

Find the periods of functions

Question # 1

$$\sin 3x$$

As $\sin$ is a periodic function and its period is 2π

$$\sin 3x = \sin (3x + 2\pi)$$

$$\sin 3x = \sin 3\left(x + \frac{2\pi}{3}\right)$$

$$\sin 3x = \sin 3\left(x + \frac{2\pi}{3}\right)$$

Hence, $\frac{2\pi}{3}$ is a period of $\sin 3x$.

=====

(3)

$$\tan 4x$$

As $\tan$ is a periodic function and its period is π

$$\tan 4x = \tan (4x + \pi)$$

$$\tan 4x = \tan 4\left(x + \frac{\pi}{4}\right)$$

Hence, $\frac{\pi}{4}$ is a period of $\tan 4x$.

=====

(5)

$$\sin \frac{x}{3}$$

As $\sin$ is a periodic function and its period is 2π

$$\cos 2x$$

As $\cos$ is a periodic function and its period is 2π

$$\cos 2x = \cos (2x + 2\pi)$$

$$\cos 2x = \cos 2\left(x + \frac{2\pi}{2}\right)$$

$$\cos 2x = \cos 2(x + \pi)$$

Hence, π is a period of $\cos 2x$.

=====

(4)

$$\cot \frac{x}{2}$$

As $\cot$ is a periodic function and its period is π

$$\cot \frac{x}{2} = \cot \left(\frac{x}{2} + \pi\right)$$

$$\cot \frac{x}{2} = \cot \left(\frac{x + 2\pi}{2}\right) \Rightarrow \cot \frac{1}{2}(x + 2\pi)$$

Hence, 2π is a period of $\cot \frac{x}{2}$

=====

(6)

$$\operatorname{cosec} \frac{x}{4}$$

As cosec is a periodic function and its period is 2π

$$\sin \frac{x}{3} = \sin \left(\frac{x}{3} + 2\pi \right)$$

$$\sin \frac{x}{3} = \sin \left(\frac{x+6\pi}{3} \right)$$

$$\sin \frac{x}{3} = \sin \frac{1}{3}(x+6\pi)$$

Hence, 6π is a period of $\sin \frac{x}{3}$

(7)

$$\sin \frac{x}{5}$$

As $\sin$ is a periodic function and its period is 2π

$$\sin \frac{x}{5} = \sin \left(\frac{x}{5} + 2\pi \right)$$

$$\sin \frac{x}{5} = \sin \left(\frac{x+10\pi}{5} \right)$$

$$\sin \frac{x}{5} = \sin \frac{1}{5}(x+10\pi)$$

Hence, 10π is a period of $\sin \frac{x}{5}$

(9)

$$\tan \frac{x}{7}$$

As $\tan$ is periodic function and its period is π

$$\tan \frac{x}{7} = \tan \left(\frac{x}{7} + \pi \right)$$

$$\tan \frac{x}{7} = \tan \frac{1}{7}(x+7\pi)$$

Hence, 7π is a period of $\tan \frac{x}{7}$

(11)

$$\sec 9x$$

As $\sec$ is a periodic function and its period is 2π

$$\sec 9x = \sec(9x+2\pi)$$

$$\sec 9x = \sec 9\left(x+\frac{2\pi}{9}\right)$$

Hence, $\frac{2\pi}{9}$ is a period of $\sec 9x$.

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \left(\frac{x}{4} + 2\pi \right)$$

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \left(\frac{x+8\pi}{4} \right)$$

$$\operatorname{cosec} = \operatorname{cosec} \frac{1}{4}(x+8\pi)$$

Hence, 8π is a period of $\operatorname{cosec} \frac{x}{4}$

(8)

$$\cos \frac{x}{6}$$

As $\cos$ is a periodic function and its period is 2π

$$\cos \frac{x}{6} = \cos \left(\frac{x}{6} + 2\pi \right)$$

$$\cos \frac{x}{6} = \cos \left(\frac{x+12\pi}{6} \right)$$

$$\cos \frac{x}{6} = \cos \frac{1}{6}(x+12\pi)$$

Hence, 12π is a period of $\cos \frac{x}{6}$

(10)

$$\cot 8x$$

As $\cot$ is a periodic function and its period is π

$$\cot 8x = \cot(8x+\pi)$$

$$\cot 8x = \cot 8\left(x+\frac{\pi}{8}\right)$$

Hence, $\frac{\pi}{8}$ is a period of $\cot 8x$.

(12)

$$\operatorname{cosec} 10x$$

As cosec is a periodic function and its period is 2π

$$\operatorname{cosec} 10x = \operatorname{cosec}(10x+2\pi)$$

$$\operatorname{cosec} 10x = \operatorname{cosec} 10\left(x+\frac{\pi}{5}\right)$$

Hence, $\frac{\pi}{5}$ is a period of $\operatorname{cosec} 10x$

(13)

$3\sin x$

As $\sin$ is a periodic function and its period is 2π .

$$3\sin x = 3\sin(x+2\pi)$$

Hence, 2π is a period of $3\sin x$.

(15)

$3\cos \frac{x}{5}$

As $\cos$ is a periodic function and its period is 2π .

$$3\cos \frac{x}{5} = 3\cos\left(\frac{x}{5} + 2\pi\right)$$

$$3\cos \frac{x}{5} = 3\cos\left(\frac{x+10\pi}{5}\right)$$

$$3\cos \frac{x}{5} = 3\cos \frac{1}{5}(x+10\pi)$$

Hence, 10π is a period of $3\cos \frac{x}{5}$

(17)

$3\sin \frac{2x}{5}$

As, $\sin$ is a periodic function and its period is 2π .

$$3\sin \frac{2x}{5} = 3\sin\left(\frac{2x}{5} + 2\pi\right)$$

$$3\sin \frac{2x}{5} = 3\sin\left(\frac{2x+10\pi}{5}\right)$$

$$3\sin \frac{2x}{5} = 3\sin \frac{2}{5}(x+5\pi)$$

Hence, 5π is a period of $3\sin \frac{2x}{5}$

(19)

Write the Do-

$y = \sin x$

$$\text{Domain} = -\infty < x < +\infty$$

$$\text{Range} = -1 \leq y \leq 1$$

(14)

$2\cos x$

As $\cos$ is a periodic function and its period is 2π .

$$2\cos x = 2\cos(x+2\pi)$$

Hence, 2π is a period of $2\cos x$.

(16)

$\cos \frac{x}{5}$

Hence, $\cos$ is a periodic function and its period is 2π .

$$\cos \frac{x}{5} = \cos\left(\frac{x}{5} + 2\pi\right)$$

$$\cos \frac{x}{5} = \cos\left(\frac{x+10\pi}{5}\right)$$

$$\cos \frac{x}{5} = \cos \frac{1}{5}(x+10\pi)$$

Hence, 10π is a period of $\cos \frac{x}{5}$.

(18)

$\tan \frac{x}{3}$

As $\tan$ is a periodic function and its period is π .

$$\tan \frac{x}{3} = \tan\left(\frac{x}{3} + \pi\right)$$

$$\tan \frac{x}{3} = \tan\left(\frac{x+3\pi}{3}\right)$$

$$\tan \frac{x}{3} = \tan \frac{1}{3}(x+3\pi)$$

Hence, 3π is a period of $\tan \frac{x}{3}$.

(20)

main and Range:

$y = \cos x$

$$\text{Domain} = -\infty < x < +\infty$$

$$\text{Range} = -1 \leq y \leq 1$$

(21)

$$y = \tan x$$

$$\text{Domain} = -\infty < x < +\infty, x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$$

$$\text{Range} = -\infty < y < +\infty$$

(22)

$$y = \operatorname{cosec} x$$

$$\text{Domain} = -\infty < x < +\infty, x \neq n\pi, n \in \mathbb{Z}$$

$$\text{Range} = y \geq 1 \text{ or } y \leq -1$$

(23)

Define periodicity:

All the six trigonometric functions repeat their values for each increase or decrease of 2π in θ . This behaviour of trigonometric function is called periodicity.

(24)

Define period of function:

Period of trigonometric function is a smallest positive number which, when added to the original circular measure of the angle give the same value of the function.

(25)

Find range and period of $3\sin x$.

Period: As $\sin$ is a periodic function so its period is 2π .

$$3\sin x = 3\sin(x + 2\pi)$$

$3\sin x$ have 2π period.

Range: $3\sin x = -1 \leq y \leq 1$

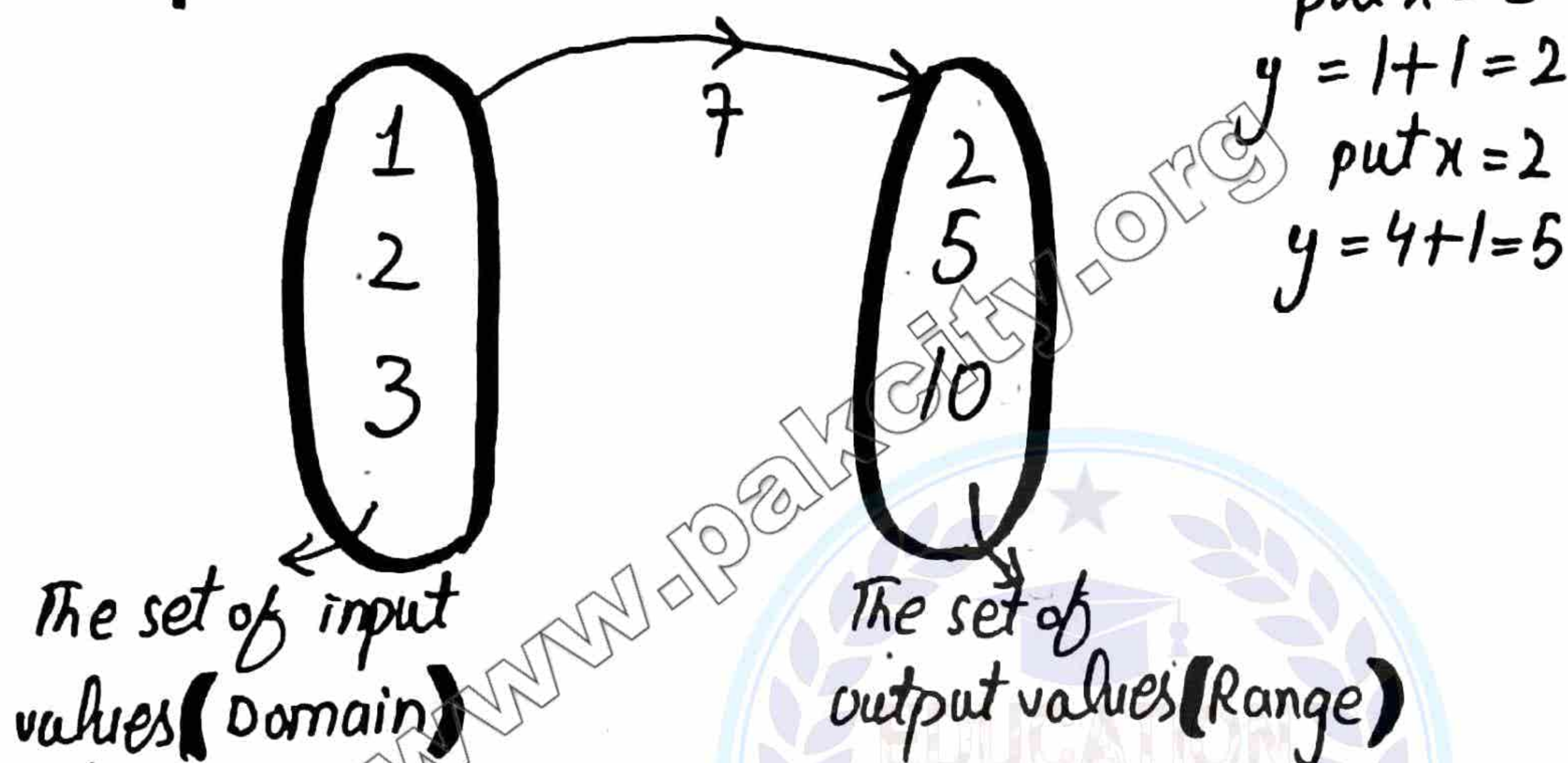
Domain of a function:

Domain is a set of all possible Input values of the function OR the values that x can take is called domain of function.

Range of function:

Range is the set of the all possible outputs values OR The values that y can take is called range of a function.

Example:-



$$y = x^2 + 1$$

put $x = 1$
 $y = 1 + 1 = 2$

put $x = 2$
 $y = 4 + 1 = 5$

put $x = 3$
 $y = 9 + 1 = 10$

Graph of sin function

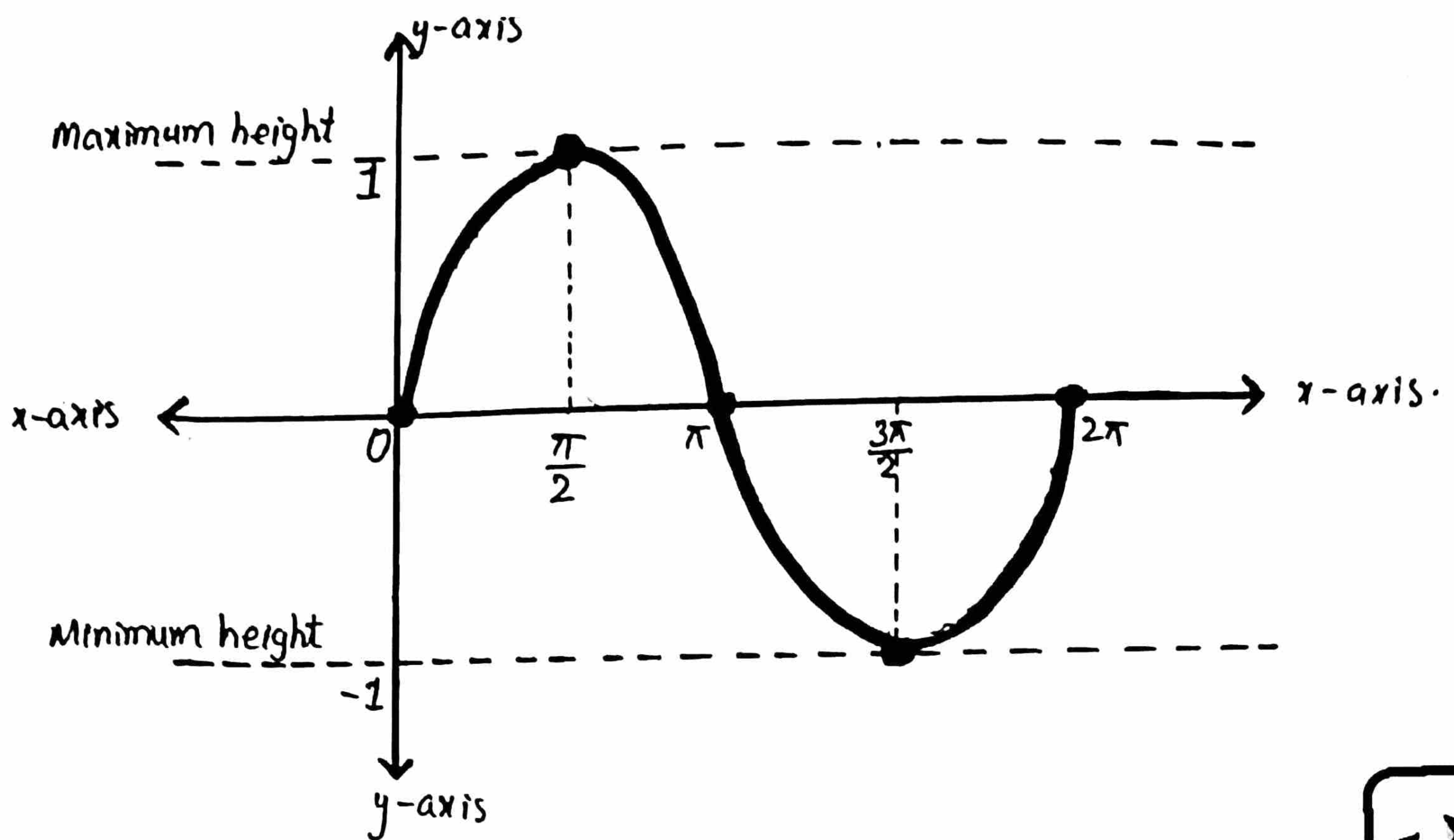
Graph of $y = \sin x$ from 0° To 360°

x	0	90°	180°	270°	360°
y	0	1	0	-1	0

Domain = set of all real numbers $\mathbb{R}$ or $(-\infty, +\infty)$.

Range = $-1 \leq y \leq 1$ or $[-1, 1]$ → closed Bracket

parenthesis

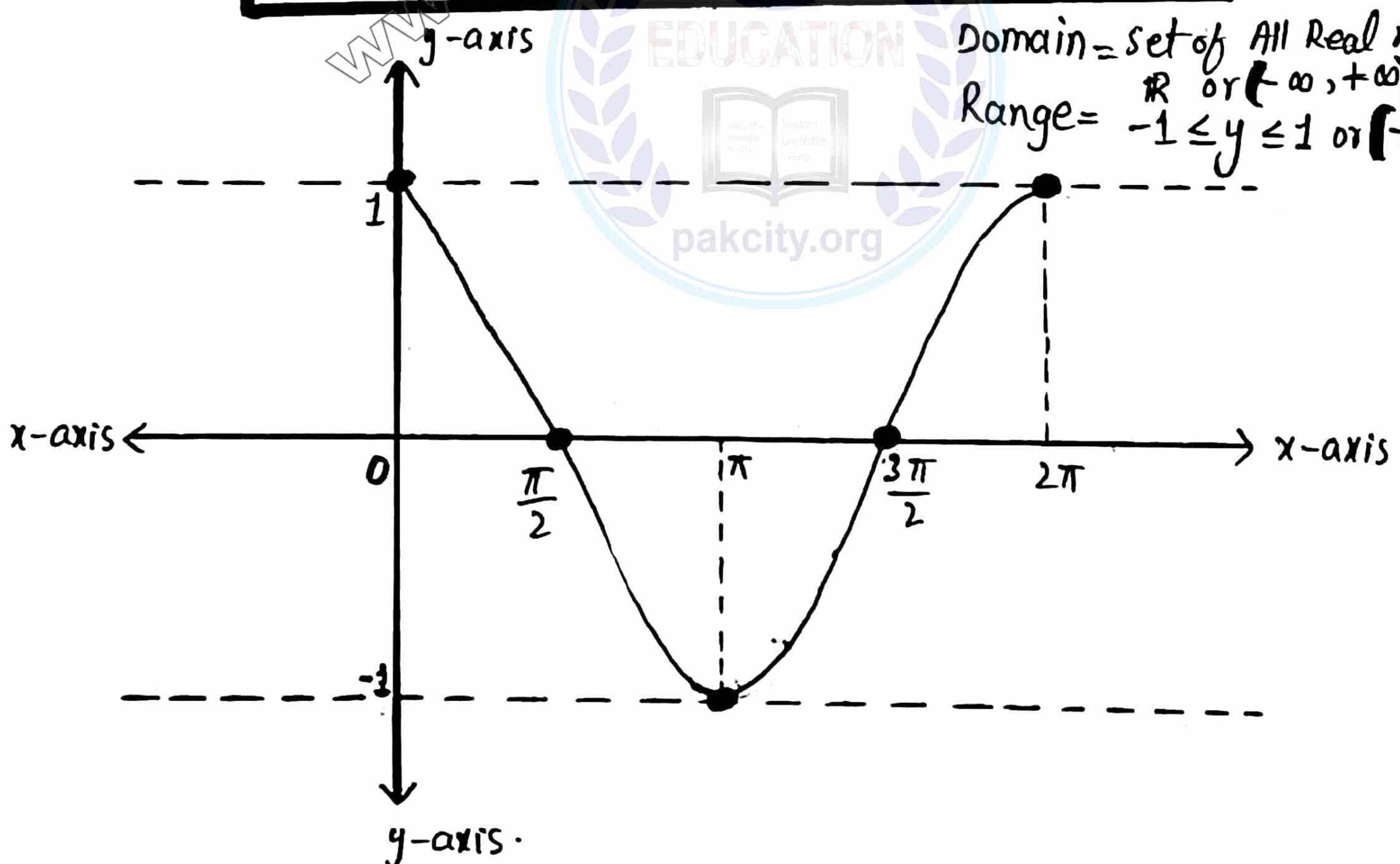


Graph of cosine function

Graph of $y = \cos x$ from 0° to 360° .

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x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	-1	0	1

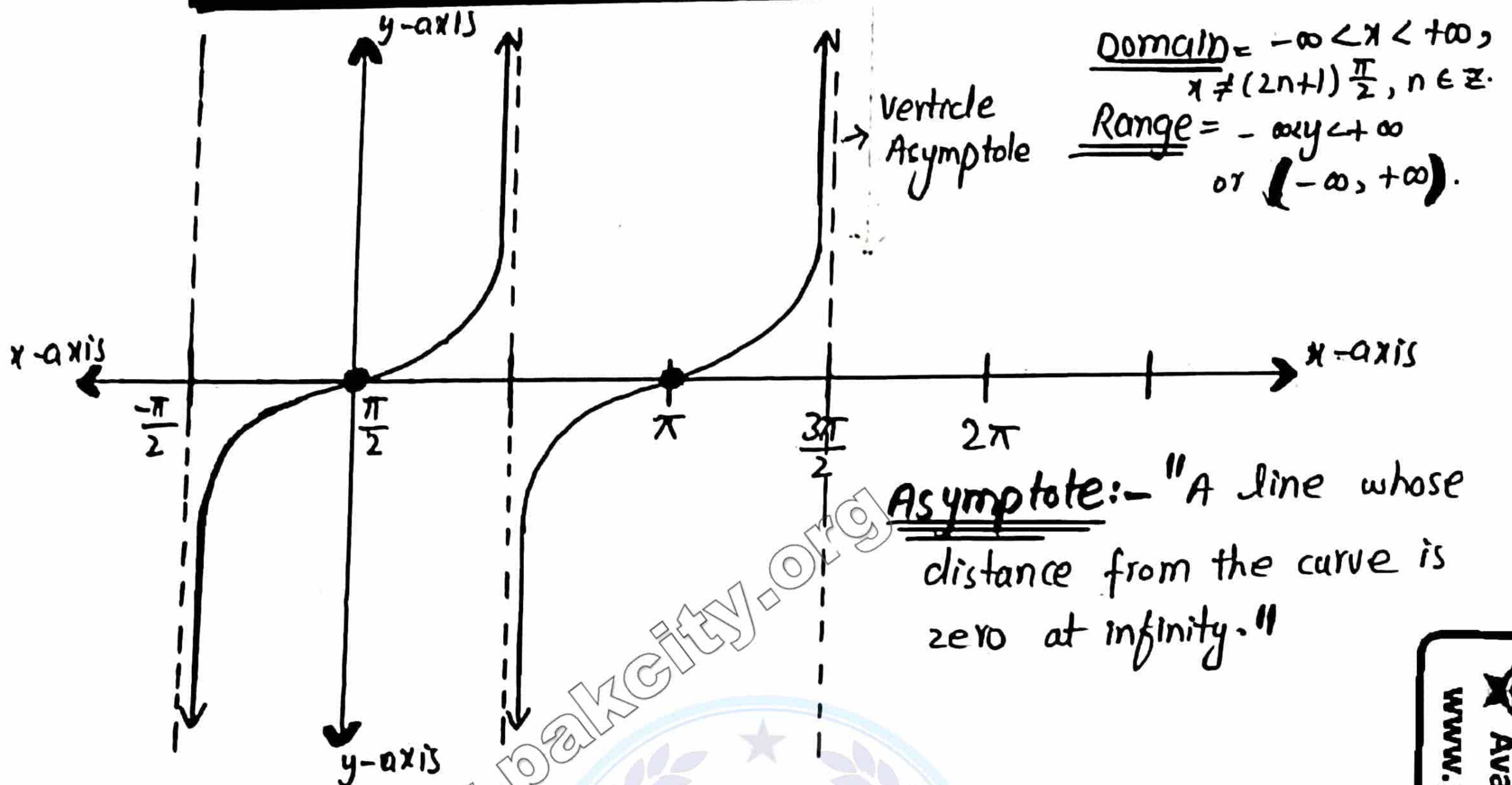


Domain = set of All Real numbers
 $\mathbb{R}$ or $(-\infty, +\infty)$
Range = $-1 \leq y \leq 1$ or $[-1, 1]$

Graph of Tangent function

Graph of $y = \tan x$ from 0 to 360° .

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	∞	0	$-\infty$	0



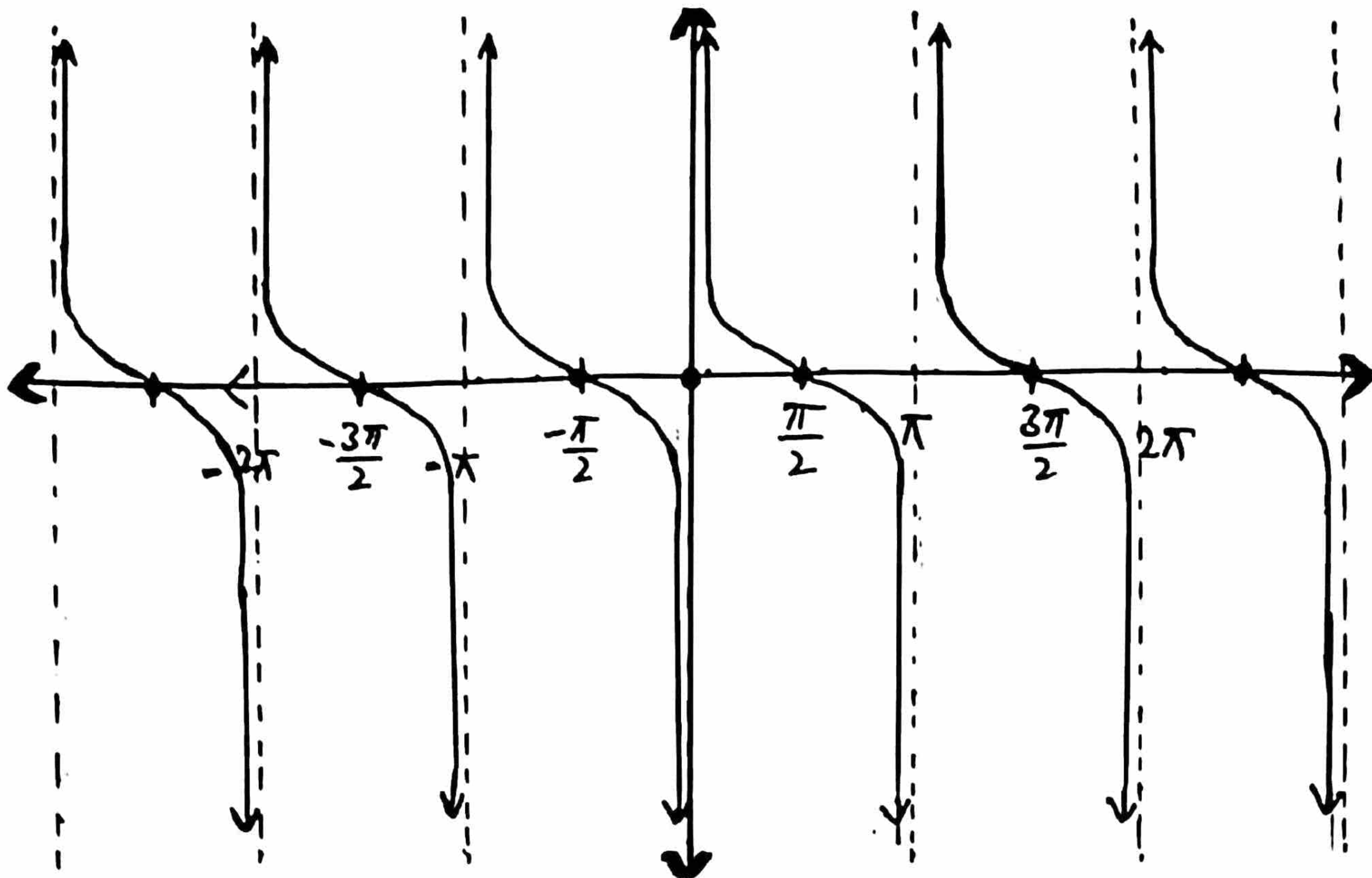
Graph of Cotangent function

Graph of $y = \cot x$ from -2π To 2π

x	0	$\pi/2$	π	$3\pi/2$	2π	$-\pi/2$	$-\pi$	$-3\pi/2$	-2π
y	∞	0	∞	0	∞	0	∞	0	∞

Domain = $-\infty < x < +\infty$, $x \neq n\pi, n \in \mathbb{Z}$

Range = $-\infty < y < +\infty$ or $(-\infty, +\infty)$



Graph of Cosecant function

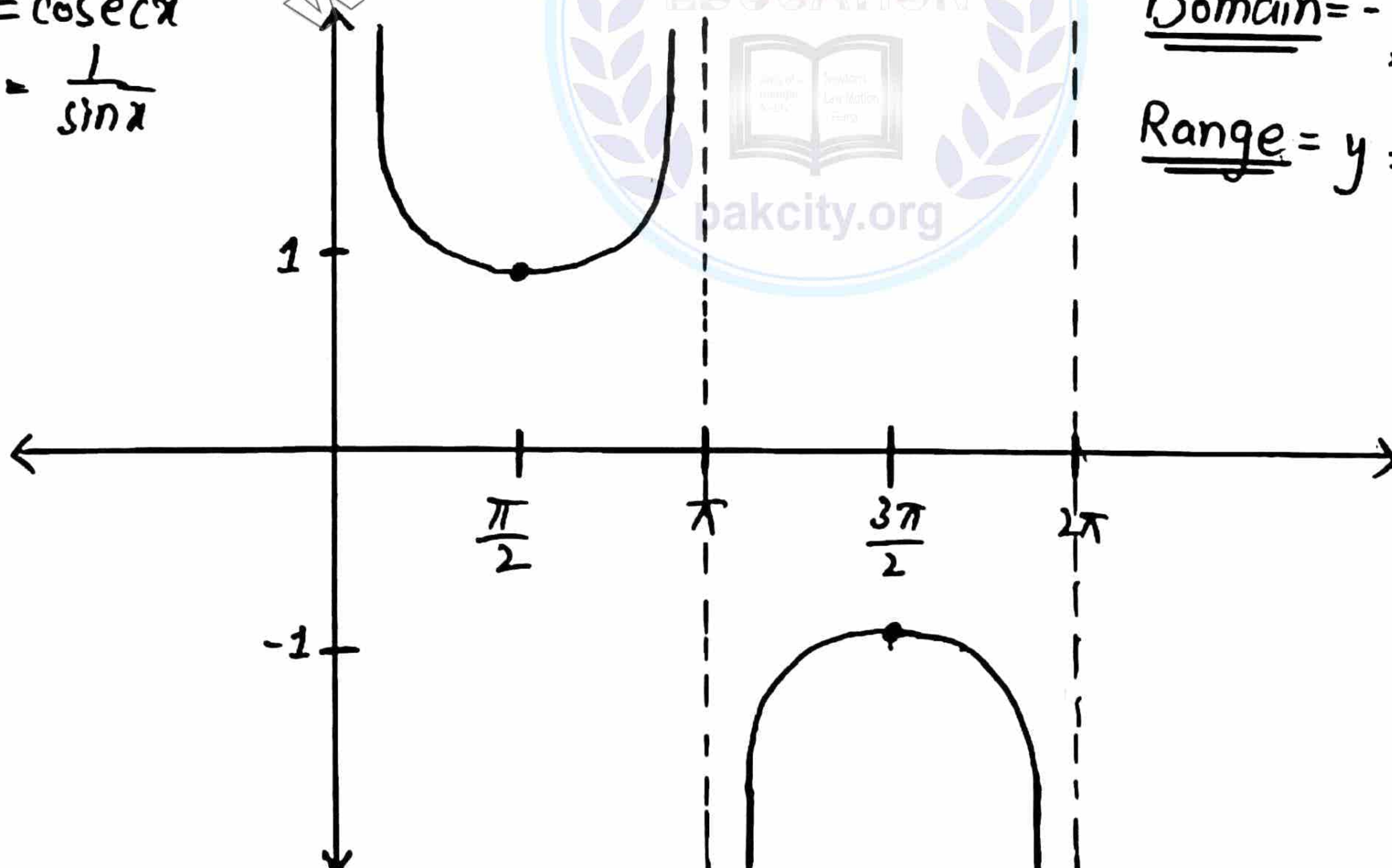
Graph of $y = \operatorname{cosec} x$ from 0 to 2π

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x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	∞	1	∞	-1	∞

$$y = \operatorname{cosec} x$$

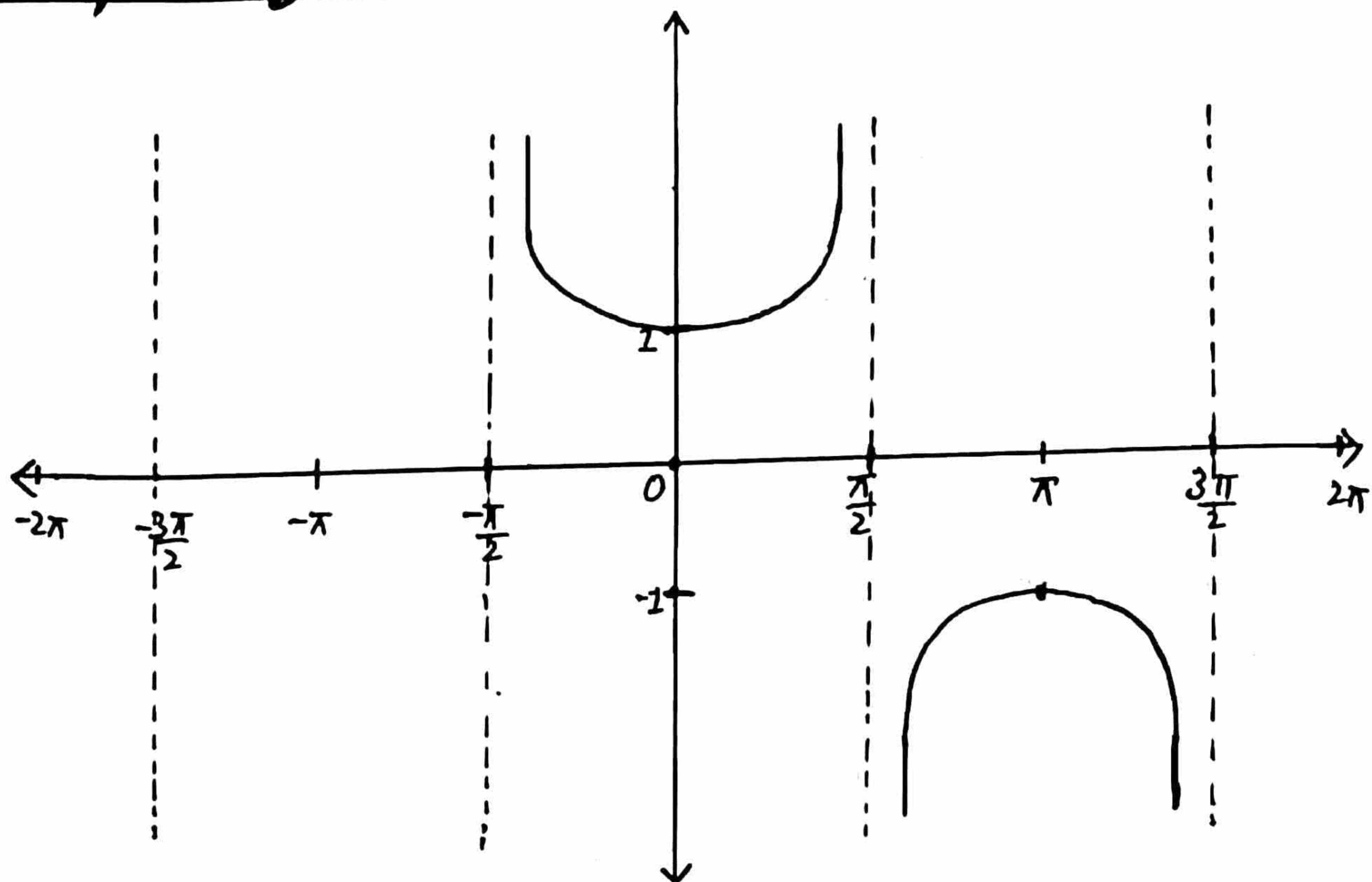
$$y = \frac{1}{\sin x}$$



Domain = $-\infty < x < +\infty$
 $x \neq n\pi$
 $n \in \mathbb{Z}$

Range = $y \geq 1$ or $y \leq -1$.

Graph of secant function:



Domain:- $-\infty < x < +\infty$, $x \neq (2n+1)\frac{\pi}{2}$.

Range:- $y \geq 1$ or $y \leq -1$.

Note:-

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✦ As we shall see the graphs of trigonometric functions will be smooth curves and none of them will be line segments or will be have sharp corners or breaks within their domains. The behaviour of the curve is called continuity. It means that the trigonometric functions are continuous, wherever, they are defined. Moreover, as the trigonometric functions are periodic so their curves repeat after a fixed intervals.

✦ The dashed lines are vertical Asymptotes in the graphs of $\tan x$, $\cot x$, $\sec x$ and $\csc x$.

✦ From the graphs of trigonometric functions we can check

their domains and ranges

★ Trigonometric functions are periodic functions
periodic functions because they repeat their values after fixed intervals

(Function)

empty sets such that:

let A and B be two non-

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(i) Domain $f = A$.

(ii) First element of no two pairs of f are equal, then f is said to be a function from A to B .

Function f is also written as: $f: A \rightarrow B$.
which is read a function from A to B .

Example:-

put $x = 1$

$$y = 1^2 = 1$$

Put $x = 2$.

$$y = 2^2 = 4.$$

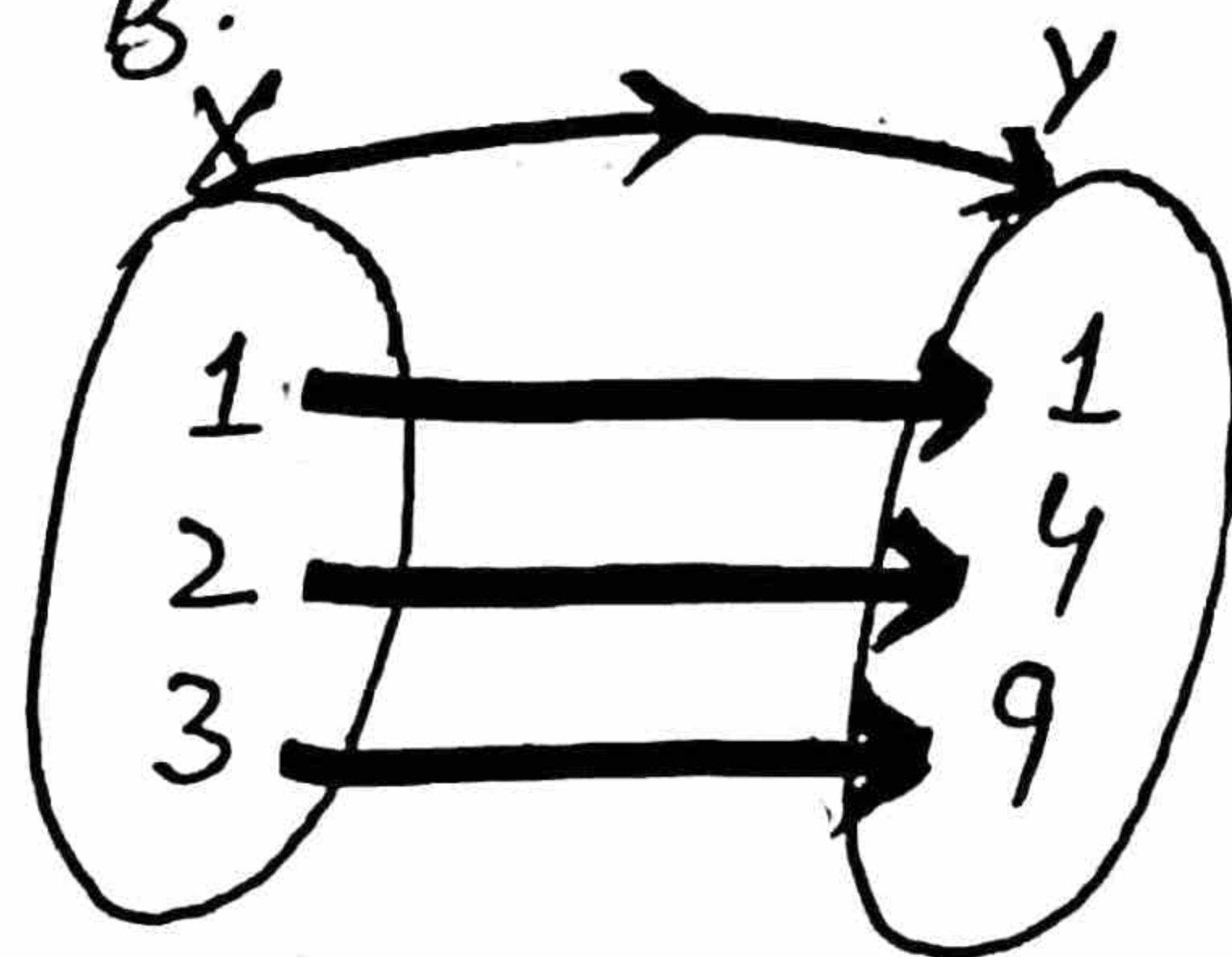
$$y = x^2$$

put $x = 3$

$$y = 9$$

Put $x = 4$

$$y = 16.$$

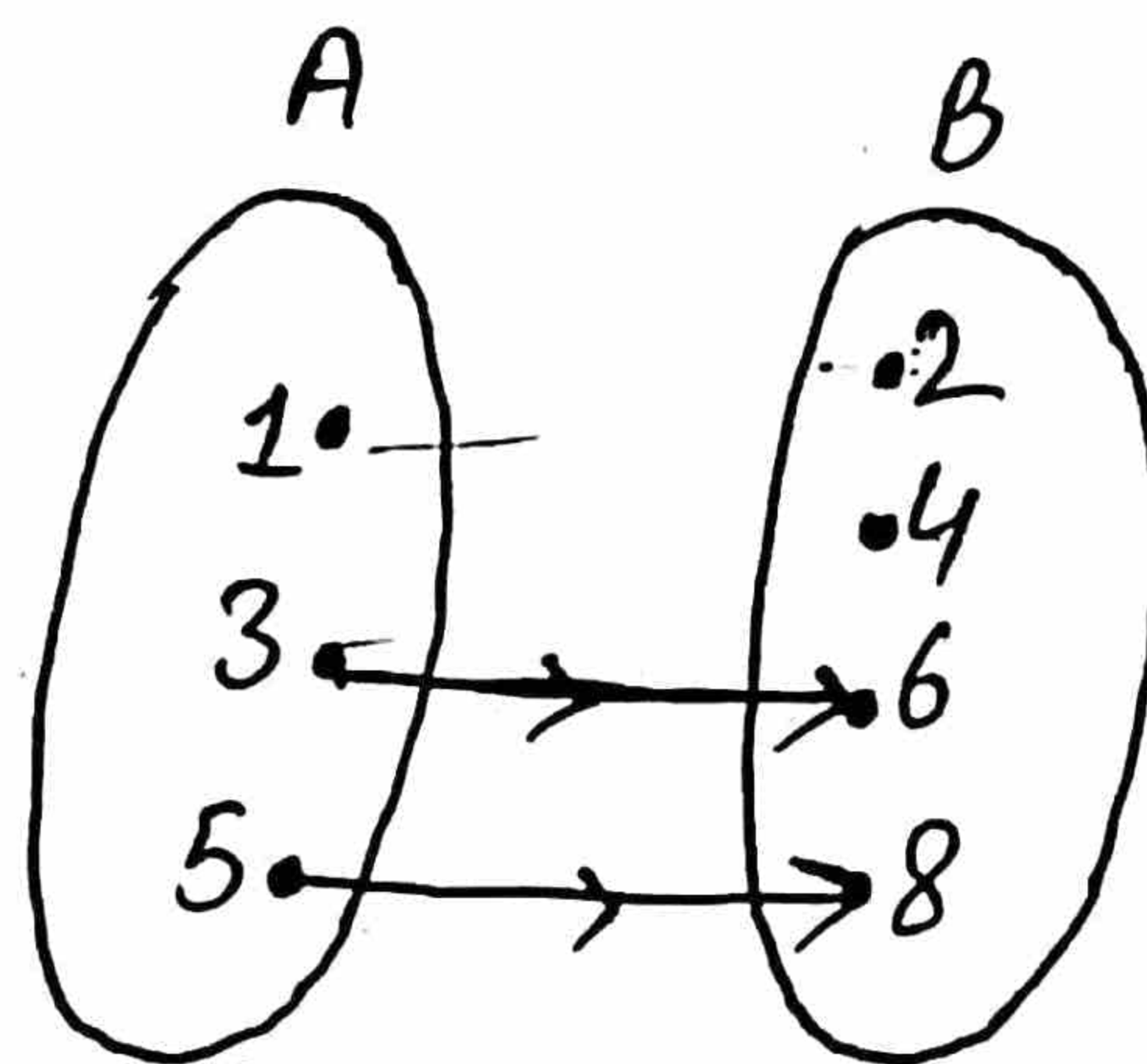


(i) Into Function:-

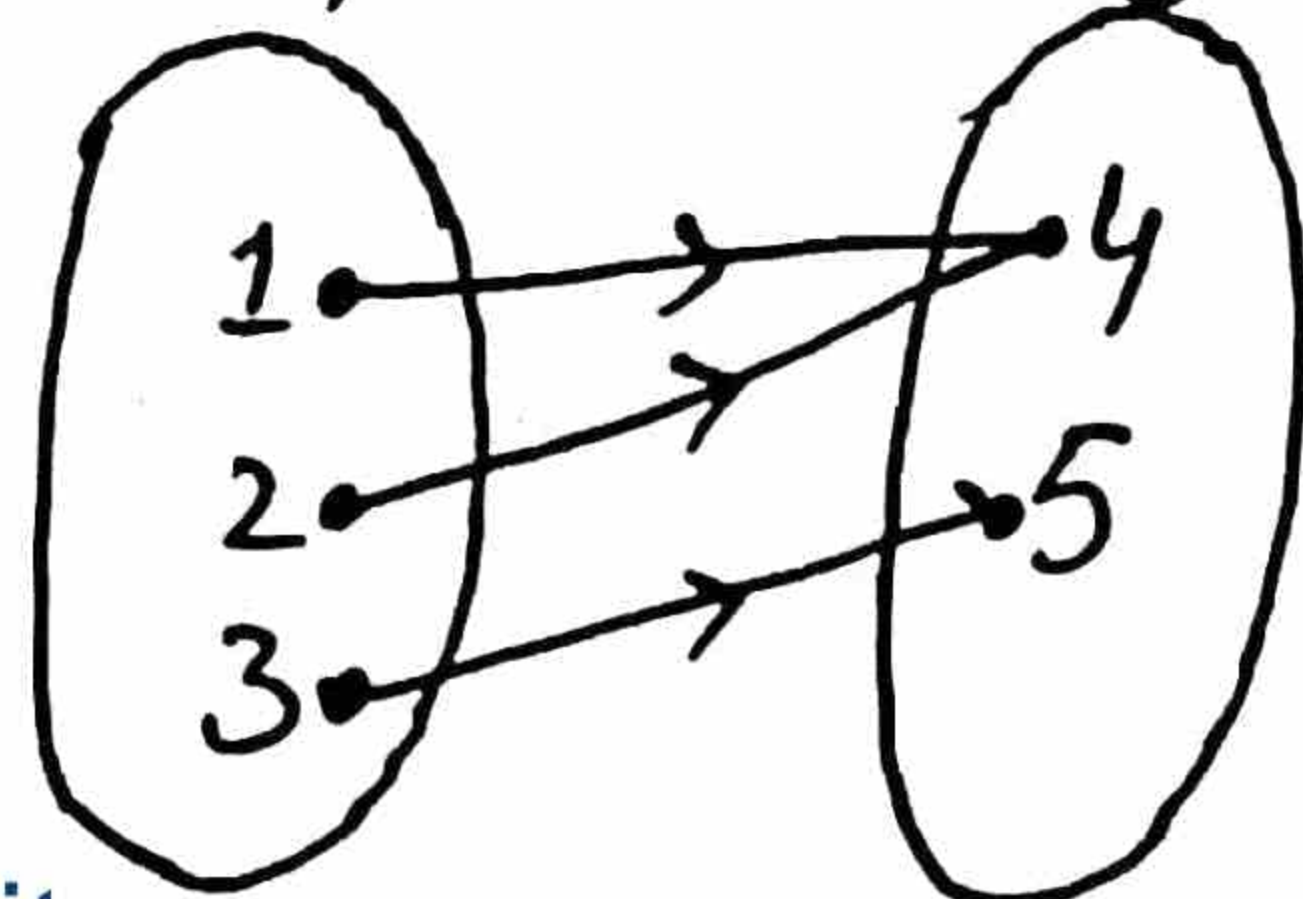
If a function $f: A \rightarrow B$ is such that $\text{Ran } f \subset B$ i.e. $\text{Ran } f \neq B$, then f is said to be a function from A into B . In figure, f is clearly a function but $\text{Ran } f \neq B$.
Therefore, f is a function from A into B .

(ii) Onto (Surjective) Function:-

If a function $f: A \rightarrow B$ is such that $\text{Ran } f = B$ i.e. every element of B is the image of the same elements of A , then



$$f = \{(3, 6), (5, 8)\}$$

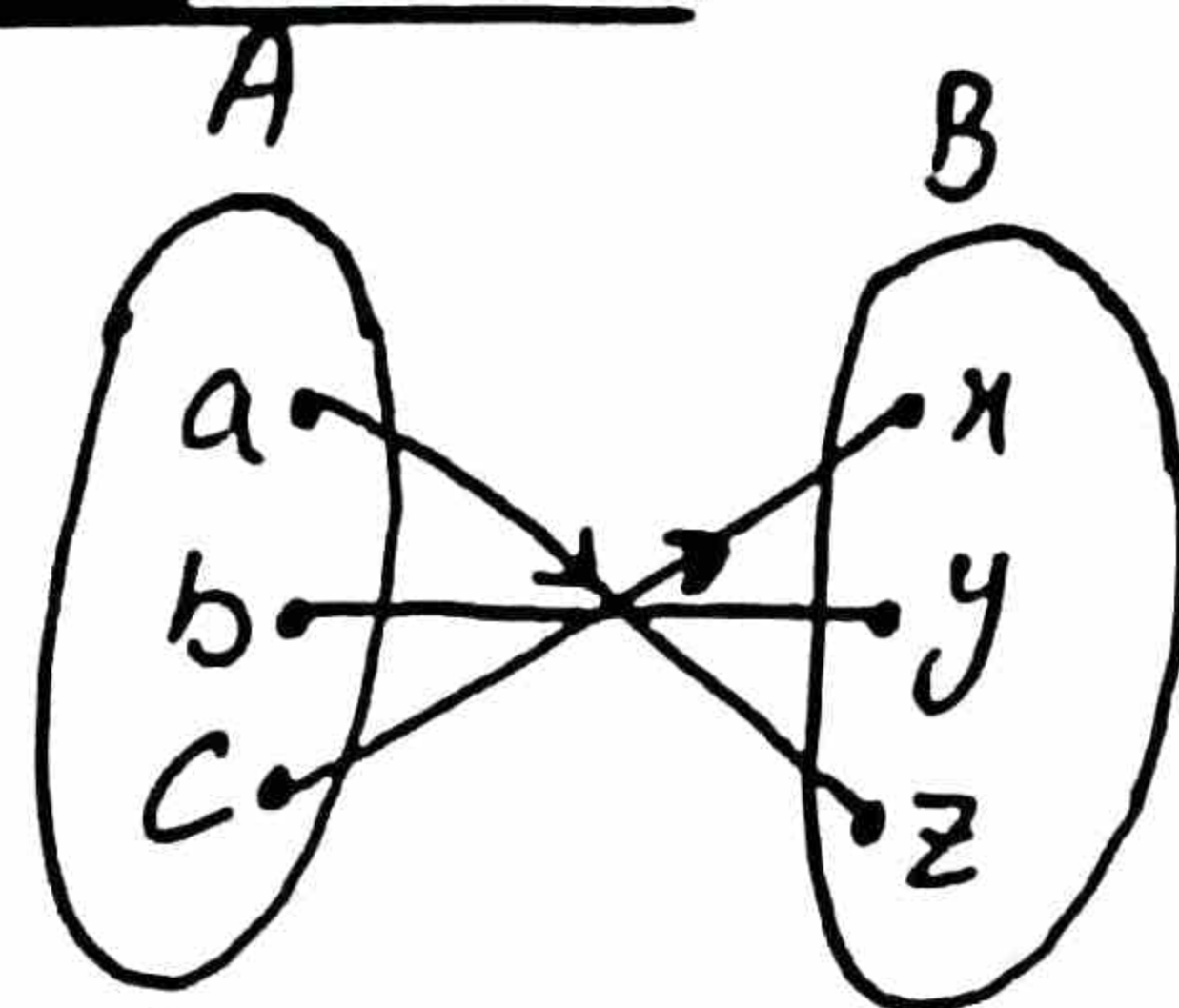


f is called Onto function or a surjective function.

$$f = \{(1,4), (2,4), (3,5)\}$$

(iii) 1-1 And Into (Injective) function:

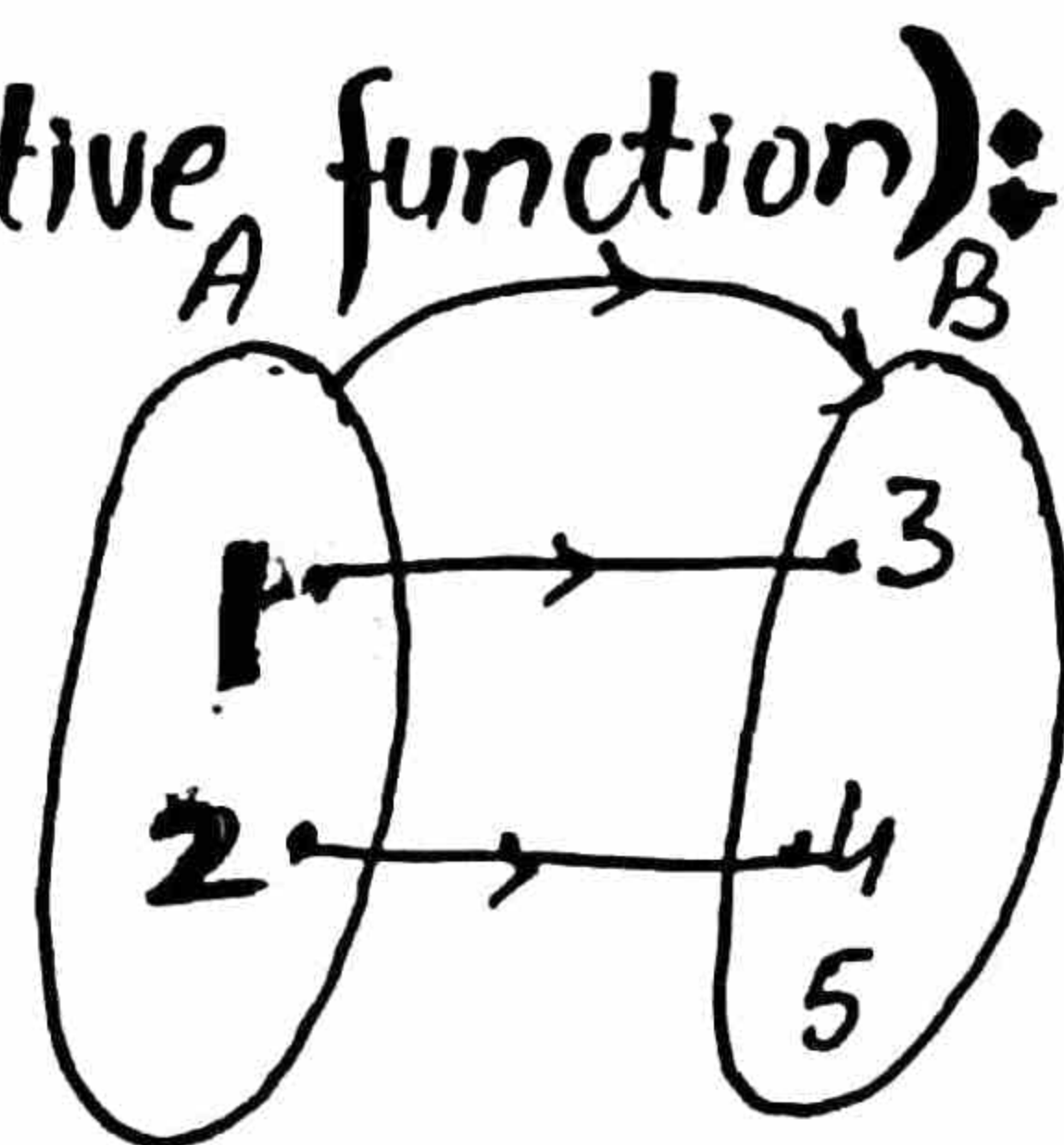
If a function f from A into B is such that second element of no two of its ordered pairs are equal, then it is called an Injective (1-1 and Into) function.



$$f = \{(a, x), (c, x), (b, y)\}$$

(iv) (1-1) and Onto Function (Bijective function):

If f is a function from A onto B such that second elements of no two of its ordered pairs are the same, then f is said to be (1-1) correspondence b/w A and B .



$$f = \{(1, 3), (2, 4)\}$$

Function	Domain	Range.
$y = \sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \tan x$	$-\infty < x < +\infty$ $x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty, x \neq n\pi$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty, x \neq \frac{(2n+1)\pi}{2}$	$y \geq 1 \text{ or } y \leq -1$
$y = \operatorname{cosec} x$	$-\infty < x < +\infty, x \neq n\pi$	$y \geq 1 \text{ or } y \leq -1$

CHAPTER #11 F.Sc Part-I

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

Domain OF Function:- Domain is the Set of all possible Input Values of a function. OR The Values that x can take is called Domain of a Function.

Range OF Function:- Range is the Set of all possible output values of a function. OR The Values that y can take is called Range of a Function.

For Example:-

The Set of Input Values (Domain)

The Set of output Values (Range)

$$y = x^2 + 1$$

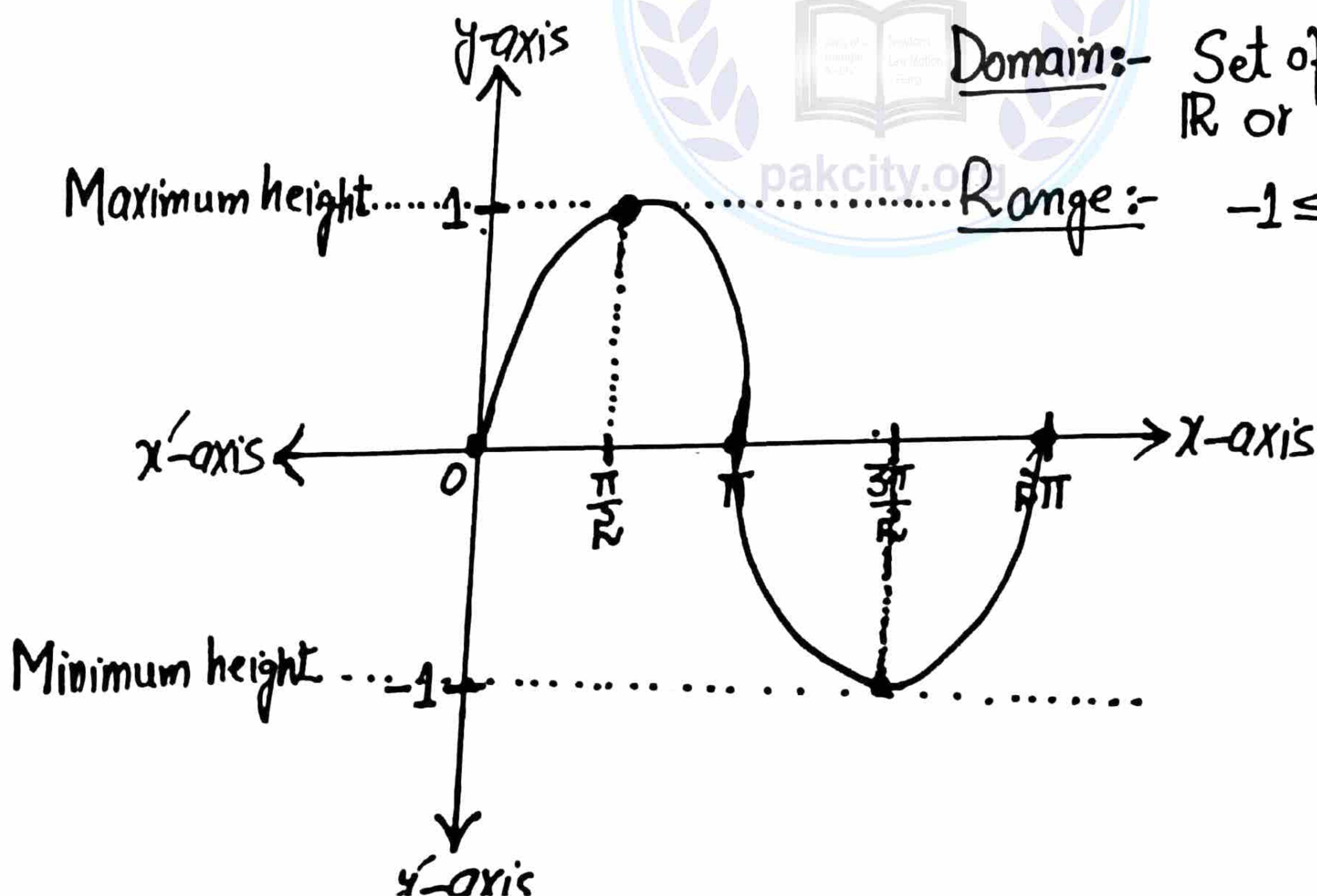
put $x = 1$
 $y = 1 + 1 = 2$
 put $x = 2$
 $y = 4 + 1 = 5$

put $x = 3$
 $y = 9 + 1 = 10$

Graph OF Sin Function:

Graph of $y = \sin x$ from 0° To 360°

x	0	90°	180°	270°	360°
y	0	1	0	-1	0



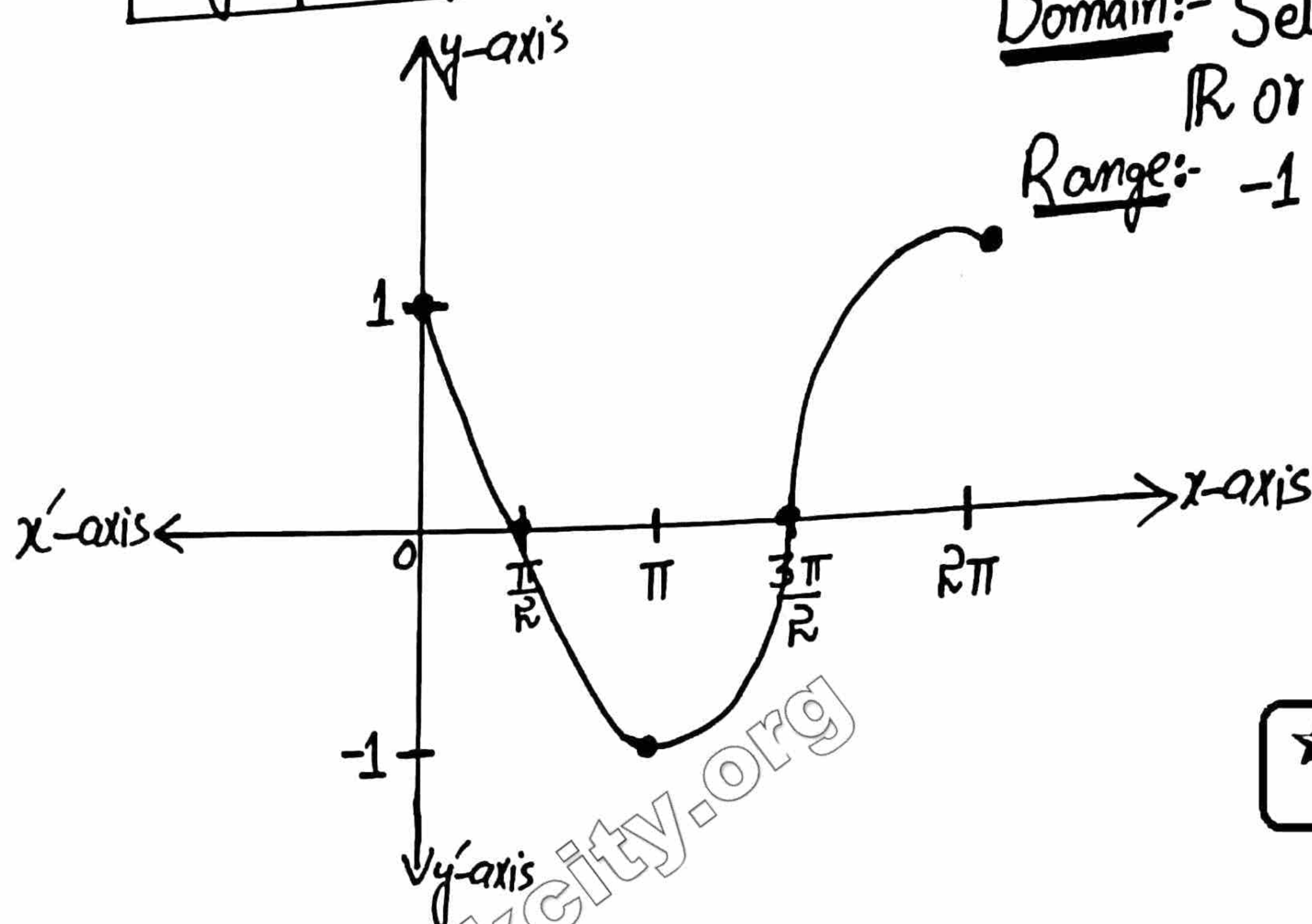
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BS(H) GCUF (2011-2015)
M.Phil Math (NTUF)
(2016-2018)
B.ed (AIU) (2015-2011)

نوٹ:-

Graph OF Cosine Function:-

Graph of $y = \cos x$ from 0° To 360°

x	0	$\pi/2$	π	$3\pi/2$	2π
y	1	0	-1	0	1



Domain:- Set of all Real Numbers
 $\mathbb{R}$ or $(-\infty, +\infty)$

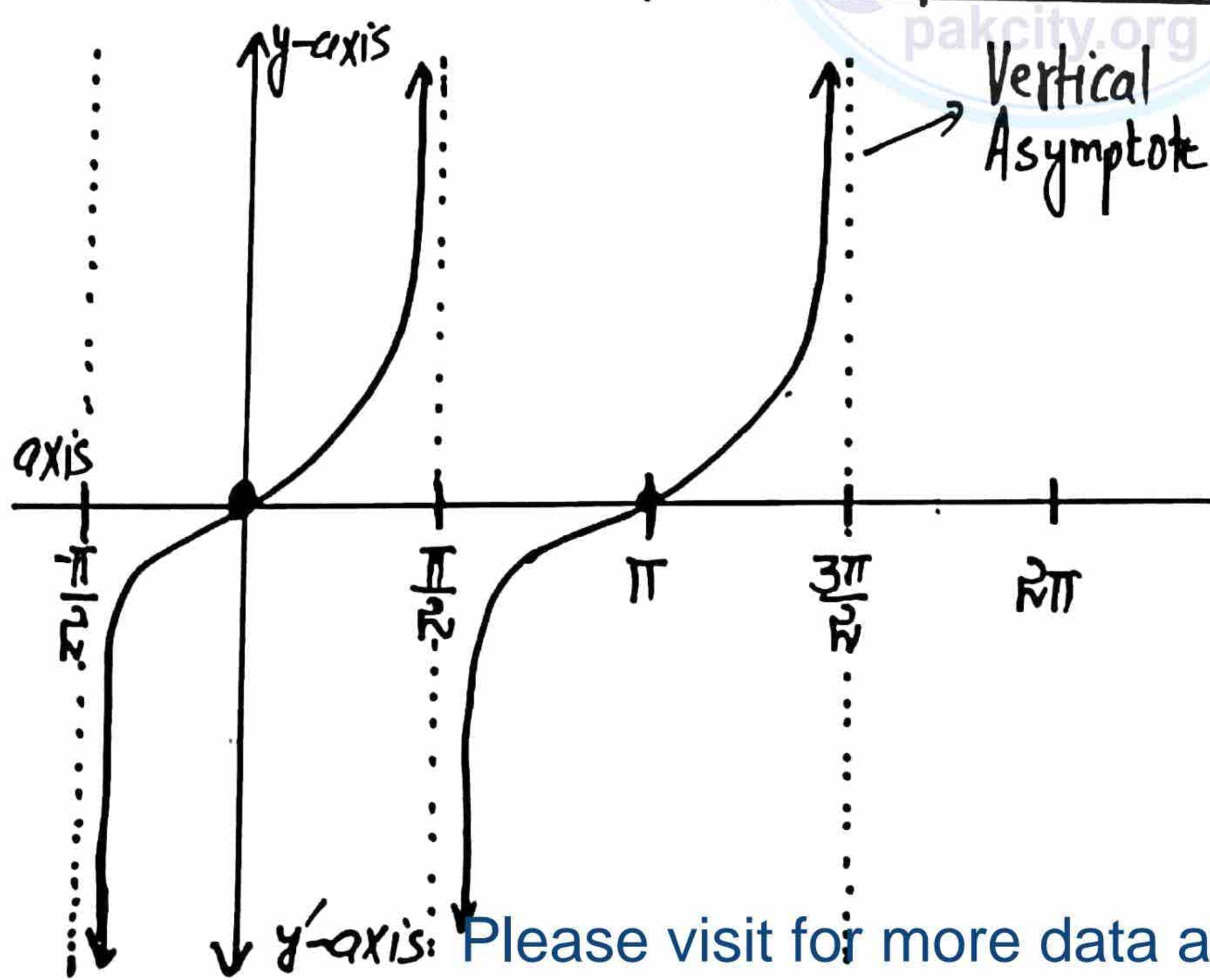
Range:- $-1 \leq y \leq 1$ or $[-1, 1]$

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Graph OF Tangent Function:-

Graph of $y = \tan x$ from 0 To 360°

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	∞	0	$-\infty$	0



Domain:- $-\infty < x < +\infty$,
 $x \neq (2n+1)\frac{\pi}{2}$
 $n \in \mathbb{Z}$

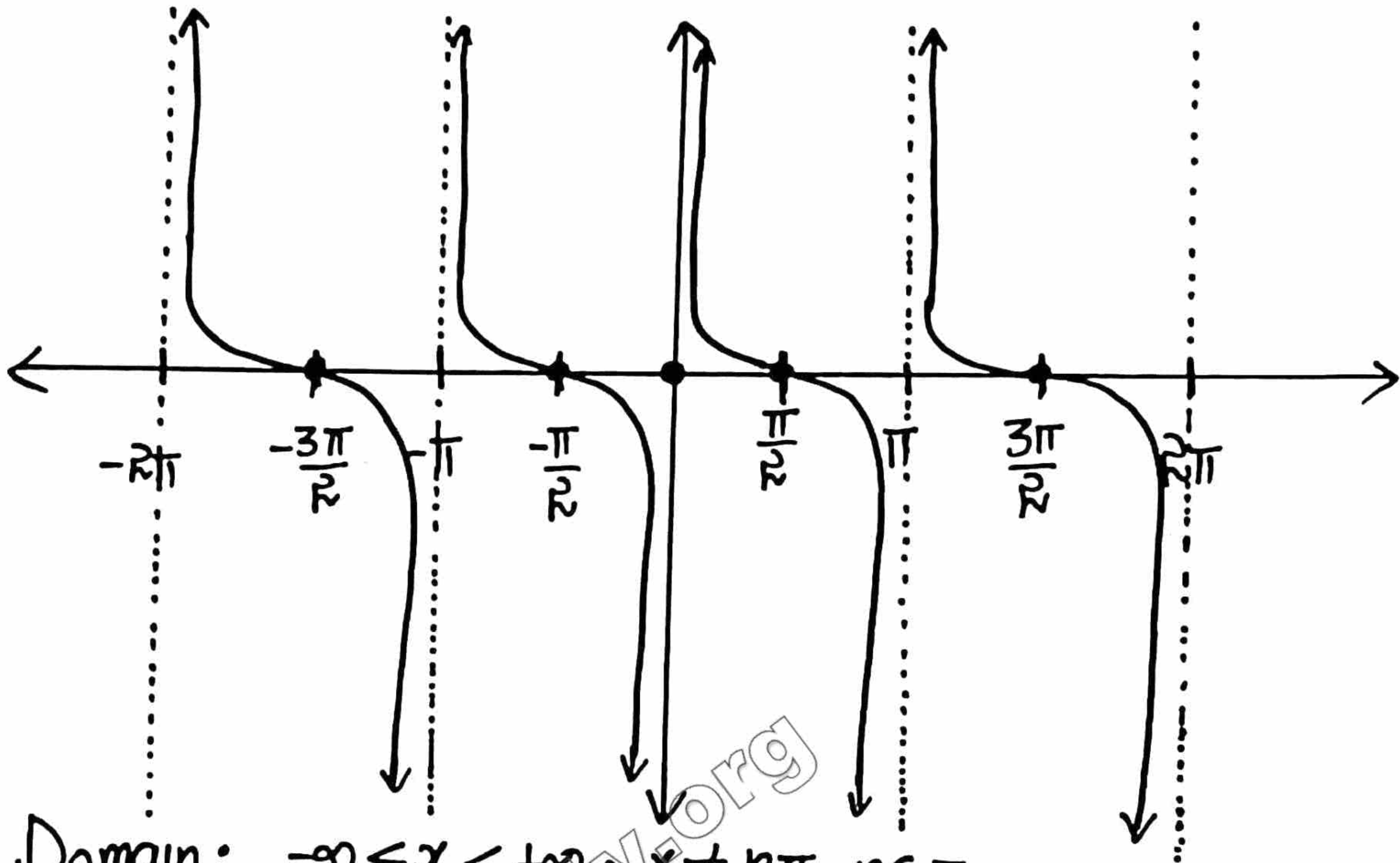
Range:- $-\infty < y < +\infty$
or $(-\infty, +\infty)$

Asymptote:- A Line whose
Distance from the Curve is
zero at Infinity.

Graph OF Cotangent Function:-

Graph of $y = \cot x$ from $-\pi$ To π

x	0	$\pi/2$	π	$3\pi/2$	2π	$-\pi/2$	$-\pi$	$-3\pi/2$	-2π
y	∞	0	∞	0	∞	0	∞	0	∞



Domain: $-\infty < x < +\infty, x \neq n\pi, n \in \mathbb{Z}$

Range: $-\infty < y < +\infty$ or $(-\infty, +\infty)$

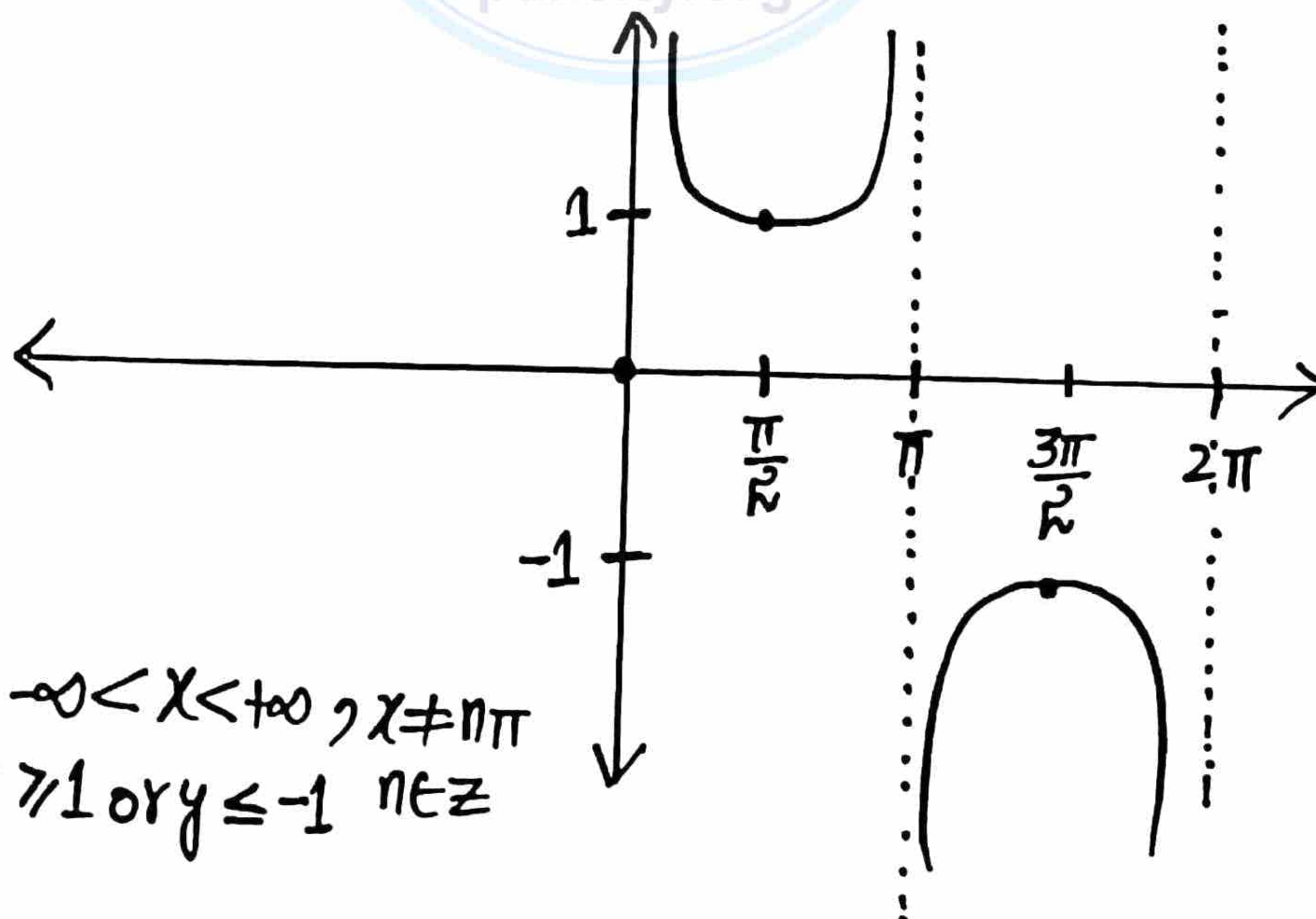
Graph OF Cosecant Function:-

Graph of $y = \csc x$ from 0 To 2π

x	0	$\pi/2$	π	$3\pi/2$	2π
y	∞	1	∞	-1	∞

$$y = \csc x$$

$$y = \frac{1}{\sin x}$$



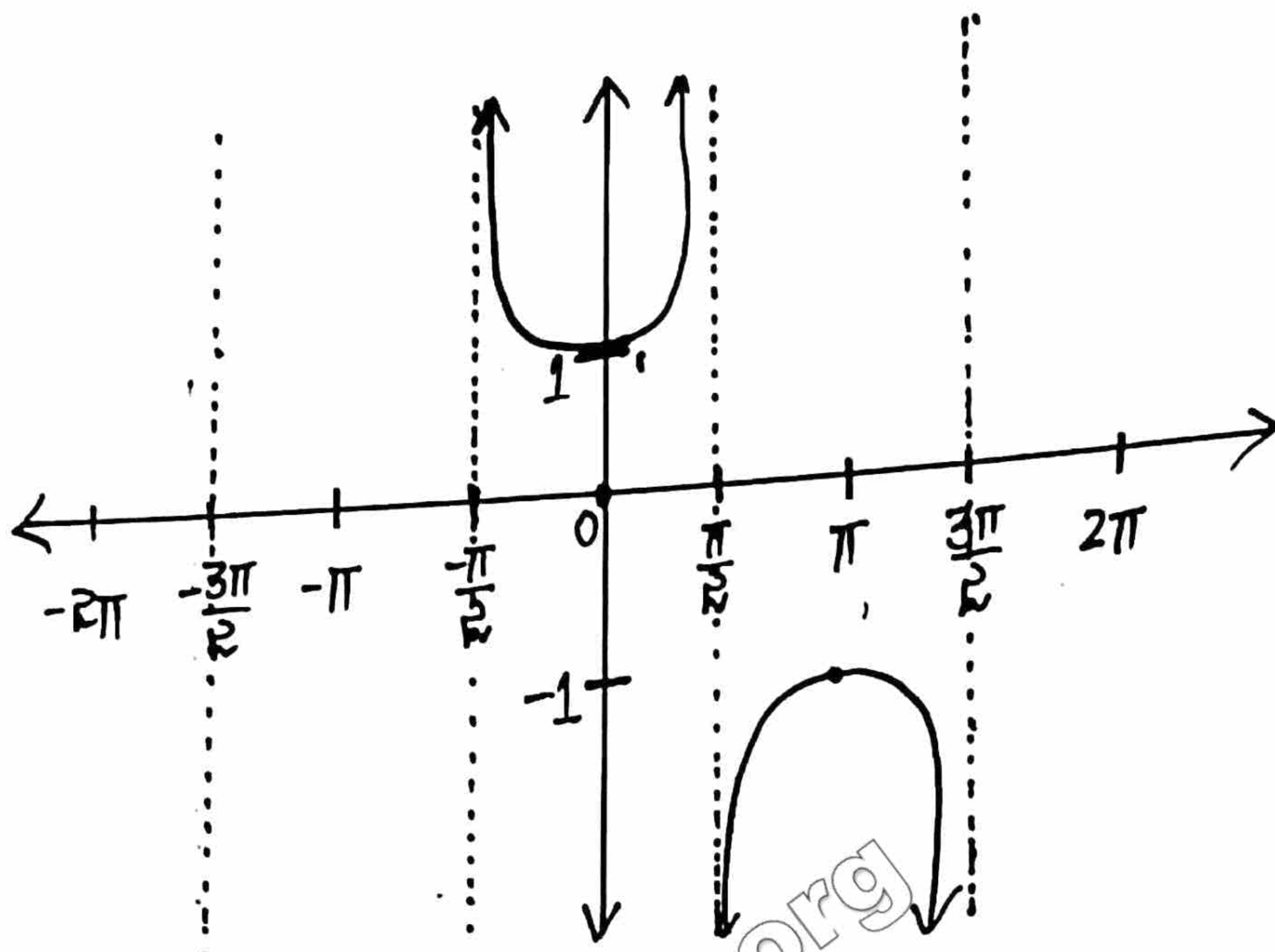
Domain: $-\infty < x < +\infty, x \neq n\pi$

Range: $y \geq 1$ or $y \leq -1, n \in \mathbb{Z}$

Graph OF Secant Function:-

Graph of $y = \sec x$ from -2π To 2π

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$-\frac{\pi}{2}$	$-\pi$	$-\frac{3\pi}{2}$	-2π
y	1	∞	-1	∞	+1	$-\infty$			



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Domain:- $-\infty < x < +\infty, x \neq (2n+1)\frac{\pi}{2}$

Range:- $y \geq 1$ or $y \leq -1$

Note:-

- ★ As we shall see the graphs of Trigonometric functions will be smooth Curves and none of them will be line Segments or will be have sharp corners or breaks within their domains. The behaviour of the Curve is called Continuity. It means that the trigonometric functions are continuous, wherever they are defined. Moreover, as the Trigonometric functions are periodic so their Curves repeat after a fixed Intervals.
- ★ The dashed lines are vertical Asymptotes in the graphs of $\tan x$, $\cot x$, $\sec x$ and $\csc x$.
- ★ From the graphs of Trigonometric functions we can check their domains and Ranges.
- ★ Trigonometric Functions are Periodic Functions because They repeat their Values after fixed Interval.

↓ y-axis:

Function:- Let A and B be Two non-empty Sets such that

(i) f is a relation from A to B that is, f is a Subset of $A \times B$.

(ii) $\text{Dom } f = A$

(iii) First element of no two pairs of f are equal, Then f is said to be a function from A to B .

The function f is also written as

$$f: A \rightarrow B$$

which is read a function from A to B .

Example:-

$$y = x^2$$

put $x = 1$

$$y = (1)^2 = 1$$

put $x = 2$

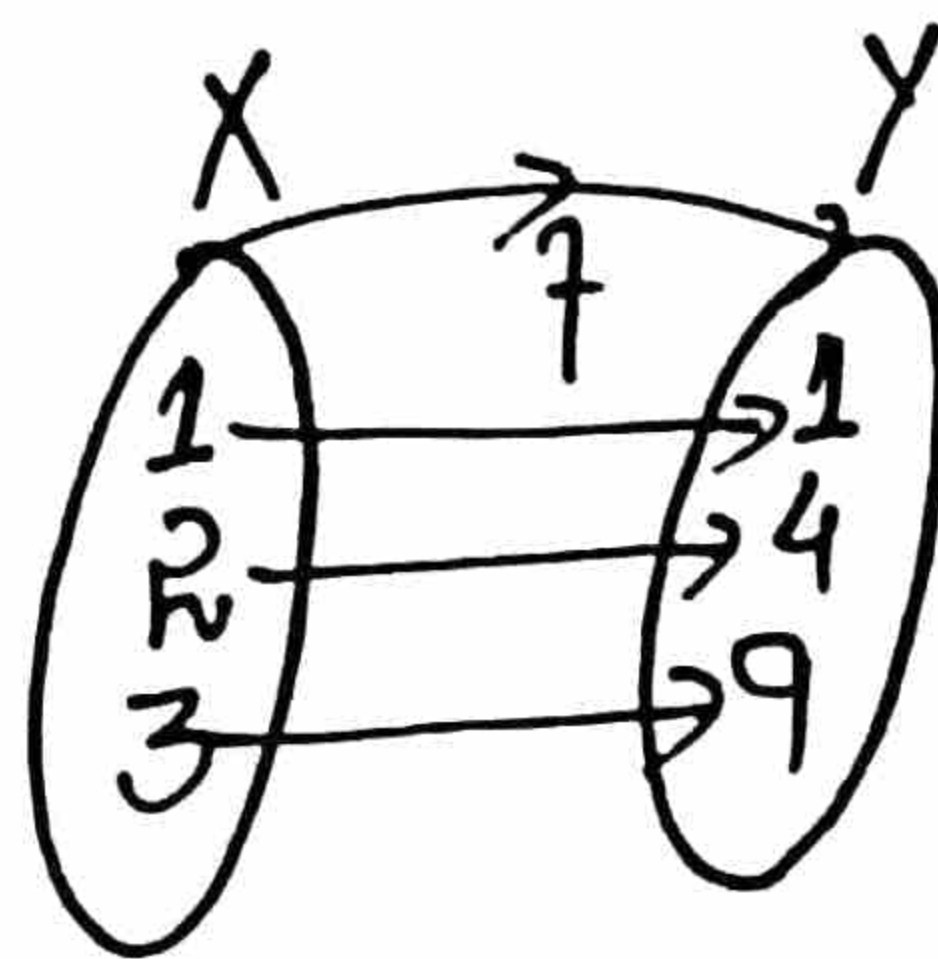
$$y = (2)^2 = 4$$

put $x = 3$

$$y = 9$$

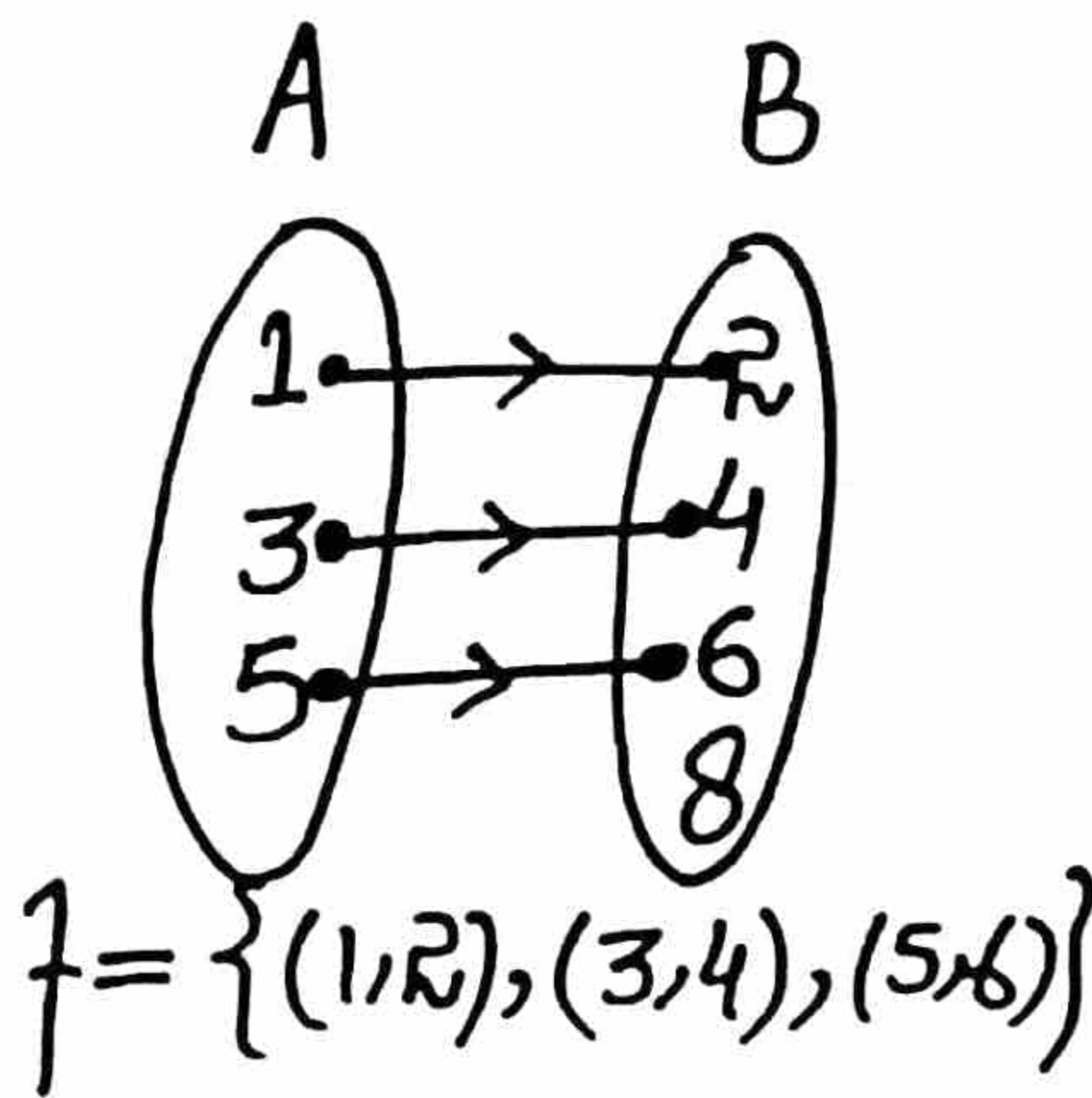
put $x = 4$

$$y = 16$$



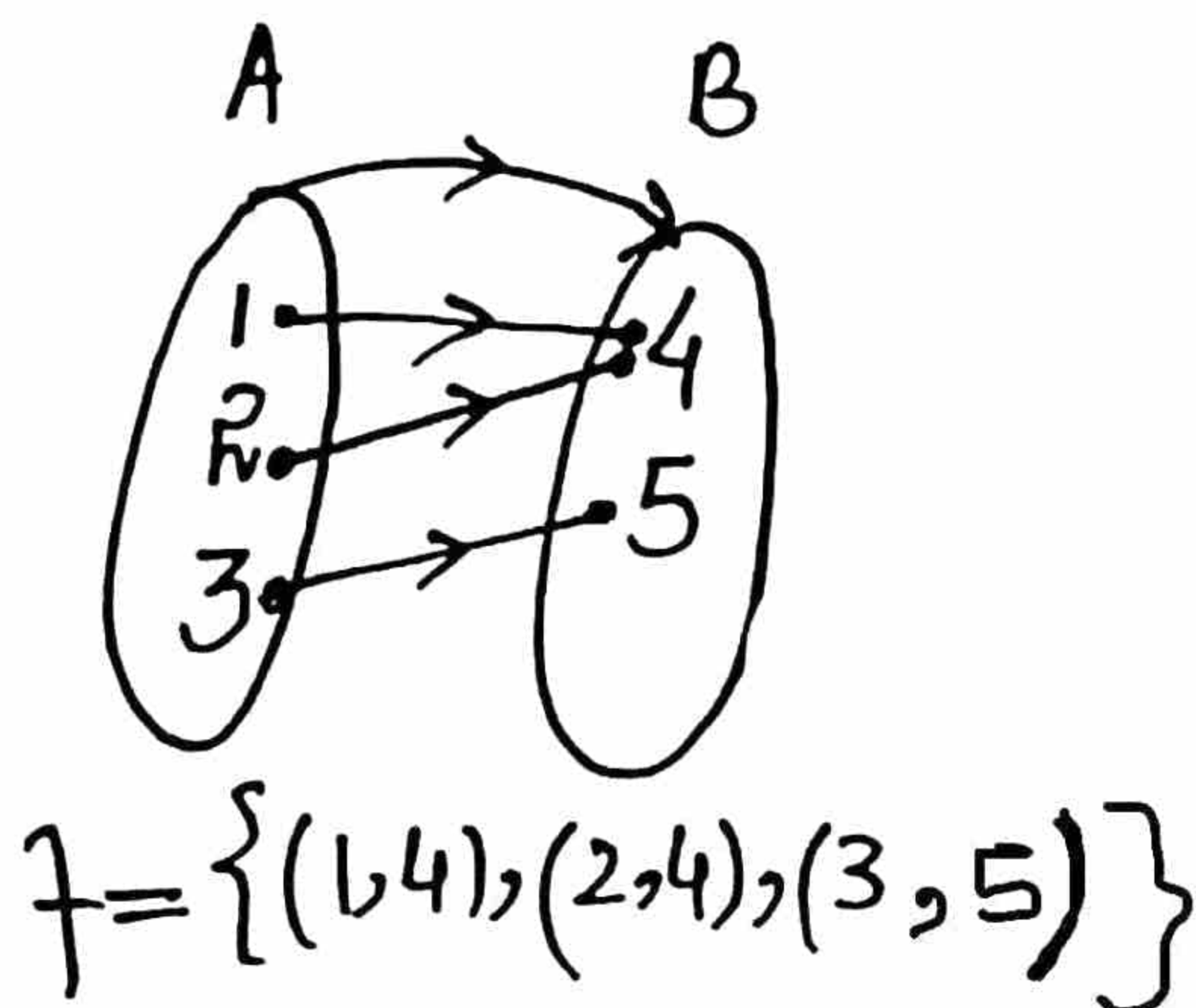
(i) Into Function:-

If a function $f: A \rightarrow B$ is such that $\text{Ran } f \subset B$ i.e. $\text{Ran } f \neq B$, Then f is said to be a function from A into B . In figure, f is clearly a function But $\text{Ran } f \neq B$. Therefore f is a function from A into B .



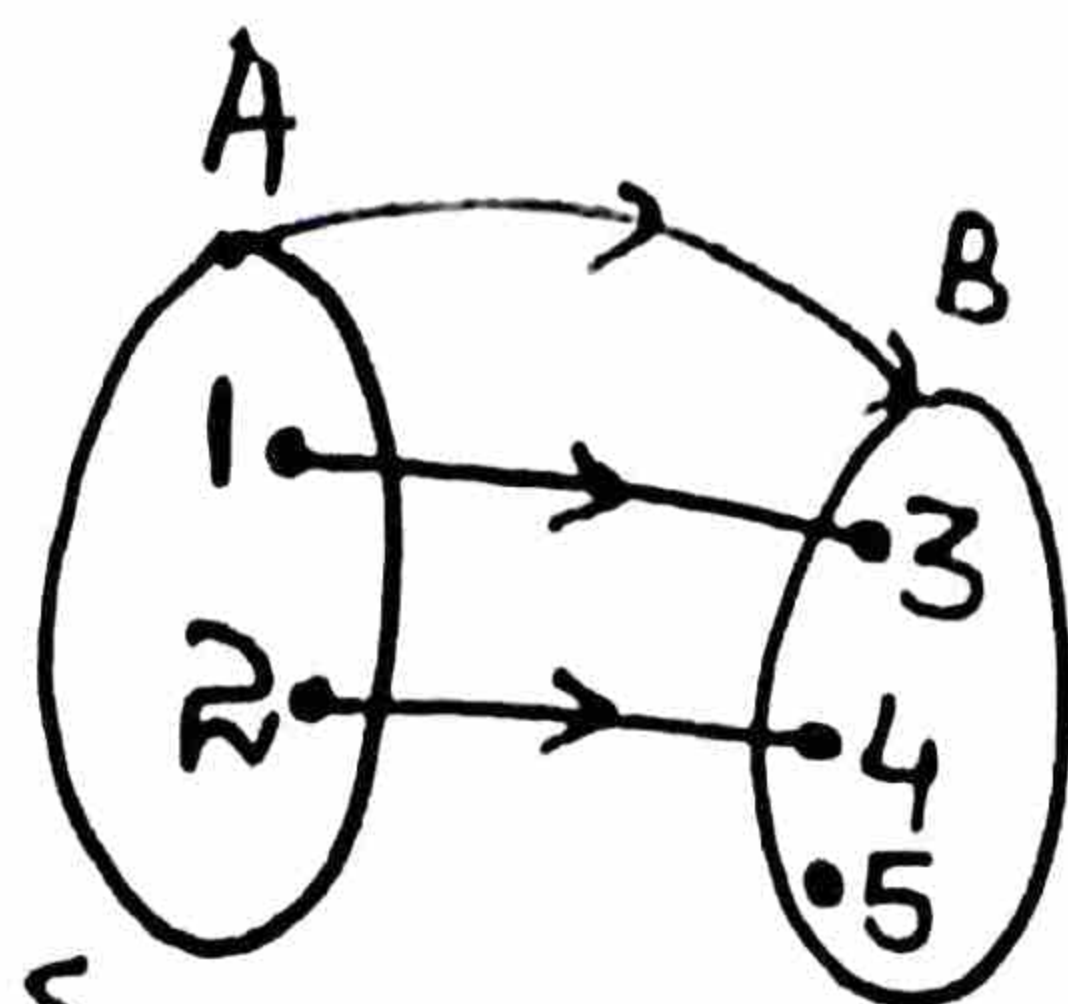
(ii) Onto (Surjective) Function:-

If a function $f: A \rightarrow B$ is such that $\text{Ran } f = B$ i.e. every element of B is the Image of Some elements of A , Then f is called Onto function or a Surjective function.



(iii) 1-1 And Into (Injective) function

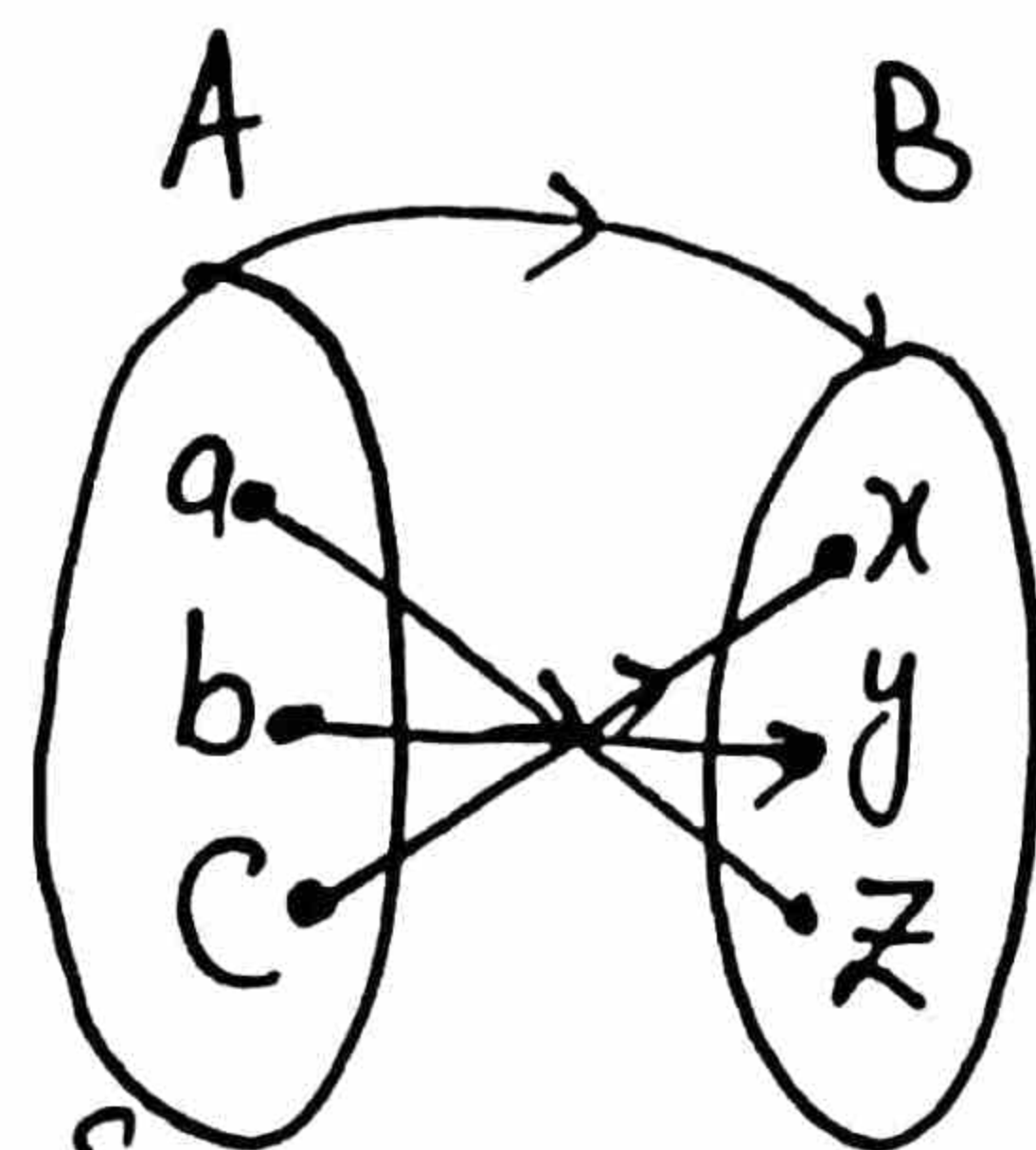
If a function f from A into B is such that second elements of no two of its ordered pairs are equal, Then it is called an Injective (1-1 and Into) function.



$$f = \{(1, 3), (2, 4)\}$$

(iv) (1-1) and Onto Function (Bijective Function):

If f is a function from A onto B such that second elements of no two of its ordered pairs are the same, Then f is said to be (1-1) function from A onto B . Such a function is also called a (1-1) Correspondance b/w A and B .



$$f = \{(a, z), (c, x), (b, y)\}$$

Functions	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \tan x$	$-\infty < x < +\infty, x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty, x \neq n\pi$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty, x \neq \frac{(2n+1)\pi}{2}$	$y \geq 1 \text{ or } y \leq -1$
$y = \csc x$	$-\infty < x < +\infty, x \neq n\pi$	$y \geq 1 \text{ or } y \leq -1$

Unit NO. 11B-

Trigonometric Functions and their graphs.

1 Definition

Period:-

Period of a trigonometric function is the smallest +ve number which, when added to

the original circular measure of the angle, gives the same value of the function.

* Exercise 11.1

Q.NO. 18-

$$\sin 3x$$

Period of $\sin$ is 2π

$$= \sin (3x + 2\pi)$$

$$= \sin 3 \left(x + \frac{2\pi}{3} \right)$$

Period of $\sin 3x$ is $\frac{2\pi}{3}$

Q.NO. 20-

$$\cos 2x$$

Period of $\cos$ is 2π

$$= \cos (2x + 2\pi)$$

$$= \cos 2 \left(x + \pi \right)$$

Period of $\cos 2x$ is π .

Q.NO. 30-

$$\tan 4x$$

Period of $\tan$ is π

$$= \tan (4x + \pi)$$

$$= \tan 4 \left(x + \frac{\pi}{4} \right)$$

Period of $\tan 4x$ is $\frac{\pi}{4}$

Q.NO.4B-

$$\cot \frac{x}{2}$$

Period of $\cot$ is π

$$= \cot \left(\frac{x}{2} + \pi \right)$$

$$= \cot \left(\frac{x + 2\pi}{2} \right)$$

$$= \cot \frac{1}{2} (x + 2\pi)$$

Period of $\cot \frac{x}{2}$ is 2π

Q.NO.5B-

$$\sin \frac{x}{3}$$

Period of $\sin$ is 2π

$$= \sin \left(\frac{x}{3} + 2\pi \right)$$

$$= \sin \left(\frac{x + 6\pi}{3} \right)$$

$$= \sin \frac{1}{3} (x + 6\pi)$$

Period of $\sin \frac{x}{3}$ is 6π .

Q. NO. 58-

$\operatorname{cosec} \frac{x}{4}$

Period of cosec is 2π

$$= \operatorname{cosec} \left(\frac{x}{4} + 2\pi \right)$$

$$= \operatorname{cosec} \left(\frac{x + 8\pi}{4} \right)$$

$$= \operatorname{cosec} \frac{1}{4} (x + 8\pi)$$

Period of $\operatorname{cosec} \frac{x}{4}$ is 8π .

Q. NO. 70-

$\sin \frac{x}{5}$

Period of $\sin$ is 2π

$$= \sin \left(\frac{x}{5} + 2\pi \right)$$

$$= \sin \left(\frac{x + 10\pi}{5} \right)$$

$$= \sin \frac{1}{5} (x + 10\pi)$$

Period of $\sin \frac{x}{5}$ is 10π .

Q.NO.88-

$$\cos \frac{x}{6}$$

Period of $\cos$ is 2π .

$$= \cos \left(\frac{x}{6} + 2\pi \right)$$

$$= \cos \left(\frac{x + 12\pi}{6} \right)$$

$$= \cos \frac{1}{6} (x + 12\pi)$$

Period of $\cos \frac{x}{6}$ is 12π .

Q.NO.90-

$$\tan \frac{x}{7}$$

Period of $\tan$ is π .

$$= \tan \left(\frac{x}{7} + \pi \right)$$

$$= \tan \left(\frac{x + 7\pi}{7} \right)$$

$$= \tan \frac{1}{7} (x + 7\pi)$$

Period of $\tan \frac{x}{7}$ is 7π .

Q.NO. 10:-

$\cot 8x$

Period of $\cot$ is π .

$$= \cot (8x + \pi)$$

$$= \cot 8 \left(x + \frac{\pi}{8} \right)$$

Period of $\cot 8x$ is $\frac{\pi}{8}$.

Q.NO. 11:-

$\sec 9x$

Period of $\sec$ is 2π

$$= \sec (9x + 2\pi)$$

$$= \sec 9 \left(x + \frac{2\pi}{9} \right)$$

Period of $\sec 9x$ is $\frac{2\pi}{9}$.

Q.NO. 12:-

$\operatorname{cosec} 10x$

Period of cosec is 2π .

$$= \operatorname{cosec} (10x + 2\pi)$$

$$= \operatorname{cosec} 10 \left(x + \frac{2\pi}{10} \right)$$

$$= \operatorname{cosec} 10 \left(x + \frac{\pi}{5} \right)$$

Period of $\operatorname{cosec} 10x$ is $\frac{\pi}{5}$

Q.NO.13:-

$$3 \sin x$$

Period of $\sin$ is 2π .

$$= 3 \sin (x + 2\pi)$$

Period of $3 \sin x$ is 2π .

Q.NO.14:-

$$2 \cos x$$

Period of $\cos$ is 2π

$$= 2 \cos (x + 2\pi)$$

Period of $2 \cos x$ is 2π .

Q.NO.15:-

$$3 \cos \frac{x}{5}$$

Period of $\cos$ is 2π .

$$= 3 \cos \left(\frac{x}{5} + 2\pi \right)$$

$$= 3 \cos \left(\frac{x + 10\pi}{5} \right)$$

$$= 3 \cos \frac{1}{5} (x + 10\pi)$$

Period of $3 \cos \frac{x}{5}$ is 10π .

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