Chapter 10

Optical Instruments

Introduction:

We shall discuss such optical instruments which are based on the principles of reflection and refractions as magnifying glass, compound microscope and telescopes. In addition to it, magnification and resolving powers of optical instruments, spectrometer and method for the measurement of speed of light will also be discussed.

10.1. Least Distance of Distinct Vison Near Point

Definition:

The minimum distance from the eye at which an object appears to be distinct is called

the least distance of distinct vision or near point.

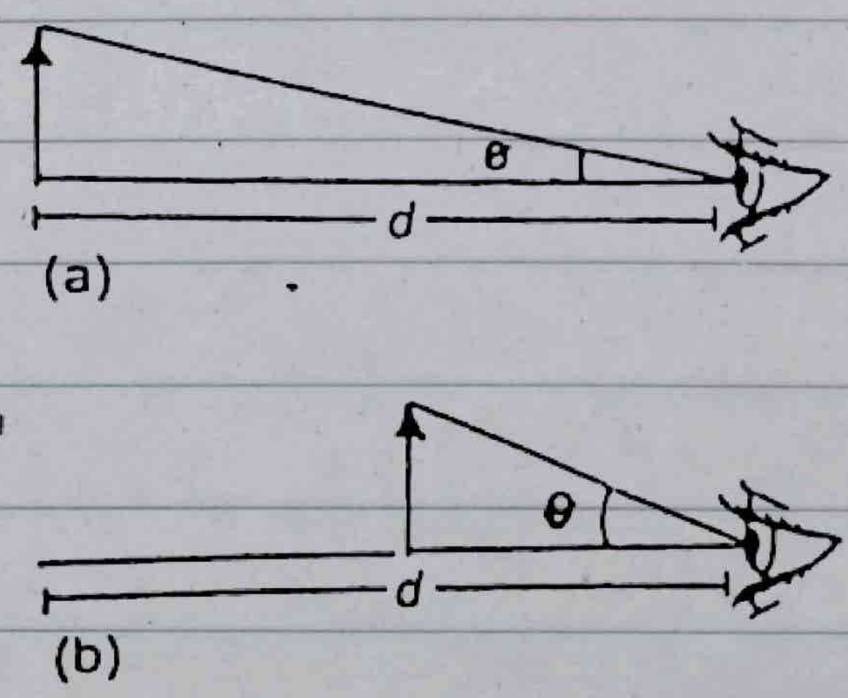
The distance is denoted by 'd'. Its value is about 25 cm from the eye of a normal person, i.e. = d= 25 cm.

A point nearest to the eye at which an object or its image is most clearly visible to the normal human eye is called the near point.

Explanation :

Suppose and object is placed at the least distance of distinct vision d as shown in fig. A normal person can see the object clearly. If it is held closer to the eye than this distance, the image will be fuzzy and blurred. In this case, the image on the eye would be much larger. Now, we can see the fine details of the object by using a converging lens (or convex lens.) Though the near point 'd' is not the same for every human body, but conventionally, This distance is taken as 25 cm

Diagram



from the normal eye. In human beings the location of the near point changes with age.

10.2: Magnifying Power and Resolving Power of Optical Instruments

Magnification:

"The ratio of the size of the image to the size of object is called Magnification." It is denoted by M.

$$M = \frac{\text{Size of image}}{\text{Size of object}}$$

$$M = \frac{I}{O}$$

It is a ratio. So it has no units.
- It is also defined as

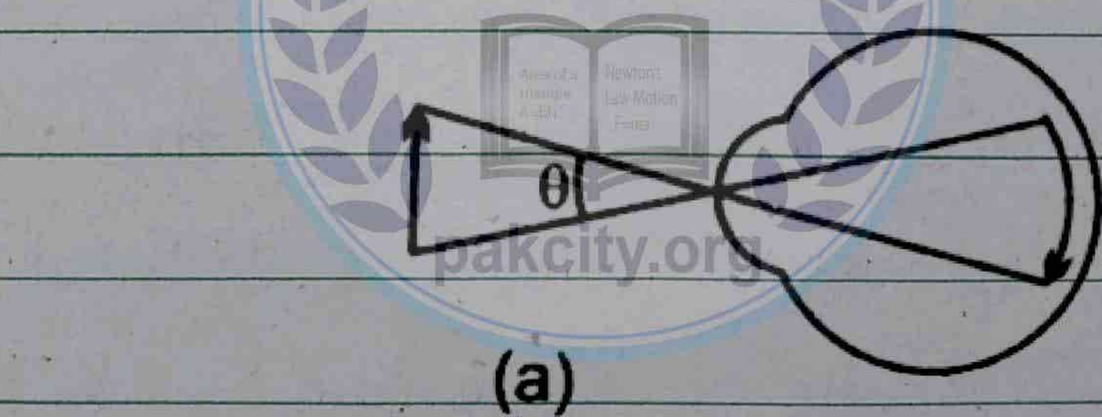
$$M = \frac{\text{Distance of the image from the lines.}}{\text{Distance of the object from the lines}}$$

$$M = \frac{q}{p}$$

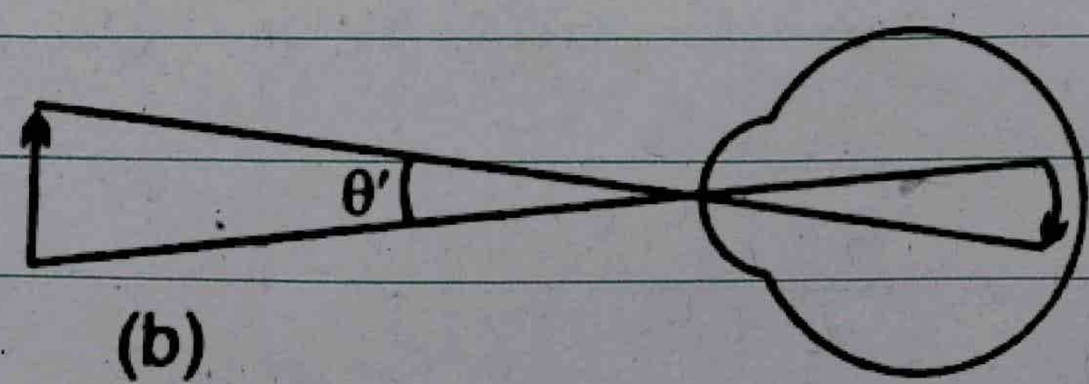
$$M = \frac{I}{O}$$

$$M = \frac{q}{p}$$

diagram



(a)



(b)

Explanation :

The apparent size of an object depends upon
(i) its actual size
(ii) angle made by it at the eye.
(It is called visual angle).

As the object is brought closer to the eye, its visual angle becomes larger and its apparent size will also be larger
In fig $\theta > \theta'$.

Magnifying Power OR Angular Magnification :-

" It is defined on the ratio of the angle made by the images as seen through an optical instrument to the angle made by the object at the unaided (naked) eye."

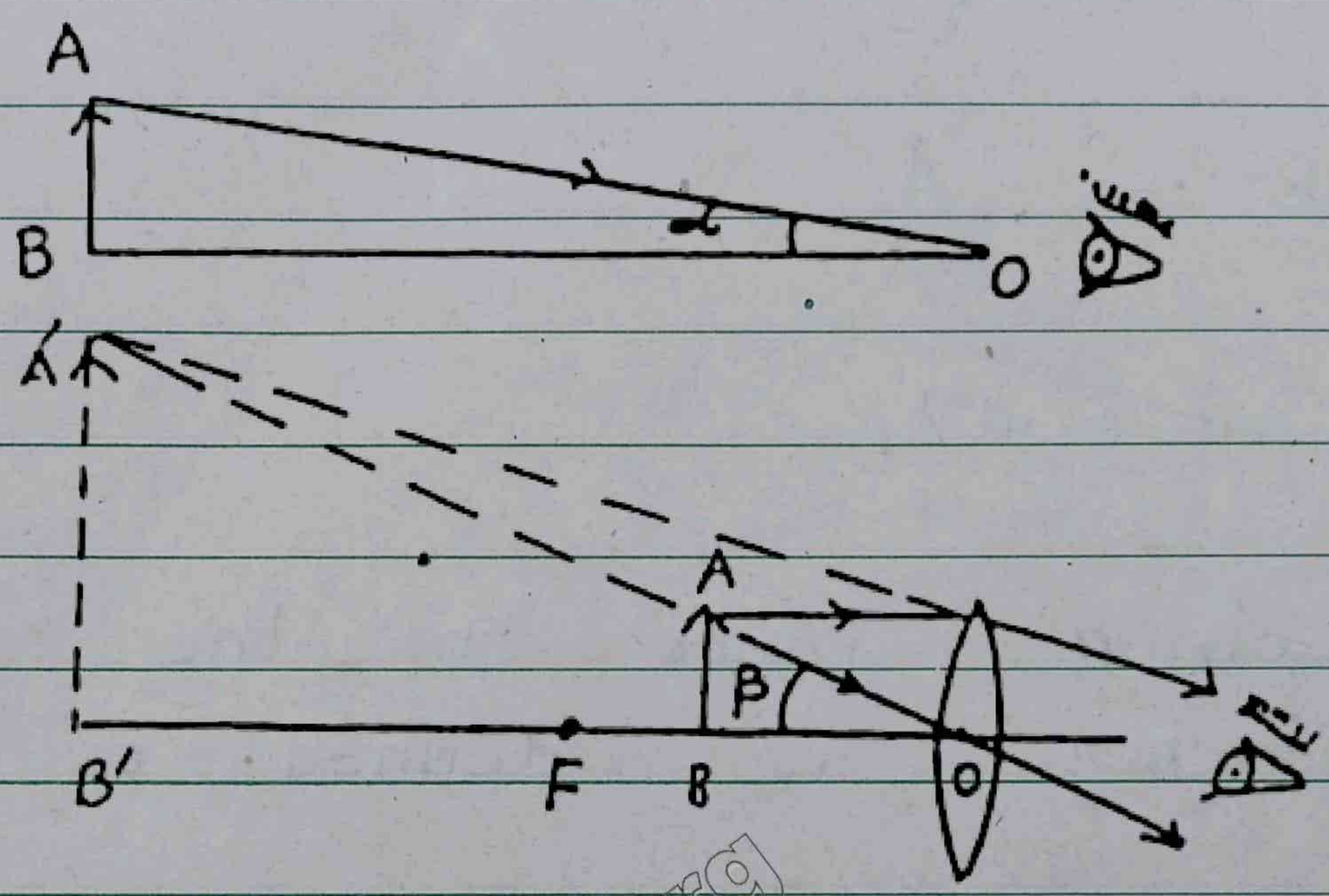
$$M = \frac{B}{\alpha}$$

B = Angle made at the eye when the object is seen through a

lens.

α = Angle made by the object at the unaided eye.

Diagram



Resolving Power:-

"Th ability of an instrument to reveal the minor details of the object under examination."

It is expressed as

$$\alpha_{min} = 1.22 \frac{\lambda}{D}$$

This was given by RALEIGH

Where

λ = Wavelength of light

D = Diameter of lens.

(i) Resolving Power R of a grating is given by

$$R = \frac{\lambda}{\lambda_2 - \lambda_1}$$

$$R = \frac{\lambda}{\Delta\lambda}$$

Where $\lambda \simeq \lambda_1 \approx \lambda_2$

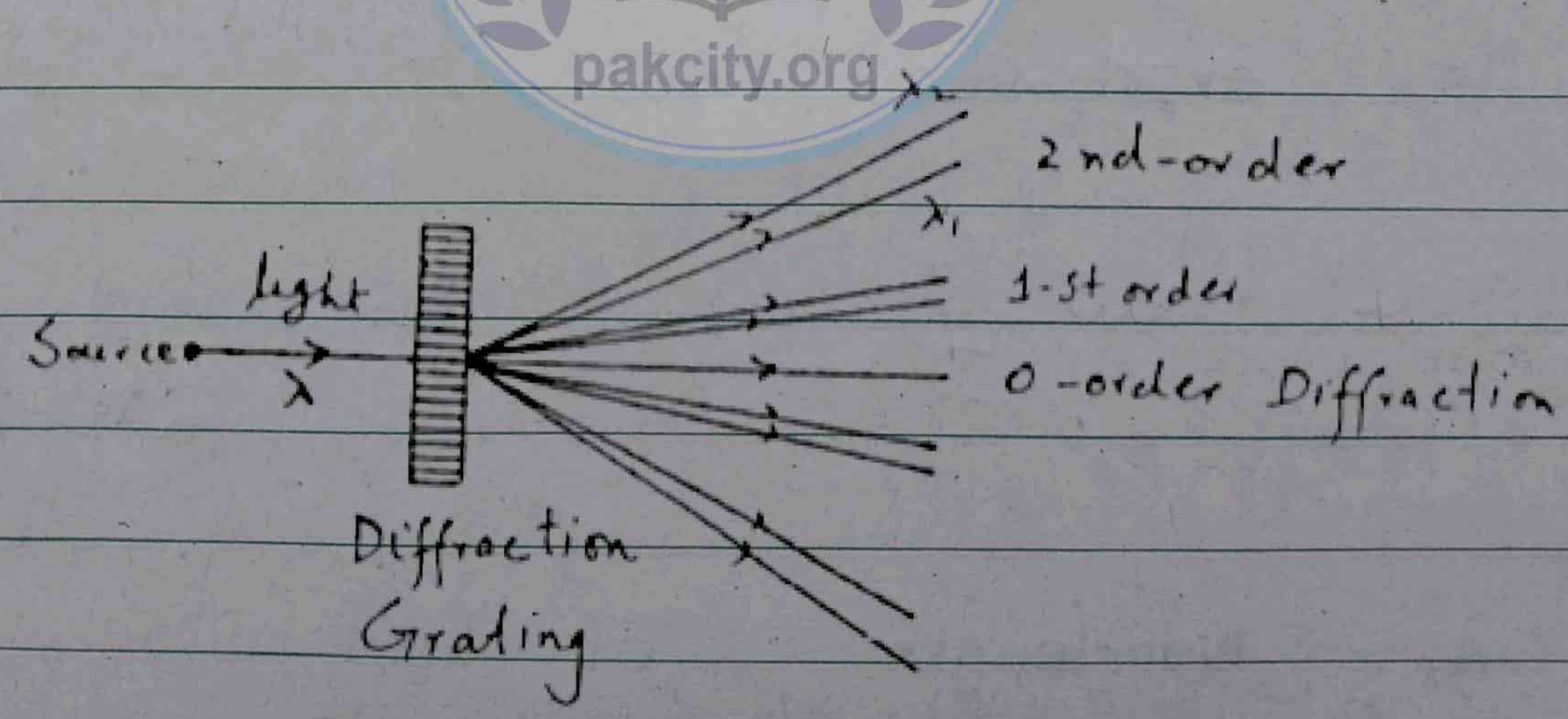
(ii) Resolving Power in the mth order diffraction is defined as

$$R = N \times M$$

Where

N = Number of rulings (lines) on the grating

m = Order of the diffraction.



10.3 Simple Microscope.

Difinition:-

" A simple convex lens used to see magnified image of a small object is called simple microscope."

Principle :-

When an object is placed inside the Focus F of a convex lens, its magnified, virtual and erect image is formed on the same side of the lens.

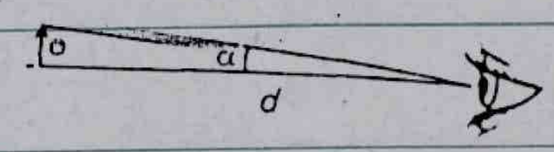
Magnification:-

α = Angle made by the object O at the eye

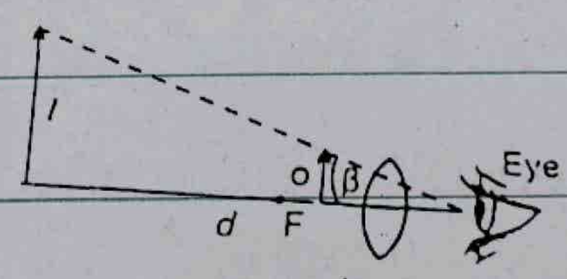
β = Angle made by the manified image I at the eye, when it is seen through the convex lens.

Both the object and image are at least distance of distinct visiond.

The angular magnification is



(a)



(b)

$M = \frac{\beta}{\alpha}$, If angles are small then

In fig (a) $\alpha = \tan \alpha$

$= \frac{O}{d}$

In fig (b) $\beta = \tan \beta \frac{I}{d}$

$$M = \frac{\frac{I}{d}}{\frac{O}{d}}$$

$$M = \frac{I}{d} \times \frac{d}{O}$$

$$= \frac{I}{O}$$

But

$$M = \frac{I}{O}$$

$$M = \frac{q}{p}$$

$M = \frac{q}{p}$ But $q = d$

$$M = \frac{d}{p} \longrightarrow (1)$$

Using Lens Formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

As the image is virtual $q = -d$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{-d} \quad \text{or} \quad \frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiply both sides by d.

$$\frac{d}{f} = \frac{d}{p} - \frac{\cancel{d}}{\cancel{d}}$$

$$\frac{d}{f} = \frac{d}{p} - 1$$

$$1 + \frac{d}{f} = \frac{d}{p}$$

$$\frac{d}{p} = 1 + \frac{d}{f}$$

$$M = 1 + \frac{d}{f}$$

For higher (larger) magnification focal length f of the convex lens should be small.

10.4 Compoud Microscope

Definition :-

" It is used to see highly magnified image of a very small object."

Cunstruction :-

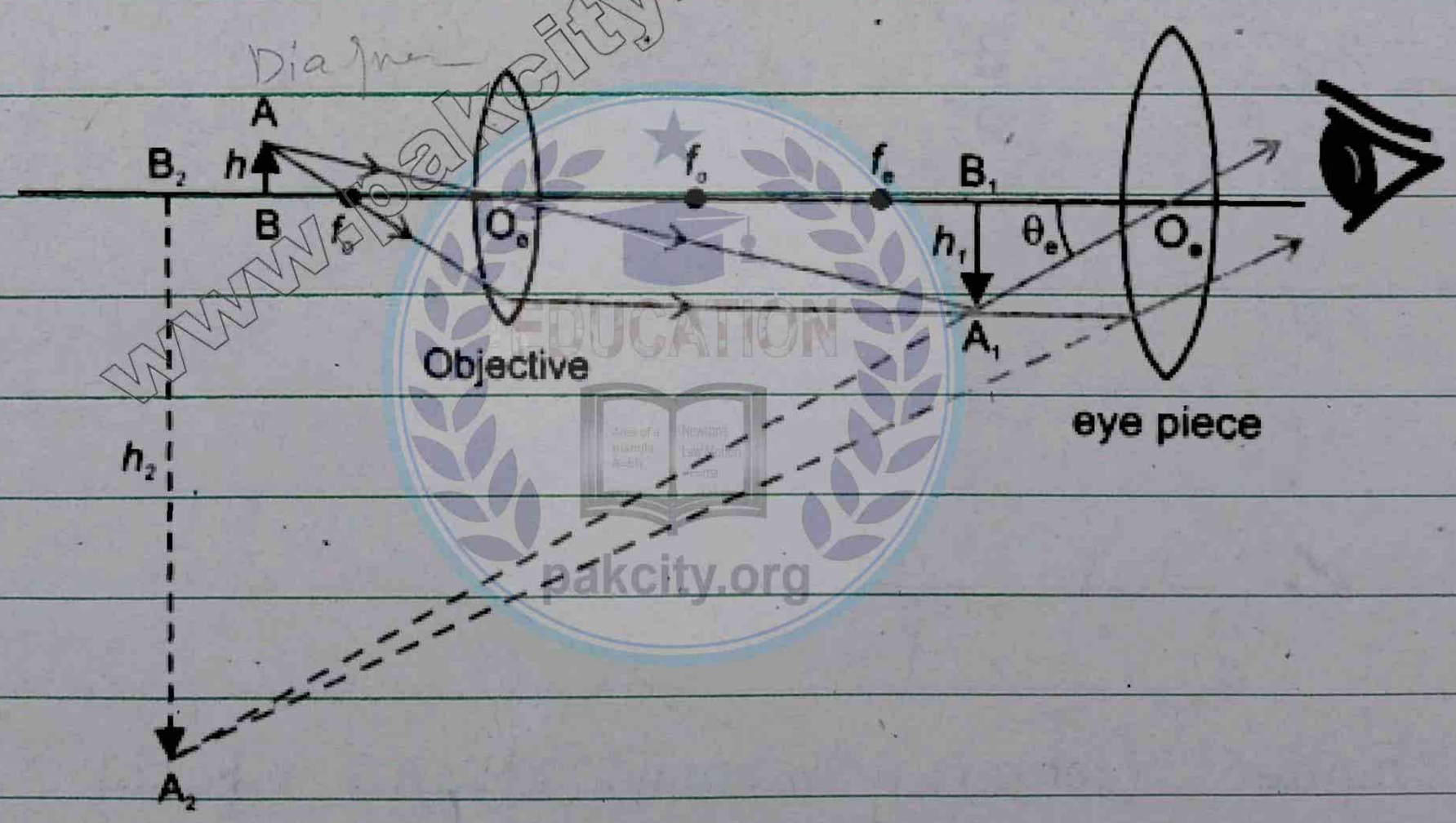
It consis of two lenses

(i) Objective

It is convex lens of very small focal length. f_o.

(ii) Eye - Piece

It is a convex lens of relatively large focal length. f_e.



Working:-

(i) A object of hieght h is placed just beyond the principal Focus F_o of the

Objective. Its real magnified and inverted image h_1 is formed within the Focus F_e of the eye-piece.

(ii) The eye-piece works as a simple microscope so that the image h_1 is further magnified. In normal adjustment of microscope, the final image h_2 is formed at a least distance of distinct vision, d.

Magnifying Power :-

The Angular magnification of Compound Microscope is

$$M = \frac{\tan \theta_e}{\tan \theta}$$

θ_e = Angle made by the final image of height h_2 at the eye.

θ = Angle that the object of height h would make at the eye if placed at the near point.

$$\tan \theta_e = \frac{h_2}{d}$$

$$\tan\theta = \frac{h}{d}$$

$$M = \frac{\frac{h_2}{d}}{\frac{h}{d}}$$

diagm

$$M = \frac{h_2}{\cancel{d}} \times \frac{\cancel{d}}{h}$$

$$M = \frac{h_2}{h}$$

$$M = \frac{h_1}{h} \times \frac{h_2}{h_1}$$ $\times \div$ by h_1

$$M = M_1 \times M_2$$

Where

$$M_1 = \frac{h_1}{h}$$

= Magnification produced by the objective.

$$M_2 = \frac{h_2}{h_1}$$

= Magnification produced by the eye-piece.

But

$$M_1 = \frac{v}{p}$$

$$M_2 = 1 + \frac{d}{f_e}$$ (Eye-piece acts as a simple microscop)

$$M = \frac{a}{p}\left(1 + \frac{d}{f_e}\right)$$

This is total Magnification of Compound Microscope.

A wider objective and use of blue light of shorter wave length which produces less diffraction allows more details to be seen.

10-5 Astronomical Telescope

Definition:-

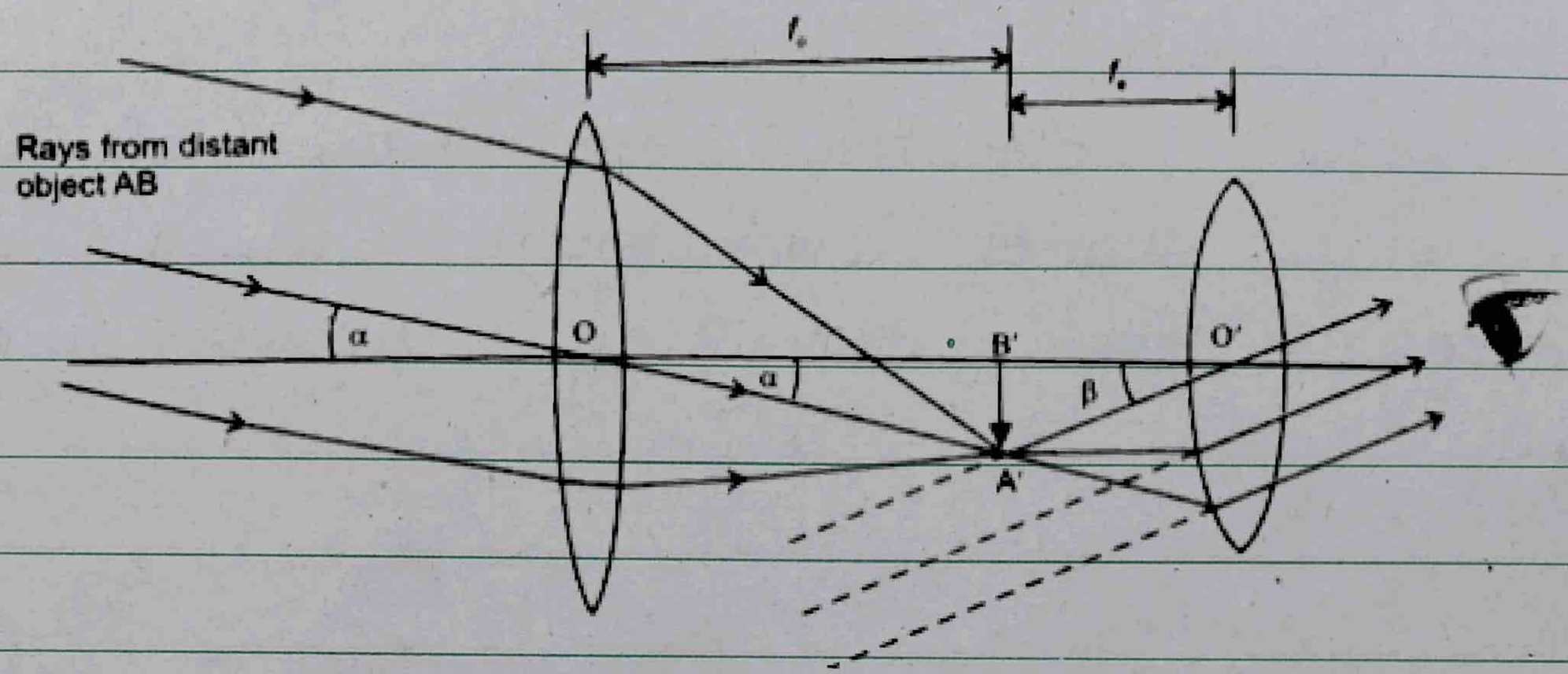
" It is an optical instrument used for viewing distant Reavenly bodies."

Principle:-

(i) When a distant object is seen through the unaided eye visual angle is very small.

(ii) When a distant object is is viewed through a telle scope, the visual angle made by the image on

the eye is large. So the image will be larger in size.



Construction:-

It consists of two lenses.

1- Objective :-

It is a convex lens of large focal length $= f_e$. and large operture (diameter)

2- Eye - Piece :-

It is a convex lens of small focal length $= f_e$ and small operture.

Working:-

Rays coming from a distant object AB are nearly

parallel. Objective forms a small image A' B' at its focus F_o. This real image A' B' acts as an object for the eye-piece which is virtual, Magnified and inverted.

Normal Adjustment :-

When the image A' B' lies at the focus of both objective and eye-piece, the rays after refraction through the eye-piece become parallel and final image appears to be formed at infinity. This is called normal adjustment for Telescope

Magnifying Power :-

$$\alpha = \tan\alpha = \frac{A'B'}{OB'}$$

$$= \frac{A'B'}{f_o} \quad \text{In } \Delta\ A'OB'$$

$$\beta = \tan\beta = \frac{A'B'}{O'B'}$$

$$= \frac{A'B'}{f_e} \quad \text{In } \Delta\ A'O'B$$

$$M = \frac{\beta}{\alpha} = \frac{\frac{A'B'}{f_e}}{\frac{A'B'}{f_o}}$$

$$M = \frac{\cancel{A'B}}{f_e} \times \frac{f_o}{\cancel{A'B'}}$$

$$\boxed{M = \frac{f_o}{f_e}}$$

$$M = \frac{\text{Focal length of Objective}}{\text{Focal length of Eye-piece}}$$

For higher Magnificatin :-

f_o should be large
f_e should be small

L = Length of the telescope
L = Distance between the objective and Eye-piece.

$$\boxed{L = f_o + f_e}$$

To collect more light rays from a distant object the objective should have large aperture. (diameter).
Astronomical telescope cannot be used for terrestrial purpose because final image in Inverted.

10.6 Spectrometer

Defination:

" It is an optical instrument which is used to study the spectrum of light coming from different sources."

Uses

(i) With the help of a spectrometer the deviation of light by a glass prism and the refractive index of the glass prism is measured.

(ii) It is used to measure wavelength of light with a diffraction grating.

Construction

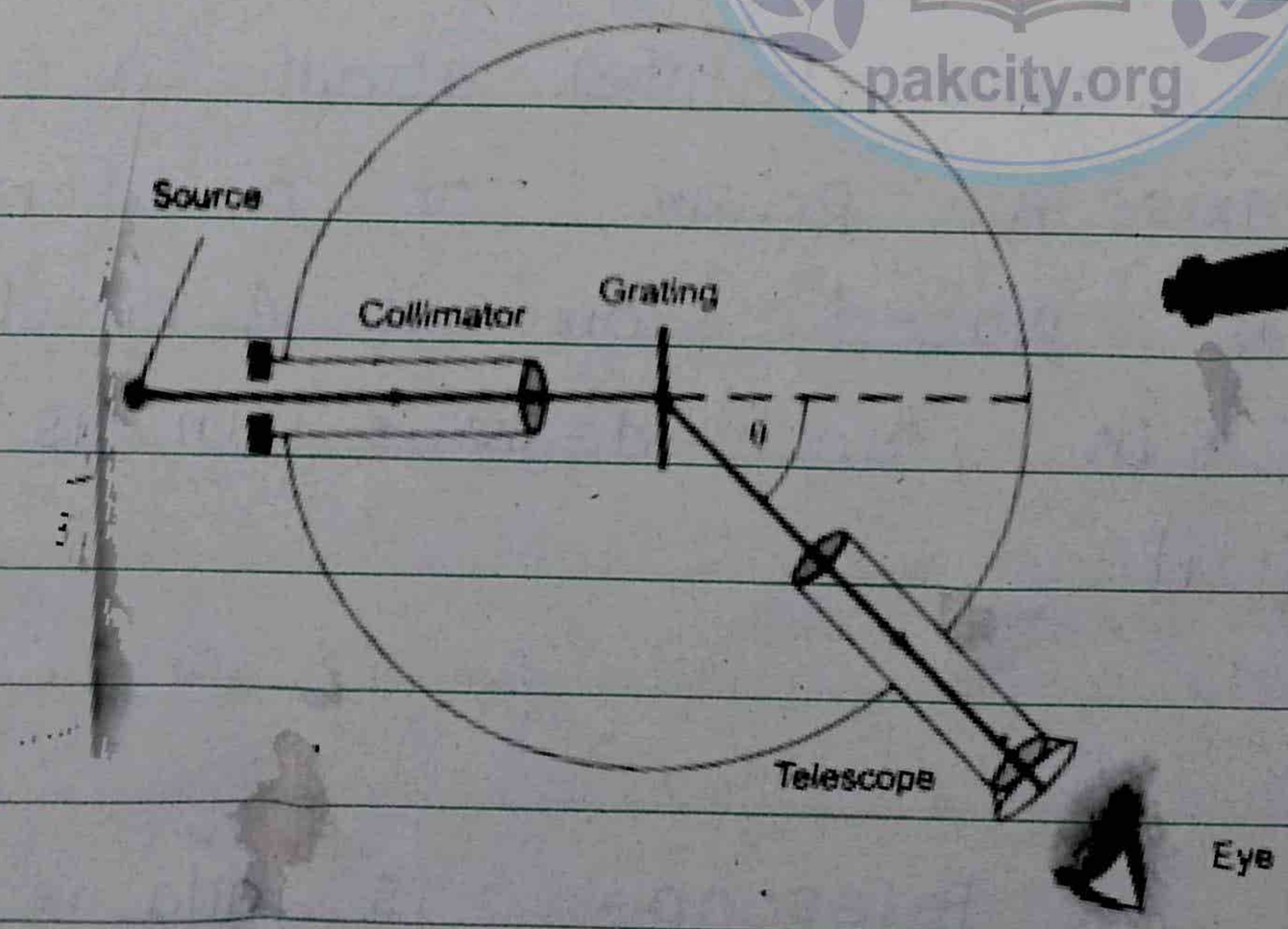


Fig. 10.6 (b)
Spectrometer.

Spectrometer has three parts.

(i) Collimator
(ii) Turntable
(iii) Telescope

(i) Collimator

It consist of a fixed metallic tube with a convex lens at one end an adjustable slit (that can be moved in and out of tube) at the other end. When the slit is at the focus of the convex lens, the rays coming out of the lens become parallel. For this reason, It is called collimator.

(ii) Turntable

It is a circular metallic tube which can turn (rotate) about a fixed vertical axis. A prism or a diffraction grating is placed on it. A circular scale graduated in half-degrees is attached with it.

(iii) Telescope

A telescope is attached

with vernier scale and it can rotate about the same vertical axis.

Working

(i) Befor using a spectrometer, the collimator is so adjusted that parallal rays of light energy out of convex lens. The telescope is adjusted in such a way that the rays of light entering it are focused at the cross wire near the eye-piece.

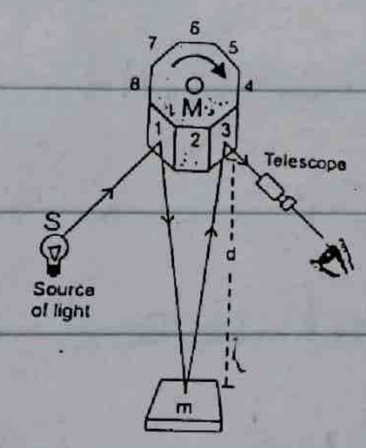
(ii) The refracting edge of the prism must be parallal to the axis of rotation of the tellescope, so that the turn table is levelled. This can be done by using levelling screws.

10.7 Speed of Light

Galileo was the first person who tried to measure the speed of light, but could not succeed. Speed of light is accurately measured by Michelso's Experiment.

Experimental Set Up

1- Source of light.
2- 8- sided polished mirror M mounted (fixed) on the shaft of a motor whose speed of rotation can be varied.
3- A plane reflecting mirror m at a distance d from M.
4- Telescope.



Working

(i) Suppose the mirror M is at rest the position shown in fig. A beam of light from the face 1 of the mirror M falls on the plane mirror m. This beam of light is reflected back on the face 3 of mirror M. On reflecting from face 3 it enters the telescope.

(ii) If the mirror M is rotated clockwise, initially the source 5 will not be visible through the telescope. But at a certain frequency of rotation of mirror M, the source S becomes visible. In this condition, the time taken by the light in moving from M to m and back to M

is equal to the time taken by the face 3 to take the position of face 2.

(iii) The distance covered by the light is

$$\boxed{S = d + d \quad S = 2d}$$

Let

f = frequency of rotation of mirror M.

Time to complete f revolution = 1 sec

Time to complete 1 revolution = $\frac{1}{f}$

Time to complete $\frac{1}{8}$ revolution = $\frac{1}{8} \times \frac{1}{f}$

So, $t = \frac{1}{8f}$

As

$S = vt$

$V = \frac{S}{t}$

$C = \frac{S}{t}$ C = Speed of light

$C = \frac{2d}{\frac{1}{8f}}$

$C = 2d \times 8f$

$C = 16\ fd$

$C = 2.99792458 \times 10^8\ m/s$ (In vacuum)

$C \simeq 3 \times 10^8\ m/s$ (In vacuum)

(i) In a medium (material such as glass, water etc) other than vaccum, the speed of light is less than c.

$C = 3 \times 10^8\ m/s$

$C = 3 \times 10^5\ km/s.$

$C = 300000\ km/s$

(ii) In a medium the speed of light depends upon the refractive index of the medium.

(iii) Speed of light in air is approximately equal to that in vacuum.

Refractive Index $n = \dfrac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

$$n = \frac{c}{v}$$

10.8 Introduction to Fibre Optics

For hundreds of year man has communicated using flashes of reflected sunlight by day and lanterns by night. Navy signal men still use powerful blinker lights to transmit coded messages to other ships during periods of radio-silence.

Graham Bell invented a device known as "Photophone" and then telephone. Bell was able to transmit a voice message via beam of light.

Transmission of light is now beign used in communication technology. The use of light as a transmission carrier wave in fibre optics has several advantages over radiowave carriers such as.

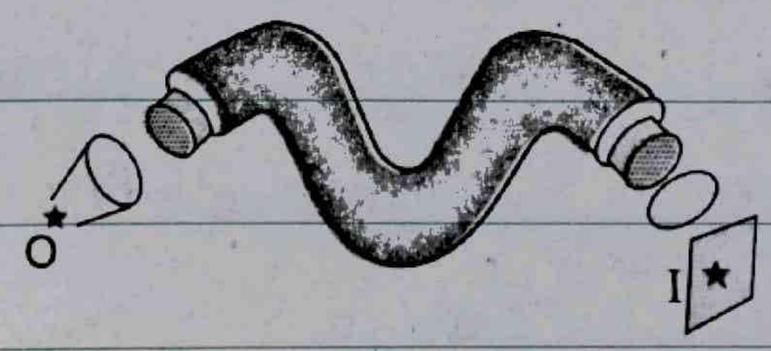
(i) Wide Band width (large capacity to transfer information)

(ii) Immunity from electromagnetic interference.

(iii) The use of fiber optics tools in industry is now very common and their importance as diagnostics tools in

in medicine has bean proved.

(iv) The fibre optic system is m ch thinner and light weight.



(v) An optical fibre with its protective case may be 6.0 mm in diameter and yet it can replace a 7.62 cm diameter bundle of copper wires now used to cary. The same amount of signals.

10.9 Fibre Optic Principles

In fibre optics light is confined with in the fibre. This is done by two diffrent ways.

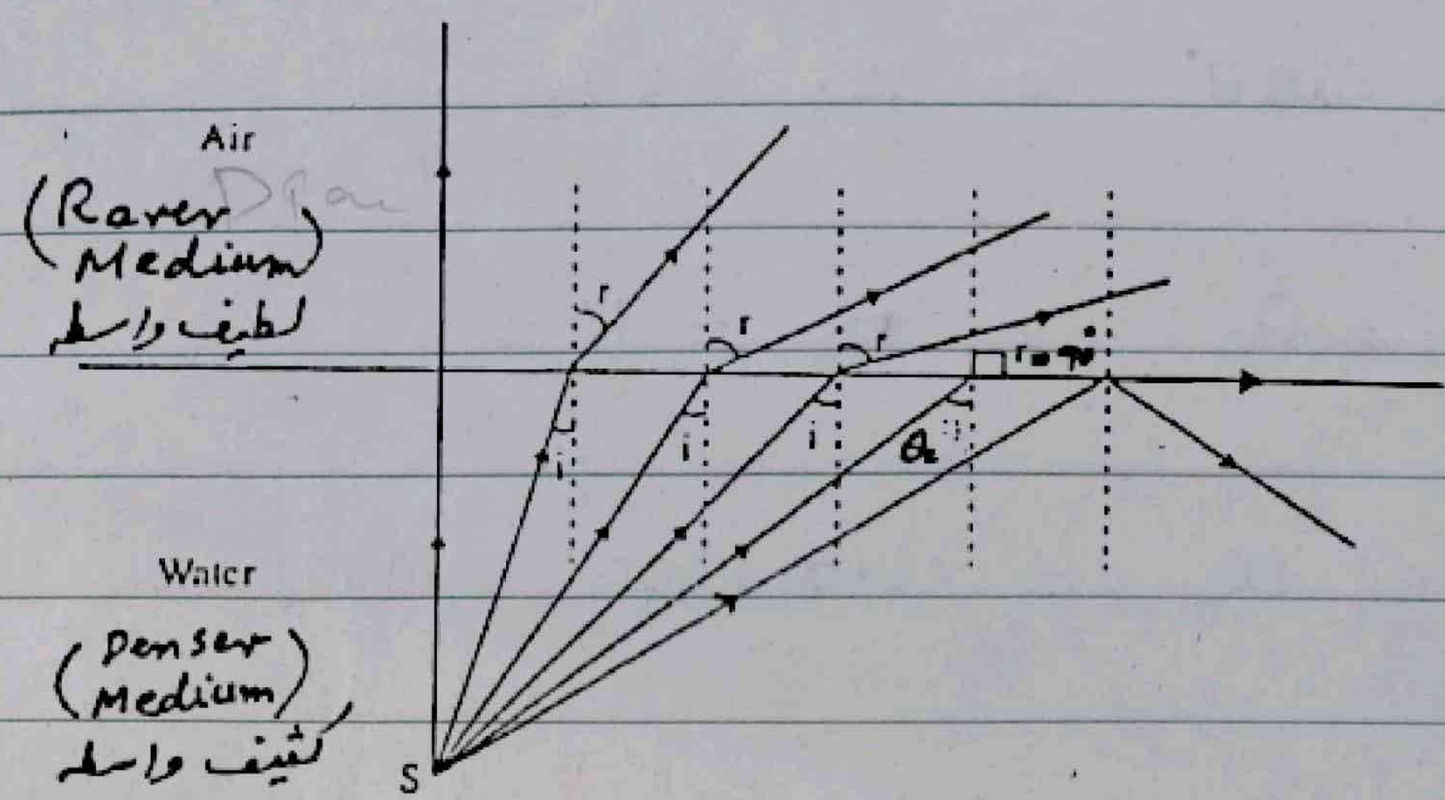
(i) Total Internal Reflection

(ii) Continuous Refraction

(i) Total Internal Reflection

when light enters from denser medium to rarer medium, it bends away

from the normal



In this case the angle of refraction is always greater than the angle of incidence.

If we increase the angle of incidence, then a stage may come when the angle of refraction is 90°.

Critical Angle $= \theta_c$

"The angle of incidence for which the angle of refraction is 90°, this angle of incidence is called critical angle."

In the denser medium if the angle of incidence is greater than critical angle, then the light is totally reflected back in the denser medium. This is called total internal reflection.

By Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \theta_c \quad , \quad \theta_2 = 90^\circ$$

$$n_1 = 1.5 \text{ (for glass)}$$

$$n_2 = 1.0 \text{ (for air)}$$

$$n_1 \sin\theta_c = n_2 \sin 90^\circ$$

$$\sin\theta_c = \frac{n_2}{n_1} \times 1$$

$$= \left(\frac{1.0}{1.5}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$\theta_c = 41.8^\circ \simeq 42^\circ$$

$$\theta_c = 42^\circ \text{ (for glass)}$$

Conditions of total Internal Reflection

(i) Ray of light should enter from denser to rarer medium.

(ii) The angle of incidence should be greater than the critical angle.

Diagram

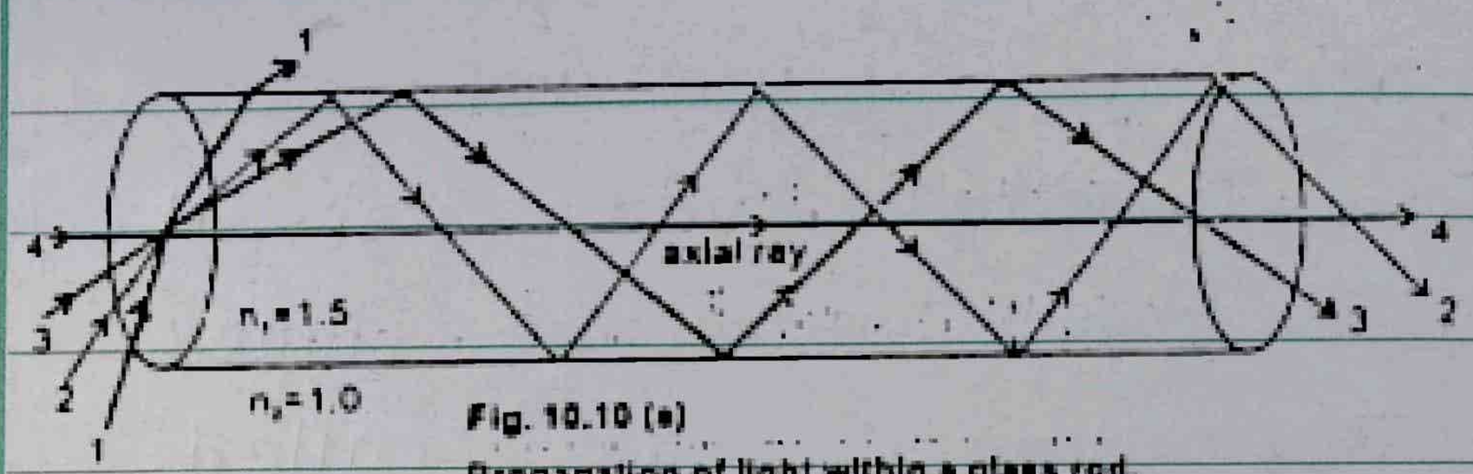


Fig. 10.10 (a)
Propagation of light within a glass rod.

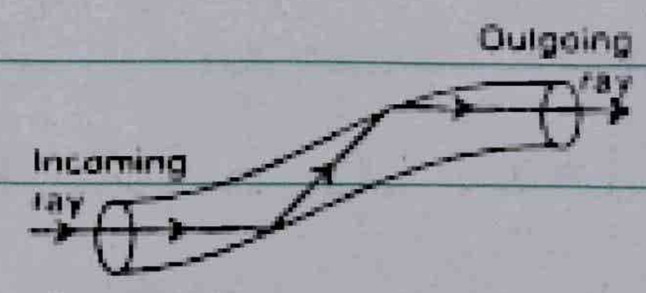


Fig. 10.10 (b)
Light propagation within a flexible glass fibre.

Consider a round glass rod.

(i) Ray -1 falls at angle 30° (less than the critical angle) at the glass-air boundary. It will escape from the glass rod. This ray would be lost.

(ii) Ray -2 falls at 42° it will be reflected back into the rod (total internal reflection).

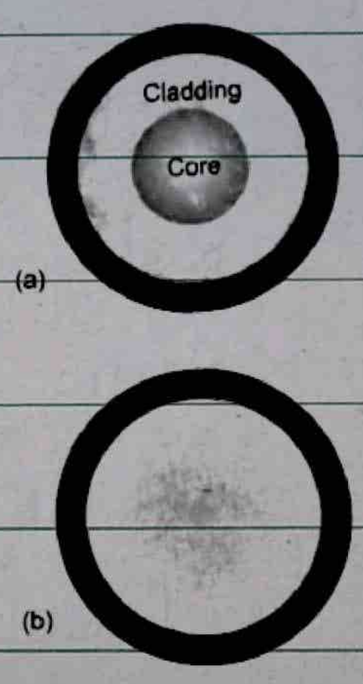
(iii) Ray-3 falls at 60°, It will also be reflected back into the rod.

(iv) Ray-4 (axial ray) will travel down this stright rod.

Continuous Refraction

In this method, light is continuou-

-sly refracted with in the fibre. For this purpose central part of the fibre has a high refractive index (i.e. high Density). and over it is a layer of lower refractive index (lens density). This layer is called cladding. This type of fibre is called Multi-Mode step Index Fibre. Its view is show in fig



Now a days, a new type of optical fibre is use in which the central cone has high refractive index high density and its density gradually decreases tow its periphery. This type of optical fibre is called Multi Mode Graded Index Fibre.

Its view is shown in Fig. In both of these fibres the propagation of light signals is due to continuous refraction and total internal reflection.

10.10 Types of Optical Fibres

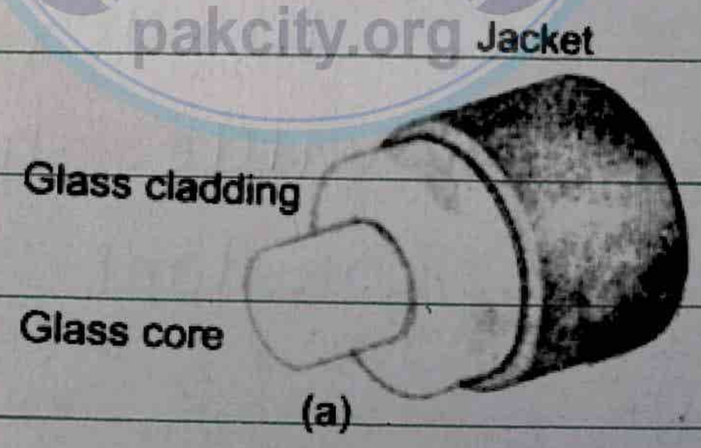
Three types

(i) Single - mode Step Index Fibre.
(ii) Multi - mode Step Index Fibre.
(iii) Multi - mode graded Index Fibre.

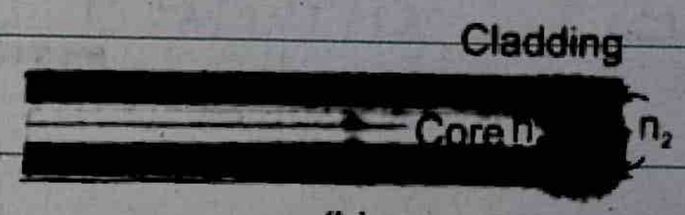
(i) Single - Mode Step Index Fibre

Single mode or mono mode step index fibre has a very thin core (central part) of about 5 μm ($= 5 \times 10^{-6}$ m) diameter and has a larger cladding (of glass or plactic) as shown in fig.

Laser source has to be used to second light signals through it. It can carry more than 14 TV chanels or 14000 phone calls.



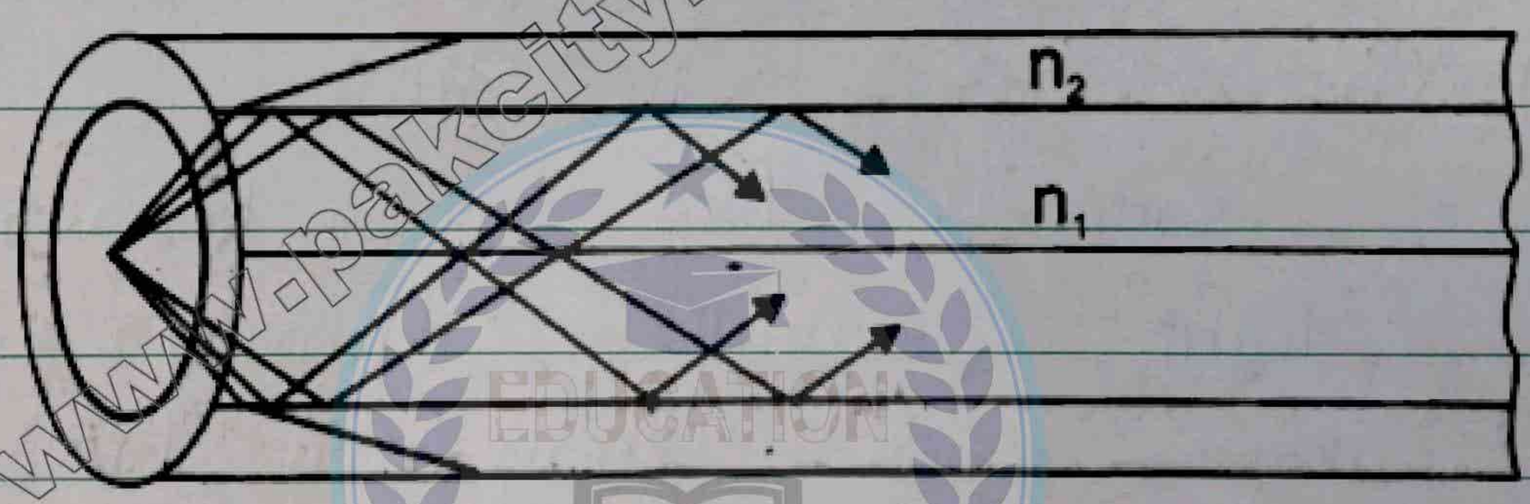
(a)



(b)

(ii) Multi-Mode Step Index Fibre.

Multimode step index fibre has a core of relatively larger diameter such as 50μm. It is used to carry white light, but due to dispersion effect it is useful for a short distance only. The fibre core has a constant refractive index n_1 such as 1.52 from its centre to boundary with the cladding as shown in fig



The refractive index the changes to a lower value n_2 such as 1.48 which remains constant throughout the cladding.

(iii) Multi-Mode Graded Index Fibre.

Multi mode Graded Index Fibre has

a core which ranges in diameter from 50 to 1000 μm. It is a core of relatively higher refraticve index and the refractive index decreases gradually from the middle to the outer surface of the fibre.

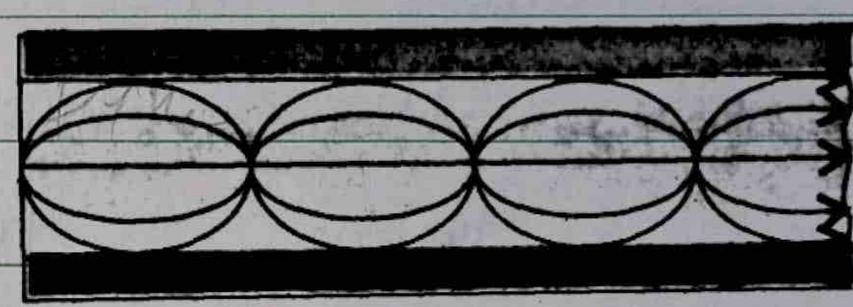


Fig. 10.15

Light propogation through Multi-mode graded-index fibre

This type of fiber is useful for long distance applications which white light is used.

10.11 Signal Transmission And Conversion to Sound

Diagram

A Fiber optics communication system

consist of three major components.

1- A transmitter that converts electrical signals to light signals

2- An optical fibre for guiding the signals

3- A receiver that captures the light signals at the other end of the fibre and recover them to electrical signals.

The light source in the transmitter can be either a semiconductor laser or a 'light emitting divde' LED.

The Lasers and L.E.D used in this application are tiny units in order to match the size of fibres.

The transmit information by light waves, it is necessary to modulate the light waves.

The most common method of modulation is called "digital modulation" in which the laser or LED is flashed on and off at an extremely fast rate. A pulse of light represents the number 1 (one) and absence of

light represents 0(zero). So instead of flashes of light traveling in optical fibre ones (1) and zero (0_s) are traveling.

Digital Modulation is expressed in bits (binary digits) or mega bits (10^6 bits) per second, where a bit is a 1 or zero.

At the end of the fibre a photo divde converts the light signal into electrical signals which are then amplified and decoded.

During the propugation of digital light pulses, signal losses occur by absorption and scattering, So amplifiers called repeaters (R) are placed at places along the fibre cable.

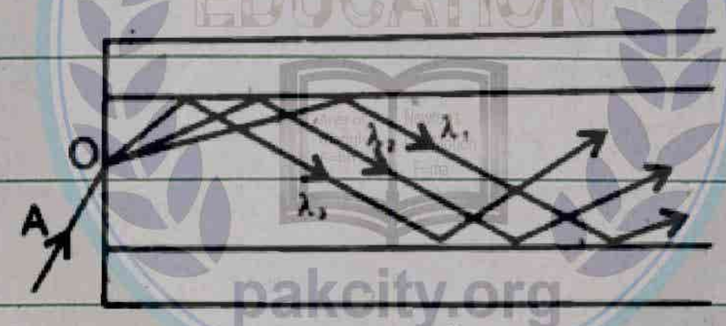
10.12 Losses of Power

When a light signal travels along fibre, by multiple reflection. Some light is absorbed due to impurition in the glass. Some of it

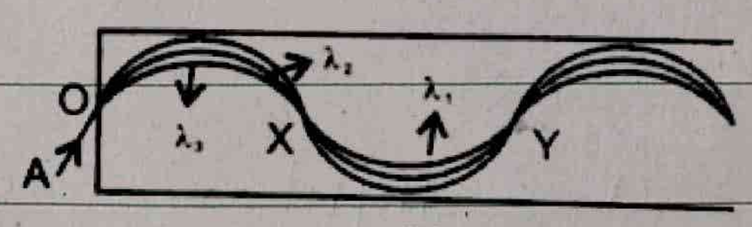
Scattered by group of atoms which are formed at places such as joints when fibre are joined together.

The information received at the other end of fibre can be inaccur due to dispersion spreading of light of the different wave lengths λ_1, λ_2 λ_3.

λ_1 meets the core-cladding at the critical angle and λ_2, λ_3 at slightly greater angles. As the light paths have different lengths, So the light of different wave lengths reaches the other end of the fibre at different times. Therefore, the signal received is distorted.



(a)



(b)

The disadvantage of the step index fibr (in fig) can be reduced by

using a graded index fibre. As shown in fig the different wave-lengths still take the different paths and are totally internally reflected at different layers but still they are at the same points like X and Y etc.

It is possible because the speed of light the wavelength λ_1 is inversely proportional to the refracti index. So the wavelength λ_1 is travels a longer path than λ_2 or λ_3 but at a greater speed.

Inspite of different dispersion, all the wavelengths arrive at the other end of fibre at the same time.

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QUESTIONS

10.1 What do you understand by linear magnification and angular magnification? Explain how a convex lens is used as a magnifier?

10.2 Explain the difference between angular magnification and resolving power of an optical instrument. What limits the magnification of an optical instrument?

10.3 Why would it be advantageous to use blue light with a compound microscope?

10.4 One can buy a cheap microscope for use by the children. The images seen in such a microscope have coloured edges. Why is this so?

10.5 Describe with the help of diagrams, how (a) a single biconvex lens can be used as a magnifying glass. (b) biconvex lenses can be arranged to form a microscope.

10.6 If a person was looking through a telescope at the full moon, how would the appearance of the moon be changed by covering half of the objective lens.

10.7 A magnifying glass gives a five times enlarged image at a distance of 25 cm from the lens. Find, by ray diagram, the focal length of the lens.

10.8 Identify the correct answer.

(i) The resolving power of a compound microscope depends on;

a. Length of the microscope.
b. The diameter of the objective lens.
c. The diameter of the eyepiece.
d. The position of an observer's eye with regard to the eye lens.

(ii) The resolving power of an astronomical telescope depends on:

a. The focal length of the objective lens.
b. The least distance of distinct vision of the observer.
c. The focal length of the eye lens.
d. The diameter of the objective lens.

10.9 Draw sketches showing the different light paths through a single-mode and a multi mode fibre. Why is the single-mode fibre preferred in telecommunications?

10.10 How the light signal is transmitted through the optical fibre?

10.11 How the power is lost in optical fibre through dispersion? Explain.

NUMERICAL PROBLEMS

10.1 A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of the observer is 25 cm and the lens is held close to the eye, calculate (i) the distance of the object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity?

[Ans: (i) 4.2 cm (ii) 6.0 ; 5.0]

10.2 A telescope objective has focal length 96 cm and diameter 12 cm. Calculate the focal length and minimum diameter of a simple eye piece lens for use with the telescope, if the linear magnification required is 24 times and all the light transmitted by the objective from a distant point on the telescope axis is to fall on the eye piece.

(Ans: f_e = 4.0 cm, dia = 0.50 cm)

Chapter 10

Questions

Q – 10.1

Answer :

Linear Magnification = Ratio of size of the image to the size of Object.

$$M = \frac{I}{O}$$

$$= \frac{q}{p}$$

Angular Magnification = " Ratio of the angle made by the image as Seen through the optical device to the angle made by the object at the unaided eye. "

A convex lens of short focal length can be used as a magnifier, when an object is placed inside the focus of the lens. The image formed is erect

virtual and magnified.

Q 10.2

Answer:

Angular magnification

The magnifying power or angular magnification is defined as the ratio of the angles subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.

Resolving Power

The resolving power of an instrument is its ability to reveal the minor details of the object under examination.

$$\alpha_{min} = 1.22 \frac{\lambda}{D}$$

The magnification of the optical instrument is limited due to defects

of the lenses such as chromatic aberrations and spherical aberrations.

Q - 10.3

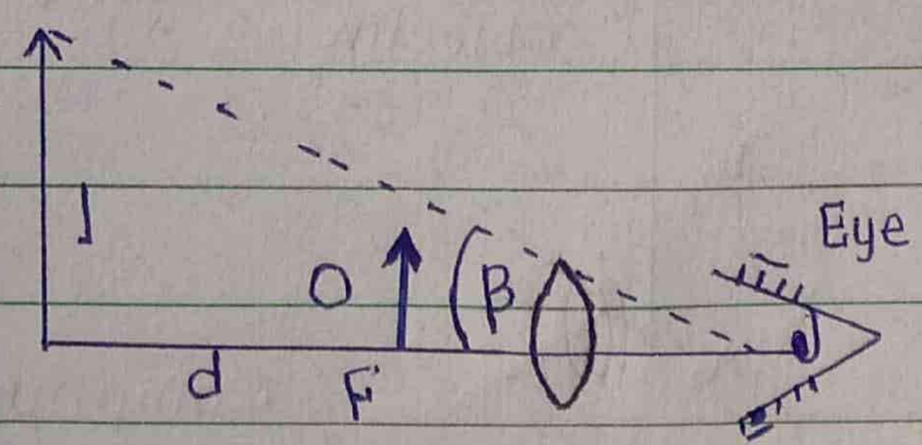
Answer

An objective lens of large aperture and use of blue light of short focal length produces less diffraction and increases its resolving power. Thus, more details of the object can be seen by the eye.

Q - 10.4

Answer

The image seen in the cheap microscope has coloured edges. This is due to the defect of the lens called chromatic aberration. Due to the prism like structure of a lens, it disperses white light into colours. So coloured image is seen.

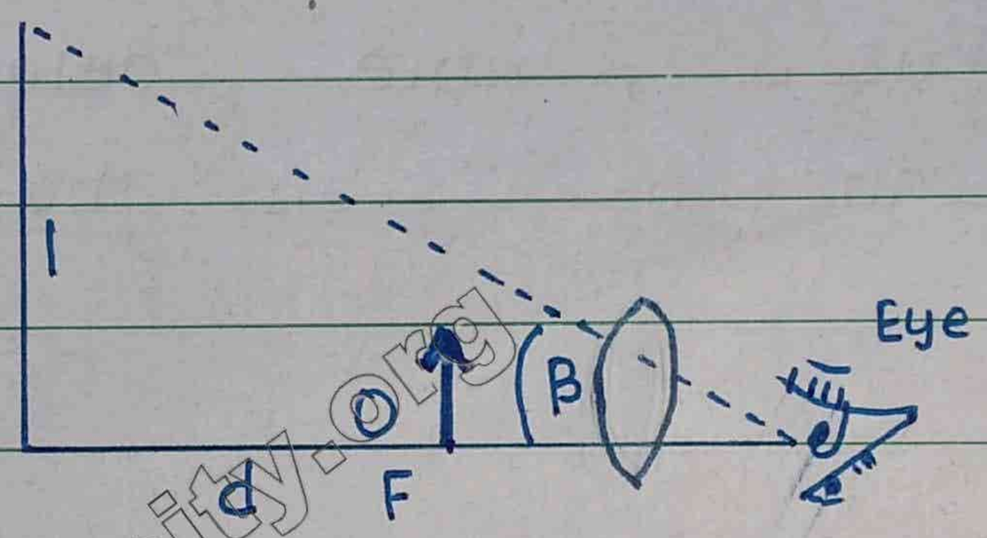


Simple Microscope

Q - 10.5

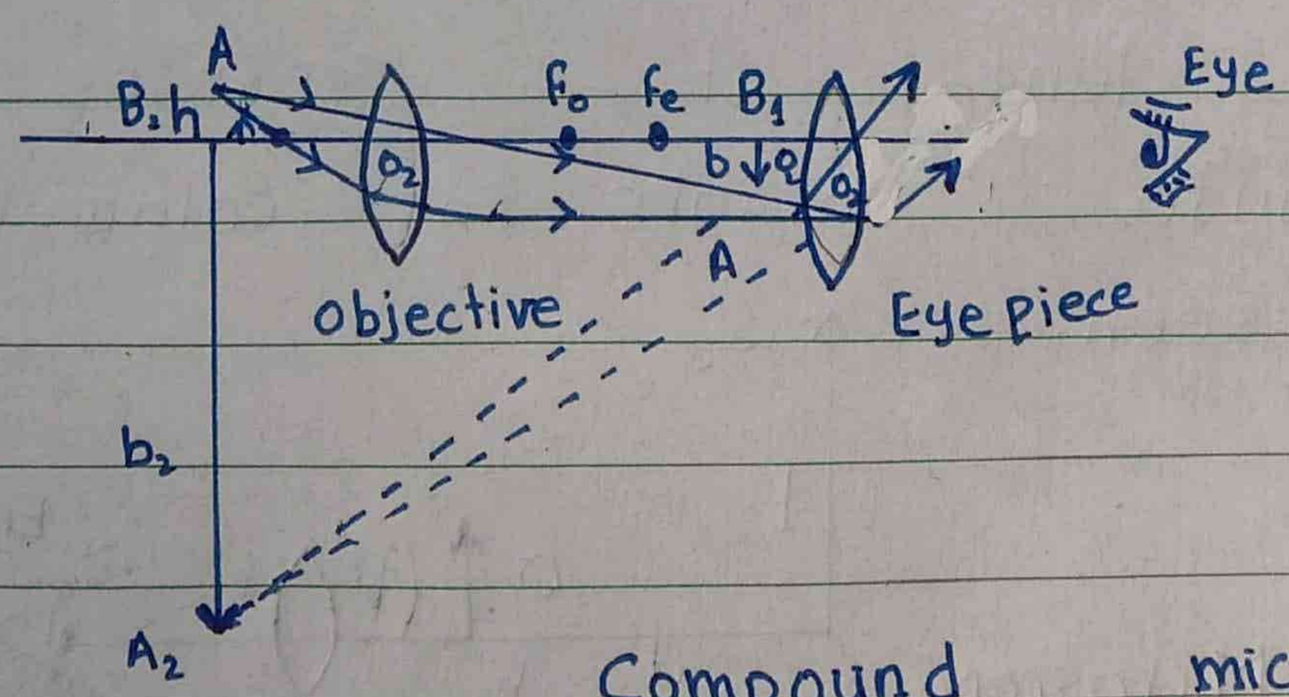
Answer

(a) A biconex lens is used as a magnifying glass is shown in the fig. The object is placed between the lens and focus.



Magnifying glass

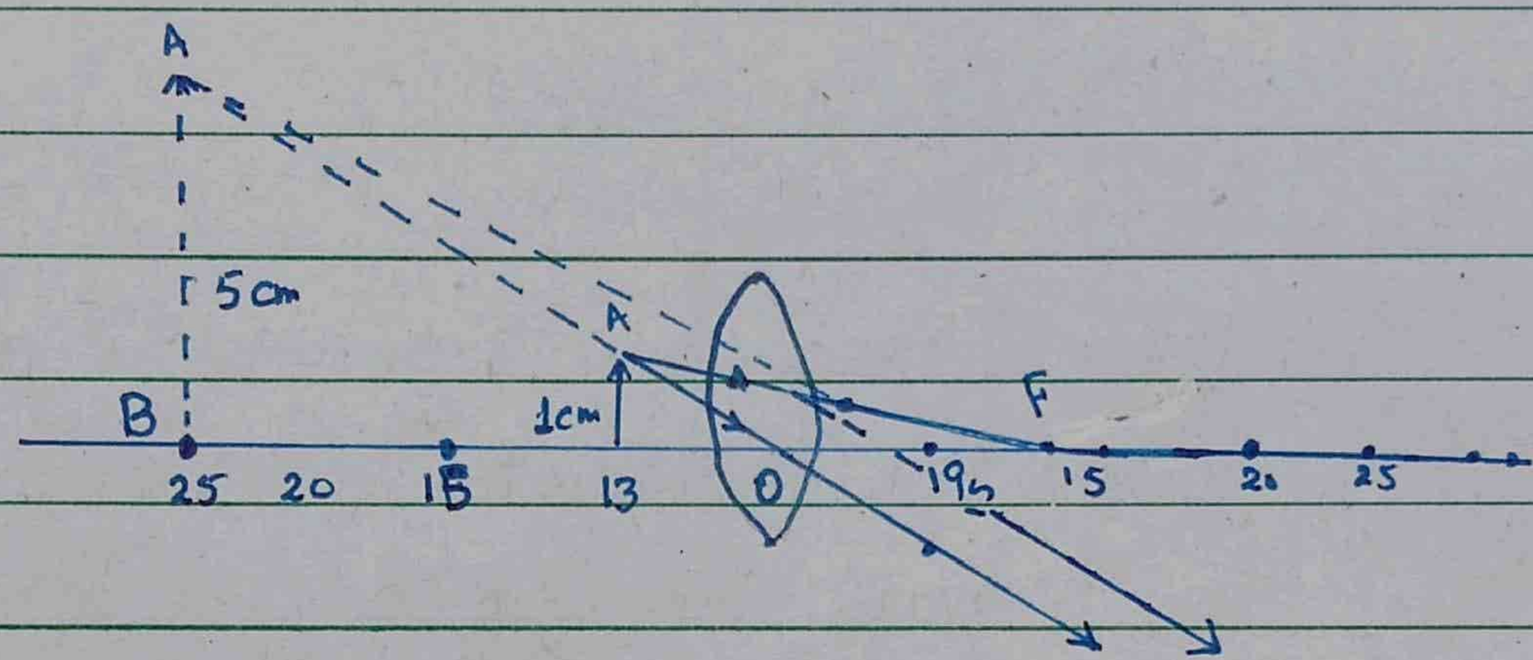
(b) Ray diagram of two biconvex lenses arranged to make a compound microscope.



Compound microscope

Q – 10.6

Answer



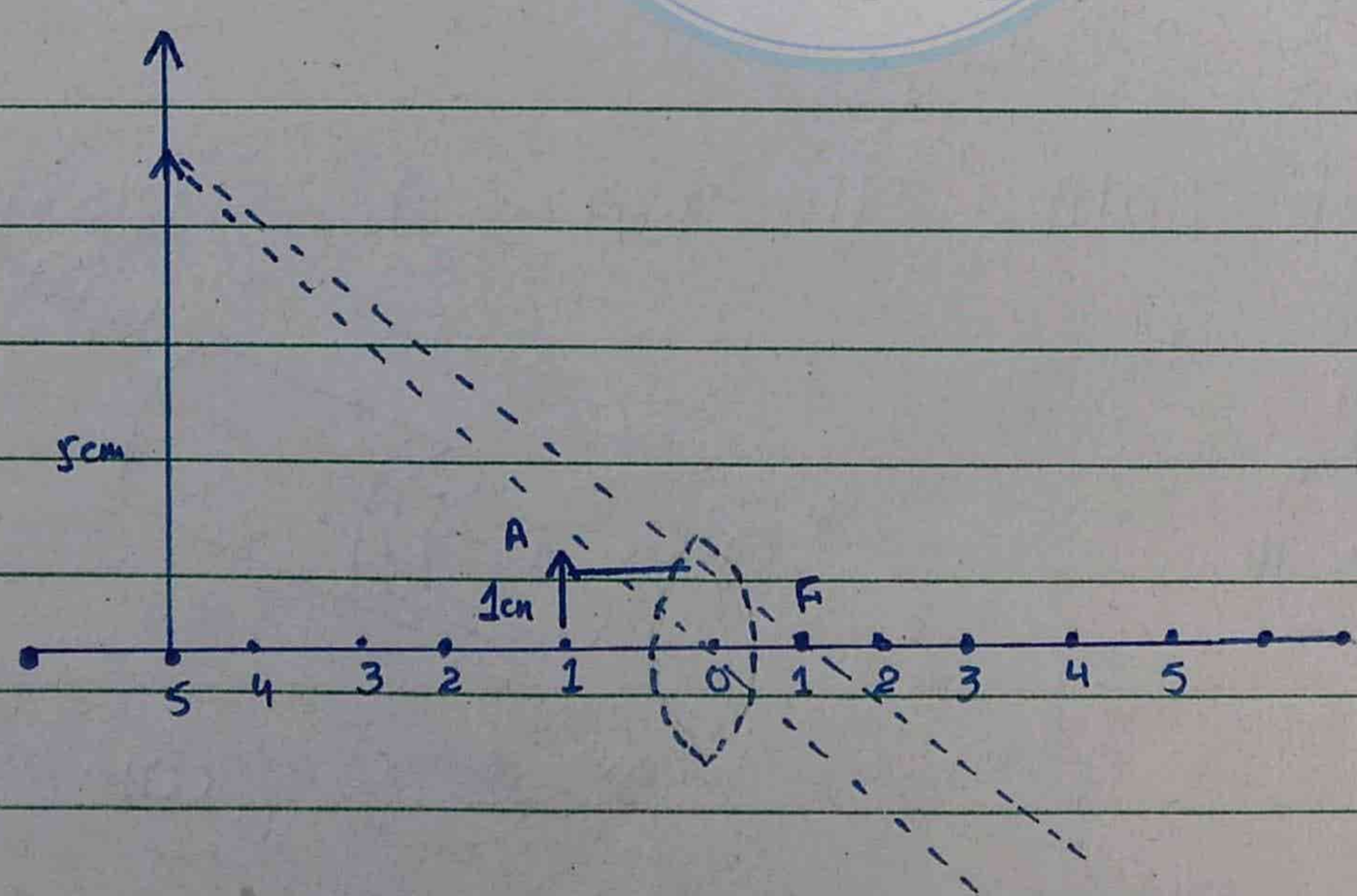
Q – 10.7

Answer

Scale :- along x- axis

5cm = 1cm

25 cm = 5cm



We measure the distance "OF" which is equal to focal length f.
Its value is 6.2 cm
$f = 6.2$ cm

Q - 10.8

Answer

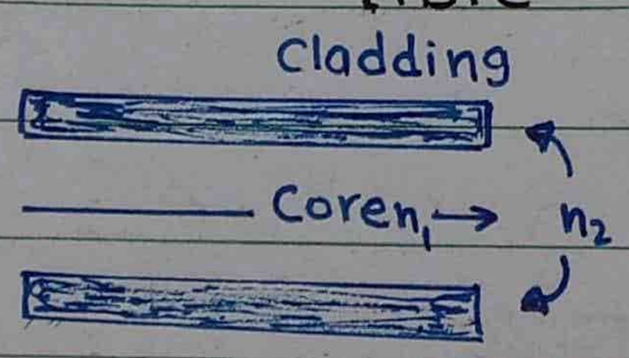
(i) The resolving power of a lens of diameter 'D' is given by 'amin $= \frac{1.22\lambda}{D}$

According to this formula
(i) Part (b) is correct answer
(ii) Part (d) is correct answer

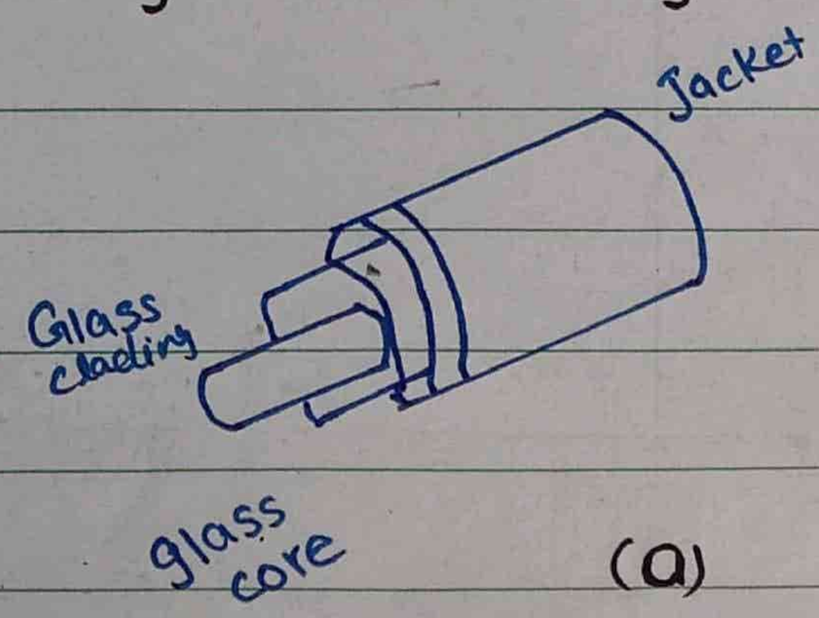
Q - 10.9

Answer

(a) Light path through a single-mode fibre.

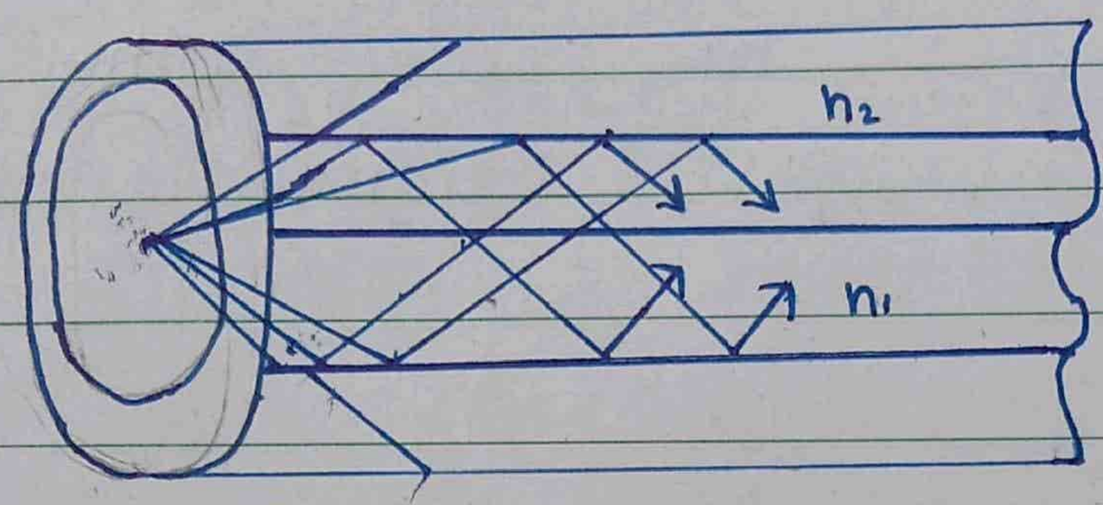


(b)



(a)

(b) Light Phaths through multi-mode fiber



Preference of single-mode fibre in tele-communications

A single mode step index fibre has a very thin core. It requires strong monochromatic light source such as laser light which is used to send light signals through it. Transmission is free from dispersion defects which limit range in multimode fibre. It can carry more than 14 TV channels or 14000 phone calls simultaneously.

Q — 10.10

Answer

An optical fibre comunication system consists of three major components.

(i) A transmitter that conves

electrical signals into light signals.

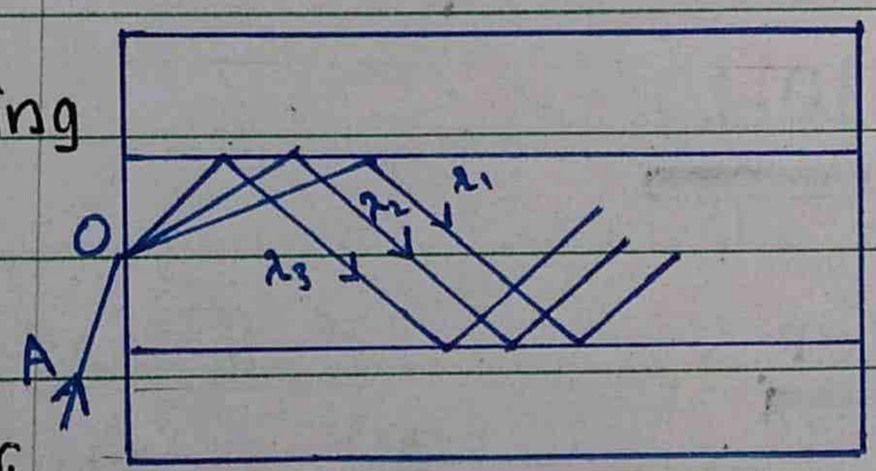
(ii) An optical fibre for guiding the signals by total internal reflection and continous refraction.

(iii) A receiver at the other end which converts light signals to electrical signals.

Q – 10.11

Answer

When the source of light signal is not perfectly monochromatic, then the narrow band of wave-lengths will disperse in different directions. When wavelengths λ_1 λ_2 and λ_3 as shown in fig. As λ_1 meets the core and cladding at the critical angle and λ_2 and λ_3 are at slightly greater angles. The light paths have thus different lengths. In the presence of power losses due to dispersion and spreading of the light signal the information received at the other end of a fibre will be faculty and distorted.



10.3 A telescope is made of an objective of focal length 20 cm and an eye piece of 5.0 cm, both convex lenses. Find the angular magnification.

(Ans: 4.0)

10.4 A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of focal length 5.0 cm. (i) Where is the final image formed? (ii) Calculate the angular magnification.

[Ans: (i) infinity (ii) 20]

10.5 A point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0 cm . A second thin convex lens of focal length 16.0 cm is placed coaxial with the first and 26.0 cm from it on the side away from the object. Find the position of the final image produced by the two lenses.

(Ans: 16 cm from second lens)

10.6 A compound microscope has lenses of focal length 1.0 cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.

(Ans: 8.7 cm, 47)

10.7 Sodium light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective is 0.90 cm, (i) find the limiting angle of resolution, (ii) using visible light of any wavelength, what is the maximum limit of resolution for this microscope.

[Ans: (i) 8.0×10^{-5} rad, (ii) 5.4×10^{-5} rad]

10.8 An astronomical telescope having magnifying power of 5 consist of two thin lenses 24 cm apart. Find the focal lengths of the lenses.

[Ans: 20 cm, 4 cm]

10.9 A glass light pipe in air will totally internally reflect a light ray if its angle of incidence is at least 39°. What is the minimum angle for total internal reflection if pipe is in water? (Refractive Index of water = 1.33)

[Ans: 57°]

10.10 The refractive index of the core and cladding of an optical fibre are 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air of a ray which enters the fibre and is incident at the critical angle on the interface.

[Ans: (i) 61°, (ii) 51°]

Chapter 10

Problems

Problem 10.1

Solution

$f = 5cm$

$d = 25cm$

$p = ?$

$M = ?$

Image at infinity

$M = ?$

(i) $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

(ii) $$M = 1 + \frac{d}{f}$$

(iii) $$M = d/p$$

Since $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{q}$$

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{p} = \frac{1}{5} + \frac{1}{25}$$

$$= \frac{5+1}{25}$$

$$= \frac{6}{25}$$

$$p = \frac{25}{6}$$

$$\boxed{p = 4.2\ cm}$$

(ii) $$M = 1 + \frac{d}{f}$$

$$= 1 + \frac{25}{5}$$

$$= 1 + 5$$

$$\boxed{M = 6}$$

(iii) $$p = f$$

As

$$M = \frac{d}{p}$$

So,

$$M = \frac{d}{f}$$

Putting the values,

$$M = \frac{25}{5}$$

$$\boxed{M = 5}$$

Problem 10.2

Solution

$f_e = ?$

$f_o = 96\ cm$

$d_e = ?$

$d_o = 12cm$

$M = 24$

For a telescope

$$M = \frac{f_o}{f_e}$$

$$f_e = \frac{f_o}{M}$$

$$f_e = \frac{96cm}{24}$$

$$f_e = 4\,cm$$

For the all light rays incident on the objective to pass through the eye-piece condition is

$$\frac{d_o}{d_e} = \frac{f_o}{f_e}$$

$$d_e = d_o \times \frac{f_e}{f_o}$$

$$d_e = 12\,cm \times \frac{4\,cm}{96\,cm}$$

$$d_e = 0.5\,cm$$

Problem 10.3

Solution

$f_o = 20\,cm$

$f_e = 5.0\,cm$

$M = ?$

$$M = \frac{f_o}{f_e}$$

$$M = \frac{20\,cm}{5.0\,cm}$$

$$\boxed{M = 4}$$

Problem 10.4

Solution

$f_o = 100\ cm$

$f_e = 5.0\ cm$

(i) Where the final image formed = ?

(ii) M = ?

(i) For astronomical telescope to be in normal adjustment final image in formed at infinity

(ii) $$M = \frac{f_o}{f_e}$$

$$M = \frac{100cm}{5.0\ cm}$$

$$\boxed{M = 20}$$

Problem 10.5

Solution

$P = 3.6\ cm$

$L = 26\ cm$

$f = 3.0\ cm$

$f_1 = 16.0\ cm$

$q_1 = ?$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Putting the values

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{3} - \frac{1}{3.6}$$

$$\frac{1}{q} = \frac{3.6 - 3}{10.8}$$

$$\frac{1}{q} = \frac{0.6}{10.8}$$

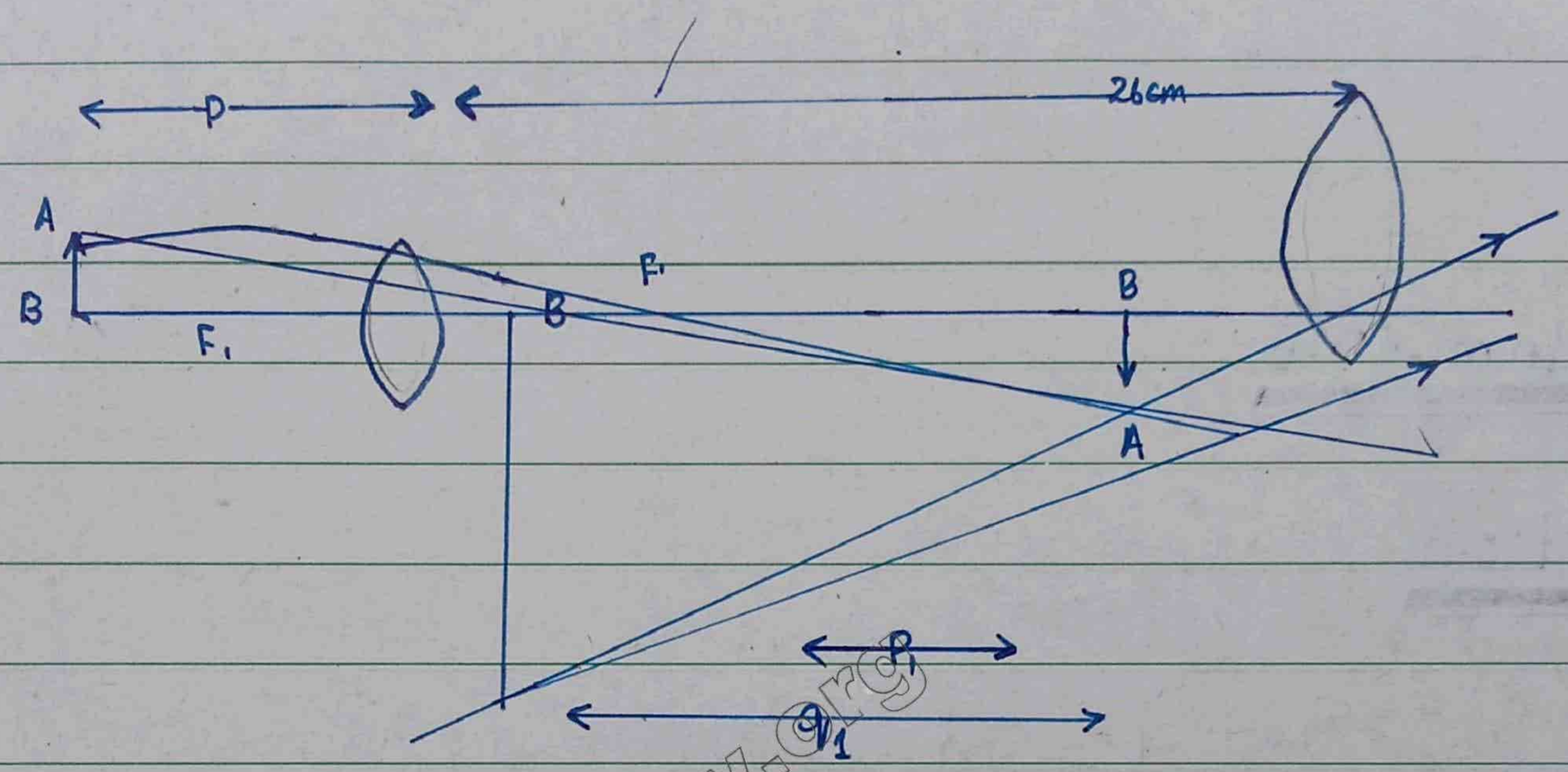
$$q = \frac{10.8}{0.6}$$

$$q = 18\ cm$$

$P_1 = L - q$

$= 26 - 18$

$P_1 = 8\,cm$



$P_1 = 8\,cm$

$f_1 = 16\,cm$

$q_1 = ?$

10th

$$\frac{1}{f_1} = \frac{1}{P_1} + \frac{1}{q_1}$$

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{P_1}$$

$$\frac{1}{q_1} = \frac{1}{16} - \frac{1}{8}$$

$$\frac{1}{q_1} = \frac{8-16}{128}$$

$$\frac{1}{q_1} = \frac{-8}{128}$$

$$\frac{1}{q_1} = \frac{-1}{16}$$

$$\boxed{q_1 = -16 \text{ cm}}$$

Problem-10.6

Solution

$$L = q_o + P_e$$

$$f_o = 1.0 \text{ cm}$$

$$f_e = 3.0 \text{ cm}$$

$$P_o = 1.2 \text{ cm}$$

$$q_o = -25 \text{ cm}$$

-ve sign for virtual image

$$L = ?$$

$$M = ?$$

$$d = 25 \text{ cm}$$

$q_o = ?$

$P_e = ?$

For objective $q_o = ?$

$$\frac{1}{f} = \frac{1}{P_e} + \frac{1}{q_o}$$

$$\frac{1}{1} = \frac{1}{1.2} + \frac{1}{q_o}$$

$$\frac{1}{q_o} = \frac{1}{1} - \frac{1}{1.2}$$

$$\frac{1}{q_o} = \frac{1.2 - 1}{1.2}$$

$$\frac{1}{q_o} = \frac{0.2}{1.2}$$

$$q_o = \frac{1.2}{.2}$$

$$q_o = 6\ cm$$

For Eye-piece $P_e = ?$

$$\frac{1}{f_e} = \frac{1}{P_e} + \frac{1}{q_e}$$

$$\frac{1}{3} = \frac{1}{P_e} + \frac{1}{-25}$$

$$\frac{1}{3} = \frac{1}{P_e} - \frac{1}{25}$$

$$\frac{1}{P_e} = \frac{1}{3} + \frac{1}{25}$$

$$\frac{1}{P_e} = \frac{25+3}{3 \times 25}$$

$$P_e = \frac{75}{28}$$

$$P_e = 2.678 \text{ cm}$$

$$P_e = 2.7 \text{ cm}$$

$$L = q_o + P_e$$

$$L = 6\text{cm} + 2.7\text{cm}$$

$$\boxed{L = 8.7 \text{ cm}}$$

$$M = \frac{a}{P}\left(1 + \frac{d}{f_e}\right)$$

$$M = \frac{a_o}{P_o}\left(1 + \frac{d}{f_e}\right)$$

$$M = \frac{6\,\cancel{cm}}{1.2\,\cancel{cm}}\left(1 + \frac{25\,\cancel{cm}}{3\,\cancel{cm}}\right)$$

$$M = 5\,(1 + 8.33)$$

$$= 5\,(9.33)$$

$$M = 46.7$$

$$\boxed{M \simeq 47}$$

Problem 10.7

Solution

$\lambda = 589 \text{ nm}$

$\lambda = 589 \times 10^{-9} \text{ m}$

$D = 0.90 \text{ cm}$

$D = \frac{0.90}{100} \text{ m}$

$D = .9 \times 10^{-2} \text{ m}$

(i) $\alpha_{min} = ?$

(ii) $\alpha'_{min} = ?$

$\alpha_{min} = 1.22 \frac{\lambda}{D}$

$\alpha_{min} = 1.22 \times \frac{589 \times 10^{-9}}{0.90 \times 10^{-2}}$

$\alpha_{min} = 7.9 \times 10^{-9} \text{ rad}$

$\boxed{\alpha_{min} \cong 8 \times 10^{-5} \text{ rad}}$

For maximum limit of resolution α'_{min} the shortest wavelength of visible light should be used. It is

$$\lambda = 400 \text{ nm}$$

$$\lambda = 400 \times 10^{-9} \text{ m}$$

For vivlel colour

$$\alpha'_{min} = 1.22 \frac{\lambda}{D}$$

$$= 1.22 \frac{400 \times 10^{-9}}{0.9 \times 10^{-2} \text{ m}}$$

$$\boxed{\alpha'_{min} = 5.4 \times 10^{-5} \text{ rad}}$$

Problem 10.8

Solution

$$M = 5$$

$$L = 24 \text{ cm}$$

$f_o = ?$

$f_e = ?$

$L = f_o + f_e$

$24 = f_o + f_e \longrightarrow (1)$

$M = \frac{f_o}{f_e}$

$5 = \frac{f_o}{f_e}$

$5 f_e = f_o$

Put $f_o = 5 f_e$ in eq (1)

$24 = 5 f_e + f_e$

$24 = 6 f_e$

$f_e = \frac{24}{6}$

$\boxed{f_e = 4\,cm}$

$f_e = 4\ cm$

$f_o = 5 f_e$

$f_o = 5 \times 4\ cm$

$$\boxed{f_o = 20\ cm}$$

Problem 10.9

Solution

Angle of incidence for glass $\theta_c = 39^\circ$

Angle of incidence for glass = ?

$$n = \frac{1}{\sin\theta_c}$$

$\because \sin 39^\circ = 0.629$

$$n = \frac{1}{0.629}$$

$$n = 1.59$$

Using Shell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 = 1.59$$

$$n_2 = 1.33$$

Putting values

$$\sin \theta_1 = \frac{1.33 \times 1}{1.59}$$

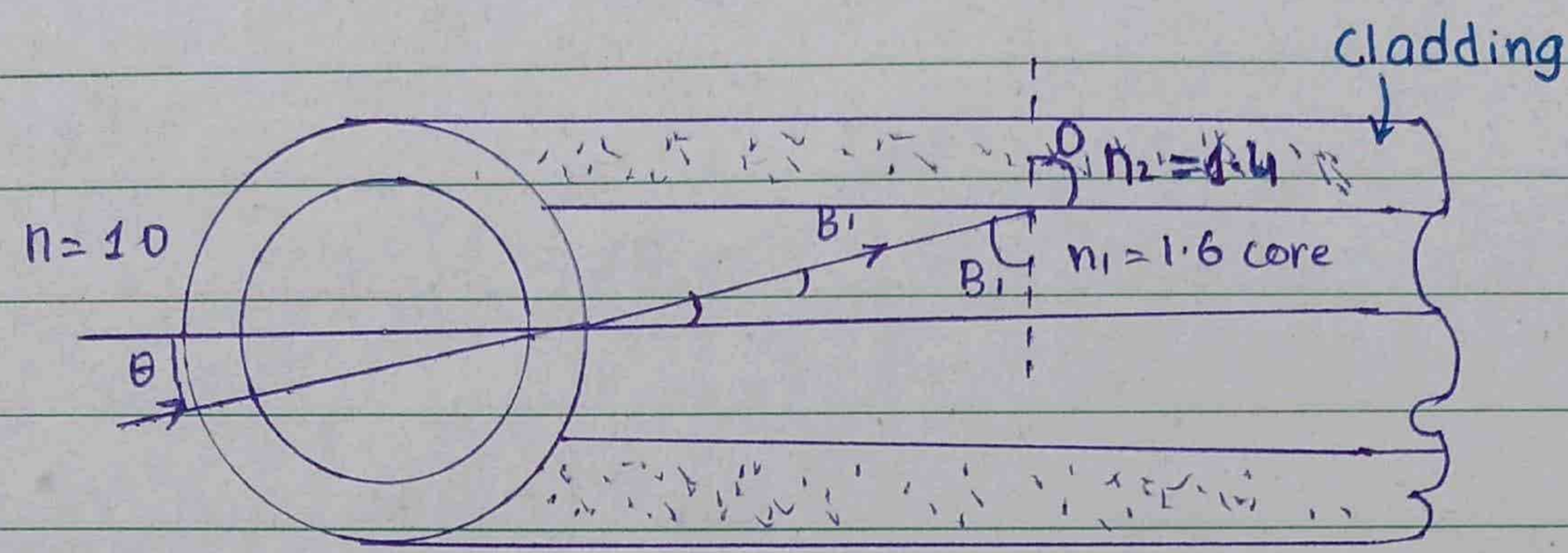
$$\sin \theta_1 = 0.84$$

$$\theta_1 = \sin^{-1}(0.84)$$

$$\boxed{\theta_1 = 57^\circ}$$

Problem 10.10

Solution



$n_1 = 1.6$ for core

$n_2 = 1.4$ for cladding

(i) $\theta = ?$

(ii) $\theta = ?$ ← Maximum Angle of incidence in air

Using Snell's law (core - cladding boundry)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

core ↗ ↖ cladding

$$\theta_1 = \theta_c = ?$$

$$\theta_2 = 90^\circ$$

$$1.6 \sin\theta_c = 1.4 \times \sin 90^\circ$$

$$\sin\theta_c = \frac{1.4}{1.6} \times 1$$

$$\sin\theta_c = 0.875$$

$$\theta_c = \sin^{-1}(0.875)$$

$$\theta_c = 61.04^\circ$$

$$\boxed{\theta_c = 61^\circ}$$

$$\theta_1 + \theta' = 90^\circ$$

$$\theta_c + \theta' = 90^\circ$$

$$61^\circ + \theta' = 90^\circ$$

$$\theta' = 90^\circ - 61$$

$$\theta' = 29^\circ$$

Now for air to core

air ↘ $n \sin\theta = n_1 \sin\theta'$ core

$1 \times \sin\theta = 1.6 \times \sin 29^\circ$

$\sin\theta = 1.6 \times (0.484)$

$\sin\theta = 0.77568$

$= 0.776$

$\theta = \sin^{-1}(0.776)$

$\theta = 50.89^\circ$

$\theta = 51^\circ$

Chapter 10

Examples

Example 10.1

Solution

$$f_o = 10 \text{ mm}$$

$$f_e = 25.0 \text{ mm}$$

$$P = 10.5 \text{ m}$$

$$L = P + q = ?$$

$$M = ?$$

$$\frac{1}{f} = \frac{1}{P} + \frac{1}{q}$$

$$\frac{1}{f_o} = \frac{1}{P} + \frac{1}{q}$$

Putting the value

$$\frac{1}{10} = \frac{1}{10.5} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{10} - \frac{1}{10.5}$$

$$\frac{1}{q} = \frac{10.5 - 10}{105}$$

$$\frac{1}{q} = \frac{0.5}{105}$$

$$q = \frac{105}{0.5}$$

$$q = 210 \text{ mm}$$

$$d = -250 \text{ mm}$$

Using formula

$$\frac{1}{f_e} = \frac{1}{p} + \frac{1}{q}$$

$$L = 22.7 + 210$$

$$= 232.7 \text{ mm}$$

$$\boxed{L \approx 233 \text{ mm}}$$

$$M_1 = \frac{q}{p}$$

$$= \frac{210}{10.5}$$

$$M_1 = 20$$

$$M_2 = \frac{q}{p}$$

$$= \frac{-250}{22.7}$$

$$M_2 = -11.0$$

$$M = m_1 \times m_2$$

$$= 20 \times (-11.0)$$

$$\boxed{M = -220}$$

as

$$q = d$$

$$= -250 \text{ mm}$$

Putting Values

$$\frac{1}{25} = \frac{1}{P} + \frac{1}{250}$$

$$\frac{1}{25} = \frac{1}{P} - \frac{1}{250}$$

$$\frac{1}{P} = \frac{1}{25} + \frac{1}{250}$$

$$\frac{1}{P} = \frac{250+25}{6250}$$

$$\frac{1}{P} = \frac{275}{6250}$$

$$\therefore \quad P = 22.7 \text{ mm}$$

$$L = P + q$$

Example 10.2

Solution

$n_1 = 1.50$

$n_2 = 1.48$

$\theta_c = ?$

$\theta = ?$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Using Snell's Law

$\theta_1 = \theta_c$

$\theta_2 = 90^\circ$

Putting values

$1.50 \sin\theta_c = 1.48 \times 1$

$\sin\theta_c = \frac{1.48 \times 1}{1.50}$

$\sin\theta_c = 0.987$

$$\theta_c = \sin^{-1}(0.987)$$

$$= 80.6^\circ$$

$$\boxed{\theta_c = 80.6}$$

$$\theta' = 90^\circ - \theta_c$$

$$\theta' = 90^\circ - 80.6^\circ$$

$$\theta' = 9.4^\circ$$

Using Snell's Law

$$n_1 \sin\theta' = n \sin\theta$$

$$n = 1.0$$

Putting values

$$1.50 \sin 9.4^\circ = 1.0 \sin\theta$$

$$1.50 \times 0.163 = 1.0 \sin\theta$$

$$\sin\theta = 1.50 \times 1.63$$

$$= 0.2445$$

$$\theta = \sin^{-1}(0.2445)$$

$$\boxed{\theta\ 14.2^\circ}$$