soon